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$K^{\pm}$

$I(J^P) = \frac{1}{2}(0^-)$

THE CHARGED KAON MASS

Revised 1994 by T.G. Trippe (LBNL).

The average of the six charged kaon mass measurements which we use in the Particle Listings is

$$m_{K^{\pm}} = 493.677 \pm 0.013 \text{ MeV (S = 2.4)} , \qquad (1)$$

where the error has been increased by the scale factor S. The large scale factor indicates a serious disagreement between different input data. The average before scaling the error is

$$m_{K^{\pm}} = 493.677 \pm 0.005 \text{ MeV} ,$$
$$\chi^2 = 22.9 \text{ for 5 D.F., Prob. } = 0.04\% , \qquad (2)$$

where the high χ^2 and correspondingly low χ^2 probability further quantify the disagreement.

The main disagreement is between the two most recent and precise results,

$$\begin{aligned} m_{K^{\pm}} &= 493.696 \pm 0.007 \text{ MeV} && \text{DENISOV 91} \\ m_{K^{\pm}} &= \underline{493.636 \pm 0.011 \text{ MeV}} \text{ (S = 1.5)} && \text{GALL 88} \\ \text{Average} &= 493.679 \pm 0.006 \text{ MeV} && \end{aligned}$$
$$\chi^2 = 21.2 \text{ for 1 D.F., Prob. } = 0.0004\% , \qquad (3)$$

both of which are measurements of x-ray energies from kaonic atoms. Comparing the average in Eq. (3) with the overall average in Eq. (2), it is clear that DENISOV 91 and GALL 88 dominate the overall average, and that their disagreement is responsible for most of the high χ^2.

The GALL 88 measurement was made using four different kaonic atom transitions, K^- Pb ($9 \rightarrow 8$), K^- Pb ($11 \rightarrow 10$), K^- W ($9 \rightarrow 8$), and K^- W ($11 \rightarrow 10$). The $m_{K^\pm}$ values they obtain from each of these transitions is shown in the Particle Listings and in Fig. 1. Their K^- Pb ($9 \rightarrow 8$) $m_{K^\pm}$is below and somewhat inconsistent with their other three transitions. The average of their four measurements is

$$m_{K^\pm} = 493.636 \pm 0.007 \;, \qquad \chi^2 = 7.0 \text{ for 3 D.F., Prob. } = 7.2\% \;. \tag{4}$$

This is a low but acceptable χ^2 probability so, to be conservative, GALL 88 scaled up the error on their average by S=1.5 to obtain their published error ±0.011 shown in Eq. (3) above and used in the Particle Listings average.

The ideogram in Fig. 1 shows that the DENISOV 91 measurement and the GALL 88 K^- Pb ($9 \rightarrow 8$) measurement yield two well-separated peaks. One might suspect the GALL 88 K^- Pb ($9 \rightarrow 8$) measurement since it is responsible both for the internal inconsistency in the GALL 88 measurements and the disagreement with DENISOV 91.

To see if the disagreement could result from a systematic problem with the K^- Pb ($9 \rightarrow 8$) transition, we have separated the CHENG 75 data, which also used K^- Pb, into its separate transitions. Figure 1 shows that the CHENG 75 and GALL 88 K^- Pb ($9 \rightarrow 8$) values are consistent, suggesting the possibility of a common effect such as contaminant nuclear γ rays near the K^- Pb ($9 \rightarrow 8$) transition energy, although the CHENG 75 errors are too large to make a strong conclusion. The average of all 13 measurements has a χ^2 of 52.6 as shown in Fig. 1 and the first line of Table 1, yielding an unacceptable χ^2 probability of 0.00005%. The second line of Table 1 excludes

