THE Z BOSON

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Precision measurements at the Z-boson resonance using electron–positron colliding beams began in 1989 at the SLC and at LEP. During 1989–95, the four CERN experiments made high-statistics studies of the Z. The availability of longitudinally polarized electron beams at the SLC since 1993 enabled a precision determination of the effective electroweak mixing angle $\sin^2\overline{\theta}_W$ that is competitive with the CERN results on this parameter.

The Z-boson properties reported in this section may broadly be categorized as:

- The standard 'lineshape' parameters of the Z consisting of its mass, M_Z, its total width, Γ_Z, and its partial decay widths, Γ(hadrons), and $\Gamma(\ell\overline{\ell})$ where $\ell = e, \mu, \tau, \nu$;
- Z asymmetries in leptonic decays and extraction of Z couplings to charged and neutral leptons;
- The b- and c-quark-related partial widths and charge asymmetries which require special techniques;
- Determination of Z decay modes and the search for modes that violate known conservation laws;
- Average particle multiplicities in hadronic Z decay;
- Z anomalous couplings.

Details on Z-parameter determination and the study of $Z \to b\overline{b}, c\overline{c}$ at LEP and SLC are given in this note.

The standard 'lineshape' parameters of the Z are determined from an analysis of the production cross sections of these final states in e^+e^- collisions. The $Z \to \nu\overline{\nu}(\gamma)$ state is identified directly by detecting single photon production and indirectly by subtracting the visible partial widths from the total width. Inclusion in this analysis of the forward-backward asymmetry of charged leptons, $A_{FB}^{(0,\ell)}$, of the τ polarization, $P(\tau)$, and its forward-backward asymmetry, $P(\tau)^{fb}$, enables the separate determination of the effective vector ($\overline{g}_V$) and axial vector ($\overline{g}_A$) couplings of the Z to these leptons and the ratio ($\overline{g}_V/\overline{g}_A$) which is related to the effective electroweak mixing angle $\sin^2\overline{\theta}_W$ (see the "Electroweak Model and Constraints on New Physics" Review).

Determination of the b- and c-quark-related partial widths and charge asymmetries involves tagging the b and c quarks. Traditionally this was done by requiring the presence of a prompt lepton in the event with high momentum and high transverse momentum (with respect to the accompanying jet). Precision vertex measurement with high-resolution detectors enabled one to do impact parameter and lifetime tagging. Neural-network techniques have also been used to classify events as b or non-b on a statistical basis using event–shape variables. Finally, the presence of a charmed meson (D/D^*) has been used to tag heavy quarks.

Z-parameter determination

LEP was run at energy points on and around the Z mass (88–94 GeV) constituting an energy 'scan.' The shape of the cross-section variation around the Z peak can be described by a Breit-Wigner *ansatz* with an energy-dependent total width [1–3]. The **three** main properties of this distribution, viz., the **position** of the peak, the **width** of the distribution, and the **height** of the peak, determine respectively the values of M_Z, Γ_Z, and $\Gamma(e^+e^-) \times \Gamma(f\overline{f})$, where $\Gamma(e^+e^-)$ and $\Gamma(f\overline{f})$ are the electron and fermion partial widths of the Z. The quantitative determination of these parameters is done by writing analytic expressions for these cross sections in terms of the parameters and fitting the calculated cross sections to the measured ones by varying these parameters, taking properly into account all the errors. Single-photon exchange (σ^0_γ) and γ-Z interference ($\sigma^0_{\gamma Z}$) are included, and the large (∼25 %) initial-state radiation (ISR) effects are taken into account by convoluting the analytic expressions over a 'Radiator Function' [1–5] $H(s, s')$. Thus for the process $e^+e^- \to f\overline{f}$:

$$\sigma_f(s) = \int H(s, s')\ \sigma^0_f(s')\ ds' \qquad (1)$$

$$\sigma^0_f(s) = \sigma^0_Z + \sigma^0_\gamma + \sigma^0_{\gamma Z} \qquad (2)$$

