39. CROSS-SECTION FORMULAE FOR SPECIFIC PROCESSES

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39.1. Leptoproduction

See section on Structure Functions (Sec. 16 of this *Review*).

39.2. e^+e^- annihilation

For pointlike, spin-1/2 fermions, the differential cross section in the c.m. for $e^+e^- \to f\overline{f}$ via single photon annihilation is (θ is the angle between the incident electron and the produced fermion; $N_c = 1$ if f is a lepton and $N_c = 3$ if f is a quark).

$$\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4s}\beta\big[1 + \cos^2\theta + (1-\beta^2)\sin^2\theta\big]Q_f^2 \ , \tag{39.1}$$

where β is the velocity of the final state fermion in the c.m. and Q_f is the charge of the fermion in units of the proton charge. For $\beta \to 1$,

$$\sigma = N_c \frac{4\pi\alpha^2}{3s} Q_f^2 = N_c \frac{86.8 Q_f^2 \ nb}{s} \ . \tag{39.2}$$

where s is in GeV^2 units.

At higher energies, the Z^0 (mass M_Z and width Γ_Z) must be included. If the mass of a fermion f is much less than the mass of the Z^0, then the differential cross section for $e^+e^- \to f\overline{f}$ is

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \ & N_c \frac{\alpha^2}{4s}\Big\{(1+\cos^2\theta)\left[Q_f^2 - 2\chi_1 v_e v_f Q_f + \chi_2 (a_e^2 + v_e^2)(a_f^2 + v_f^2)\right] \\ & + 2\cos\theta\left[-2\chi_1 a_e a_f Q_f + 4\chi_2 a_e a_f v_e v_f\right]\Big\}\end{aligned} \tag{39.3}$$

where

$$\begin{aligned}\chi_1 &= \frac{1}{16\sin^2\theta_W\cos^2\theta_W}\frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \ , \\ \chi_2 &= \frac{1}{256\sin^4\theta_W\cos^4\theta_W}\frac{s^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \ , \\ a_e &= -1 \ , \\ v_e &= -1 + 4\sin^2\theta_W \ , \\ a_f &= 2T_{3f} \ , \\ v_f &= 2T_{3f} - 4Q_f\sin^2\theta_W \ ,\end{aligned} \tag{39.4}$$

where $T_{3f} = 1/2$ for u, c and neutrinos, while $T_{3f} = -1/2$ for d, s, b, and negatively charged leptons.

At LEP II it may be possible to produce the orthodox Higgs boson, H, (see the mini-review on Higgs bosons) in the reaction $e^+e^- \to HZ^0$, which proceeds dominantly through a virtual Z^0. The Standard Model prediction for the cross section [3] is

$$\sigma(e^+e^- \to HZ^0) = \frac{\pi\alpha^2}{24} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2 + 3M_Z^2}{(s - M_Z^2)^2} \cdot \frac{1 - 4\sin^2\theta_W + 8\sin^4\theta_W}{\sin^4\theta_W \cos^4\theta_W} . \tag{39.5}$$

where K is the c.m. momentum of the produced H or Z^0. Near the production threshold, this formula needs to be corrected for the finite width of the Z^0.

39.3. Two-photon process at e^+e^- colliders

When an e^+ and an e^- collide with energies E_1 and E_2, they emit dn_1 and dn_2 virtual photons with energies ω_1 and ω_2 and 4-momenta q_1 and q_2. In the equivalent photon approximation, the cross section for $e^+e^- \to e^+e^-X$ is related to the cross section for $\gamma\gamma \to X$ by (Ref. 1)

$$d\sigma_{e^+e^-\to e^+e^-X}\,(s) = dn_1\, dn_2\, d\sigma_{\gamma\gamma\to X}\,(W^2) \tag{39.6}$$

where $s = 4E_1E_2$, $W^2 = 4\omega_1\omega_2$ and

$$dn_i = \frac{\alpha}{\pi}\left[1 - \frac{\omega_i}{E_i} + \frac{\omega_i^2}{2E_i^2} - \frac{m_e^2\omega_i^2}{(-q_i^2)E_i^2}\right]\frac{d\omega_i}{\omega_i}\,\frac{d(-q_i^2)}{(-q_i^2)} . \tag{39.7}$$