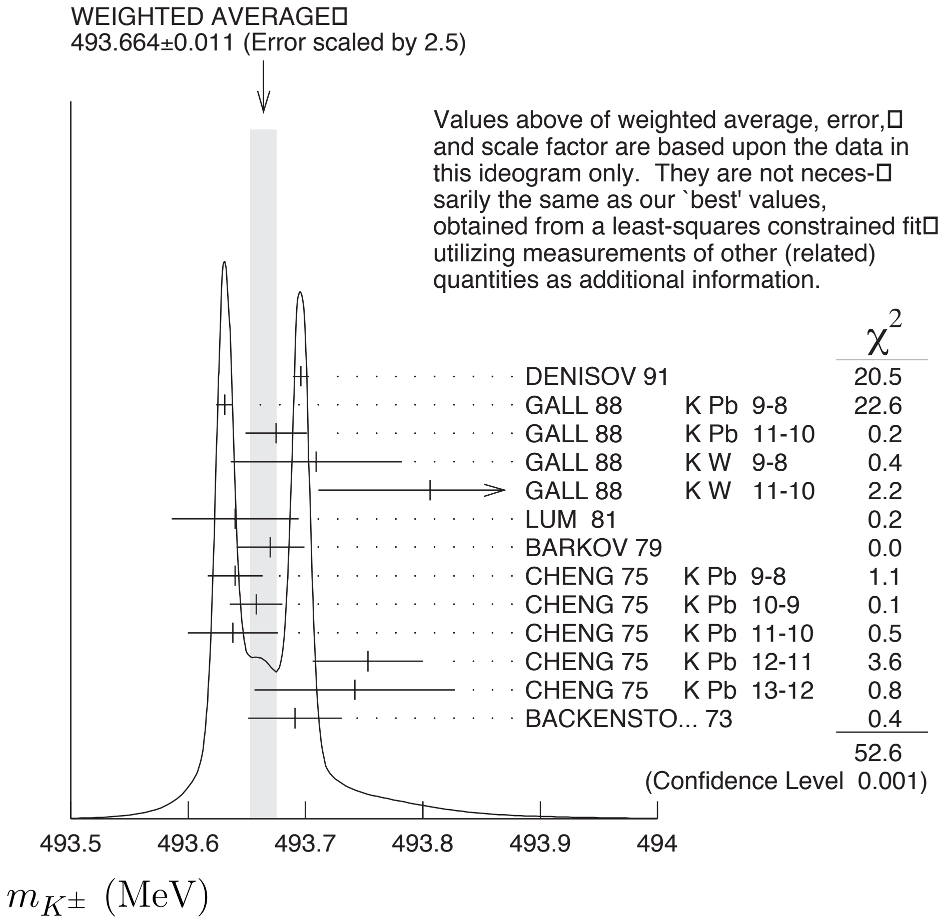


Figure 1: Ideogram of $m_{K^\pm}$ mass measurements. GALL 88 and CHENG 75 measurements are shown separately for each transition they measured.

both the GALL 88 and CHENG 75 measurements of the K^-Pb $(9 \rightarrow 8)$ transition and yields a χ^2 probability of 43%. The third [fourth] line of Table 1 excludes only the GALL 88 K^-Pb $(9 \rightarrow 8)$ [DENISOV 91] measurement and yields a χ^2 probability of 20% [8.6%]. Table 1 shows that removing both measurements of the K^-Pb $(9 \rightarrow 8)$ transition produces the most consistent set of data, but that excluding only the

GALL 88 K^-Pb ($9 \rightarrow 8$) transition or DENISOV 91 also produces acceptable probabilities.

Table 1: $m_{K^\pm}$ averages for some combinations of Fig. 1 data.

$m_{K^\pm}$ (MeV)	χ^2	D.F.	Prob. (%)	Measurements used
493.664 ± 0.004	52.6	12	0.00005	all 13 measurements
493.690 ± 0.006	10.1	10	43	no K^-Pb(9→8)
493.687 ± 0.006	14.6	11	20	no GALL 88 K^-Pb(9→8)
493.642 ± 0.006	17.8	11	8.6	no DENISOV 91

Yu.M. Ivanov, representing DENISOV 91, has estimated corrections needed for the older experiments because of improved ^{192}Ir and ^{198}Au calibration γ-ray energies. He estimates that CHENG 75 and BACKENSTOSS 73 $m_{K^\pm}$ values could be raised by about 15 keV and 22 keV, respectively. With these estimated corrections, Table 1 becomes Table 2. The last line of Table 2 shows that if such corrections are assumed, then GALL 88 K^-Pb ($9 \rightarrow 8$) is inconsistent with the rest of the data even when DENISOV 91 is excluded. Yu.M. Ivanov warns that these are rough estimates. Accordingly, we do not use Table 2 to reject the GALL 88 K^-Pb ($9 \rightarrow 8$) transition, but we note that a future reanalysis of the CHENG 75 data could be useful because it might provide supporting evidence for such a rejection.

Table 2: $m_{K^\pm}$ averages for some combinations of Fig. 1 data after raising CHENG 75 and BACKENSTOSS 73 values by 0.015 and 0.022 MeV respectively.

$m_{K^\pm}$ (MeV)	χ^2	D.F.	Prob. (%)	Measurements used
493.666 ± 0.004	53.9	12	0.00003	all 13 measurements
493.693 ± 0.006	9.0	10	53	no K^- Pb(9→8)
493.690 ± 0.006	11.5	11	40	no GALL 88 K^- Pb(9→8)
493.645 ± 0.006	23.0	11	1.8	no DENISOV 91

The GALL 88 measurement uses a Ge semiconductor spectrometer which has a resolution of about 1 keV, so they run the risk of some contaminant nuclear γ rays. Studies of γ rays following stopped π^- and Σ^- absorption in nuclei (unpublished) do not show any evidence for contaminants according to GALL 88 spokesperson, B.L. Roberts. The DENISOV 91 measurement uses a crystal diffraction spectrometer with a resolution of 6.3 eV for radiation at 22.1 keV to measure the 4f-3d transition in K^- ^{12}C. The high resolution and the light nucleus reduce the probability for overlap by contaminant γ rays, compared with the measurement of GALL 88. The DENISOV 91 measurement is supported by their high-precision measurement of the 4d-2p transition energy in π^- ^{12}C, which is good agreement with the calculated energy.