$$\sigma_Z^0 = \frac{12\pi}{M_Z^2} \; \frac{\Gamma(e^+e^-)\Gamma(f\overline{f})}{\Gamma_Z^2} \; \frac{s\ \Gamma_Z^2}{(s-M_Z^2)^2\ +\ s^2\Gamma_Z^2/M_Z^2} \quad (3)$$

$$\sigma_\gamma^0 = \frac{4\pi\alpha^2(s)}{3s} \; Q_f^2 N_c^f \quad (4)$$

$$\sigma_{\gamma Z}^0 = -\frac{2\sqrt{2}\alpha(s)}{3} \; (Q_f G_F N_c^f \mathcal{G}_V^e \mathcal{G}_V^f) \times \frac{(s-M_Z^2)M_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \quad (5)$$

where Q_f is the charge of the fermion, $N_c^f = 3(1)$ for quark (lepton) and $\mathcal{G}_V^f$ is the neutral vector coupling of the Z to the fermion-antifermion pair $f\overline{f}$.

Since $\sigma_{\gamma Z}^0$ is expected to be much less than σ_Z^0, the LEP Collaborations have generally calculated the interference term in the framework of the Standard Model. This fixing of $\sigma_{\gamma Z}^0$ leads to a tighter constraint on M_Z and consequently a smaller error on its fitted value.

In the above framework, the QED radiative corrections have been explicitly taken into account by convoluting over the ISR and allowing the electromagnetic coupling constant to run [9]: $\alpha(s) = \alpha/(1-\Delta\alpha)$. On the other hand, weak radiative corrections that depend upon the assumptions of the electroweak theory and on the values of $M_{\rm top}$ and $M_{\rm Higgs}$ are accounted for by **absorbing them into the couplings**, which are then called the *effective* couplings $\mathcal{G}_V$ and $\mathcal{G}_A$ (or alternatively the effective parameters of the $\star$ scheme of Kennedy and Lynn [10]).

$\mathcal{G}_V^f$ and $\mathcal{G}_A^f$ are complex numbers with a small imaginary part. As experimental data does not allow simultaneous extraction of both real and imaginary parts of the effective couplings, the convention $g_A^f = {\rm Re}(\mathcal{G}_A^f)$ and $g_V^f = {\rm Re}(\mathcal{G}_V^f)$ is used and the imaginary parts are added in the fitting code [4].

Defining

$$A_f = 2\frac{g_V^f \cdot g_A^f}{(g_V^f)^2 + (g_A^f)^2} \quad (6)$$

the lowest-order expressions for the various lepton-related asymmetries on the Z pole are [6–8] $A_{FB}^{(0,\ell)} = (3/4)A_e A_f$, $P(\tau) = -A_\tau$, $P(\tau)^{fb} = -(3/4)A_e$, $A_{LR} = A_e$. The full analysis takes into account the energy dependence of the asymmetries.

Experimentally A_{LR} is defined as $(\sigma_L - \sigma_R)/(\sigma_L + \sigma_R)$ where $\sigma_{L(R)}$ are the $e^+e^- \to Z$ production cross sections with left-(right)-handed electrons.

The definition of the partial decay width of the Z to $f\overline{f}$ includes the effects of QED and QCD final state corrections as well as the contribution due to the imaginary parts of the couplings:

$$\Gamma(f\overline{f}) = \frac{G_F M_Z^3}{6\sqrt{2}\pi}\ N_c^f(\left|\mathcal{G}_A^f\right|^2 R_A^f + \left|\mathcal{G}_V^f\right|^2 R_V^f) + \Delta_{ew/\mathrm{QCD}} \qquad (7)$$

where R_V^f and R_A^f are radiator factors to account for final state QED and QCD corrections as well as effects due to nonzero fermion masses, and $\Delta_{ew/\mathrm{QCD}}$ represents the non-factorizable electroweak/QCD corrections.

