D^0–$\overline{D}^0$ MIXING

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Standard Model contributions to D^0–$\overline{D}^0$ mixing are strongly suppressed by CKM and GIM factors. Thus the observation of D^0–$\overline{D}^0$ mixing might be evidence for physics beyond the Standard Model. See Bigi [1] for a review of D^0–$\overline{D}^0$ mixing, and see Nelson [2] for a recent compilation of mixing predictions.

Formalism: The time evolution of the D^0–$\overline{D}^0$ system is described by the Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}, \tag{1}$$

where the $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance requires $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of these matrices describe the dispersive and absorptive parts of D^0–$\overline{D}^0$ mixing.

The two eigenstates D_1 and D_2 of the effective Hamiltonian matrix $(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma})$ are given by

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle\,. \tag{2}$$

The corresponding eigenvalues are

$$\lambda_{1,2} \equiv m_{1,2} - \tfrac{i}{2}\Gamma_{1,2} = \left(M - \tfrac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \tfrac{i}{2}\Gamma_{12}\right)\,, \tag{3}$$

where

$$\left|\frac{q}{p}\right|^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\,. \tag{4}$$

We extend the formalism from our note on "B^0–$\overline{B}^0$ mixing" [3]. In addition to the 'right-sign' instantaneous decay amplitudes $\overline{A}_f \equiv \langle f|H|\overline{D}^0\rangle$ and $A_{\overline{f}} \equiv \langle\overline{f}|H|D^0\rangle$ for CP conjugate final states f and $\overline{f}$, we include the 'wrong-sign' amplitudes $\overline{A}_{\overline{f}} \equiv \langle\overline{f}|H|\overline{D}^0\rangle$ and $A_f \equiv \langle f|H|D^0\rangle$.

It is usual to normalize the wrong-sign decay distributions to the integrated rate of right-sign decays and to express time in units of the precisely measured D^0 mean life, $\overline{\tau}_{D^0} = 1/\Gamma = 2/(\Gamma_1 + \Gamma_2)$. We denote the resulting decay distributions as $r(t)$

and $\overline{r}(t)$. Starting from a pure $|D^0\rangle$ or $|\overline{D}^0\rangle$ state at $t=0$, the time dependence to the wrong-sign final states is then

$$r(t) = \frac{\left|\langle f|H|D^0(t)\rangle\right|^2}{\left|\overline{A}_f\right|^2} = \left|\frac{q}{p}\right|^2 \left|g_+(t)\,\chi_f^{-1} + g_-(t)\right|^2 , \qquad (5)$$

$$\overline{r}(t) = \frac{\left|\langle \overline{f}|H|\overline{D}^0(t)\rangle\right|^2}{\left|A_{\overline{f}}\right|^2} = \left|\frac{p}{q}\right|^2 \left|g_+(t)\,\chi_{\overline{f}} + g_-(t)\right|^2 , \qquad (6)$$

where

$$\chi_f = \frac{q\overline{A}_f}{pA_f} \qquad (7)$$

and

$$g_\pm(t) = \frac{1}{2}\left(e^{-iz_1 t} \pm e^{-iz_2 t}\right) , \quad z_{1,2} = \frac{\lambda_{1,2}}{\Gamma} . \qquad (8)$$

Note that a change of the relative phase of D^0 and $\overline{D}^0$ cancels between q/p and $\overline{A}_f/A_f$ and leaves χ_f invariant.

Since D^0–$\overline{D}^0$ mixing is a small effect, the identification tag of the initial particle as a D^0 or a $\overline{D}^0$ must be extremely accurate. The usual tag is the charge of the distinctive slow pion in the decay sequence $D^{*+} \to D^0\pi^+$ or $D^{*-} \to \overline{D}^0\pi^-$. In current experiments, mis-tags occur at a rate of about one per thousand events. Another tag of sufficient accuracy is identification of one of the D's from $\psi(3770) \to D^0\overline{D}^0$.

