RARE KAON DECAYS

Revised March 2002 by L. Littenberg (BNL) and G. Valencia, (Iowa State University).

A. Introduction: There are several useful reviews on rare kaon decays and related topics [1–14]. The current activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters
3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+ \nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focused on the decays $K_L \to \pi^0 \ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+ \ell^+ \ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Other reactions of this type are $K_L \to \pi^0 \gamma\gamma$ and $K_L \to \ell^+ \ell^- \gamma$. The former is important in understanding a CP-conserving contribution to $K_L \to \pi^0 \ell^+ \ell^-$, whereas the latter could shed light on long distance contributions to $K_L \to \mu^+ \mu^-$.

B. Explicit violations of the Standard Model: Most of the activity here is in searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 4.7 \times 10^{-12} (148\ \mathrm{TeV}/M_X)^4$ [5]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of over 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV, along with the expected near-future progress. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to parity-odd couplings and the latter is sensitive to parity-even couplings. There have also been recent limits placed on lepton-number violating kaon decays [15,16]. Related searches in μ and τ processes are discussed in our section "Tests of Conservation Laws".

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.	(Near-) future aim
$K^+ \to \pi^+ e^- \mu^+$	2.8×10^{-11}	BNL-865	01/17	9×10^{-12}
$K^+ \to \pi^+ e^+ \mu^-$	5.2×10^{-10}	BNL-865	01/15	
$K_L \to \mu e$	4.7×10^{-12}	BNL-871	98/18	
$K_L \to \pi^0 e\mu$	4.4×10^{-10}	KTeV	01/19	3×10^{-10}

Physics beyond the SM is also pursued through the search for $K^+ \to \pi^+ X^0$, where X^0 is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, *etc.*). The 90% CL upper limit on this process has recently been improved to 5.9×10^{-11} [20].

C. Measurements of Standard Model parameters: Until 1997, searches for $K^+ \to \pi^+ \nu\overline{\nu}$ were motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [21] and long-distance contributions are known to be quite small [2,22]. Since then, BNL-787 has observed two candidate events [23,20], yielding a branching ratio of $(1.57^{+1.75}_{-0.82}) \times 10^{-10}$ [20]. At this level, this reaction becomes interesting from the point of view of constraining SM parameters. An upgrade to the experiment to collect roughly an order of magnitude more sensitivity is in progress [24], and a new experiment with a sensitivity goal of $\sim 10^{-12}$/event has recently been given scientific approval at FNAL [25]. In the future this mode may provide grounds for precision tests of the flavor structure of the standard model [26]. The branching ratio can be written in terms of the very well-measured K_{e3} rate as [2]:

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 \mathrm{B}(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4\theta_W} \times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^{\ell} + V_{ts}^* V_{td} X(m_t)|^2 \qquad (1)$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [27]. In Eq. (1) the Inami-Lim function $X(m_t)$ is of order 1 [28], and X^{ℓ}_{NL} is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. QCD corrections, which mainly affect X^{ℓ}_{NL}, are known at next-to-leading order [12,29] and lead to a residual error of $< 10\%$ for the decay amplitude. Evaluating the constants in Eq. (1), one can cast this result in terms of the CKM parameters A, ρ and η (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [12]

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) \approx 1.0 \times 10^{-10} A^4 [\eta^2 + (\rho_o - \rho)^2] \qquad (2)$$

where $\rho_o \equiv 1 + (\frac{2}{3}X^{e}_{NL} + \frac{1}{3}X^{\tau}_{NL})/(A^2 V^4_{us} X(m_t)) \approx 1.4$. Thus, $\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})$ determines a circle in the ρ, η plane with center $(\rho_o, 0)$ and radius $\approx \dfrac{1}{A^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.0\times 10^{-10}}}$.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter ρ, given by [12]:

$$\mathrm{B_{SD}}(K_L \to \mu^+\mu^-) \approx 1.6 \times 10^{-9} A^4 (\rho'_o - \rho)^2 \qquad (3)$$

where ρ'_o depends on the charm quark mass and is approximately 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is calculated in terms of the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B_{abs}}(K_L \to \mu^+\mu^-) = (7.07 \pm 0.18) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$ [30]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. In order to use this mode to constrain ρ it is, therefore, necessary to know the real part of the long-distance contribution. Unlike the absorptive part, the real part of the long-distance contribution cannot be derived from the measured rate for $K_L \to \gamma\gamma$. At present it is not possible to compute this long-distance component reliably, and therefore it is not possible to constrain ρ from this mode in a model independent way [31]. Several models exist to estimate this long-distance component [32,33] that are sufficient to place rough bounds on new physics from the measured rate for $K_L \to \mu^+\mu^-$ [34]. The decay $K_L \to e^+e^-$ is completely dominated by long distance physics and is easier to estimate. The result, $B(K_L \to e^+e^-) \sim 9\times10^{-12}$ [31,33], is in good agreement with the recent measurement [35]. It is expected that studies of the reactions $K_L \to \ell^+\ell^-\gamma$ [36], and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ [37,16], currently under active study by the KTeV and NA48 experiments, will improve our understanding of the long distance effects in $K_L \to \mu^+\mu^-$ (the current data is often parameterized in terms of α^*_K, discussed at the end of the Form Factors section of the K^0_L Particle Properties Listing in this edition).

