RARE KAON DECAYS

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A. Introduction: There are several useful reviews on rare kaon decays and related topics [1–14]. The current activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters
3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+ \nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focused on the decays $K_L \to \pi^0 \ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+\ell^+\ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Other reactions of this type are $K_L \to \pi^0\gamma\gamma$ and $K_L \to \ell^+\ell^-\gamma$. The former is important in understanding a CP-conserving contribution to $K_L \to \pi^0\ell^+\ell^-$, whereas the latter could shed light on long distance contributions to $K_L \to \mu^+\mu^-$.

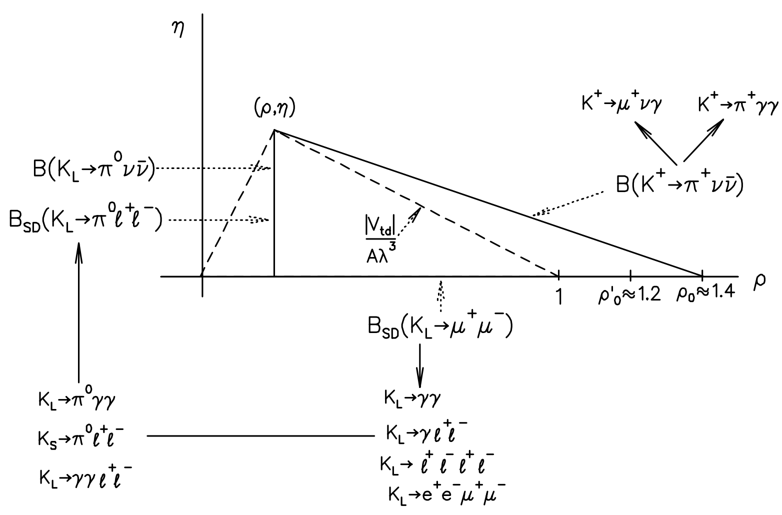

Figure 1: Role of rare kaon decays in determining the unitarity triangle. The solid arrows point to auxiliary modes needed to interpret the main results.

The interplay between Categories 2-4 can be illustrated in Fig. 1. The modes $K \to \pi\nu\overline{\nu}$ are the cleanest ones theoretically. They can provide accurate determinations of certain CKM parameters (shown in the figure). In combination with alternate determinations of these parameters they also constrain new interactions. The modes $K_L \to \pi^0 e^+ e^-$ and $K_L \to \mu^+\mu^-$ are also sensitive to CKM parameters. However, they suffer from a series of hadronic uncertainties that can be addressed, at least in part, through a systematic study of the additional modes indicated in the figure.

B. Explicit violations of the Standard Model: Most of the activity here is in searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 4.7 \times 10^{-12} (148\ \mathrm{TeV}/M_X)^4$ [5]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of over 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to parity-odd couplings and the latter is sensitive to parity-even couplings. Limits on certain lepton-number violating kaon decays [15,16] also exist. Related searches in μ and τ processes are discussed in our section "Tests of Conservation Laws".

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.
$K^+ \to \pi^+ e^- \mu^+$	1.2×10^{-11}	BNL-865 (prelim.)	2003/Ref. 17
$K^+ \to \pi^+ e^+ \mu^-$	5.2×10^{-10}	BNL-865	2001/Ref. 15
$K_L \to \mu e$	4.7×10^{-12}	BNL-871	1998/Ref. 18
$K_L \to \pi^0 e \mu$	3.4×10^{-10}	KTeV (prelim.)	2003/Ref. 19

Physics beyond the SM is also pursued through the search for $K^+ \to \pi^+ X^0$, where X^0 is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, *etc.*). The 90% CL upper limit on this process is 5.9×10^{-11} [20].

C. Measurements of Standard Model parameters: Until 1997, searches for $K^+ \to \pi^+ \nu\overline{\nu}$ were motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [21] and long-distance contributions are known to be quite small [2,22]. Since then, BNL-787 has observed two candidate events [20,23], yielding a branching ratio of $(1.57^{+1.75}_{-0.82}) \times 10^{-10}$ [20]. At this level, this reaction becomes interesting from the point of view of constraining SM parameters. An upgrade to the experiment to collect roughly an order of magnitude more sensitivity is in progress [24]. A new experiment with a sensitivity goal of $\sim 10^{-12}$/event was given scientific approval at FNAL [25] in 2001, but as of the end of 2003 its outlook is less certain. In the future this mode may provide grounds for precision tests of the flavor structure of the Standard Model [26]. The branching ratio can be written in terms of the very well-measured K_{e3} rate as [2]:

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 \mathrm{B}(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4 \theta_W} \times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^\ell + V_{ts}^* V_{td} X(m_t)|^2 \qquad (1)$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [27]. In Eq. (1) the Inami-Lim function $X(m_t)$ is of order 1 [28], and X_{NL}^ℓ is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. QCD corrections, which mainly affect X_{NL}^ℓ, are known at next-to-leading order [12,29] and lead to a residual error of $< 10\%$ for the decay amplitude. Evaluating the constants in Eq. (1), one can cast this result in terms of the CKM parameters A, ρ and η (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [12]