S-matrix approach to the Z

While practically all experimental analyses of LEP/SLC data have followed the 'Breit-Wigner' approach described above, an alternative S-matrix-based analysis is also possible. The Z, like all unstable particles, is associated with a complex pole in the S matrix. The pole position is process independent and gauge invariant. The mass, $\overline{M}_Z$, and width, $\overline{\Gamma}_Z$, can be defined in terms of the pole in the energy plane via [11–14]

$$\overline{s} = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z \qquad (8)$$

leading to the relations

$$\begin{aligned}\overline{M}_Z &= M_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \\ &\approx M_Z\ - 34.1\ \mathrm{MeV} \qquad (9)\\ \overline{\Gamma}_Z &= \Gamma_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \\ &\approx \Gamma_Z\ - 0.9\ \mathrm{MeV}\ . \qquad (10)\end{aligned}$$

Some authors [15] choose to define the Z mass and width via

$$\overline{s} = (\overline{M}_Z - \frac{i}{2}\overline{\Gamma}_Z)^2 \qquad (11)$$

which yields $\overline{M}_Z \approx M_Z - 26$ MeV, $\overline{\Gamma}_Z \approx \Gamma_Z - 1.2$ MeV.

The L3 and OPAL Collaborations at LEP (ACCIARRI 00Q and ACKERSTAFF 97C) have analyzed their data using the S–matrix approach as defined in Eq. (8), in addition to the conventional one. They observe a downward shift in the Z mass as expected.

Handling the large-angle e^+e^- final state

Unlike other $f\overline{f}$ decay final states of the Z, the e^+e^- final state has a contribution not only from the s-channel but also from the t-channel and s-t interference. The full amplitude is not amenable to fast calculation, which is essential if one has to carry out minimization fits within reasonable computer time. The usual procedure is to calculate the non-s channel part of the cross section separately using the Standard Model programs ALIBABA [16] or TOPAZ0 [17] with the measured value of M_{top}, and $M_{\text{Higgs}} = 150$ GeV and add it to the s-channel cross section calculated as for other channels. This leads to two additional sources of error in the analysis: firstly, the theoretical calculation in ALIBABA itself is known to be accurate to $\sim 0.5\%$, and secondly, there is uncertainty due to the error on M_{top} and the unknown value of M_{Higgs} (100–1000 GeV). These errors are propagated into the analysis by including them in the systematic error on the e^+e^- final state. As these errors are common to the four LEP experiments, this is taken into account when performing the LEP average.

Errors due to uncertainty in LEP energy determination [18–23]

The systematic errors related to the LEP energy measurement can be classified as:

- The absolute energy scale error;
- Energy-point-to-energy-point errors due to the non-linear response of the magnets to the exciting currents;
- Energy-point-to-energy-point errors due to possible higher-order effects in the relationship between the dipole field and beam energy;

- Energy reproducibility errors due to various unknown uncertainties in temperatures, tidal effects, corrector settings, RF status, *etc.*

Precise energy calibration was done outside normal data taking using the resonant depolarization technique. Run-time energies were determined every 10 minutes by measuring the relevant machine parameters and using a model which takes into account all the known effects, including leakage currents produced by trains in the Geneva area and the tidal effects due to gravitational forces of the Sun and the Moon. The LEP Energy Working Group has provided a covariance matrix from the determination of LEP energies for the different running periods during 1993–1995 [18].

Choice of fit parameters

The LEP Collaborations have chosen the following primary set of parameters for fitting: M_Z, Γ_Z, $\sigma^0_{\rm hadron}$, R(lepton), $A^{(0,\ell)}_{FB}$, where $R(\text{lepton}) = \Gamma(\text{hadrons})/\Gamma(\text{lepton})$, $\sigma^0_{\rm hadron} = 12\pi\Gamma(e^+e^-)\Gamma(\text{hadrons})/M^2_Z\Gamma^2_Z$. With a knowledge of these fitted parameters and their covariance matrix, any other parameter can be derived. The main advantage of these parameters is that they form the **least correlated** set of parameters, so that it becomes easy to combine results from the different LEP experiments.