While we suspect that the GALL 88 K^- Pb ($9 \rightarrow 8$) measurements could be the problem, we are unable to find clear grounds for rejecting it. Therefore, we retain their measurement in the average and accept the large scale factor until further information can be obtained from new measurements and/or from reanalysis of GALL 88 and CHENG 75 data.

We thank B.L. Roberts (Boston Univ.) and Yu.M. Ivanov (Petersburg Nuclear Physics Inst.) for their extensive help in understanding this problem.

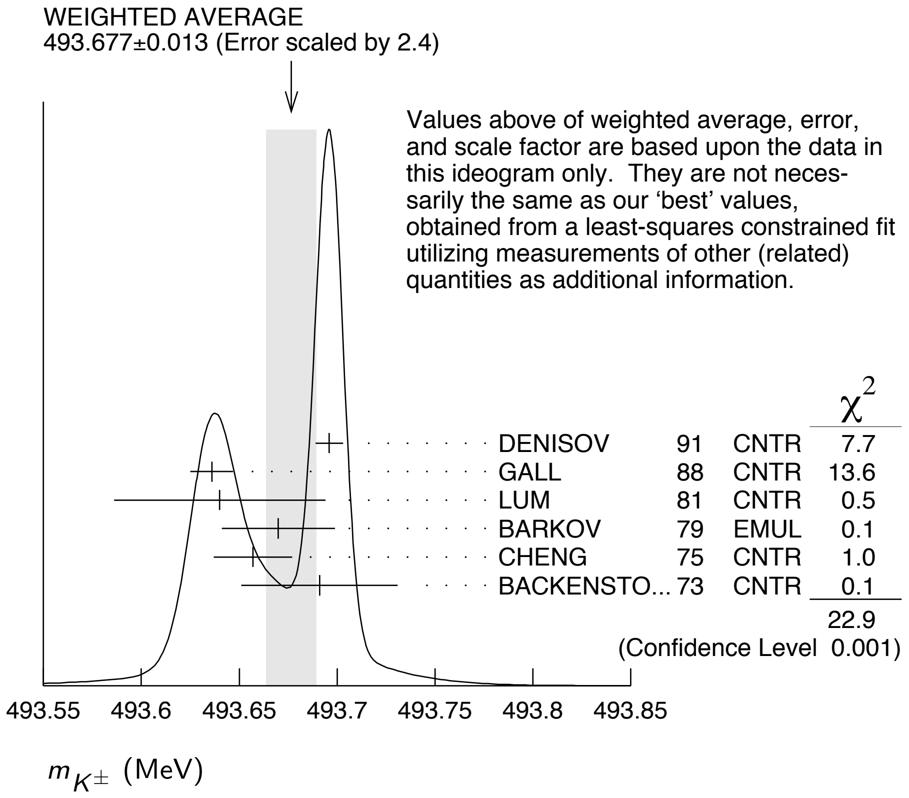
$K^{\pm}$ MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
493.677±0.016 OUR FIT Error includes scale factor of 2.8.					
493.677±0.013 OUR AVERAGE Error includes scale factor of 2.4. See the ideogram below.					
493.696±0.007	[1] DENISOV	91	CNTR	−	Kaonic atoms
493.636±0.011	[2] GALL	88	CNTR	−	Kaonic atoms
493.640±0.054	LUM	81	CNTR	−	Kaonic atoms
493.670±0.029	BARKOV	79	EMUL	±	$e^+ e^- \rightarrow K^+ K^-$
493.657±0.020	[2] CHENG	75	CNTR	−	Kaonic atoms
493.691±0.040	BACKENSTO...	73	CNTR	−	Kaonic atoms
• • • We do not use the following data for averages, fits, limits, etc. • • •					
493.631±0.007	GALL	88	CNTR	−	K^- Pb (9→ 8)
493.675±0.026	GALL	88	CNTR	−	K^- Pb (11→ 10)
493.709±0.073	GALL	88	CNTR	−	K^- W (9→ 8)
493.806±0.095	GALL	88	CNTR	−	K^- W (11→ 10)
493.640±0.022±0.008	[3] CHENG	75	CNTR	−	K^- Pb (9→ 8)
493.658±0.019±0.012	[3] CHENG	75	CNTR	−	K^- Pb (10→ 9)
493.638±0.035±0.016	[3] CHENG	75	CNTR	−	K^- Pb (11→ 10)
493.753±0.042±0.021	[3] CHENG	75	CNTR	−	K^- Pb (12→ 11)
493.742±0.081±0.027	[3] CHENG	75	CNTR	−	K^- Pb (13→ 12)

[1] Error increased from 0.0059 based on the error analysis in IVANOV 92.

