



$$J = \frac{1}{2}$$

## $\mu$ MASS (atomic mass units u)

The primary determination of a muon's mass comes from measuring the ratio of the mass to that of a nucleus, so that the result is obtained in u (atomic mass units). The conversion factor to MeV is more uncertain than the mass of the muon in u. In this datablock we give the result in u, and in the following datablock in MeV.

VALUE (u)	DOCUMENT ID	TECN	COMMENT
<b>0.1134289264 ± 0.0000000030</b>	MOHR	05	RVUE 2002 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •			
0.1134289168 ± 0.0000000034	<sup>1</sup> MOHR	99	RVUE 1998 CODATA value
0.113428913 ± 0.0000000017	<sup>2</sup> COHEN	87	RVUE 1986 CODATA value
<sup>1</sup> MOHR 99 make use of other 1998 CODATA entries below.			
<sup>2</sup> COHEN 87 make use of other 1986 CODATA entries below.			

## $\mu$ MASS

2002 CODATA gives the conversion factor from u (atomic mass units, see the above datablock) as 931.494 043 (80). Earlier values use the then-current conversion factor. The conversion error dominates the masses given below.

VALUE (MeV)	DOCUMENT ID	TECN	CHG	COMMENT
<b>105.6583692 ± 0.0000094</b>	MOHR	05	RVUE	2002 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
105.6583568 ± 0.0000052	MOHR	99	RVUE	1998 CODATA value
105.658353 ± 0.000016	<sup>3</sup> COHEN	87	RVUE	1986 CODATA value
105.658386 ± 0.000044	<sup>4</sup> MARIAM	82	CNTR +	
105.65836 ± 0.00026	<sup>5</sup> CROWE	72	CNTR	
105.65865 ± 0.00044	<sup>6</sup> CRANE	71	CNTR	
<sup>3</sup> Converted to MeV using the 1998 CODATA value of the conversion constant, 931.494013 ± 0.0000037 MeV/u.				
<sup>4</sup> MARIAM 82 give $m_\mu/m_e = 206.768259(62)$ .				
<sup>5</sup> CROWE 72 give $m_\mu/m_e = 206.7682(5)$ .				
<sup>6</sup> CRANE 71 give $m_\mu/m_e = 206.76878(85)$ .				

**$\mu$  MEAN LIFE  $\tau$** 

Measurements with an error  $> 0.001 \times 10^{-6}$  s have been omitted.

<u>VALUE (<math>10^{-6}</math> s)</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>
<b>2.19703 <math>\pm</math> 0.00004 OUR AVERAGE</b>			
2.197078 $\pm$ 0.000073	BARDIN	84	CNTR +
2.197025 $\pm$ 0.000155	BARDIN	84	CNTR -
2.19695 $\pm$ 0.00006	GIOVANETTI	84	CNTR +
2.19711 $\pm$ 0.00008	BALANDIN	74	CNTR +
2.1973 $\pm$ 0.0003	DUCLOS	73	CNTR +

 **$\tau_{\mu^+}/\tau_{\mu^-}$  MEAN LIFE RATIO**

A test of *CPT* invariance.

<u>VALUE</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>COMMENT</u>
<b>1.000024 <math>\pm</math> 0.000078</b>	BARDIN	84	CNTR
• • • We do not use the following data for averages, fits, limits, etc. • • •			
1.0008 $\pm$ 0.0010	BAILEY	79	CNTR Storage ring
1.000 $\pm$ 0.001	MEYER	63	CNTR Mean life $\mu^+/\mu^-$

 **$(\tau_{\mu^+} - \tau_{\mu^-}) / \tau_{\text{average}}$** 

A test of *CPT* invariance. Calculated from the mean-life ratio, above.

<u>VALUE</u>	<u>DOCUMENT ID</u>
<b><math>(2 \pm 8) \times 10^{-5}</math> OUR EVALUATION</b>	

 **$\mu/p$  MAGNETIC MOMENT RATIO**

This ratio is used to obtain a precise value of the muon mass and to reduce experimental muon Larmor frequency measurements to the muon magnetic moment anomaly. Measurements with an error  $> 0.00001$  have been omitted. By convention, the minus sign on this ratio is omitted. CODATA values were fitted using their selection of data, plus other data from multiparameter fits.

<u>VALUE</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
<b>3.183345118 <math>\pm</math> 0.000000089</b>	MOHR	05	RVUE	2002 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
3.18334513 $\pm$ 0.00000039	LIU	99	CNTR +	HFS in muonium
3.18334539 $\pm$ 0.00000010	MOHR	99	RVUE	1998 CODATA value
3.18334547 $\pm$ 0.00000047	COHEN	87	RVUE	1986 CODATA value
3.1833441 $\pm$ 0.0000017	KLEMP	82	CNTR +	Precession strob
3.1833461 $\pm$ 0.0000011	MARIAM	82	CNTR +	HFS splitting
3.1833448 $\pm$ 0.0000029	CAMANI	78	CNTR +	See KLEMP 82
3.1833403 $\pm$ 0.0000044	CASPERSON	77	CNTR +	HFS splitting
3.1833402 $\pm$ 0.0000072	COHEN	73	RVUE	1973 CODATA value
3.1833467 $\pm$ 0.0000082	CROWE	72	CNTR +	Precession phase

# THE MUON ANOMALOUS MAGNETIC MOMENT

Updated March 2006 by A. Höcker (CERN) and W.J. Marciano (BNL)

The Dirac equation predicts a muon magnetic moment,  $\vec{M} = g_\mu \frac{e}{2m_\mu} \vec{S}$ , with gyromagnetic ratio  $g_\mu = 2$ . Quantum loop effects lead to a small calculable deviation from  $g_\mu = 2$ , parameterized by the anomalous magnetic moment

$$a_\mu \equiv \frac{g_\mu - 2}{2} . \quad (1)$$

That quantity can be accurately measured and, within the Standard Model (SM) framework, precisely predicted. Hence, comparison of experiment and theory tests the SM at its quantum loop level. A deviation in  $a_\mu^{\text{exp}}$  from the SM expectation would signal effects of new physics, with current sensitivity reaching up to mass scales of  $\mathcal{O}(\text{TeV})$  [1, 2].

The recently completed experiment E821 at Brookhaven National Lab (BNL) studied the precession of  $\mu^+$  and  $\mu^-$  in a constant external magnetic field as they circulated in a confining storage ring. It found [3]

$$\begin{aligned} a_{\mu^+}^{\text{exp}} &= 11\,659\,203(6)(5) \times 10^{-10} , \\ a_{\mu^-}^{\text{exp}} &= 11\,659\,214(8)(3) \times 10^{-10} , \end{aligned} \quad (2)$$

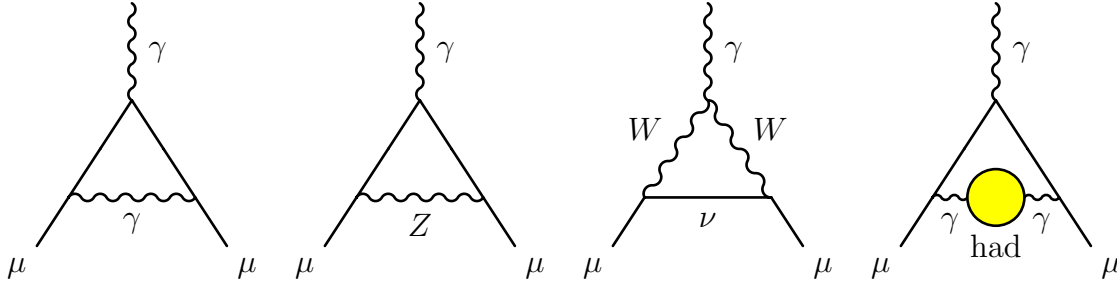
where the first errors are statistical and the second systematic. Assuming CPT invariance and taking into account correlations between systematic errors, one finds for their average [3]

$$a_\mu^{\text{exp}} = 11\,659\,208.0(5.4)(3.3) \times 10^{-10} . \quad (3)$$

These results represent about a factor of 14 improvement over the classic CERN experiments of the 1970's [4].

The SM prediction for  $a_\mu^{\text{SM}}$  is generally divided into three parts (see Fig. 1 for representative Feynman diagrams)

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{Had}} . \quad (4)$$



**Figure 1:** Representative diagrams contributing to  $a_\mu^{\text{SM}}$ . From left to right: first order QED (Schwinger term), lowest-order weak, lowest-order hadronic.

The QED part includes all photonic and leptonic ( $e, \mu, \tau$ ) loops starting with the classic  $\alpha/2\pi$  Schwinger contribution. It has now been computed through 4 loops and estimated at the 5-loop level [5]

$$a_\mu^{\text{QED}} = \frac{\alpha}{2\pi} + 0.76585741(3) \left(\frac{\alpha}{\pi}\right)^2 + 24.0505096(4) \left(\frac{\alpha}{\pi}\right)^3 + 131.01(1) \left(\frac{\alpha}{\pi}\right)^4 + 663(20) \left(\frac{\alpha}{\pi}\right)^5 + \dots \quad (5)$$

Employing  $\alpha^{-1} = 137.0359988(5)$ , determined [5] from the electron  $a_e$  measurement, leads to

$$a_\mu^{\text{QED}} = 116\,584\,719.0(0.1)(0.4) \times 10^{-11} , \quad (6)$$

where the errors result from uncertainties in the coefficients of Eq.(5) and in  $\alpha$  (see the reviews in [2] and [6]). Although the uncertainty in  $\alpha$  is already very small, an experiment underway at Harvard aims to reduce the error on  $a_e$  from which it is derived by a factor of 15 [7].

Loop contributions involving heavy  $W^\pm, Z$  or Higgs particles are collectively labeled as  $a_\mu^{\text{EW}}$ . They are suppressed by at least a factor of  $\frac{\alpha}{\pi} \frac{m_\mu^2}{m_W^2} \simeq 4 \times 10^{-9}$ . At 1-loop order [8]

$$a_\mu^{\text{EW}}[\text{1-loop}] = \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2} \left[ \frac{5}{3} + \frac{1}{3} (1 - 4 \sin^2 \theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_W^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right) \right], \quad (7)$$

$$= 194.8 \times 10^{-11}, \quad \text{for } \sin^2 \theta_W \equiv 1 - \frac{M_W^2}{M_Z^2} \simeq 0.223. \quad (8)$$

Two-loop corrections are relatively large and negative [9]

$$a_\mu^{\text{EW}}[\text{2-loop}] = -40.7(1.0)(1.8) \times 10^{-11}, \quad (9)$$

where the errors stem from quark triangle loops and the assumed Higgs mass range  $m_H = 150_{-40}^{+100}$  GeV. The 3-loop leading logarithms are negligible [9,10],  $\mathcal{O}(10^{-12})$ , implying in total

$$a_\mu^{\text{EW}} = 154(1)(2) \times 10^{-11}. \quad (10)$$

Hadronic (quark and gluon) loop contributions to  $a_\mu^{\text{SM}}$  give rise to its main theoretical uncertainties. At present, those effects are not calculable from first principles, but such an approach may become possible as lattice QCD matures. Instead, one currently relies on a dispersion relation approach to

evaluate the lowest-order (*i.e.*,  $\mathcal{O}(\alpha^2)$ ) hadronic vacuum polarization contribution  $a_\mu^{\text{Had}}[LO]$  from corresponding cross section measurements [11]

$$a_\mu^{\text{Had}}[LO] = \frac{1}{3} \left( \frac{\alpha}{\pi} \right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R^{(0)}(s) , \quad (11)$$

where  $K(s)$  is a QED kernel function [12], and where  $R^{(0)}(s)$  denotes the ratio of the bare\* cross section for  $e^+e^-$  annihilation into hadrons to the pointlike muon-pair cross section at center-of-mass energy  $\sqrt{s}$ . The function  $K(s) \sim 1/s$  in Eq. (11) gives a strong weight to the low-energy part of the integral. Hence,  $a_\mu^{\text{Had}}[LO]$  is dominated by the  $\rho(770)$  resonance.