After integration (including that over q_i^2 in the region $m_e^2\omega_i^2/E_i(E_i - \omega_i) \le -q_i^2 \le (-q^2)_{\max}$), the cross section is

$$\begin{aligned}\sigma_{e^+e^-\to e^+e^-X}\,(s) = &\frac{\alpha^2}{\pi^2}\int_{z_{th}}^1 \frac{dz}{z}\left[f(z)\left(\ln\frac{(-q^2)_{\max}}{m_e^2 z} - 1\right)^2\right.\\ &\left. - \tfrac{1}{3}\left(\ln\frac{1}{z}\right)^3\right]\sigma_{\gamma\gamma\to X}\,(zs) \; ;\\ f(z) = &\left(1 + \tfrac{1}{2}z\right)^2 \ln\frac{1}{z} - \tfrac{1}{2}(1-z)(3+z) \; ;\\ z = &\frac{W^2}{s} .\end{aligned} \tag{39.8}$$

The quantity $(-q^2)_{\max}$ depends on properties of the produced system X, in particular, $(-q^2)_{\max} \sim m_\rho^2$ for hadron production ($X = h$) and $(-q^2)_{\max} \sim W^2$ for lepton pair production ($X = \ell^+\ell^-$, $\ell = e, \mu, \tau$).

For production of a resonance of mass m_R and spin $J \ne 1$

$$\begin{aligned}\sigma_{e^+e^-\to e^+e^-R}\,(s) = &(2J+1)\frac{8\alpha^2\Gamma_{R\to\gamma\gamma}}{m_R^3}\\ &\times\left[f(m_R^2/s)\left(\ln\frac{sm_V^2}{m_e^2m_R^2} - 1\right)^2 - \frac{1}{3}\left(\ln\frac{s}{m_R^2}\right)^3\right]\end{aligned} \tag{39.9}$$

where m_V is the mass that enters into the form factor of the $\gamma\gamma \to R$ transition: $m_V \sim m_\rho$ for $R = \pi^0, \eta, f_2(1270), \ldots$, $m_V \sim m_R$ for $R = c\overline{c}$ or $b\overline{b}$ resonances.

39.4. Inclusive hadronic reactions

One-particle inclusive cross sections $Ed^3\sigma/d^3p$ for the production of a particle of momentum p are conveniently expressed in terms of rapidity (see above) and the momentum p_T transverse to the beam direction (defined in the center-of-mass frame)

$$E\frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi\, dy\, p_T\, dp_T} \ . \tag{39.10}$$

In the case of processes where p_T is large or the mass of the produced particle is large (here large means greater than 10 GeV), the parton model can be used to calculate the rate. Symbolically

$$\sigma_{\text{hadronic}} = \sum_{ij} \int f_i(x_1,\, Q^2)\; f_j(x_2,\, Q^2)\; dx_1\; dx_2\; \widehat{\sigma}_{\text{partonic}} \ , \tag{39.11}$$

where $f_i(x,\, Q^2)$ is the parton distribution introduced above and Q is a typical momentum transfer in the partonic process and $\widehat{\sigma}$ is the partonic cross section. Some examples will help to clarify. The production of a W^+ in pp reactions at rapidity y in the center-of-mass frame is given by

$$\begin{aligned}\frac{d\sigma}{dy} = \frac{G_F\,\pi\sqrt{2}}{3} \times \tau \Big[&\cos^2\theta_c \Big(u(x_1\,,\; M_W^2)\; \overline{d}\;(x_2, M_W^2) \\ &\qquad + \; u(x_2\,,\; M_W^2)\; \overline{d}\;(x_1, M_W^2) \Big) \\ &+ \; \sin^2\theta_c \Big(u(x_1\,,\; M_W^2)\; \overline{s}\;(x_2\,,\; M_W^2) \\ &\qquad + \; s(x_2, M_W^2)\; \overline{u}\;(x_1, M_W^2) \Big) \Big] \ ,\end{aligned} \tag{39.12}$$

where $x_1 = \sqrt{\tau}\; e^y$, $x_2 = \sqrt{\tau}\; e^{-y}$, and $\tau = M_W^2/s$. Similarly the production of a jet in pp (or $p\overline{p}$) collisions is given by

$$\begin{aligned}\frac{d^3\sigma}{d^2p_T\; dy} = \sum_{ij} \int &f_i(x_1\,,\; p_T^2)\; f_j(x_2, p_T^2) \\ \times &\left[\widehat{s}\; \frac{d\widehat{\sigma}}{d\,\widehat{t}} \right]_{ij} dx_1\; dx_2\; \delta(\,\widehat{s} + \widehat{t} + \widehat{u}) \ ,\end{aligned} \tag{39.13}$$