[2] This value is the authors' combination of all of the separate transitions listed for this paper.

[3] The CHENG 75 values for separate transitions were calculated from their Table 7 transition energies. The first error includes a 20% systematic error in the noncircular contaminant shift. The second error is due to a ±5 eV uncertainty in the theoretical transition energies.



$m_{K^+} - m_{K^-}$

Test of *CPT*.

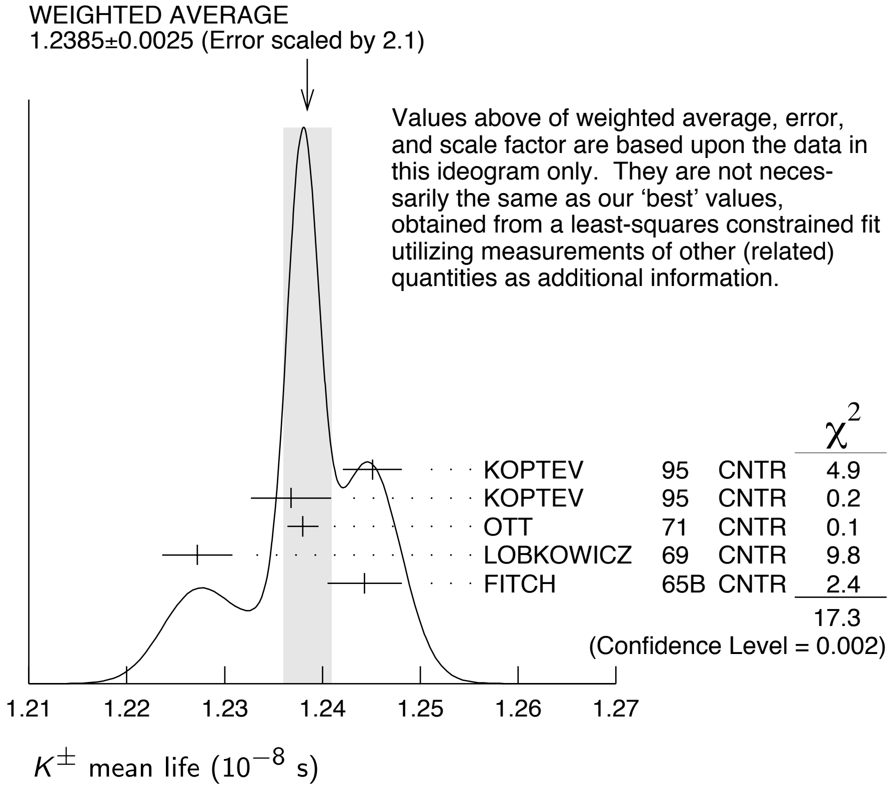
VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
−0.032±0.090	1.5M	[4] FORD	72	ASPK	±

[4] FORD 72 uses $m_{\pi^+} - m_{\pi^-} = +28 \pm 70$ keV.

$K^\pm$ MEAN LIFE

VALUE (10^{-8} s)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
1.2385±0.0024 OUR FIT Error includes scale factor of 2.0.						
1.2385±0.0025 OUR AVERAGE Error includes scale factor of 2.1. See the ideogram below.						
1.2451±0.0030	250k	KOPTEV	95	CNTR		*K* at rest, U target
1.2368±0.0041	150k	KOPTEV	95	CNTR		*K* at rest, Cu target
1.2380±0.0016	3M	OTT	71	CNTR	+	*K* at rest
1.2272±0.0036		LOBKOWICZ	69	CNTR	+	*K* in flight
1.2443±0.0038		FITCH	65B	CNTR	+	*K* at rest
• • • We do not use the following data for averages, fits, limits, etc. • • •						
1.2415±0.0024	400k	[5] KOPTEV	95	CNTR		*K* at rest
1.221 ±0.011		FORD	67	CNTR	±	
1.231 ±0.011		BOYARSKI	62	CNTR	+	

[5] KOPTEV 95 report this weighted average of their U-target and Cu-target results, where they have weighted by $1/\sigma$ rather than $1/\sigma^2$.



$(\tau_{K^+} - \tau_{K^-})\,/\,\tau_{\text{average}}$

This quantity is a measure of *CPT* invariance in weak interactions.