Currently, the available  $\sigma(e^+e^- \rightarrow \text{hadrons})$  data give a leading order hadronic vacuum polarization (representative) contribution of [13]

$$a_\mu^{\text{Had}}[LO] = 6\,963(62)(36) \times 10^{-11} , \quad (12)$$

where the errors correspond to experimental, dominated by systematic uncertainties, and QED radiative corrections to the data.

Alternatively, one can use precise vector spectral functions from  $\tau \rightarrow \nu_\tau + \text{hadrons}$  decays [14] that can be related to isovector  $e^+e^- \rightarrow \text{hadrons}$  cross sections by isospin rotation. When isospin-violating corrections (from QED and  $m_d - m_u \neq 0$ ) are applied, one finds [13]

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\* The bare cross section is defined as the measured cross section corrected for initial-state radiation, electron-vertex loop contributions and vacuum-polarization effects in the photon propagator. However, QED effects in the hadron vertex and final state, as photon radiation, must be included.

$$a_{\mu}^{\text{Had}}[LO] = 7\,110(50)(8)(28) \times 10^{-11} \, (\tau) \, , \quad (13)$$

where the errors are statistical and systematic, and where the last error is an estimate for the uncertainty in the isospin-breaking corrections. The discrepancy between the  $e^+e^-$  and  $\tau$ -based determinations of  $a_{\mu}^{\text{Had}}[LO]$  is currently unexplained. It may be indicative of problems with one or both data sets. It may also suggest the need for additional isospin-violating corrections to the  $\tau$  data. Preliminary new low-energy  $e^+e^-$  and  $\tau$  data may help to resolve this discrepancy and should reduce the hadronic uncertainty.

Higher order,  $\mathcal{O}(\alpha^3)$ , hadronic contributions are obtained from the same  $e^+e^- \rightarrow \text{hadrons}$  data [14–16] along with model-dependent estimates of the hadronic light-by-light scattering contribution motivated by large- $N_C$  QCD [17]. Following [2], one finds

$$a_{\mu}^{\text{Had}}[NLO] = 22(35) \times 10^{-11} \, , \quad (14)$$

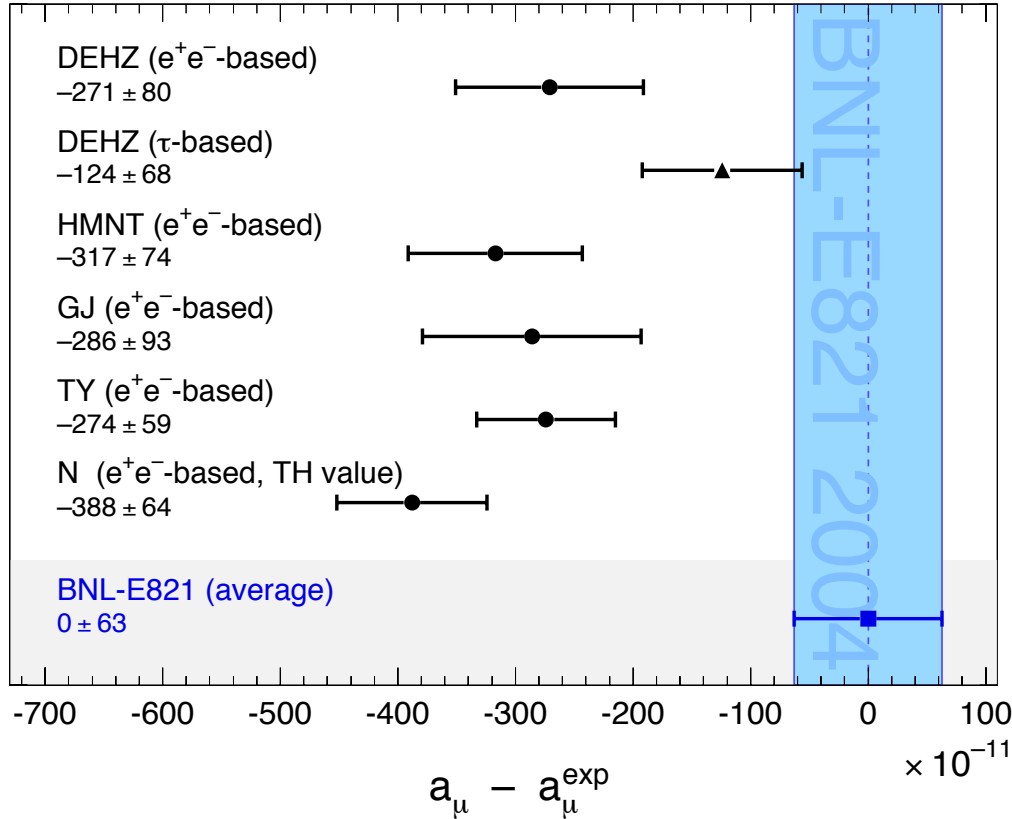
where the error is dominated by hadronic light-by-light uncertainties.

Adding Eqs. (6), (10), (12), and (14) gives the representative  $e^+e^-$  data-based SM prediction (which includes recent changes in the QED and hadronic light by light contributions)

$$a_{\mu}^{\text{SM}} = 116\,591\,858(72)(35)(3) \times 10^{-11} \, . \quad (15)$$

The difference between experiment and theory

$$\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 22(10) \times 10^{-10} \, , \quad (16)$$



**Figure 2:** Compilation of recently published results for  $a_\mu$  (in units of  $10^{-11}$ ), subtracted by the central value of the experimental average (3). The shaded band indicates the experimental error. The SM predictions are taken from: DEHZ [13], HMNT [16], GJ [18], TY [19], N [20]. Note that the quoted errors do not include the uncertainty on the subtracted experimental value. To obtain for each theory calculation a result equivalent to Eq. (16), one has to add the errors from theory and experiment in quadrature. See full-color version on color pages at end of book.

(with all errors combined in quadrature) represents an interesting but not compelling discrepancy of 2.2 times the estimated  $1\sigma$  error. Using the recent estimates for the hadronic contribution



compiled in Fig. 2, this discrepancy can exhibit up to  $3\sigma$ . Those larger discrepancies arise in part because the published results illustrated there have not been updated to include more recent evaluations of the QED [5] and hadronic light-by-light [2,17] contributions. Switching to  $\tau$  data reduces the discrepancy by about a factor of 3, assuming the isospin-violating corrections are under control within the estimated uncertainties.

An alternate interpretation is that  $\Delta a_\mu$  may be a new physics signal with supersymmetric particle loops as the leading candidate explanation. Such a scenario is quite natural, since generically, supersymmetric models predict [1] an additional contribution to  $a_\mu^{\text{SM}}$

$$a_\mu^{\text{SUSY}} \simeq \pm 130 \times 10^{-11} \cdot \left( \frac{100 \text{ GeV}}{m_{\text{SUSY}}} \right)^2 \tan\beta, \quad (17)$$

where  $m_{\text{SUSY}}$  is a representative supersymmetric mass scale, and  $\tan\beta \simeq 3\text{--}40$  is a potential enhancement factor. Supersymmetric particles in the mass range 100–500 GeV could be the source of the deviation  $\Delta a_\mu$ . If so, those particles could be directly observed at the next generation of high energy colliders. New physics effects [1] other than supersymmetry could also explain a non-vanishing  $\Delta a_\mu$ .

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### $\mu$ MAGNETIC MOMENT ANOMALY

The parity-violating decay of muons in a storage ring is observed. The difference frequency  $\omega_a$  between the muon spin precession and the orbital angular frequency  $(e/m_\mu c)\langle B \rangle$  is measured, as is the free proton NMR frequency  $\omega_p$ , thus determining the ratio  $R = \omega_a/\omega_p$ . Given the magnetic moment ratio  $\lambda = \mu_\mu/\mu_p$  (from hyperfine structure in muonium),  $(g-2)/2 = R/(\lambda - R)$ .

$$\mu_\mu/(e\hbar/2m_\mu) - 1 = (g_\mu - 2)/2$$

VALUE (units $10^{-10}$ )	DOCUMENT ID	TECN	CHG	COMMENT
<b>11659208 ± 6</b>	BENNETT	04	MUG2	Average $\mu^+$ and $\mu^-$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
11659214 ± 8 ± 3	BENNETT	04	MUG2	— Storage ring
11659204 ± 7 ± 5	BENNETT	02	MUG2	+ Storage ring
11659202 ± 14 ± 6	BROWN	01	MUG2	+ Storage ring
11659191 ± 59	BROWN	00	MUG2	+ Storage ring
11659100 ± 110	<sup>7</sup> BAILEY	79	CNTR	+ Storage ring
11659360 ± 120	<sup>7</sup> BAILEY	79	CNTR	— Storage ring
11659230 ± 85	<sup>7</sup> BAILEY	79	CNTR	± Storage ring
11620000 ± 5000	CHARPAK	62	CNTR	+ Storage ring

<sup>7</sup> BAILEY 79 values recalculated by HUGHES 99 using the COHEN 87  $\mu/p$  magnetic moment. The improved MOHR 99 value does not change the result.

$$(g_{\mu^+} - g_{\mu^-}) / g_{\text{average}}$$

A test of *CPT* invariance.

VALUE (units $10^{-8}$ )	DOCUMENT ID
<b>−2.6 ± 1.6</b>	BAILEY 79

**$\mu$  ELECTRIC DIPOLE MOMENT**

A nonzero value is forbidden by both  $T$  invariance and  $P$  invariance.

VALUE ( $10^{-19}$ ecm)	DOCUMENT ID	TECN	CHG	COMMENT
<b><math>3.7 \pm 3.4</math></b>	<sup>8</sup> BAILEY	78	CNTR	$\pm$ Storage ring
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$8.6 \pm 4.5$	BAILEY	78	CNTR	$+$ Storage rings
$0.8 \pm 4.3$	BAILEY	78	CNTR	$-$ Storage rings

<sup>8</sup> This is the combination of the two BAILEY 78 results given below.