Thus, the most general fit carried out to cross section and asymmetry data determines the **nine parameters**: M_Z, Γ_Z, $\sigma^0_{\rm hadron}$, $R(e)$, $R(\mu)$, $R(\tau)$, $A^{(0,e)}_{FB}$, $A^{(0,\mu)}_{FB}$, $A^{(0,\tau)}_{FB}$. Assumption of lepton universality leads to a **five-parameter fit** determining M_Z, Γ_Z, $\sigma^0_{\rm hadron}$, R(lepton), $A^{(0,\ell)}_{FB}$.

Combining results from LEP and SLC experiments

With steady increase in statistics over the years and improved understanding of the common systematic errors between LEP experiments, the procedures for combining results have evolved continuously [24]. The Line Shape Sub-group of the LEP Electroweak Working Group investigated the effects of these common errors and devised a combination procedure for the precise determination of the Z parameters from LEP experiments [25]. Using these procedures this note also gives the results after combining the final parameter sets from the four experiments and these are the results quoted as the fit results in the Z listings below. Transformation of variables leads to values of derived parameters like partial decay widths and branching ratios to hadrons and leptons. Finally, transforming the LEP combined nine parameter set to (M_Z, Γ_Z, $\sigma^\circ_{\rm hadron}$, g^f_A, g^f_V, $f = e, \mu, \tau$) using the average values of lepton asymmetry parameters (A_e, A_μ, A_τ) as constraints, leads to the best fitted values of the vector and axial-vector couplings (g_V, g_A) of the charged leptons to the Z.

Brief remarks on the handling of common errors and their magnitudes are given below. The identified common errors are those coming from

(a) LEP energy calibration uncertainties, and

(b) the theoretical uncertainties in (i) the luminosity determination using small angle Bhabha scattering, (ii) estimating the non-s channel contribution to large angle Bhabha scattering, (iii) the calculation of QED radiative effects, and (iv) the parametrization of the cross section in terms of the parameter set used.

Common LEP energy errors

All the collaborations incorporate in their fit the full LEP energy error matrix as provided by the LEP energy group for their intersection region [18]. The effect of these errors is separated out from that of other errors by carrying out fits with energy errors scaled up and down by $\sim 10\%$ and redoing the fits. From the observed changes in the overall error matrix the covariance matrix of the common energy errors is determined. Common LEP energy errors lead to uncertainties on M_Z, Γ_Z, and $\sigma^\circ_{\rm hadron}$ of 1.7, 1.2 MeV, and 0.011 nb respectively.

Common luminosity errors

BHLUMI 4.04 [26] is used by all LEP collaborations for small angle Bhabha scattering leading to a common uncertainty in their measured cross sections of 0.061% [27]. BHLUMI does not include a correction for production of light fermion pairs. OPAL explicitly correct for this effect and reduce their luminosity uncertainty to 0.054% which is taken fully correlated with the other experiments. The other three experiments among themselves have a common uncertainty of 0.061%.

Common non-s channel uncertainties

The same standard model programs ALIBABA [16] and TOPAZ0 [17] are used to calculate the non-s channel contribution to the large angle Bhabha scattering [28]. As this contribution is a function of the Z mass, which itself is a variable in the fit, it is parametrized as a function of M_Z by each collaboration to properly track this contribution as M_Z varies in the fit. The common errors on R_e and $A_{FB}^{0,e}$ are 0.024 and 0.0014 respectively and are correlated between them.

Common theoretical uncertainties: QED

There are large initial state photon and fermion pair radiation effects near the Z resonance for which the best currently available evaluations include contributions up to $\mathcal{O}(\alpha^3)$. To estimate the remaining uncertainties different schemes are incorporated in the standard model programs ZFITTER [5], TOPAZ0 [17] and MIZA [29]. Comparing the different options leads to error estimates of 0.3 and 0.2 MeV on M_Z and Γ_Z respectively and of 0.02% on $\sigma^\circ_{\text{hadron}}$.