VALUE (%)	*DOCUMENT ID*		*TECN*
0.11 ±0.09 OUR AVERAGE	Error includes scale factor of 1.2.		
0.090±0.078	LOBKOWICZ	69	CNTR
0.47 ±0.30	FORD	67	CNTR

RARE KAON DECAYS

(Revised November 2005 by L. Littenberg, BNL and G. Valencia, Iowa State University)

A. Introduction: There are several useful reviews on rare kaon decays and related topics [1–14]. Activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters

3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \rightarrow \mu e$. Category 2 includes processes such as $K^+ \rightarrow \pi^+\nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focused on the decays $K_L \rightarrow \pi^0\ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \rightarrow \pi^+\ell^+\ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Category 4 also includes $K_L \rightarrow \pi^0\gamma\gamma$ and $K_L \rightarrow \ell^+\ell^-\gamma$. The former is important in understanding a CP-conserving contribution to $K_L \rightarrow \pi^0\ell^+\ell^-$, whereas the latter could shed light on long distance contributions to $K_L \rightarrow \mu^+\mu^-$.

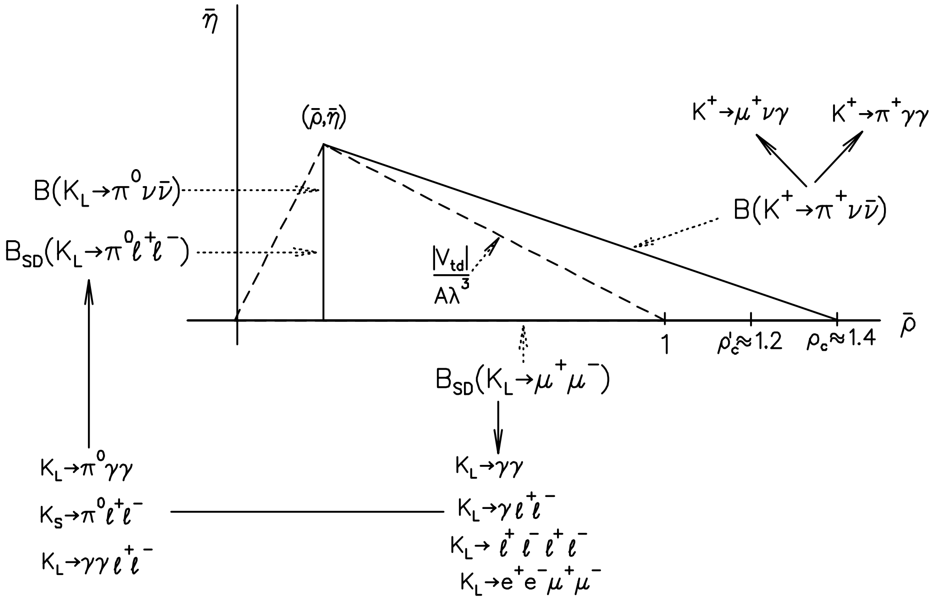


Figure 1: Role of rare kaon decays in determining the unitarity triangle. The solid arrows point to auxiliary modes needed to interpret the main results, or potential backgrounds to them.

The interplay between Categories 2-4 can be illustrated in Fig. 1. The modes $K \to \pi\nu\overline{\nu}$ are the cleanest ones theoretically. They can provide accurate determinations of certain CKM parameters (shown in the figure). In combination with alternate determinations of these parameters they also constrain new interactions. The modes $K_L \to \pi^0 e^+ e^-$ and $K_L \to \mu^+\mu^-$ are also sensitive to CKM parameters. However, they suffer from a series of hadronic uncertainties that can be addressed, at least in part, through a systematic study of the additional modes indicated in the figure.

B. Explicit violations of the Standard Model: Much activity has focussed on searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 4.7 \times 10^{-12} (148\ \mathrm{TeV}/M_X)^4$ [5]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of over 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to parity-odd couplings and the latter is sensitive to parity-even couplings. Limits on certain lepton-number violating kaon decays [15,16] also exist. Related searches in μ and τ processes are discussed in our section "Tests of Conservation Laws".

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.
$K^+ \to \pi^+ e^- \mu^+$	1.2×10^{-11}	BNL-865	2003/Ref. 17
$K^+ \to \pi^+ e^+ \mu^-$	5.2×10^{-10}	BNL-865	2001/Ref. 15
$K_L \to \mu e$	4.7×10^{-12}	BNL-871	1998/Ref. 18
$K_L \to \pi^0 e\mu$	3.4×10^{-10}	KTeV (prelim.)	2003/Ref. 19

Physics beyond the SM is also pursued through the search for $K^+ \to \pi^+ X^0$, where X^0 is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, *etc.*). The 90% CL upper limit on this process is 5.9×10^{-11} [20].