**MUON-ELECTRON CHARGE RATIO ANOMALY  $q_{\mu^+}/q_{e^-} + 1$** 

VALUE	DOCUMENT ID	TECN	CHG	COMMENT
<b><math>(1.1 \pm 2.1) \times 10^{-9}</math></b>	<sup>9</sup> MEYER	00	CNTR	$+$ 1s–2s muonium interval

<sup>9</sup> MEYER 00 measure the 1s–2s muonium interval, and then interpret the result in terms of muon-electron charge ratio  $q_{\mu^+}/q_{e^-}$ .

 **$\mu^-$  DECAY MODES**

$\mu^+$  modes are charge conjugates of the modes below.

Mode	Fraction ( $\Gamma_i/\Gamma$ )	Confidence level
$\Gamma_1$ $e^- \bar{\nu}_e \nu_\mu$	$\approx 100\%$	
$\Gamma_2$ $e^- \bar{\nu}_e \nu_\mu \gamma$	[a] $(1.4 \pm 0.4) \%$	
$\Gamma_3$ $e^- \bar{\nu}_e \nu_\mu e^+ e^-$	[b] $(3.4 \pm 0.4) \times 10^{-5}$	

**Lepton Family number ( $LF$ ) violating modes**

$\Gamma_4$ $e^- \nu_e \bar{\nu}_\mu$	$LF$	[c] $< 1.2$	$\%$	90%
$\Gamma_5$ $e^- \gamma$	$LF$	$< 1.2$	$\times 10^{-11}$	90%
$\Gamma_6$ $e^- e^+ e^-$	$LF$	$< 1.0$	$\times 10^{-12}$	90%
$\Gamma_7$ $e^- 2\gamma$	$LF$	$< 7.2$	$\times 10^{-11}$	90%

[a] This only includes events with the  $\gamma$  energy  $> 10$  MeV. Since the  $e^- \bar{\nu}_e \nu_\mu$  and  $e^- \bar{\nu}_e \nu_\mu \gamma$  modes cannot be clearly separated, we regard the latter mode as a subset of the former.

[b] See the Particle Listings below for the energy limits used in this measurement.

[c] A test of additive vs. multiplicative lepton family number conservation.

$\mu^-$  BRANCHING RATIOS $\Gamma(e^- \bar{\nu}_e \nu_\mu \gamma)/\Gamma_{\text{total}}$  $\Gamma_2/\Gamma$ 

VALUE	EVTS	DOCUMENT ID	TECN	COMMENT
<b>0.014 ± 0.004</b>		CRITTENDEN 61	CNTR	$\gamma$ KE > 10 MeV
• • • We do not use the following data for averages, fits, limits, etc. • • •				
	862	BOGART	67	CNTR $\gamma$ KE > 14.5 MeV
0.0033 ± 0.0013		CRITTENDEN 61	CNTR	$\gamma$ KE > 20 MeV
	27	ASHKIN	59	CNTR

 $\Gamma(e^- \bar{\nu}_e \nu_\mu e^+ e^-)/\Gamma_{\text{total}}$  $\Gamma_3/\Gamma$ 

<u>VALUE (units 10<sup>-5</sup>)</u>	<u>EVTs</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>	
<b>3.4±0.2±0.3</b>	7443	<sup>10</sup> BERTL	85	SPEC	+	SINDRUM
• • • We do not use the following data for averages, fits, limits, etc. • • •						
2.2±1.5	7	<sup>11</sup> CRITTENDEN	61	HLBC	+	$E(e^+e^-)>10$ MeV
2	1	<sup>12</sup> GUREVICH	60	EMUL	+	
1.5±1.0	3	<sup>13</sup> LEE	59	HBC	+	

<sup>10</sup> BERTL 85 has transverse momentum cut  $p_T > 17$  MeV/c. Systematic error was increased by us.

<sup>11</sup> CRITTENDEN 61 count only those decays where total energy of either  $(e^+, e^-)$  combination is >10 MeV.

<sup>12</sup> GUREVICH 60 interpret their event as either virtual or real photon conversion.  $e^+$  and  $e^-$  energies not measured.

<sup>13</sup> In the three LEE 59 events, the sum of energies  $E(e^+) + E(e^-) + E(e^+)$  was 51 MeV, 55 MeV, and 33 MeV.

 $\Gamma(e^- \nu_e \bar{\nu}_\mu)/\Gamma_{\text{total}}$  $\Gamma_4/\Gamma$ 

Forbidden by the additive conservation law for lepton family number. A multiplicative law predicts this branching ratio to be 1/2. For a review see NEMETHY 81.

VALUE	CL%	DOCUMENT ID	TECN	CHG	COMMENT	
< 0.012	90	<sup>14</sup> FREEDMAN	93	CNTR	+	$\nu$ oscillation search
• • • We do not use the following data for averages, fits, limits, etc. • • •						
< 0.018	90	KRAKAUER	91B	CALO	+	
< 0.05	90	<sup>15</sup> BERGSMA	83	CALO		$\overline{\nu}_{\mu} e \rightarrow \mu^{-} \overline{\nu}_e$
< 0.09	90	JONKER	80	CALO		See BERGSMA 83
-0.001 ± 0.061		WILLIS	80	CNTR	+	
0.13 ± 0.15		BLIETSCHAU	78	HLBC	±	Avg. of 4 values
< 0.25	90	EICHTEN	73	HLBC	+	

<sup>14</sup> FREEDMAN 93 limit on  $\bar{\nu}_e$  observation is here interpreted as a limit on lepton family number violation.

<sup>15</sup> BERGSMA 83 gives a limit on the inverse muon decay cross-section ratio  $\sigma(\bar{\nu}_\mu e^- \rightarrow \mu^- \bar{\nu}_e)/\sigma(\nu_\mu e^- \rightarrow \mu^- \nu_e)$ , which is essentially equivalent to  $\Gamma(e^- \nu_e \bar{\nu}_\mu)/\Gamma_{\text{total}}$  for small values like that quoted.

$\Gamma(e^- \gamma) / \Gamma_{\text{total}}$  $\Gamma_5 / \Gamma$ 

Forbidden by lepton family number conservation.

<u>VALUE (units 10<sup>-11</sup>)</u>	<u>CL%</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>	
<b>&lt; 1.2</b>	90	BROOKS	99	SPEC	+	LAMPF
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●						
< 1.2	90	AHMED	02	SPEC	+	MEGA
< 4.9	90	BOLTON	88	CBOX	+	LAMPF
<100	90	AZUELOS	83	CNTR	+	TRIUMF
< 17	90	KINNISON	82	SPEC	+	LAMPF
<100	90	SCHAAF	80	ELEC	+	SIN

 $\Gamma(e^- e^+ e^-) / \Gamma_{\text{total}}$  $\Gamma_6 / \Gamma$ 

Forbidden by lepton family number conservation.

<u>VALUE (units 10<sup>-12</sup>)</u>	<u>CL%</u>		<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>	
<b>&lt; 1.0</b>	90	<sup>16</sup>	BELLEGARDT	88	SPEC	+	SINDRUM
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●							
< 36	90		BARANOV	91	SPEC	+	ARES
< 35	90		BOLTON	88	CBOX	+	LAMPF
< 2.4	90	<sup>16</sup>	BERTL	85	SPEC	+	SINDRUM
<160	90	<sup>16</sup>	BERTL	84	SPEC	+	SINDRUM
<130	90	<sup>16</sup>	BOLTON	84	CNTR		LAMPF

<sup>16</sup> These experiments assume a constant matrix element. $\Gamma(e^- 2\gamma) / \Gamma_{\text{total}}$  $\Gamma_7 / \Gamma$ 

Forbidden by lepton family number conservation.

<u>VALUE (units 10<sup>-11</sup>)</u>	<u>CL%</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>	
<b>&lt; 7.2</b>	90	BOLTON	88	CBOX	+	LAMPF
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●						
< 840	90	<sup>17</sup> AZUELOS	83	CNTR	+	TRIUMF
<5000	90	<sup>18</sup> BOWMAN	78	CNTR		DEPOMMIER 77 data

<sup>17</sup> AZUELOS 83 uses the phase space distribution of BOWMAN 78.<sup>18</sup> BOWMAN 78 assumes an interaction Lagrangian local on the scale of the inverse  $\mu$  mass.**LIMIT ON  $\mu^- \rightarrow e^-$  CONVERSION**

Forbidden by lepton family number conservation.

 $\sigma(\mu^- {}^{32}\text{S} \rightarrow e^- {}^{32}\text{S}) / \sigma(\mu^- {}^{32}\text{S} \rightarrow \nu_\mu {}^{32}\text{P}^*)$ 

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
<b>&lt; <math>7 \times 10^{-11}</math></b>	90	BADERT...	80	STRC SIN
• • • We do not use the following data for averages, fits, limits, etc. • • •				
< $4 \times 10^{-10}$	90	BADERT...	77	STRC SIN

 $\sigma(\mu^- \text{Cu} \rightarrow e^- \text{Cu}) / \sigma(\mu^- \text{Cu} \rightarrow \text{capture})$ 

VALUE	CL%	DOCUMENT ID	TECN
• • • We do not use the following data for averages, fits, limits, etc. • • •			
< $1.6 \times 10^{-8}$	90	BRYMAN	72 SPEC

**$\sigma(\mu^- \text{Ti} \rightarrow e^- \text{Ti}) / \sigma(\mu^- \text{Ti} \rightarrow \text{capture})$** 

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
$<4.3 \times 10^{-12}$	90	<sup>19</sup> DOHMEN 93	SPEC	SINDRUM II

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<4.6 \times 10^{-12}$	90	AHMAD 88	TPC	TRIUMF
$<1.6 \times 10^{-11}$	90	BRYMAN 85	TPC	TRIUMF

<sup>19</sup>DOHMEN 93 assumes  $\mu^- \rightarrow e^-$  conversion leaves the nucleus in its ground state, a process enhanced by coherence and expected to dominate.

 **$\sigma(\mu^- \text{Pb} \rightarrow e^- \text{Pb}) / \sigma(\mu^- \text{Pb} \rightarrow \text{capture})$** 

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
$<4.6 \times 10^{-11}$	90	HONECKER 96	SPEC	SINDRUM II

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<4.9 \times 10^{-10}$	90	AHMAD 88	TPC	TRIUMF
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**LIMIT ON  $\mu^- \rightarrow e^+$  CONVERSION**

Forbidden by total lepton number conservation.