Common theoretical uncertainties: parametrization of lineshape and asymmetries

To estimate uncertainties arising from ambiguities in the model-independent parametrization of the differential cross-section near the Z resonance, results from TOPAZ0 and ZFITTER were compared by using ZFITTER to fit the cross sections and asymmetries calculated using TOPAZ0. The resulting uncertainties on M_Z, Γ_Z, $\sigma^\circ_{\text{hadron}}$, $R(\text{lepton})$ and $A_{FB}^{0,\ell}$ are 0.1 MeV, 0.1 MeV, 0.001 nb, 0.004, and 0.0001 respectively.

Thus the overall theoretical errors on M_Z, Γ_Z, $\sigma^\circ_{\text{hadron}}$ are 0.3 MeV, 0.2 MeV, and 0.008 nb respectively; on each $R(\text{lepton})$ is 0.004 and on each $A_{FB}^{0,\ell}$ is 0.0001. Within the set of three $R(\text{lepton})$'s and the set of three $A_{FB}^{0,\ell}$'s the respective errors are fully correlated.

All the theory related errors mentioned above utilize standard model programs which need the Higgs mass and running electromagnetic coupling constant as inputs; uncertainties on these inputs will also lead to common errors. All LEP collaborations used the same set of inputs for standard model calculations: $M_Z = 91.187$ GeV, the Fermi constant $G_F = (1.16637 \pm 0.00001) \times 10^{-5}$ GeV^{-2} [30], $\alpha^{(5)}(M_Z) = 1/128.877 \pm 0.090$ [31], $\alpha_s(M_Z) = 0.119$ [32], $M_{\rm top} = 174.3 \pm 5.1$ GeV [32] and $M_{\rm Higgs}$ = 150 GeV. The only observable effect, on M_Z, is due to the variation of $M_{\rm Higgs}$ between 100–1000 GeV (due to the variation of the γ/Z interference term which is taken from the standard model): M_Z changes by +0.23 MeV per unit change in $\log_{10} M_{\rm Higgs}$/GeV, which is not an error but a correction to be applied once $M_{\rm Higgs}$ is determined. The effect is much smaller than the error on M_Z (±2.1 MeV).

Methodology of combining the LEP experimental results

The LEP experimental results actually used for combination are slightly modified from those published by the experiments (which are given in the Listings below). This has been done in order to facilitate the procedure by making the inputs more consistent. These modified results are given explicitly in Ref. 25. The main differences compared to the published results are

(a) consistent use of ZFITTER 6.23 and TOPAZ0. The published ALEPH results used ZFITTER 6.10. (b) use of the combined energy error matrix which makes a difference of 0.1 MeV on the M_Z and Γ_Z for L3 only as at that intersection the RF modeling uncertainties are the largest.

Thus, nine-parameter sets from all four experiments with their covariance matrices are used together with all the common errors correlations. A grand covariance matrix, V, is constructed and a combined nine-parameter set is obtained by minimizing $\chi^2 = \Delta^T V^{-1} \Delta$, where Δ is the vector of residuals of the combined parameter set to the results of individual experiments.

Study of $Z \rightarrow b\overline{b}$ and $Z \rightarrow c\overline{c}$

In the sector of c- and b-physics the LEP experiments have measured the ratios of partial widths $R_b = \Gamma(Z \rightarrow b\overline{b})/\Gamma(Z \rightarrow \text{hadrons})$ and $R_c = \Gamma(Z \rightarrow c\overline{c})/\Gamma(Z \rightarrow \text{hadrons})$ and the forward-backward (charge) asymmetries $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$. The final state coupling parameters A_b and A_c have been obtained from the left-right forward-backward asymmetry at SLD.

Several of the analyses have also determined other quantities, in particular the semileptonic branching ratios, B($b \to \ell^-$), B($b \to c \to \ell^+$), and B($c \to \ell^+$), the average $B^0\overline{B}^0$ mixing parameter $\overline{\chi}$ and the probabilities for a c–quark to fragment into a D^+, a D_s, a D^{*+} , or a charmed baryon. The latter measurements do not concern properties of the Z boson and hence they do not appear in the listing below. However, for completeness, we will report at the end of this minireview their values as obtained fitting the data contained in the Z section. All these quantities are correlated with the electroweak parameters, and since the mixture of b hadrons is different from the one at the $\Upsilon(4S)$, their values might differ from those measured at the $\Upsilon(4S)$.