C. Measurements of Standard Model parameters:

Until 1997, searches for $K^+ \to \pi^+ \nu\overline{\nu}$ were motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [21] and long-distance contributions are known to be quite small [2,22,23]. Since then, BNL-787 has observed two candidate events [20,24], and BNL-949 has observed one more, yielding a branching ratio of $(1.47^{+1.30}_{-0.89}) \times 10^{-10}$ [25]. At this level, this reaction becomes interesting from the point of view of constraining SM parameters. A new experiment with a sensitivity goal of $\sim 10^{-12}$/event was proposed [26] at CERN in 2005. In the future this mode may provide grounds for precision tests of the flavor structure of the Standard Model [27]. The branching ratio can be written in terms of the very well-measured K_{e3} rate as [2]:

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 \mathrm{B}(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4\theta_W}$$

$$\times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^{\ell} + V_{ts}^* V_{td} X(m_t)|^2 \qquad (1)$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [28]. In Eq. (1) the Inami-Lim function $X(m_t)$ is of order 1 [29], and X_{NL}^{ℓ} is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. QCD corrections, which mainly affect X_{NL}^{ℓ}, lead to a residual error of $< 5\%$ for the decay amplitude [12,23,30,31]. Evaluating the constants in Eq. (1), one can cast this result in terms of the CKM parameters λ, V_{cb}, $\bar{\rho}$ and $\bar{\eta}$ (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [12]

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) \approx 1.6 \times 10^{-5} |V_{cb}|^4 [\sigma\bar{\eta}^2 + (\rho_c - \bar{\rho})^2] \qquad (2)$$

where $\rho_c \equiv 1 + (\frac{2}{3}X_{NL}^{e} + \frac{1}{3}X_{NL}^{\tau})/(|V_{cb}|^2 X(m_t)) \approx 1.4$ and $\sigma \equiv 1/(1 - \frac{1}{2}\lambda^2)^2$. Thus, $\mathrm{B}(K^+ \to \pi^+\nu\overline{\nu})$ determines an ellipse in the $\bar{\rho}$, $\bar{\eta}$ plane with center $(\rho_c, 0)$ and semi-axes $\approx \frac{1}{|V_{cb}|^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.6\times10^{-5}}}$ and $\frac{1}{\sigma|V_{cb}|^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.6\times10^{-5}}}$. Current constraints on the CKM parameters lead to a predicted branching ratio $(8.0 \pm 1.1) \times 10^{-11}$ [31], near the lower end of the BNL-787 measurement.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter $\bar{\rho}$, given by [12]:

$$\mathrm{B}_{\mathrm{SD}}(K_L \to \mu^+\mu^-) \approx 2.7 \times 10^{-4} |V_{cb}|^4 (\rho_c' - \bar{\rho})^2 \qquad (3)$$

where ρ_c' depends on the charm quark mass and is approximately 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is determined by the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B}_{\mathrm{abs}}(K_L \to \mu^+\mu^-) = (6.64 \pm 0.07) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (6.87 \pm 0.11) \times 10^{-9}$ [32]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. The latter cannot be derived directly from experiment [33] but can be estimated with certain assumptions [34,35]. The decay $K_L \to e^+e^-$ is completely dominated by long distance physics and is easier to estimate. The result, $B(K_L \to e^+e^-) \sim 9 \times 10^{-12}$ [33,36], is in good agreement with the BNL-871 measurement, $(8.7^{+5.7}_{-4.1}) \times 10^{-12}$ [37].

D. Searches for direct CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [2,38,39]. In the Standard Model this mode is dominated by an intermediate top-quark state and does not suffer from the small uncertainty associated with the charm-quark intermediate state that affects the mode $K^+ \to \pi^+\nu\overline{\nu}$. The branching ratio is given approximately by Ref. 12:

$$\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \approx 7.6 \times 10^{-5}|V_{cb}|^4\bar{\eta}^2 \ . \tag{4}$$

With current constraints on the CKM parameters this leads to a predicted branching ratio $(3.0 \pm 0.6) \times 10^{-11}$ [40]. The current published experimental upper bound is $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \leq 5.9 \times 10^{-7}$ [41]. The 90% CL bound on $K^+ \to \pi^+\nu\bar{\nu}$ provides a nearly model independent bound $B(K_L \to \pi^0\nu\bar{\nu}) < 1.4 \times 10^{-9}$ [42]. KEK-391a [43], which began data-taking in early 2004, aims to reach this level, and has presented a preliminary result of $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \leq 2.86 \times 10^{-7}$ [44]. A Letter of Intent for an experiment to reach the $\sim 5 \times 10^{-13}$/event level has been submitted to the J-PARC PAC [45].

There has been much theoretical work on possible contributions to rare K decays beyond the SM. While in the simplest case of the MSSM with no new sources of flavor or CP violation the main effect is a suppression of the rare K decays [2,21,46], substantial enhancements are possible in more general SUSY models [47]. A comprehensive discussion can be found in Refs. [40] and [48].