 **$\sigma(\mu^- {}^{32}\text{S} \rightarrow e^+ {}^{32}\text{Si}^*) / \sigma(\mu^- {}^{32}\text{S} \rightarrow \nu_\mu {}^{32}\text{P}^*)$** 

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
$<9 \times 10^{-10}$	90	BADERT... 80	STRC	SIN

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<1.5 \times 10^{-9}$	90	BADERT... 78	STRC	SIN
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 **$\sigma(\mu^- {}^{127}\text{I} \rightarrow e^+ {}^{127}\text{Sb}^*) / \sigma(\mu^- {}^{127}\text{I} \rightarrow \text{anything})$** 

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
$<3 \times 10^{-10}$	90	<sup>20</sup> ABELA 80	CNTR	Radiochemical tech.

<sup>20</sup>ABELA 80 is upper limit for  $\mu^- e^+$  conversion leading to particle-stable states of <sup>127</sup>Sb. Limit for total conversion rate is higher by a factor less than 4 (G. Backenstoss, private communication).

 **$\sigma(\mu^- \text{Cu} \rightarrow e^+ \text{Co}) / \sigma(\mu^- \text{Cu} \rightarrow \nu_\mu \text{Ni})$** 

VALUE	CL%	DOCUMENT ID	TECN
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• • • We do not use the following data for averages, fits, limits, etc. • • •

$<2.6 \times 10^{-8}$	90	BRYMAN 72	SPEC
$<2.2 \times 10^{-7}$	90	CONFORTO 62	OSPK

 **$\sigma(\mu^- \text{Ti} \rightarrow e^+ \text{Ca}) / \sigma(\mu^- \text{Ti} \rightarrow \text{capture})$** 

VALUE	CL%	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
$<3.6 \times 10^{-11}$	90	1 <sup>21,22</sup>	KAULARD 98	SPEC	—	SINDRUM II

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<1.7 \times 10^{-12}$	90	1 <sup>22,23</sup>	KAULARD 98	SPEC	—	SINDRUM II
$<4.3 \times 10^{-12}$	90	<sup>23</sup>	DOHMEN 93	SPEC		SINDRUM II
$<8.9 \times 10^{-11}$	90	<sup>21</sup>	DOHMEN 93	SPEC		SINDRUM II
$<1.7 \times 10^{-10}$	90	<sup>24</sup>	AHMAD 88	TPC		TRIUMF

- <sup>21</sup> This limit assumes a giant resonance excitation of the daughter Ca nucleus (mean energy and width both 20 MeV).  
<sup>22</sup> KAULARD 98 obtained these same limits using the unified classical analysis of FELDMAN 98.  
<sup>23</sup> This limit assumes the daughter Ca nucleus is left in the ground state. However, the probability of this is unknown.  
<sup>24</sup> Assuming a giant-resonance-excitation model.

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## LIMIT ON MUONIUM $\rightarrow$ ANTIMUONIUM CONVERSION

Forbidden by lepton family number conservation.

$$R_g = G_C / G_F$$

The effective Lagrangian for the  $\mu^+ e^- \rightarrow \mu^- e^+$  conversion is assumed to be

$$\mathcal{L} = 2^{-1/2} G_C [\bar{\psi}_\mu \gamma_\lambda (1 - \gamma_5) \psi_e] [\bar{\psi}_\mu \gamma_\lambda (1 - \gamma_5) \psi_e] + \text{h.c.}$$

The experimental result is then an upper limit on  $G_C/G_F$ , where  $G_F$  is the Fermi coupling constant.

<u>VALUE</u>	<u>CL%</u>	<u>EVTs</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>	
< <b>0.0030</b>	90	1	<sup>25</sup> WILLMANN	99	SPEC	+	$\mu^+$ at 26 GeV/c
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●							
< 0.14	90	1	<sup>26</sup> GORDEEV	97	SPEC	+	JINR phasotron
< 0.018	90	0	<sup>27</sup> ABELA	96	SPEC	+	$\mu^+$ at 24 MeV
< 6.9	90		NI	93	CBOX		LAMPF
< 0.16	90		MATTHIAS	91	SPEC		LAMPF
< 0.29	90		HUBER	90B	CNTR		TRIUMF
<20	95		BEER	86	CNTR		TRIUMF
<42	95		MARSHALL	82	CNTR		

<sup>25</sup> WILLMANN 99 quote both probability  $P_{M\bar{M}} < 8.3 \times 10^{-11}$  at 90%CL in a 0.1 T field and  $R_g = G_C/G_F$ .

<sup>26</sup> GORDEEV 97 quote limits on both  $f = G_{MM}/G_F$  and the probability  $W_{MM} < 4.7 \times 10^{-7}$  (90% CL).

<sup>27</sup> ABELA 96 quote both probability  $P_{M\bar{M}} < 8 \times 10^{-9}$  at 90% CL and  $R_g = G_C/G_F$ .

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## MUON DECAY PARAMETERS

Revised January 2006 by W. Fetscher and H.-J. Gerber (ETH Zürich).

**Introduction:** All measurements in direct muon decay,  $\mu^- \rightarrow e^- + 2 \text{ neutrals}$ , and its inverse,  $\nu_\mu + e^- \rightarrow \mu^- + \text{neutral}$ , are successfully described by the “V-A interaction”, which is a particular case of a local, derivative-free, lepton-number-conserving, four fermion interaction [1]. As shown below, within this framework, the Standard Model assumptions, such



as the  $V$ - $A$  form and the nature of the neutrals ( $\nu_\mu$  and  $\bar{\nu}_e$ ), and hence the doublet assignments  $(\nu_e e^-)_L$  and  $(\nu_\mu \mu^-)_L$ , have been determined from experiments [2,3]. All considerations on muon decay are valid for the leptonic tau decays  $\tau \rightarrow \ell + \nu_\tau + \bar{\nu}_e$  with the replacements  $m_\mu \rightarrow m_\tau$ ,  $m_e \rightarrow m_\ell$ .

**Parameters:** The differential decay probability to obtain an  $e^\pm$  with (reduced) energy between  $x$  and  $x + dx$ , emitted in the direction  $\hat{\mathbf{x}}_3$  at an angle between  $\vartheta$  and  $\vartheta + d\vartheta$  with respect to the muon polarization vector  $\mathbf{P}_\mu$ , and with its spin parallel to the arbitrary direction  $\hat{\boldsymbol{\zeta}}$ , neglecting radiative corrections, is given by

$$\begin{aligned} \frac{d^2\Gamma}{dx d\cos\vartheta} = & \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2} \\ & \times (F_{\text{IS}}(x) \pm P_\mu \cos\vartheta F_{\text{AS}}(x)) \\ & \times \left[ 1 + \hat{\boldsymbol{\zeta}} \cdot \mathbf{P}_e(x, \vartheta) \right] . \end{aligned} \quad (1)$$

Here,  $W_{e\mu} = \max(E_e) = (m_\mu^2 + m_e^2)/2m_\mu$  is the maximum  $e^\pm$  energy,  $x = E_e/W_{e\mu}$  is the reduced energy,  $x_0 = m_e/W_{e\mu} = 9.67 \times 10^{-3}$ , and  $P_\mu = |\mathbf{P}_\mu|$  is the degree of muon polarization.  $\hat{\boldsymbol{\zeta}}$  is the direction in which a perfect polarization-sensitive electron detector is most sensitive. The isotropic part of the spectrum,  $F_{\text{IS}}(x)$ , the anisotropic part  $F_{\text{AS}}(x)$  and the electron polarization,  $\mathbf{P}_e(x, \vartheta)$ , may be parametrized by the Michel parameters [1,4]  $\rho, \eta, \xi, \delta$ , *etc.* These are bilinear combinations of the coupling constants  $g_{e\mu}^\gamma$ , which occur in the matrix element (given below).

If the masses of the neutrinos as well as  $x_0^2$  are neglected, the energy and angular distribution of the electron in the rest frame of a muon ( $\mu^\pm$ ) measured by a polarization insensitive detector, is given by

$$\frac{d^2\Gamma}{dx d\cos\vartheta} \sim x^2 \cdot \left\{ 3(1-x) + \frac{2\rho}{3}(4x-3) + 3\eta x_0(1-x)/x \right. \\ \left. \pm P_\mu \cdot \xi \cdot \cos\vartheta \left[ 1-x + \frac{2\delta}{3}(4x-3) \right] \right\} . \quad (2)$$

Here,  $\vartheta$  is the angle between the electron momentum and the muon spin, and  $x \equiv 2E_e/m_\mu$ . For the Standard Model coupling, we obtain  $\rho = \xi\delta = 3/4$ ,  $\xi = 1$ ,  $\eta = 0$  and the differential decay rate is

$$\frac{d^2\Gamma}{dx d\cos\vartheta} = \frac{G_F^2 m_\mu^5}{192\pi^3} [3 - 2x \pm P_\mu \cos\vartheta(2x-1)] x^2 . \quad (3)$$

The coefficient in front of the square bracket is the total decay rate.

If only the neutrino masses are neglected, and if the  $e^\pm$  polarization is detected, then the functions in Eq. (1) become

$$F_{\text{IS}}(x) = x(1-x) + \frac{2}{9} \rho(4x^2 - 3x - x_0^2) + \eta \cdot x_0(1-x) \\ F_{\text{AS}}(x) = \frac{1}{3}\xi \sqrt{x^2 - x_0^2} \\ \times \left[ 1 - x + \frac{2}{3}\delta \left( 4x - 3 + \left( \sqrt{1 - x_0^2} - 1 \right) \right) \right] \\ \mathbf{P}_e(x, \vartheta) = P_{T_1} \cdot \hat{\mathbf{x}}_1 + P_{T_2} \cdot \hat{\mathbf{x}}_2 + P_L \cdot \hat{\mathbf{x}}_3 . \quad (4)$$

Here  $\hat{\mathbf{x}}_1$ ,  $\hat{\mathbf{x}}_2$ , and  $\hat{\mathbf{x}}_3$  are orthogonal unit vectors defined as follows:

$$\begin{aligned} \hat{\mathbf{x}}_3 & \text{ is along the } e \text{ momentum } \mathbf{p}_e \\ \frac{\hat{\mathbf{x}}_3 \times \mathbf{P}_\mu}{|\hat{\mathbf{x}}_2 \times \mathbf{P}_\mu|} = \hat{\mathbf{x}}_2 & \text{ is transverse to } \mathbf{p}_e \text{ and perpendicular} \\ & \text{to the “decay plane”} \\ \hat{\mathbf{x}}_2 \times \hat{\mathbf{x}}_3 = \hat{\mathbf{x}}_1 & \text{ is transverse to the } \mathbf{p}_e \text{ and in the} \\ & \text{“decay plane.”} \end{aligned}$$