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) \approx 1.0 \times 10^{-10} A^4 [\eta^2 + (\rho_o - \rho)^2] \qquad (2)$$

where $\rho_o \equiv 1 + (\frac{2}{3}X^e_{NL} + \frac{1}{3}X^\tau_{NL})/(A^2 V^4_{us} X(m_t)) \approx 1.4$. Thus, $\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})$ determines a circle in the ρ, η plane with center $(\rho_o, 0)$ and radius $\approx \dfrac{1}{A^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.0\times10^{-10}}}$. Current constraints on the CKM parameters lead to a predicted branching ratio $(7.2 \pm 2.1) \times 10^{-11}$ [30], near the lower end of the BNL-787 measurement.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter ρ, given by [12]:

$$\mathrm{B}_{\mathrm{SD}}(K_L \to \mu^+\mu^-) \approx 1.6 \times 10^{-9} A^4 (\rho'_o - \rho)^2 \tag{3}$$

where ρ'_o depends on the charm quark mass and is approximately 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is determined by the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B}_{\mathrm{abs}}(K_L \to \mu^+\mu^-) = (7.07 \pm 0.18) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$ [31]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. In order to use this mode to constrain ρ it is, therefore, necessary to know the real part of the long-distance contribution. Unlike the absorptive part, the real part of the long-distance contribution cannot be derived from the measured rate for $K_L \to \gamma\gamma$. At present it is not possible to compute this long-distance component reliably, and therefore it is not possible to constrain ρ from this mode in a model independent way [32]. Several hadronic models exist to estimate this long-distance component [33,34] and are commonly used to place rough bounds on new physics contributions to the $K_L \to \mu^+\mu^-$ rate [35,36]. The decay $K_L \to e^+e^-$ is completely dominated by long distance physics and is easier to estimate. The result, $B(K_L \to e^+e^-) \sim 9 \times 10^{-12}$ [32,34], is in good agreement with the BNL-871 measurement, $(8.7^{+5.7}_{-4.1}) \times 10^{-12}$ [37].

D. Searches for direct CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [2,38,39]. In the Standard Model this mode is dominated by an intermediate top-quark state and does not suffer from the small uncertainty associated with the charm-quark intermediate state that affects the mode $K^+ \to \pi^+ \nu\overline{\nu}$. The branching ratio is given approximately by Ref. 12:

$$\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \approx 4.1 \times 10^{-10} A^4 \eta^2 \ . \qquad (4)$$

With current constraints on the CKM parameters this leads to a predicted branching ratio $(2.8 \pm 1.0) \times 10^{-11}$ [30]. The current experimental upper bound is $\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \leq 5.9 \times 10^{-7}$ [40]. The 90% CL bound on $K^+ \to \pi^+ \nu\bar{\nu}$ provides a nearly model independent bound $B(K_L \to \pi^0 \nu\bar{\nu}) < 1.7 \times 10^{-9}$ [41]. A KEK experiment to reach the 3×10^{-10}/event level [42] is scheduled to begin data-taking in early 2004. The KOPIO [43] experiment aims to reach the 6×10^{-13}/event level for $K_L \to \pi^0 \nu\overline{\nu}$ at the BNL AGS. A Letter of Intent for an experiment of similar sensitivity has been submitted to the J-PARC PAC [44].

There has been much theoretical work on possible contributions to ϵ'/ϵ and rare K decays in supersymmetric extensions of the SM. While in the simplest case of the MSSM with no new sources of flavor or CP violation the main effect is a suppression of the rare K decays [45], substantial enhancements are possible in more general SUSY models [35,46]. Other recent predictions of the possible effects of new physics on rare kaon decays include those of large extra dimensions [47] and of effective theories with minimal flavor violation [36].

The decay $K_L \to \pi^0 e^+ e^-$ also has sensitivity to the CKM parameter η through its CP-violating component. There are both direct and indirect CP-violating amplitudes which can interfere. The direct CP-violating amplitude is short distance dominated and has been calculated in detail within the SM [9]. The indirect CP-violating amplitude can be inferred from a measurement of $K_S \to \pi^0 e^+ e^-$. The complete CP-violating contribution to the rate can be written as [50]:

$$\mathrm{B}_{\mathrm{CPV}} \approx \left(15.3 a_s^2 \pm 35 A^2 \eta |a_s| + 74.4 A^4 \eta^2\right) \times 10^{-12}. \qquad (5)$$

The parameter a_s has recently been extracted by NA48 from a measurement of the decay $K_S \to \pi^0 e^+ e^-$ with the result

$|a_s| = 1.06^{+0.26}_{-0.21} \pm 0.07$ [51]. With current constraints on the CKM parameters this implies that

$$\mathrm{B_{CPV}} \approx (17.2 \pm 9.4 + 4.7) \times 10^{-12}. \tag{6}$$

The indirect CP violation is larger than the direct CP violation. While the sign of the interference is *a priori* unknown, arguments in favor of a positive sign have been put forward in Ref. 52.