The decay $K_L \to \pi^0 e^+ e^-$ also has sensitivity to the CKM parameter η through its CP-violating component. There are both direct and indirect CP-violating amplitudes which can interfere. The direct CP-violating amplitude is short distance dominated and has been calculated in detail within the SM [9]. The indirect CP-violating amplitude can be inferred from a measurement of $K_S \to \pi^0 e^+ e^-$. The complete CP-violating contribution to the rate can be written as [49]:

$$\mathrm{B}_{\mathrm{CPV}} \approx 10^{-12} \Bigg[15.7 |a_S|^2 \pm 1.45 \left(\frac{|V_{cb}|^2 \bar{\eta}}{10^{-4}} \right) |a_S| + 0.129 \left(\frac{|V_{cb}|^2 \bar{\eta}}{10^{-4}} \right)^2 \Bigg] \tag{5}$$

where the three terms correspond to the indirect CP violation, the interference, and the direct CP violation respectively. The parameter a_S has recently been extracted by NA48 from a measurement of the decay $K_S \to \pi^0 e^+ e^-$ with the result $|a_S| = 1.06^{+0.26}_{-0.21} \pm 0.07$ [50]. With current constraints on the CKM parameters this implies that

$$\mathrm{B}_{\mathrm{CPV}} \approx (17.2 \pm 9.4 + 4.7) \times 10^{-12}. \tag{6}$$

The indirect CP violation is larger than the direct CP violation. While the sign of the interference is *a priori* unknown, arguments in favor of a positive sign have been put forward in Ref. 51 and Ref. 52. NA48 has also obtained the value $a_s = 1.54^{+0.40}_{-0.32} \pm 0.06$ [53] from a measurement of the $K_S \to \pi^0\mu^+\mu^-$ rate, in agreement with the value extracted from $K_S \to \pi^0 e^+e^-$

This mode also has a CP-conserving component dominated by a two-photon intermediate state that is still subject to a sizable uncertainty. This CP-conserving component can be decomposed into an absorptive and a dispersive part. The absorptive part can be extracted from the measurement of the low $m_{\gamma\gamma}$ region of the $K_L \to \pi^0\gamma\gamma$ spectrum. The rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$ in $K_L \to \pi^0\gamma\gamma$ are well described in chiral perturbation theory in terms of three (a priori) unknown parameters [54,55]. Both KTeV and NA48 have studied the mode $K_L \to \pi^0\gamma\gamma$ reporting conflicting results. KTeV finds $B(K_L \to \pi^0\gamma\gamma) = (1.68 \pm 0.07_{\text{stat}} \pm 0.08_{\text{sys}}) \times 10^{-6}$ [56], whereas NA48 finds $B(K_L \to \pi^0\gamma\gamma) = (1.36 \pm 0.03_{\text{stat}} \pm 0.03_{\text{sys}} \pm 0.03_{\text{norm}}) \times 10^{-6}$ [57]. Furthermore, the NA48 data indicates a negligible rate in the low $m_{\gamma\gamma}$ region suggesting a very small CP-conserving component $B_{\text{CP}}(K_L \to \pi^0 e^+e^-) \sim \mathcal{O}(10^{-13})$ [51,55,57]. KTeV, on the other hand, reports a larger rate in the low $m_{\gamma\gamma}$ region, which suggests a larger $B_{\text{CP}}(K_L \to \pi^0 e^+e^-)$ between $1 - 2 \times 10^{-12}$ [56]. In addition to this difference between the two experiments, there remains some model dependence in the estimate of the dispersive part of the CP-conserving $K_L \to \pi^0 e^+e^-$ [51].

The related process, $K_L \to \pi^0\gamma e^+e^-$, is potentially an additional background in some region of phase space [58]. This process has been observed with a branching ratio of $(2.34 \pm 0.35_{\text{stat}} \pm 0.13_{\text{sys}}) \times 10^{-8}$ [59].

The decay $K_L \rightarrow \gamma\gamma e^+e^-$ constitutes the dominant background to $K_L \rightarrow \pi^0 e^+e^-$. It was first observed by BNL-845 [60] and subsequently confirmed with a much larger sample by FNAL-799 [61]. It has been estimated that this background will enter at about the 10^{-10} level [62,63], comparable to or larger than the signal level. Because of this, the observation of $K_L \rightarrow \pi^0 e^+e^-$ at the SM level will depend on background subtraction with good statistics. Possible alternative strategies are discussed in Ref. 51 and references cited therein.

The 90% CL upper bound for the process $K_L \rightarrow \pi^0 e^+e^-$ is 2.8×10^{-10} [63]. For the closely related muonic process, the published upper bound is $\mathrm{B}(K_L \rightarrow \pi^0\mu^+\mu^-) \leq 3.8 \times 10^{-10}$ [64] compared with the SM prediction of $(1.5 \pm 0.3) \times 10^{-11}$ [65] (assuming positive interference between the direct- and indirect-CP violating components). KTeV has additional data corresponding to about a factor 1.3 in sensitivity for the latter reaction that is under analysis.