The components of  $\mathbf{P}_e$  then are given by

$$\begin{aligned}
 P_{T_1}(x, \vartheta) &= P_\mu \sin \vartheta \cdot F_{T_1}(x) / (F_{IS}(x) \pm P_\mu \cos \vartheta \cdot F_{AS}(x)) \\
 P_{T_2}(x, \vartheta) &= P_\mu \sin \vartheta \cdot F_{T_2}(x) / (F_{IS}(x) \pm P_\mu \cos \vartheta \cdot F_{AS}(x)) \\
 P_L(x, \vartheta) &= \left( \pm F_{IP}(x) + P_\mu \cos \vartheta \right. \\
 &\quad \left. \times F_{AP}(x) \right) / (F_{IS}(x) \pm P_\mu \cos \vartheta \cdot F_{AS}(x)) ,
 \end{aligned}$$

where

$$\begin{aligned}
 F_{T_1}(x) &= \frac{1}{12} \left\{ -2 \left[ \xi'' + 12(\rho - \frac{3}{4}) \right] (1-x)x_0 \right. \\
 &\quad \left. - 3\eta(x^2 - x_0^2) + \eta''(-3x^2 + 4x - x_0^2) \right\} \\
 F_{T_2}(x) &= \frac{1}{3} \sqrt{x^2 - x_0^2} \left\{ 3 \frac{\alpha'}{A} (1-x) + 2 \frac{\beta'}{A} \sqrt{1 - x_0^2} \right\} \\
 F_{IP}(x) &= \frac{1}{54} \sqrt{x^2 - x_0^2} \left\{ 9\xi' \left( -2x + 2 + \sqrt{1 - x_0^2} \right) \right. \\
 &\quad \left. + 4\xi(\delta - \frac{3}{4})(4x - 4 + \sqrt{1 - x_0^2}) \right\} \\
 F_{AP}(x) &= \frac{1}{6} \left\{ \xi''(2x^2 - x - x_0^2) + 4(\rho - \frac{3}{4})(4x^2 - 3x - x_0^2) \right. \\
 &\quad \left. + 2\eta''(1-x)x_0 \right\} . \tag{5}
 \end{aligned}$$

For the experimental values of the parameters  $\rho$ ,  $\xi$ ,  $\xi'$ ,  $\xi''$ ,  $\delta$ ,  $\eta$ ,  $\eta''$ ,  $\alpha/A$ ,  $\beta/A$ ,  $\alpha'/A$ ,  $\beta'/A$ , which are not all independent, see the Data Listings below. Experiments in the past have also been analyzed using the parameters  $a$ ,  $b$ ,  $c$ ,  $a'$ ,  $b'$ ,  $c'$ ,  $\alpha/A$ ,  $\beta/A$ ,  $\alpha'/A$ ,  $\beta'/A$  (and  $\eta = (\alpha - 2\beta)/2A$ ), as defined by Kinoshita and Sirlin [5]. They serve as a model-independent summary of all possible measurements on the decay electron (see Listings below). The relations between the two sets of parameters are

$$\begin{aligned}
\rho - \frac{3}{4} &= \frac{3}{4}(-a + 2c)/A , \\
\eta &= (\alpha - 2\beta)/A , \\
\eta'' &= (3\alpha + 2\beta)/A , \\
\delta - \frac{3}{4} &= \frac{9}{4} \cdot \frac{(a' - 2c')/A}{1 - [a + 3a' + 4(b + b') + 6c - 14c']/A} , \\
1 - \xi \frac{\delta}{\rho} &= 4 \frac{[(b + b') + 2(c - c')]/A}{1 - (a - 2c)/A} , \\
1 - \xi' &= [(a + a') + 4(b + b') + 6(c + c')]/A , \\
1 - \xi'' &= (-2a + 20c)/A ,
\end{aligned}$$

where

$$A = a + 4b + 6c . \quad (6)$$

The differential decay probability to obtain a *left-handed*  $\nu_e$  with (reduced) energy between  $y$  and  $y + dy$ , neglecting radiative corrections as well as the masses of the electron and of the neutrinos, is given by [6]

$$\frac{d\Gamma}{dy} = \frac{m_\mu^5 G_F^2}{16\pi^3} \cdot Q_L^{\nu_e} \cdot y^2 \left\{ (1 - y) - \omega_L \cdot \left(y - \frac{3}{4}\right) \right\} . \quad (7)$$

Here,  $y = 2 E_{\nu_e}/m_\mu$ .  $Q_L^{\nu_e}$  and  $\omega_L$  are parameters.  $\omega_L$  is the neutrino analog of the spectral shape parameter  $\rho$  of Michel. Since in the Standard Model,  $Q_L^{\nu_e} = 1$ ,  $\omega_L = 0$ , the measurement of  $d\Gamma/dy$  has allowed a null-test of the Standard Model (see Listings below).

**Matrix element:** All results in direct muon decay (energy spectra of the electron and of the neutrinos, polarizations, and angular distributions) and in inverse muon decay (the reaction cross section) at energies well below  $m_W c^2$  may be parametrized

in terms of amplitudes  $g_{\varepsilon\mu}^\gamma$  and the Fermi coupling constant  $G_F$ , using the matrix element

$$\frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g_{\varepsilon\mu}^\gamma \langle \bar{e}_\varepsilon | \Gamma^\gamma | (\nu_e)_n \rangle \langle \bar{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle. \quad (8)$$

We use the notation of Fetscher *et al.* [2], who in turn use the sign conventions and definitions of Scheck [7]. Here,  $\gamma = S, V, T$  indicates a scalar, vector, or tensor interaction; and  $\varepsilon, \mu = R, L$  indicate a right- or left-handed chirality of the electron or muon. The chiralities  $n$  and  $m$  of the  $\nu_e$  and  $\bar{\nu}_\mu$  are then determined by the values of  $\gamma, \varepsilon$ , and  $\mu$ . The particles are represented by fields of definite chirality [8].

As shown by Langacker and London [9], explicit lepton-number nonconservation still leads to a matrix element equivalent to Eq. (8). They conclude that it is not possible, even in principle, to test lepton-number conservation in (leptonic) muon decay if the final neutrinos are massless and are not observed.

The ten complex amplitudes  $g_{\varepsilon\mu}^\gamma$  ( $g_{RR}^T$  and  $g_{LL}^T$  are identically zero) and  $G_F$  constitute 19 independent (real) parameters to be determined by experiment. The Standard Model interaction corresponds to one single amplitude  $g_{LL}^V$  being unity and all the others being zero.

The (direct) muon decay experiments are compatible with an arbitrary mix of the scalar and vector amplitudes  $g_{LL}^S$  and  $g_{LL}^V$  – in the extreme even with purely scalar  $g_{LL}^S = 2, g_{LL}^V = 0$ . The decision in favour of the Standard Model comes from the quantitative observation of inverse muon decay, which would be forbidden for pure  $g_{LL}^S$  [2].

**Experimental determination of  $V-A$ :** In order to determine the amplitudes  $g_{\varepsilon\mu}^\gamma$  uniquely from experiment, the

following set of equations, where the left-hand sides represent experimental results, has to be solved.

$$\begin{aligned}
a &= 16(|g_{RL}^V|^2 + |g_{LR}^V|^2) + |g_{RL}^S + 6g_{RL}^T|^2 + |g_{LR}^S + 6g_{LR}^T|^2 \\
a' &= 16(|g_{RL}^V|^2 - |g_{LR}^V|^2) + |g_{RL}^S + 6g_{RL}^T|^2 - |g_{LR}^S + 6g_{LR}^T|^2 \\
\alpha &= 8\text{Re} \left\{ g_{RL}^V(g_{LR}^{S*} + 6g_{LR}^{T*}) + g_{LR}^V(g_{RL}^{S*} + 6g_{RL}^{T*}) \right\} \\
\alpha' &= 8\text{Im} \left\{ g_{LR}^V(g_{RL}^{S*} + 6g_{RL}^{T*}) - g_{RL}^V(g_{LR}^{S*} + 6g_{LR}^{T*}) \right\} \\
b &= 4(|g_{RR}^V|^2 + |g_{LL}^V|^2) + |g_{RR}^S|^2 + |g_{LL}^S|^2 \\
b' &= 4(|g_{RR}^V|^2 - |g_{LL}^V|^2) + |g_{RR}^S|^2 - |g_{LL}^S|^2 \\
\beta &= -4\text{Re} \left\{ g_{RR}^V g_{LL}^{S*} + g_{LL}^V g_{RR}^{S*} \right\} \\
\beta' &= 4\text{Im} \left\{ g_{RR}^V g_{LL}^{S*} - g_{LL}^V g_{RR}^{S*} \right\} \\
c &= \frac{1}{2} \left\{ |g_{RL}^S - 2g_{RL}^T|^2 + |g_{LR}^S - 2g_{LR}^T|^2 \right\} \\
c' &= \frac{1}{2} \left\{ |g_{RL}^S - 2g_{RL}^T|^2 - |g_{LR}^S - 2g_{LR}^T|^2 \right\}
\end{aligned}$$

and

$$\begin{aligned}
Q_L^{\nu_e} &= 1 - \left\{ \frac{1}{4}|g_{LR}^S|^2 + \frac{1}{4}|g_{LL}^S|^2 + |g_{RR}^V|^2 + |g_{RL}^V|^2 + 3|g_{LR}^T|^2 \right\} \\
\omega_L &= \frac{3}{4} \frac{\{|g_{RR}^S|^2 + 4|g_{LR}^V|^2 + |g_{RL}^S + 2g_{RL}^T|^2\}}{|g_{RL}^S|^2 + |g_{RR}^S|^2 + 4|g_{LL}^V|^2 + 4|g_{LR}^V|^2 + 12|g_{RL}^T|^2} .
\end{aligned}$$

It has been noted earlier by C. Jarlskog [10], that certain experiments observing the decay electron are especially informative if they yield the  $V$ - $A$  values. The complete solution is now found as follows. Fetscher *et al.* [2] introduced four probabilities  $Q_{\varepsilon\mu}(\varepsilon, \mu = R, L)$  for the decay of a  $\mu$ -handed muon into an  $\varepsilon$ -handed electron and showed that there exist upper bounds on  $Q_{RR}$ ,  $Q_{LR}$ , and  $Q_{RL}$ , and a lower bound on  $Q_{LL}$ . These probabilities are given in terms of the  $g_{\varepsilon\mu}^\gamma$ 's by

$$Q_{\varepsilon\mu} = \frac{1}{4}|g_{\varepsilon\mu}^S|^2 + |g_{\varepsilon\mu}^V|^2 + 3(1 - \delta_{\varepsilon\mu})|g_{\varepsilon\mu}^T|^2, \quad (9)$$

where  $\delta_{\varepsilon\mu} = 1$  for  $\varepsilon = \mu$ , and  $\delta_{\varepsilon\mu} = 0$  for  $\varepsilon \neq \mu$ . They are related to the parameters  $a$ ,  $b$ ,  $c$ ,  $a'$ ,  $b'$ , and  $c'$  by

$$\begin{aligned} Q_{RR} &= 2(b + b')/A, \\ Q_{LR} &= [(a - a') + 6(c - c')]/2A, \\ Q_{RL} &= [(a + a') + 6(c + c')]/2A, \\ Q_{LL} &= 2(b - b')/A, \end{aligned} \quad (10)$$

with  $A = 16$ . In the Standard Model,  $Q_{LL} = 1$  and the others are zero.