Semileptonic decays: We expand $r(t)$ and $\overline{r}(t)$ to second order in time for modes where the ratio of decay amplitudes $R_D = |A_f/\overline{A}_f|^2$ is very small. In semileptonic decays, $A_f = \overline{A}_{\overline{f}} = 0$ in the Standard Model. We define reduced mixing amplitudes x and y by

$$x \equiv \frac{2M_{12}}{\Gamma} = \frac{m_1 - m_2}{\Gamma} = \frac{\Delta m}{\Gamma} \qquad (9)$$

and

$$y \equiv \frac{\Gamma_{12}}{\Gamma} = \frac{\Gamma_1 - \Gamma_2}{2\Gamma} = \frac{\Delta\Gamma}{2\Gamma} . \qquad (10)$$

In these, the first equality holds in the limit of CP conservation; and now the subscripts 1 and 2 indicate the CP-even and CP-odd eigenstates, respectively. Then, in the limit of weak mixing, where $|ix+y| \ll 1$, $r(t)$ is given by

$$r(t) = |g_-(t)|^2 \left|\frac{q}{p}\right|^2 \approx \frac{e^{-t}}{4}(x^2+y^2)t^2 \left|\frac{q}{p}\right|^2 . \qquad (11)$$

For $\overline{r}(t)$ one replaces q/p by p/q, and in the limit of CP conservation, $r(t) = \overline{r}(t)$ and

$$R_M = \int_0^\infty r(t)dt \approx \frac{1}{2}(x^2 + y^2)\,. \tag{12}$$

Wrong-sign decays to hadronic non-CP eigenstates: Consider the final state $f = K^+\pi^-$, where A_f is doubly Cabibbo suppressed, and the ratio of decay amplitudes is

$$\frac{A_f}{\overline{A}_f} = -\sqrt{R_D}e^{-i\delta} \sim O(\tan^2\theta_c)\,. \tag{13}$$

The minus sign originates from the sign of V_{us} with respect to V_{cd}, and δ is a strong phase difference between doubly Cabibbo suppressed and Cabibbo-favored decay amplitudes.

We characterize the violation of CP in the mixing amplitude, decay amplitude, and the interference between those two processes, by the real-valued parameters A_M, A_D, and ϕ. In general $\chi_{\overline{f}}$ and χ_f^{-1} are two independent complex numbers. We adopt a parameterization similar to that of Nir [4] and CLEO [5] and express these quantities in a way that is convenient to describe the three types of CP violation:

$$\left|\frac{q}{p}\right| = 1 + A_M\,, \tag{14}$$

$$\chi_f^{-1} \equiv \frac{pA_f}{q\overline{A}_f} = \frac{-\sqrt{R_D}(1+A_D)}{(1+A_M)}e^{-i(\delta+\phi)}\,, \tag{15}$$

$$\chi_{\overline{f}} \equiv \frac{q\overline{A}_{\overline{f}}}{pA_{\overline{f}}} = \frac{-\sqrt{R_D}(1+A_M)}{(1+A_D)}e^{-i(\delta-\phi)}\,. \tag{16}$$

To leading order,

$$r(t) = e^{-t} \times \Bigg[R_D(1+A_D)^2 + \sqrt{R_D}(1+A_M)(1+A_D)y'_-t + \frac{(1+A_M)^2R_M}{2}t^2\Bigg]\,, \tag{17}$$

$$\overline{r}(t) = e^{-t} \times \left[\frac{R_D}{(1+A_D)^2} + \frac{\sqrt{R_D}}{(1+A_D)(1+A_M)}y'_+t + \frac{R_M}{2(1+A_M)^2}t^2\right], \tag{18}$$

where

$$y'_{\pm} \equiv y' \cos\phi \pm x' \sin\phi = y\cos(\delta \mp \phi) - x\sin(\delta \mp \phi) \quad (19)$$

$$y' \equiv y\cos\delta - x\sin\delta, \;\; x' \equiv x\cos\delta + y\sin\delta\,, \quad (20)$$

and R_D and R_M are the doubly Cabibbo-suppressed decay and mixing rates, respectively, relative to the time-integrated right-sign rate. Comparing the terms in Eq. (17) and Eq. (18) probes the three fundamental types of CP violation. In the limit of CP conservation, A_M, A_D, and ϕ are all zero, and $r(t) = \overline{r}(t)$. Eq. (17) and Eq. (18) become

$$r(t) = e^{-t}\left(R_D + \sqrt{R_D}y't + \frac{1}{2}R_M t^2\right)\,, \quad (21)$$

$$R = \int_0^\infty r(t)dt = R_D + \sqrt{R_D}y' + R_M\,, \quad (22)$$

where R is the time-integrated wrong-sign rate relative to the time-integrated right-sign rate.

For multibody final states, Eqs. (13)–(22) are applicable for one point in the Dalitz space. Although x and y do not vary across the Dalitz space, knowledge of the resonant substructure is needed to extrapolate the strong phase difference δ to a different point. Both the sign and magnitude of x and y are believed to be experimentally accessible through the study of the time-dependent resonant substructure in decay modes such as $D^0 \to K_S\pi^+\pi^-$ [6].