All the above quantities are correlated to each other since:

- Several analyses (for example the lepton fits) determine more than one parameter simultaneously;
- Some of the electroweak parameters depend explicitly on the values of other parameters (for example R_b depends on R_c);
- Common tagging and analysis techniques produce common systematic uncertainties.

The LEP Electroweak Heavy Flavour Working Group has developed [33] a procedure for combining the measurements taking into account known sources of correlation. The combining procedure determines twelve parameters: the four parameters of interest in the electroweak sector, R_b, R_c, $A_{FB}^{b\overline{b}}$, and $A_{FB}^{c\overline{c}}$ and, in addition, B($b \to \ell^-$), B($b \to c \to \ell^+$), B($c \to \ell^+$), $\overline{\chi}$, $f(D^+)$, $f(D_s)$, $f(c_{\text{baryon}})$ and $P(c \to D^{*+}) \times$ B($D^{*+} \to \pi^+ D^0$), to take into account their correlations with the electroweak parameters. Before the fit both the peak and off-peak asymmetries are translated to the common energy $\sqrt{s} = 91.26$ GeV using the predicted energy dependence from ZFITTER [5].

Summary of the measurements and of the various kinds of analysis

The measurements of R_b and R_c fall into two classes. In the first, named single-tag measurement, a method for selecting b and c events is applied and the number of tagged events is counted. The second technique, named double-tag measurement, is based on the following principle: if the number of events with a single hemisphere tagged is N_t and with both hemispheres tagged is N_{tt}, then given a total number of N_{had} hadronic Z decays one has:

$$\frac{N_t}{2N_{\text{had}}} = \varepsilon_b R_b + \varepsilon_c R_c + \varepsilon_{uds}(1 - R_b - R_c) \tag{12}$$

$$\frac{N_{tt}}{N_{\text{had}}} = \mathcal{C}_b \varepsilon_b^2 R_b + \mathcal{C}_c \varepsilon_c^2 R_c + \mathcal{C}_{uds} \varepsilon_{uds}^2 (1 - R_b - R_c) \tag{13}$$

where ε_b, ε_c, and ε_{uds} are the tagging efficiencies per hemisphere for b, c, and light quark events, and $\mathcal{C}_q \neq 1$ accounts for the fact that the tagging efficiencies between the hemispheres may be correlated. In tagging the b one has $\varepsilon_b \gg \varepsilon_c \gg \varepsilon_{uds}$, $C_b \approx 1$. Neglecting the c and uds background and the hemisphere correlations, these equations give:

$$\varepsilon_b = 2N_{tt}/N_t \tag{14}$$

$$R_b = N_t^2/(4N_{tt}N_{\text{had}}) \ . \tag{15}$$

The double-tagging method has thus the great advantage that the tagging efficiency is directly derived from the data, reducing the systematic error of the measurement. The backgrounds, dominated by $c\overline{c}$ events, obviously complicate this simple picture, and their level must still be inferred by other means. The rate of charm background in these analyses depends explicitly on the value of R_c. The correlations in the tagging efficiencies between the hemispheres (due for instance to correlations in momentum between the b hadrons in the two hemispheres) are small but nevertheless lead to further systematic uncertainties.

The measurements in the b- and c-sector can be essentially grouped in the following categories:

- Lifetime (and lepton) double-tagging measurements of R_b. These are the most precise measurements of R_b and obviously dominate the combined result. The main sources of systematics come from the charm contamination and from estimating the hemisphere b-tagging efficiency correlation. The charm rejection has been improved (and hence the systematic errors reduced) by using either the information of the secondary vertex invariant mass or the information from the energy of all particles at the secondary vertex and their rapidity;
- Analyses with $D/D^{*\pm}$ to measure R_c. These measurements make use of several different tagging techniques (inclusive/exclusive double tag, exclusive double tag, reconstruction of all weakly decaying charmed states) and no assumptions are made on the energy dependence of charm fragmentation;
- Lepton fits which use hadronic events with one or more leptons in the final state to measure $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$. Each analysis usually gives several other electroweak parameters. The dominant sources of systematics are due to lepton identification, to other semileptonic branching ratios and to the modeling of the semileptonic decay;
- Measurements of $A_{FB}^{b\overline{b}}$ using lifetime tagged events with a hemisphere charge measurement. Their contribution to the combined result has roughly the same weight as the lepton fits;
- Analyses with $D/D^{*\pm}$ to measure $A_{FB}^{c\overline{c}}$ or simultaneously $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$;
- Measurements of A_b and A_c from SLD, using several tagging methods (lepton, kaon, D/D^*, and vertex mass). These quantities are directly extracted from a measurement of the left–right forward–backward asymmetry in $c\overline{c}$ and $b\overline{b}$ production using a polarized electron beam.

Averaging procedure

All the measurements are provided by the LEP Collaborations in the form of tables with a detailed breakdown of the systematic errors of each measurement and its dependence on other electroweak parameters.

The averaging proceeds via the following steps:

- Define and propagate a consistent set of external inputs such as branching ratios, hadron lifetimes, fragmentation models *etc.* All the measurements are also consistently checked to ensure that all use a common set of assumptions (for instance since the QCD corrections for the forward–backward asymmetries are strongly dependent on the experimental conditions, the data are corrected before combining);
- Form the full (statistical and systematic) covariance matrix of the measurements. The systematic correlations between different analyses are calculated from the detailed error breakdown in the measurement tables. The correlations relating several measurements made by the same analysis are also used;
- Take into account any explicit dependence of a measurement on the other electroweak parameters. As an example of this dependence we illustrate the case of the double-tag measurement of R_b, where c-quarks constitute the main background. The normalization of the charm contribution is not usually fixed by the data and the measurement of R_b depends on the assumed value of R_c, which can be written as:

$$R_b = R_b^{\rm meas} + a(R_c)\frac{(R_c - R_c^{\rm used})}{R_c} \ , \qquad (16)$$

where $R_b^{\rm meas}$ is the result of the analysis which assumed a value of $R_c = R_c^{\rm used}$ and $a(R_c)$ is the constant which gives the dependence on R_c;
- Perform a χ^2 minimization with respect to the combined electroweak parameters.

After the fit the average peak asymmetries $A^{c\overline{c}}_{FB}$ and $A^{b\overline{b}}_{FB}$ are corrected for the energy shift from 91.26 GeV to M_Z and for QED (initial state radiation), γ exchange, and γZ interference effects to obtain the corresponding pole asymmetries $A^{0,c}_{FB}$ and $A^{0,b}_{FB}$.

This averaging procedure, using the fourteen parameters described above and applied to the data contained in the Z particle listing below, gives the following results:

$$
\begin{aligned}
R_b^0 &= 0.21650 \pm 0.00072 \\
R_c^0 &= 0.1682 \pm 0.0047 \\
A^{0,b}_{FB} &= 0.1002 \pm 0.0019 \\
A^{0,c}_{FB} &= 0.0716 \pm 0.0036 \\
A_b &= 0.928 \pm 0.031 \\
A_c &= 0.666 \pm 0.036 \\
B(b \to \ell^-) &= 0.1057 \pm 0.0021 \\
\mathrm{B}(b \to c \to \ell^+) &= 0.0807 \pm 0.0018 \\
B(c \to \ell^+) &= 0.0985 \pm 0.0034 \\
\overline{\chi} &= 0.1185 \pm 0.0043 \\
f(D^+) &= 0.236 \pm 0.016 \\
f(D_s) &= 0.119 \pm 0.025 \\
f(c_{\mathrm{baryon}}) &= 0.090 \pm 0.022 \\
P(c \to D^{*+}) \times \mathrm{B}(D^{*+} \to \pi^+ D^0) &= 0.1650 \pm 0.0056
\end{aligned}
$$

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