Since the upper bounds on  $Q_{RR}$ ,  $Q_{LR}$ , and  $Q_{RL}$  are found to be small, and since the helicity of the  $\nu_\mu$  in pion decay is known from experiment [11,12] to very high precision to be  $-1$  [13], the cross section  $S$  of *inverse* muon decay, normalized to the  $V$ - $A$  value, yields [2]

$$|g_{LL}^S|^2 \leq 4(1 - S) \quad (11)$$

and

$$|g_{LL}^V|^2 = S. \quad (12)$$

Thus the Standard Model assumption of a pure  $V$ - $A$  leptonic charged weak interaction of  $e$  and  $\mu$  is derived (within errors) from experiments at energies far below mass of the  $W^\pm$ : Eq. (12) gives a lower limit for  $V$ - $A$ , and Eqs. (9) and (11) give upper limits for the other four-fermion interactions. The existence of such upper limits may also be seen from  $Q_{RR} + Q_{RL} = (1 - \xi')/2$  and  $Q_{RR} + Q_{LR} = \frac{1}{2}(1 + \xi/3 - 16 \xi\delta/9)$ . Table 1 gives the current experimental limits on the magnitudes of the  $g_{\varepsilon\mu}^\gamma$ 's.

More stringent limits on the six coupling constants  $g_{LR}^S$ ,  $g_{LR}^V$ ,  $g_{LR}^T$ ,  $g_{RL}^S$ ,  $g_{RL}^V$ , and  $g_{RL}^T$  have been derived from upper limits on the neutrino mass [16]. Limits on the “charge retention” coordinates, as used in the older literature (*e.g.*, Ref. 17), are given by Burkard *et al.* [18].

**Table 1.** Coupling constants  $g_{\varepsilon\mu}^\gamma$ . Ninety-percent confidence level experimental limits. The limits on  $|g_{LL}^S|$  and  $|g_{LL}^V|$  are from Ref. 14, and the others from a general analysis of muon decay measurements [15]. The experimental uncertainty on the muon polarization in pion decay is included. Note that, by definition,  $|g_{\varepsilon\mu}^S| \leq 2$ ,  $|g_{\varepsilon\mu}^V| \leq 1$  and  $|g_{\varepsilon\mu}^T| \leq 1/\sqrt{3}$ .

$ g_{RR}^S  < 0.067$	$ g_{RR}^V  < 0.034$	$ g_{RR}^T  \equiv 0$
$ g_{LR}^S  < 0.088$	$ g_{LR}^V  < 0.036$	$ g_{LR}^T  < 0.025$
$ g_{LR}^S  < 0.417$	$ g_{LR}^V  < 0.104$	$ g_{LR}^T  < 0.104$
$ g_{LL}^S  < 0.550$	$ g_{LL}^V  > 0.960$	$ g_{LL}^T  \equiv 0$
$ g_{LR}^S + 6g_{LR}^T  < 0.143$	$ g_{RL}^S + 6g_{RL}^T  < 0.418$	
$ g_{LR}^S + 2g_{LR}^T  < 0.108$	$ g_{RL}^S + 2g_{RL}^T  < 0.417$	
$ g_{LR}^S - 2g_{LR}^T  < 0.070$	$ g_{RL}^S - 2g_{RL}^T  < 0.418$	

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## $\mu$ DECAY PARAMETERS

### $\rho$ PARAMETER

( $V-A$ ) theory predicts  $\rho = 0.75$ .

VALUE	EVTs	DOCUMENT ID	TECN	CHG	COMMENT
<b>0.7509 <math>\pm</math> 0.0010</b>	<b>OUR AVERAGE</b>				
0.75080 $\pm$ 0.00032 $\pm$ 0.00100	6G	<sup>28</sup> MUSSER	05	SPEC +	surface $\mu^+$ at TRIUMF
0.7518 $\pm$ 0.0026		DERENZO	69	RVUE	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.72 $\pm$ 0.06 $\pm$ 0.08		AMORUSO	04	ICAR	Liquid Ar TPC
0.762 $\pm$ 0.008	170k	<sup>29</sup> FRYBERGER	68	ASPK +	25–53 MeV $e^+$
0.760 $\pm$ 0.009	280k	<sup>29</sup> SHERWOOD	67	ASPK +	25–53 MeV $e^+$
0.7503 $\pm$ 0.0026	800k	<sup>29</sup> PEOPLES	66	ASPK +	20–53 MeV $e^+$

<sup>28</sup> The quoted systematic error includes a contribution of 0.00023 (added in quadrature) from the dependence on the Michel parameter  $\eta$ .

<sup>29</sup>  $\eta$  constrained = 0. These values incorporated into a two parameter fit to  $\rho$  and  $\eta$  by DERENZO 69.

### $\eta$ PARAMETER

( $V-A$ ) theory predicts  $\eta = 0$ .

<u>VALUE</u>	<u>EVTs</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
<b>0.001 ±0.024</b>	<b>OUR AVERAGE</b>	Error includes scale factor of 2.0. See the ideogram below.			
0.071 ±0.037 ±0.005	30M	DANNEBERG	05	CNTR +	7–53 MeV e <sup>+</sup>
−0.007 ±0.013	5.3M	<sup>30</sup> BURKARD	85B	FIT +	9–53 MeV e <sup>+</sup>
−0.12 ±0.21	6346	DERENZO	69	HBC +	1.6–6.8 MeV e <sup>+</sup>

• • • We do not use the following data for averages, fits, limits, etc. • • •

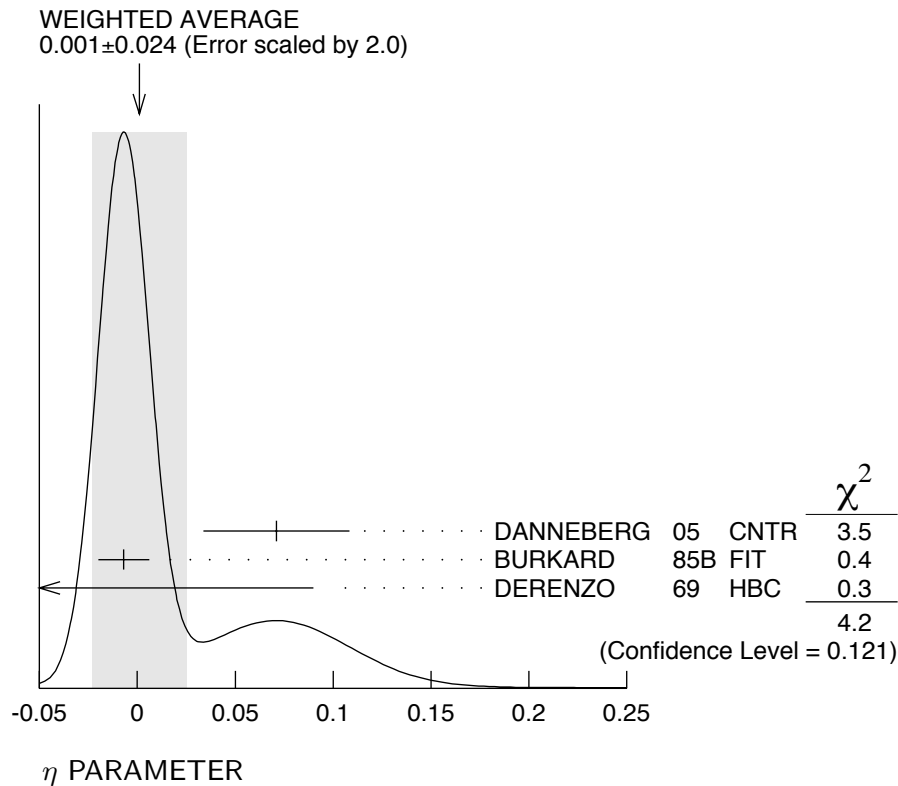
$-0.0021 \pm 0.0070 \pm 0.0010$	30M	<sup>31</sup> DANNEBERG	05	CNTR	+	7–53 MeV $e^+$
$-0.012 \pm 0.015 \pm 0.003$	5.3M	<sup>31</sup> BURKARD	85B	CNTR	+	9–53 MeV $e^+$
$0.011 \pm 0.081 \pm 0.026$	5.3M	BURKARD	85B	CNTR	+	9–53 MeV $e^+$
$-0.7 \pm 0.5$	170k	<sup>32</sup> FRYBERGER	68	ASPK	+	25–53 MeV $e^+$
$-0.7 \pm 0.6$	280k	<sup>32</sup> SHERWOOD	67	ASPK	+	25–53 MeV $e^+$
$0.05 \pm 0.5$	800k	<sup>32</sup> PEOPLES	66	ASPK	+	20–53 MeV $e^+$
$-2.0 \pm 0.9$	9213	<sup>33</sup> PLANO	60	HBC	+	Whole spec- trum

<sup>30</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

<sup>31</sup>  $\alpha = \alpha' = 0$  assumed.

<sup>32</sup>  $\rho$  constrained = 0.75.

<sup>33</sup> Two parameter fit to  $\rho$  and  $\eta$ ; PLANO 60 discounts value for  $\eta$ .



## $\delta$ PARAMETER

(V–A) theory predicts  $\delta = 0.75$ .

<u>VALUE</u>	<u>EVS</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>	
<b>0.7495 ±0.0012</b>	<b>OUR AVERAGE</b>					
0.74964±0.00066±0.00112	6G	GAPONENKO 05	SPEC	+	surface $\mu^+$ at TRIUMF	
0.7486 ±0.0026 ±0.0028	<sup>34</sup> BALKE	88	SPEC	+	Surface $\mu^+$ 's	
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●						
	<sup>35</sup> VOSSLER	69				
0.752 ±0.009	490k	FRYBERGER	68	ASPK	+	25–53 MeV $e^+$
0.782 ±0.031		KRUGER	61			
0.78 ±0.05	8354	PLANO	60	HBC	+	Whole spec- trum

<sup>34</sup> BALKE 88 uses  $\rho = 0.752 \pm 0.003$ .

<sup>35</sup> VOSSLER 69 has measured the asymmetry below 10 MeV. See comments about radiative corrections in VOSSLER 69.

## **|( $\xi$ PARAMETER) $\times(\mu$ LONGITUDINAL POLARIZATION)|**

(V-A) theory predicts  $\xi = 1$ , longitudinal polarization = 1.