Decays to CP eigenstates: When the final state f is a CP eigenstate, there is no distinction between f and $\overline{f}$, and $A_f = A_{\overline{f}}$ and $\overline{A}_{\overline{f}} = \overline{A}_f$. We denote final states with CP eigenvalues ± 1 by $f_\pm$. In analogy with Eqs. (5)–(6), the time dependence of decays to CP eigenstates is then

$$\begin{aligned} r_\pm(t) &= \frac{\left|\langle f_\pm|H|D^0(t)\rangle\right|^2}{\left|\overline{A}_\pm\right|^2} \\ &= \frac{1}{4}\left|h_\pm(t)\left(\frac{A_\pm}{\overline{A}_\pm} \pm \frac{q}{p}\right) + h_\mp(t)\left(\frac{A_\pm}{\overline{A}_\pm} \mp \frac{q}{p}\right)\right|^2, \\ &\propto \frac{1}{|p|^2}\left|h_\pm(t) + \eta_\pm h_\mp(t)\right|^2, \end{aligned} \quad (23)$$

$$\overline{r}_{\pm}(t) = \frac{\left|\langle f_{\pm}|H|\overline{D}^0(t)\rangle\right|^2}{\left|A_{\pm}\right|^2} \propto \frac{1}{|q|^2}\left|h_{\pm}(t) - \eta_{\pm} h_{\mp}(t)\right|^2, \tag{24}$$

where

$$h_{\pm}(t) = g_{+}(t) \pm g_{-}(t) = e^{-iz_{\pm}t}\,, \tag{25}$$

$$\eta_{\pm} \equiv \frac{pA_{\pm} \mp q\overline{A}_{\pm}}{pA_{\pm} \pm q\overline{A}_{\pm}} = \frac{1 \mp \chi_{\pm}}{1 \pm \chi_{\pm}}\,, \tag{26}$$

and the variable $\eta_{\pm}$ describes CP violation. This $\eta_{\pm}$ can receive contributions from each of the three fundamental types of CP violation.

The quantity y is accessible experimentally, by comparing the lifetime measured, for example, with decays to non-CP eigenstates such as $D^0 \to K^- \pi^+$, with that measured with decays to a CP eigenstate such as $D^0 \to K^+ K^-$; see Bergmann [7]. A positive y would make K^+K^- decays appear to have a shorter lifetime than $K^-\pi^+$ decays.

In the limit of weak mixing, where $|ix + y| \ll 1$, and small CP violation, where $|A_M|$, $|A_D|$, and $|\sin\phi| \ll 1$, the time dependence of decays to CP eigenstates is proportional to a single exponential:

$$r_{\pm}(t) \propto e^{-[1 \pm \frac{q}{p}(y\cos\phi - x\sin\phi)]t}\,, \tag{27}$$

$$\overline{r}_{\pm}(t) \propto e^{-[1 \pm \frac{p}{q}(y\cos\phi + x\sin\phi)]t}\,, \tag{28}$$

$$r_{\pm}(t) + \overline{r}_{\pm}(t) \propto e^{-(1 \pm y_{CP})t}\,, \tag{29}$$

where

$$\begin{aligned} y_{CP} \equiv & y\cos\phi\left[\frac{1}{2}\left(\frac{q}{p} + \frac{p}{q}\right) + \frac{A_{\text{prod}}}{2}\left(\frac{q}{p} - \frac{p}{q}\right)\right] \\ & - x\sin\phi\left[\frac{1}{2}\left(\frac{q}{p} - \frac{p}{q}\right) + \frac{A_{\text{prod}}}{2}\left(\frac{q}{p} + \frac{p}{q}\right)\right]\ , \end{aligned} \tag{30}$$

and

$$A_{\text{prod}} \equiv \frac{N(D^0) - N(\overline{D}^0)}{N(D^0) + N(\overline{D}^0)} \tag{31}$$

is defined as the production asymmetry of the D^0 and $\overline{D}^0$. Note that deviations from the lifetime measured in non-CP eigenstates do not require $y \neq 0$ but can be due to $x\sin\phi \neq 0$.

This possibility is distinguished by a relative sign difference between the D^0 and $\overline{D}^0$ samples.

In the limit of CP conservation, $A_\pm = \pm\overline{A}_\pm$, $\eta_\pm = 0$, $y = y_{CP}$, and

$$r_\pm(t)\left|\overline{A}_\pm\right|^2 = \overline{r}_\pm(t)\left|A_\pm\right|^2 = e^{-(1\pm y_{CP})t} \,. \qquad (32)$$

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