This mode also has a CP-conserving component dominated by a two-photon intermediate state that cannot be computed reliably at present. This component has an absorptive part that can be, in principle, determined from a detailed analysis of $K_L \to \pi^0\gamma\gamma$. To understand the rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$ in $K_L \to \pi^0\gamma\gamma$ within chiral perturbation theory it is necessary to go beyond leading order, and this introduces three (a priori) unknown parameters [53]. Both KTeV and NA48 analyze their $K_L \to \pi^0\gamma\gamma$ data assuming vector meson dominance and extract from this analysis a bound on the CP-conserving rate in $K_L \to \pi^0 e^+e^-$ using the model of Ref. [54]. The two experiments report conflicting results. The more recent NA48 data finds a negligible rate in the low $m_{\gamma\gamma}$ region suggesting a very small CP-conserving component in $K_L \to \pi^0 e^+e^-$, $B_{\rm CP}(K_L \to \pi^0 e^+e^-) = (0.47^{+0.22}_{-0.18}) \times 10^{-12}$ [55]. On the other hand KTeV reported a larger fraction of $K_L \to \pi^0\gamma\gamma$ in the low $m_{\gamma\gamma}$ region, favoring a larger CP-conserving rate $B_{\rm CP}(K_L \to \pi^0 e^+e^-)$ between $1 - 2 \times 10^{-12}$ [56]. In addition to this difference between the two experiments, there remain two other sources of uncertainty in the prediction for the CP-conserving rate in $K_L \to \pi^0 e^+e^-$. The first one arises from the way the $K_L \to \pi^0\gamma\gamma$ amplitude is parameterized [57] assuming vector meson dominance, and the second one reflects the model dependence of the form factor connecting the two modes [52]. For the $K_L \to \pi^0\gamma\gamma$ rates, KTeV finds $B(K_L \to \pi^0\gamma\gamma) = (1.68 \pm 0.07_{\rm stat} \pm 0.08_{\rm sys}) \times 10^{-6}$ [56], whereas NA48 finds $B(K_L \to \pi^0\gamma\gamma) = (1.36 \pm 0.03_{\rm stat} \pm 0.03_{\rm sys} \pm 0.03_{\rm norm}) \times 10^{-6}$ [55].

The related process, $K_L \to \pi^0\gamma e^+e^-$, is potentially an additional background in some region of phase space [58].

This process has been observed with a branching ratio of $(2.34 \pm 0.35_{\rm stat} \pm 0.13_{\rm sys}) \times 10^{-8}$ [59].

The decay $K_L \to \gamma\gamma e^+e^-$ constitutes the dominant background to $K_L \to \pi^0 e^+e^-$. It was first observed by BNL-845 [60] and subsequently confirmed with a much larger sample by FNAL-799 [61]. It has been estimated that this background will enter at the level of 3×10^{-10} [62,63], comparable to or larger than the signal level. Because of this, the observation of $K_L \to \pi^0 e^+e^-$ will depend on background subtraction with good statistics. Possible alternative strategies are discussed in Ref. 52 and references cited therein.

The published 90% CL upper bound for the process $K_L \to \pi^0 e^+e^-$ is 5.1×10^{-10} [63]. There is now a preliminary 90% CL upper bound of 2.8×10^{-10} [64]. For the closely related muonic process, the published upper bound is $\mathrm{B}(K_L \to \pi^0\mu^+\mu^-) \leq 3.8 \times 10^{-10}$ [65]. KTeV has additional data corresponding to about a factor 1.3 in sensitivity for the latter reaction that is still to be analyzed.

A recent study of $K_L \to \pi^0\mu^+\mu^-$ has indicated that it might be possible to extract the direct CP-violating contribution by a joint study of the Dalitz plot variables and the components of the μ^+ polarization [66]. The latter tends to be quite substantial so that large statistics may not be necessary.

E. Other long distance dominated modes:

The decays $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e$ or μ) have received considerable attention. The rate and spectrum have been measured for both the electron and muon modes [67,68]. Ref. 50 has proposed a parameterization inspired by chiral perturbation theory, which provides a successful description of data but indicates the presence of large corrections beyond leading order. More work is needed to fully understand the origin of these large corrections.

Much information has been recorded by KTeV and NA48 on the rates and spectrum for the Dalitz pair conversion modes $K_L \to \ell^+\ell^-\gamma$ [69], and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ [16,70–72]. All these results are used to test hadronic models and could further our understanding of the long distance component in $K_L \to \mu^+\mu^-$.

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