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
<b>1.0027<math>\pm</math>0.0079<math>\pm</math>0.0030</b>		BELTRAMI	87	CNTR	SIN, $\pi$ decay in flight

• • • We do not use the following data for averages, fits, limits, etc. • • •

1.0013 $\pm$ 0.0030 $\pm$ 0.0053		<sup>36</sup> IMAZATO	92	SPEC +	$K^+ \rightarrow \mu^+ \nu_\mu$
0.975 $\pm$ 0.015		AKHMANOV	68	EMUL	140 kG
0.975 $\pm$ 0.030	66k	GUREVICH	64	EMUL	See AKHMA- NOV 68
0.903 $\pm$ 0.027		<sup>37</sup> ALI-ZADE	61	EMUL +	27 kG
0.93 $\pm$ 0.06	8354	PLANO	60	HBC +	8.8 kG
0.97 $\pm$ 0.05	9k	BARDON	59	CNTR	Bromoform target

<sup>36</sup> The corresponding 90% confidence limit from IMAZATO 92 is  $|\xi P_\mu| > 0.990$ . This measurement is of  $K^+$  decay, not  $\pi^+$  decay, so we do not include it in an average, nor do we yet set up a separate data block for  $K$  results.

<sup>37</sup> Depolarization by medium not known sufficiently well.

## **$\xi \times (\mu$ LONGITUDINAL POLARIZATION) $\times \delta / \rho$**

VALUE	CL%	DOCUMENT ID	TECN	CHG	COMMENT
<b>&gt;0.99682</b>	90	<sup>38</sup> JODIDIO	86	SPEC +	TRIUMF

• • • We do not use the following data for averages, fits, limits, etc. • • •

>0.9966	90	<sup>39</sup> STOKER	85	SPEC +	$\mu$ -spin rotation
>0.9959	90	CARR	83	SPEC +	11 kG

<sup>38</sup> JODIDIO 86 includes data from CARR 83 and STOKER 85. The value here is from the erratum.

<sup>39</sup> STOKER 85 find  $(\xi P_\mu \delta / \rho) > 0.9955$  and  $> 0.9966$ , where the first limit is from new  $\mu$  spin-rotation data and the second is from combination with CARR 83 data. In V-A theory,  $(\delta / \rho) = 1.0$ .

## **$\xi' =$ LONGITUDINAL POLARIZATION OF $e^+$**

(V-A) theory predicts the longitudinal polarization =  $\pm 1$  for  $e^\pm$ , respectively. We have flipped the sign for  $e^-$  so our programs can average.

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
<b>1.00 <math>\pm</math>0.04 OUR AVERAGE</b>					
0.998 $\pm$ 0.045	1M	BURKARD	85	CNTR +	Bhabha + annihil
0.89 $\pm$ 0.28	29k	SCHWARTZ	67	OSPK -	Moller scattering
0.94 $\pm$ 0.38		BLOOM	64	CNTR +	Brems. transmiss.
1.04 $\pm$ 0.18		DUCLOS	64	CNTR +	Bhabha scattering
1.05 $\pm$ 0.30		BUHLER	63	CNTR +	Annihilation

## **$\xi''$ PARAMETER**

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
<b>0.65<math>\pm</math>0.36</b>	326k	<sup>40</sup> BURKARD	85	CNTR +	Bhabha + annihil

<sup>40</sup> BURKARD 85 measure  $(\xi'' - \xi\xi')/\xi$  and  $\xi'$  and set  $\xi = 1$ .

**TRANSVERSE  $e^+$  POLARIZATION IN PLANE OF  $\mu$  SPIN,  $e^+$  MOMENTUM**

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>EVTS</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
<b>7 <math>\pm</math> 8 OUR AVERAGE</b>					
6.3 $\pm$ 7.7 $\pm$ 3.4	30M	DANNEBERG	05 CNTR	+	7–53 MeV $e^+$
16 $\pm$ 21 $\pm$ 10	5.3M	BURKARD	85B CNTR	+	Annihil 9–53 MeV

**TRANSVERSE  $e^+$  POLARIZATION NORMAL TO PLANE OF  $\mu$  SPIN,  $e^+$  MOMENTUM**Zero if  $T$  invariance holds.

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>EVTS</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
<b>–2 <math>\pm</math> 8 OUR AVERAGE</b>					
–3.7 $\pm$ 7.7 $\pm$ 3.4	30M	DANNEBERG	05 CNTR	+	7–53 MeV $e^+$
7 $\pm$ 22 $\pm$ 7	5.3M	BURKARD	85B CNTR	+	Annihil 9–53 MeV

 **$\alpha/A$** 

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>EVTS</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
<b>0.4 <math>\pm</math> 4.3</b>		<sup>41</sup> BURKARD	85B FIT		

• • • We do not use the following data for averages, fits, limits, etc. • • •

15 $\pm$ 50 $\pm$ 14	5.3M	BURKARD	85B CNTR	+	9–53 MeV $e^+$
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<sup>41</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B. **$\alpha'/A$** Zero if  $T$  invariance holds.

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>EVTS</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
<b>0 <math>\pm</math> 4 OUR AVERAGE</b>					
– 3.4 $\pm$ 21.3 $\pm$ 4.9	30M	DANNEBERG	05 CNTR	+	7–53 MeV $e^+$
– 0.2 $\pm$ 4.3		<sup>42</sup> BURKARD	85B FIT		

• • • We do not use the following data for averages, fits, limits, etc. • • •

–47 $\pm$ 50 $\pm$ 14	5.3M	<sup>43</sup> BURKARD	85B CNTR	+	9–53 MeV $e^+$
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<sup>42</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.<sup>43</sup> BURKARD 85B measure  $e^+$  polarizations  $P_{T1}$  and  $P_{T2}$  versus  $e^+$  energy. **$\beta/A$** 

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>EVTS</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
<b>3.9 <math>\pm</math> 6.2</b>		<sup>44</sup> BURKARD	85B FIT		

• • • We do not use the following data for averages, fits, limits, etc. • • •

2 $\pm$ 17 $\pm$ 6	5.3M	BURKARD	85B CNTR	+	9–53 MeV $e^+$
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<sup>44</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

**$\beta'/A$** Zero if  $T$  invariance holds.

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>EVTS</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
<b>1 <math>\pm</math> 5 OUR AVERAGE</b>					
– 0.5 $\pm$ 7.8 $\pm$ 1.8	30M	DANNEBERG	05 CNTR	+	7–53 MeV $e^+$
1.5 $\pm$ 6.3		<sup>45</sup> BURKARD	85B FIT		

• • • We do not use the following data for averages, fits, limits, etc. • • •

– 1.3 $\pm$ 3.5 $\pm$ 0.6	30M	<sup>46</sup> DANNEBERG	05 CNTR	+	7–53 MeV $e^+$
17 $\pm$ 17 $\pm$ 6	5.3M	<sup>47</sup> BURKARD	85B CNTR	+	9–53 MeV $e^+$

<sup>45</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.<sup>46</sup>  $\alpha = \alpha' = 0$  assumed.<sup>47</sup> BURKARD 85B measure  $e^+$  polarizations  $P_{T_1}$  and  $P_{T_2}$  versus  $e^+$  energy. **$a/A$** 

This comes from an alternative parameterization to that used in the Summary Table (see the “Note on Muon Decay Parameters” above).

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>CL%</u>	<u>DOCUMENT ID</u>	<u>TECN</u>
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• • • We do not use the following data for averages, fits, limits, etc. • • •

<15.9	90	<sup>48</sup> BURKARD	85B FIT
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<sup>48</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B. **$a'/A$** 

This comes from an alternative parameterization to that used in the Summary Table (see the “Note on Muon Decay Parameters” above).

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>DOCUMENT ID</u>	<u>TECN</u>
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• • • We do not use the following data for averages, fits, limits, etc. • • •

5.3 $\pm$ 4.1	<sup>49</sup> BURKARD	85B FIT
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<sup>49</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B. **$(b'+b)/A$** 

This comes from an alternative parameterization to that used in the Summary Table (see the “Note on Muon Decay Parameters” above).

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>CL%</u>	<u>DOCUMENT ID</u>	<u>TECN</u>
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• • • We do not use the following data for averages, fits, limits, etc. • • •

<1.04	90	<sup>50</sup> BURKARD	85B FIT
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<sup>50</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B. **$c/A$** 

This comes from an alternative parameterization to that used in the Summary Table (see the “Note on Muon Decay Parameters” above).

<u>VALUE (units <math>10^{-3}</math>)</u>	<u>CL%</u>	<u>DOCUMENT ID</u>	<u>TECN</u>
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• • • We do not use the following data for averages, fits, limits, etc. • • •

<6.4	90	<sup>51</sup> BURKARD	85B FIT
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<sup>51</sup> Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

$c'/A$ 

This comes from an alternative parameterization to that used in the Summary Table (see the “Note on Muon Decay Parameters” above).

VALUE (units $10^{-3}$ )	DOCUMENT ID	TECN
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• • • We do not use the following data for averages, fits, limits, etc. • • •

$3.5 \pm 2.0$	<sup>52</sup> BURKARD	85B FIT
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<sup>52</sup>Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

 $\bar{\eta}$  PARAMETER

( $V-A$ ) theory predicts  $\bar{\eta} = 0$ .  $\bar{\eta}$  affects spectrum of radiative muon decay.

VALUE	DOCUMENT ID	TECN	CHG	COMMENT
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**0.02  $\pm$  0.08 OUR AVERAGE**

$-0.014 \pm 0.090$	EICHENBER...	84	ELEC	+	$\rho$ free
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$+0.09 \pm 0.14$	BOGART	67	CNTR	+	
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• • • We do not use the following data for averages, fits, limits, etc. • • •

$-0.035 \pm 0.098$	EICHENBER...	84	ELEC	+	$\rho=0.75$ assumed
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 $\mu$  REFERENCES