where the summation is over quarks, gluons, and antiquarks. Here

$$s = (p_1 + p_2)^2 \ , \tag{39.14}$$

$$t = (p_1 - p_{\text{jet}})^2 \ , \tag{39.15}$$

$$u = (p_2 - p_{\text{jet}})^2 \ , \tag{39.16}$$

p_1 and p_2 are the momenta of the incoming p and p (or $\overline{p}$) and $\widehat{s}$, $\widehat{t}$, and $\widehat{u}$ are s, t, and u with $p_1 \rightarrow x_1 p_1$ and $p_2 \rightarrow x_2\, p_2$. The partonic cross section $\widehat{s}\,[\,(d\widehat{\sigma})/(d\,\widehat{t}\,)\,]$ can be found in Ref. 2. Example: for the process $gg \rightarrow q\overline{q}$,

$$\widehat{s}\; \frac{d\sigma}{dt} = 3\alpha_s^2\; \frac{(\,\widehat{t}^{\,2} + \widehat{u}^2)}{8\widehat{s}} \; \left[\frac{4}{9\,\widehat{t}\,\widehat{u}} - \frac{1}{\widehat{s}^{\,2}} \right] \ . \tag{39.17}$$

The prediction of Eq. (39.13) is compared to data from the UA1 and UA2 collaborations in Fig. 40.1 in the Plots of Cross Sections and Related Quantities section of this *Review*.

The associated production of a Higgs boson and a gauge boson is analogous to the process $e^+e^- \to HZ^0$ in Sec. 39.2. The required parton-level cross sections [4], averaged over initial quark colors, are

$$\sigma(q_i\overline{q}_j \to W^\pm H) = \frac{\pi\alpha^2|V_{ij}|^2}{36\sin^4\theta_W} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2+3M_W^2}{(s-M_W^2)^2}$$

$$\sigma(q\overline{q} \to Z^0 H) = \frac{\pi\alpha^2(a_q^2+v_q^2)}{144\sin^4\theta_W\cos^4\theta_W} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2+3M_Z^2}{(s-M_Z^2)^2} \ .$$

Here V_{ij} is the appropriate element of the Kobayashi-Maskawa matrix and K is the c.m. momentum of the produced H. The axial and vector couplings are defined as in Sec. 39.2.

39.5. One-particle inclusive distributions

In order to describe one-particle inclusive production in e^+e^- annihilation or deep inelastic scattering, it is convenient to introduce a fragmentation function $D_i^h\,(z,Q^2)$ where $D_i^h\,(z,Q^2)$ is the number of hadrons of type h and momentum between zp and $(z+dz)p$ produced in the fragmentation of a parton of type i. The Q^2 evolution is predicted by QCD and is similar to that of the parton distribution functions [see section on Quantum Chromodynamics (Sec. 9 of this *Review*)]. The $D_i^h(z,Q^2)$ are normalized so that

$$\sum_h \int zD_i^h\,(z,Q^2)dz = 1 \ . \tag{39.18}$$

If the contributions of the Z boson and three-jet events are neglected, the cross section for producing a hadron h in e^+e^- annihilation is given by

$$\frac{1}{\sigma_{\rm had}}\frac{d\sigma}{dz} = \frac{\sum_i e_i^2\,D_i^h\,(z,Q^2)}{\sum_i e_i^2} \ , \tag{39.19}$$

where e_i is the charge of quark-type i, $\sigma_{\rm had}$ is the total hadronic cross section, and the momentum of the hadron is $zE_{\rm cm}/2$.

In the case of deep inelastic muon scattering, the cross section for producing a hadron of energy E_h is given by

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dz} = \frac{\sum_i e_i^2\,q_i(x,Q^2)\,D_i^h(z,Q^2)}{\sum_i e_i^2\,q_i(x,Q^2)} \ , \tag{39.20}$$

where $E_h = \nu z$. (For the kinematics of deep inelastic scattering, see Sec. 38.4.2 of the Kinematics section of this *Review*.) The fragmentation functions for light and heavy quarks have a different z dependence; the former peak near $z = 0$. They are illustrated in Figs. 17.5a and 17.5b in the section on "Fragmentation Functions in e^+e^- Annihilation" (Sec. 17 of this *Review*).

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