D. Searches for direct CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [2,38,39]. In the Standard Model this mode is dominated by an intermediate top-quark state and does not suffer from the uncertainty associated with the charm-quark intermediate state that affects the mode $K^+ \to \pi^+\nu\overline{\nu}$. The branching ratio is given approximately by Ref. 12:

$$\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \approx 4.1 \times 10^{-10} A^4\eta^2 \ . \tag{4}$$

With current constraints on the CKM parameters this leads to a predicted branching ratio $(2.6 \pm 1.2) \times 10^{-11}$. The current experimental upper bound is $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \leq 5.9 \times 10^{-7}$ [40]. The 90% CL bound on $K^+ \to \pi^+\nu\bar{\nu}$ provides a nearly model independent bound $B(K_L \to \pi^0\nu\bar{\nu}) < 1.7\times10^{-9}$ [41]. A KEK experiment to reach the 3×10^{-10}/event level is in preparation [42]. The KOPIO [43] proposal aims to reach the 6×10^{-13}/event level for $K_L \to \pi^0\nu\overline{\nu}$ at the BNL AGS.

There has been much theoretical work on possible contributions to ϵ'/ϵ and rare K decays in supersymmetric extensions of the SM. While in the simplest case of the MSSM with no new sources of flavor or CP-violation the main effect is a suppression of the rare K decays [44], substantial enhancements are possible in more general SUSY models [34,45].

The decay $K_L \to \pi^0 e^+ e^-$ also has sensitivity to the product $A^4\eta^2$. It has a direct CP-violating component given by [12]:

$$\mathrm{B}_{\mathrm{dir}}(K_L \to \pi^0 e^+ e^-) \approx 6.7 \times 10^{-11} A^4 \eta^2 \ . \tag{5}$$

However, like $K_L \to \mu^+\mu^-$ this mode suffers from large theoretical uncertainties due to long distance strong interaction effects.

The CP-violating component also receives an indirect contribution which is given by:

$$\mathrm{B}_{\mathrm{ind}}(K_L \to \pi^0 e^+ e^-) = |\epsilon|^2 \frac{\tau_{K_L}}{\tau_{K_S}} \mathrm{B}(K_S \to \pi^0 e^+ e^-) \ , \tag{6}$$

when interference between the direct and indirect contributions is neglected. Certain models that relate the processes $K_S \to \pi^0 e^+ e^-$ and $K^+ \to \pi^+ e^+ e^-$ have been used to predict that $B_{\mathrm{ind}}(K_L \to \pi^0 e^+ e^-)$ is less than 10^{-12} [46]. However, precise knowledge of this component awaits measurement of $K_S \to \pi^0 e^+ e^-$ [4,47]. The 90% CL upper limit, $B(K_S \to \pi^0 e^+ e^-) < 1.4 \times 10^{-7}$, recently obtained by NA48 [48] is about two orders of magnitude short of the expected level. NA48 proposes to reach $\sim 10^{-9}$/event sensitivity for this mode in their upcoming K_S run [49].

There is also a CP-conserving component dominated by a two-photon intermediate state that cannot be computed reliably at present. This component has an absorptive part that can be, in principle, determined from a detailed analysis of $K_L \to \pi^0\gamma\gamma$. To understand the rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$ in $K_L \to \pi^0\gamma\gamma$ within chiral perturbation theory it is necessary to go beyond leading order. It is possible to accommodate the existing measurements in terms of one parameter, a_V [50]. A fit to the distribution by the KTeV collaboration [51] has found $a_V = -0.72 \pm 0.05 \pm 0.06$. This value suggests that the absorptive part of the CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$ could be comparable to the direct CP-violating component [47,51]. However, a new result from NA48, $a_V = -0.46 \pm 0.03 \pm 0.03 \pm 0.02$ [52] would suggest that this contribution is smaller. A model independent prediction for the CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$ is not possible in terms of a_V alone [53]. The related process, $K_L \to \pi^0 \gamma e^+ e^-$, is potentially an additional background in some region of phase space [54]. This process has recently been observed with a branching ratio of $(2.34 \pm 0.35_{\rm stat} \pm 0.13_{\rm sys}) \times 10^{-8}$ [55].

Finally, BNL-845 observed a potential background to $K_L \to \pi^0 e^+ e^-$ from the decay $K_L \to \gamma\gamma e^+ e^-$ [56]. This has recently been confirmed with a 500-fold larger sample by FNAL-799 [57], which measured additional kinematic quantities. It has been estimated that this background will enter at the level of 3×10^{-10} [58,59], comparable to or larger than the signal level. Because of this, the observation of $K_L \to \pi^0 e^+ e^-$ will depend on background subtraction with good statistics.

The current 90% CL preliminary upper bound for the process $K_L \to \pi^0 e^+ e^-$ is 5.1×10^{-10} [59]. For the closely related muonic process, the corresponding upper bound is $\mathrm{B}(K_L \to \pi^0 \mu^+ \mu^-) \leq 3.8 \times 10^{-10}$ [60]. KTeV has collected data corresponding to about a factor 1.3 in sensitivity for both reactions which is still to be analyzed [61].

Recently, a new study of $K_L \to \pi^0 \mu^+ \mu^-$ has indicated that it might be possible to extract the direct CP-violating contribution by a joint study of the Dalitz plot variables and the components of the μ^+ polarization [62]. The latter tend to be quite substantial so that large statistics may not be necessary.

E. Other long distance dominated modes:

The decays $K^+ \to \pi^+ \ell^+ \ell^-$ ($\ell = e$ or μ) have received considerable attention. The rate and spectrum have been measured for both the electron and muon modes [63,64]. Ref. 65 has proposed a parameterization inspired by chiral perturbation theory, which provides a successful description of data but indicates the presence of large corrections beyond leading order. More work is needed to fully understand the origin of these large corrections.

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