DANNEBERG	05	PRL 94 021802	N. Danneberg <i>et al.</i>	(ETH, JAGL, PSI+)
GAPONENKO	05	PR D71 071101R	A. Gaponenko <i>et al.</i>	(TWIST Collab.)
MOHR	05	RMP 77 1	P.J. Mohr, B.N. Taylor	(NIST)
MUSSER	05	PRL 94 101805	J.R. Musser <i>et al.</i>	(TWIST Collab.)
AMORUSO	04	EPJ C33 233	S. Amoruso <i>et al.</i>	(ICARUS Collab.)
BENNETT	04	PRL 92 161802	G.W. Bennett <i>et al.</i>	(Muon(g-2) Collab.)
AHMED	02	PR D65 112002	M. Ahmed <i>et al.</i>	(MEGA Collab.)
BENNETT	02	PRL 89 101804	G.W. Bennett <i>et al.</i>	(Muon(g-2) Collab.)
BROWN	01	PRL 86 2227	H.N. Brown <i>et al.</i>	(Muon(g-2) Collab.)
BROWN	00	PR D62 091101R	H.N. Brown <i>et al.</i>	(BNL/G-2 Collab.)
MEYER	00	PRL 84 1136	V. Meyer <i>et al.</i>	
BROOKS	99	PRL 83 1521	M.L. Brooks <i>et al.</i>	(MEGA/LAMPF Collab.)
HUGHES	99	RMP 71 S133	V.W. Hughes, T. Kinoshita	
LIU	99	PRL 82 711	W. Liu <i>et al.</i>	(LAMPF Collab.)
MOHR	99	JPCRD 28 1713	P.J. Mohr, B.N. Taylor	(NIST)
Also		RMP 72 351	P.J. Mohr, B.N. Taylor	(NIST)
WILLMANN	99	PRL 82 49	L. Willmann <i>et al.</i>	
FELDMAN	98	PR D57 3873	G.J. Feldman, R.D. Cousins	
KAULARD	98	PL B422 334	J. Kaulard <i>et al.</i>	(SINDRUM-II Collab.)
GORDEEV	97	PAN 60 1164	V.A. Gordeev <i>et al.</i>	(PNPI)
		Translated from YAF 60	1291.	
ABELA	96	PRL 77 1950	R. Abela <i>et al.</i>	(PSI, ZURI, HEIDH, TBIL+)
HONECKER	96	PRL 76 200	W. Honecker <i>et al.</i>	(SINDRUM II Collab.)
DOHMEN	93	PL B317 631	C. Dohmen <i>et al.</i>	(PSI SINDRUM-II Collab.)
FREEDMAN	93	PR D47 811	S.J. Freedman <i>et al.</i>	(LAMPF E645 Collab.)
NI	93	PR D48 1976	B. Ni <i>et al.</i>	(LAMPF Crystal-Box Collab.)
IMAZATO	92	PRL 69 877	J. Imazato <i>et al.</i>	(KEK, INUS, TOKY+)
BARANOV	91	SJNP 53 802	V.A. Baranov <i>et al.</i>	(JINR)
		Translated from YAF 53	1302.	
KRAKAUER	91B	PL B263 534	D.A. Krakauer <i>et al.</i>	(UMD, UCI, LANL)
MATTHIAS	91	PRL 66 2716	B.E. Matthias <i>et al.</i>	(YALE, HEIDP, WILL+)
Also		PRL 67 932 (erratum)	B.E. Matthias <i>et al.</i>	(YALE, HEIDP, WILL+)
HUBER	90B	PR D41 2709	T.M. Huber <i>et al.</i>	(WYOM, VICT, ARIZ+)
AHMAD	88	PR D38 2102	S. Ahmad <i>et al.</i>	(TRIUM, VICT, VPI, BRCCO+)
Also		PRL 59 970	S. Ahmad <i>et al.</i>	(TRIUM, VPI, VICT, BRCCO+)
BALKE	88	PR D37 587	B. Balke <i>et al.</i>	(LBL, UCB, COLO, NWES+)
BELLGARDT	88	NP B299 1	U. Bellgardt <i>et al.</i>	(SINDRUM Collab.)
BOLTON	88	PR D38 2077	R.D. Bolton <i>et al.</i>	(LANL, STAN, CHIC+)
Also		PRL 56 2461	R.D. Bolton <i>et al.</i>	(LANL, STAN, CHIC+)
Also		PRL 57 3241	D. Grosnick <i>et al.</i>	(CHIC, LANL, STAN+)
BELTRAMI	87	PL B194 326	I. Beltrami <i>et al.</i>	(ETH, SIN, MANZ)
COHEN	87	RMP 59 1121	E.R. Cohen, B.N. Taylor	(RISC, NBS)
BEER	86	PRL 57 671	G.A. Beer <i>et al.</i>	(VICT, TRIUM, WYOM)

JODIDIO	86	PR D34 1967	A. Jodidio <i>et al.</i>	(LBL, NWES, TRIU)
Also		PR D37 237 (erratum)	A. Jodidio <i>et al.</i>	(LBL, NWES, TRIU)
BERTL	85	NP B260 1	W. Bertl <i>et al.</i>	(SINDRUM Collab.)
BRYMAN	85	PRL 55 465	D.A. Bryman <i>et al.</i>	(TRIU, CNRC, BRCO+)
BURKARD	85	PL 150B 242	H. Burkhardt <i>et al.</i>	(ETH, SIN, MANZ)
BURKARD	85B	PL 160B 343	H. Burkhardt <i>et al.</i>	(ETH, SIN, MANZ)
Also		PR D24 2004	F. Corriveau <i>et al.</i>	(ETH, SIN, MANZ)
Also		PL 129B 260	F. Corriveau <i>et al.</i>	(ETH, SIN, MANZ)
STOKER	85	PRL 54 1887	D.P. Stoker <i>et al.</i>	(LBL, NWES, TRIU)
BARDIN	84	PL 137B 135	G. Bardin <i>et al.</i>	(SACL, CERN, BGNA, FIRZ)
BERTL	84	PL 140B 299	W. Bertl <i>et al.</i>	(SINDRUM Collab.)
BOLTON	84	PRL 53 1415	R.D. Bolton <i>et al.</i>	(LANL, CHIC, STAN+)
EICHENBER...	84	NP A412 523	W. Eichenberger, R. Engfer, A. van der Schaff	
GIOVANETTI	84	PR D29 343	K.L. Giovanetti <i>et al.</i>	(WILL)
AZUELOS	83	PRL 51 164	G. Azuelos <i>et al.</i>	(MONT, TRIU, BRCO)
Also		PRL 39 1113	P. Depommier <i>et al.</i>	(MONT, BRCO, TRIU+)
BERGSMA	83	PL 122B 465	F. Bergsma <i>et al.</i>	(CHARM Collab.)
CARR	83	PRL 51 627	J. Carr <i>et al.</i>	(LBL, NWES, TRIU)
KINNISON	82	PR D25 2846	W.W. Kinnison <i>et al.</i>	(EFI, STAN, LANL)
Also		PRL 42 556	J.D. Bowman <i>et al.</i>	(LASL, EFI, STAN)
KLEMPPT	82	PR D25 652	E. Klemppt <i>et al.</i>	(MANZ, ETH)
MARIAM	82	PRL 49 993	F.G. Mariam <i>et al.</i>	(YALE, HEIDH, BERN)
MARSHALL	82	PR D25 1174	G.M. Marshall <i>et al.</i>	(BRCO)
NEMETHY	81	CNPP 10 147	P. Nemethy, V.W. Hughes	(LBL, YALE)
ABELA	80	PL 95B 318	R. Abela <i>et al.</i>	(BASL, KARLK, KARLE)
BADERT...	80	LNC 28 401	A. Badertscher <i>et al.</i>	(BERN)
Also		NP A377 406	A. Badertscher <i>et al.</i>	(BERN)
JONKER	80	PL 93B 203	M. Jonker <i>et al.</i>	(CHARM Collab.)
SCHAAF	80	NP A340 249	A. van der Schaaf <i>et al.</i>	(ZURI, ETH+)
Also		PL 72B 183	H.P. Povel <i>et al.</i>	(ZURI, ETH, SIN)
WILLIS	80	PRL 44 522	S.E. Willis <i>et al.</i>	(YALE, LBL, LASL+)
Also		PRL 45 1370	S.E. Willis <i>et al.</i>	(YALE, LBL, LASL+)
BAILEY	79	NP B150 1	J.M. Bailey	(CERN, DARE, MANZ)
BADERT...	78	PL 79B 371	A. Badertscher <i>et al.</i>	(BERN)
BAILEY	78	JPG 4 345	J.M. Bailey	(DARE, BERN, SHEF, MANZ, RMCS+)
Also		NP B150 1	J.M. Bailey	(CERN, DARE, MANZ)
BLIETSCHAU	78	NP B133 205	J. Blietschau <i>et al.</i>	(Gargamelle Collab.)
BOWMAN	78	PRL 41 442	J.D. Bowman <i>et al.</i>	(LASL, IAS, CMU+)
CAMANI	78	PL 77B 326	M. Camani <i>et al.</i>	(ETH, MANZ)
BADERT...	77	PRL 39 1385	A. Badertscher <i>et al.</i>	(BERN)
CASPERSON	77	PRL 38 956	D.E. Casperson <i>et al.</i>	(BERN, HEIDH, LASL+)
DEPOMMIER	77	PRL 39 1113	P. Depommier <i>et al.</i>	(MONT, BRCO, TRIU+)
BALANDIN	74	JETP 40 811	M.P. Balandin <i>et al.</i>	(JINR)
		Translated from ZETF 67 1631.		
COHEN	73	JPCRD 2 664	E.R. Cohen, B.N. Taylor	(RISC, NBS)
DUCLOS	73	PL 47B 491	J. Duclos, A. Magnon, J. Picard	(SACL)
EICHTEN	73	PL 46B 281	T. Eichten <i>et al.</i>	(Gargamelle Collab.)
BRYMAN	72	PRL 28 1469	D.A. Bryman <i>et al.</i>	(VPI)
CROWE	72	PR D5 2145	K.M. Crowe <i>et al.</i>	(LBL, WASH)
CRANE	71	PRL 27 474	T. Crane <i>et al.</i>	(YALE)
DERENZO	69	PR 181 1854	S.E. Derenzo	(EFI)
VOSSLER	69	NC 63A 423	C. Vossler	(EFI)
AKHMANOV	68	SJNP 6 230	V.V. Akhmanov <i>et al.</i>	(KIAE)
		Translated from YAF 6 316.		
FRYBERGER	68	PR 166 1379	D. Fryberger	(EFI)
BOGART	67	PR 156 1405	E. Bogart <i>et al.</i>	(COLU)
SCHWARTZ	67	PR 162 1306	D.M. Schwartz	(EFI)
SHERWOOD	67	PR 156 1475	B.A. Sherwood	(EFI)
PEOPLES	66	Nevis 147 unpub.	J. Peoples	(COLU)
BLOOM	64	PL 8 87	S. Bloom <i>et al.</i>	(CERN)
DUCLOS	64	PL 9 62	J. Duclos <i>et al.</i>	(CERN)
GUREVICH	64	PL 11 185	I.I. Gurevich <i>et al.</i>	(KIAE)
BUHLER	63	PL 7 368	A. Buhler-Broglin <i>et al.</i>	(CERN)
MEYER	63	PR 132 2693	S.L. Meyer <i>et al.</i>	(COLU)
CHARPAK	62	PL 1 16	G. Charpak <i>et al.</i>	(CERN)
CONFORTO	62	NC 26 261	G. Conforto <i>et al.</i>	(INFN, ROMA, CERN)
ALI-ZADE	61	JETP 13 313	S.A. Ali-Zade, I.I. Gurevich, B.A. Nikolsky	
		Translated from ZETF 40 452.		
CRITTENDEN	61	PR 121 1823	R.R. Crittenden, W.D. Walker, J. Ballam	(WISC+)
KRUGER	61	UCRL 9322 unpub.	H. Kruger	(LRL)
GUREVICH	60	JETP 10 225	I.I. Gurevich, B.A. Nikolsky, L.V. Surkova	(ITEP)
		Translated from ZETF 37 318.		

PLANO	60	PR 119 1400	R.J. Plano	(COLU)
ASHKIN	59	NC 14 1266	J. Ashkin <i>et al.</i>	(CERN)
BARDON	59	PRL 2 56	M. Bardon, D. Berley, L.M. Lederman	(COLU)
LEE	59	PRL 3 55	J. Lee, N.P. Samios	(COLU)

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