A recent study of $K_L \rightarrow \pi^0\mu^+\mu^-$ has indicated that it might be possible to extract the direct CP-violating contribution by a joint study of the Dalitz plot variables and the components of the μ^+ polarization [66]. The latter tends to be quite substantial so that large statistics may not be necessary.

E. Other long distance dominated modes:

The decays $K^+ \rightarrow \pi^+\ell^+\ell^-$ ($\ell = e$ or μ) have received considerable attention. The rate and spectrum have been measured for both the electron and muon modes [67,68]. Ref. 49 has proposed a parameterization inspired by chiral perturbation theory, which provides a successful description of data but indicates the presence of large corrections beyond leading order. More work is needed to fully understand the origin of these large corrections.

Much information has been recorded by KTeV and NA48 on the rates and spectrum for the Dalitz pair conversion modes $K_L \to \ell^+\ell^-\gamma$ [69,70], and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' =$ e or μ [16,70–72]. All these results are used to test hadronic models and could further our understanding of the long distance component in $K_L \to \mu^+\mu^-$.

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K^+ DECAY MODES

K^- modes are charge conjugates of the modes below.

Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level

Leptonic and semileptonic modes

Γ_1	$e^+ \nu_e$		$(1.55 \pm 0.07) \times 10^{-5}$	
Γ_2	$\mu^+ \nu_\mu$		(63.44 ± 0.14) %	S=1.2
Γ_3	$\pi^0 e^+ \nu_e$ Called K^+_{e3}.		(4.98 ± 0.07) %	S=1.3
Γ_4	$\pi^0 \mu^+ \nu_\mu$ Called $K^+_{\mu 3}$.		(3.32 ± 0.06) %	S=1.2
Γ_5	$\pi^0 \pi^0 e^+ \nu_e$		$(2.2 \pm 0.4) \times 10^{-5}$	
Γ_6	$\pi^+ \pi^- e^+ \nu_e$		$(4.09 \pm 0.09) \times 10^{-5}$	
Γ_7	$\pi^+ \pi^- \mu^+ \nu_\mu$		$(1.4 \pm 0.9) \times 10^{-5}$	
Γ_8	$\pi^0 \pi^0 \pi^0 e^+ \nu_e$		$< 3.5 \times 10^{-6}$	CL=90%

Hadronic modes

Γ_9	$\pi^+ \pi^0$		(20.92 ± 0.12) %	S=1.1
Γ_{10}	$\pi^+ \pi^0 \pi^0$		(1.757 ± 0.024) %	S=1.1
Γ_{11}	$\pi^+ \pi^+ \pi^-$		(5.590 ± 0.031) %	S=1.1

Leptonic and semileptonic modes with photons

Γ_{12}	$\mu^+ \nu_\mu \gamma$	[*a*,*b*]	$(6.2 \pm 0.8) \times 10^{-3}$	
Γ_{13}	$\mu^+ \nu_\mu \gamma(\mathrm{SD}^+)$	[*c*]	$< 3.0 \times 10^{-5}$	CL=90%
Γ_{14}	$\mu^+ \nu_\mu \gamma(\mathrm{SD}^+\mathrm{INT})$	[*c*]	$< 2.7 \times 10^{-5}$	CL=90%
Γ_{15}	$\mu^+ \nu_\mu \gamma(\mathrm{SD}^- + \mathrm{SD}^-\mathrm{INT})$	[*c*]	$< 2.6 \times 10^{-4}$	CL=90%
Γ_{16}	$e^+ \nu_e \gamma(\mathrm{SD}^+)$	[*c*]	$(1.52 \pm 0.23) \times 10^{-5}$	
Γ_{17}	$e^+ \nu_e \gamma(\mathrm{SD}^-)$	[*c*]	$< 1.6 \times 10^{-4}$	CL=90%
Γ_{18}	$\pi^0 e^+ \nu_e \gamma$	[*a*,*b*]	$(2.69 \pm 0.20) \times 10^{-4}$	
Γ_{19}	$\pi^0 e^+ \nu_e \gamma(\mathrm{SD})$	[*c*]	$< 5.3 \times 10^{-5}$	CL=90%
Γ_{20}	$\pi^0 \mu^+ \nu_\mu \gamma$	[*a*,*b*]	$(2.4 \pm 0.8) \times 10^{-5}$	
Γ_{21}	$\pi^0 \pi^0 e^+ \nu_e \gamma$		$< 5 \times 10^{-6}$	CL=90%

Hadronic modes with photons

Γ_{22}	$\pi^+ \pi^0 \gamma$	[*a*,*b*]	$(2.75 \pm 0.15) \times 10^{-4}$	
Γ_{23}	$\pi^+ \pi^0 \gamma(\mathrm{DE})$	[*b*,*d*]	$(4.4 \pm 0.7) \times 10^{-6}$	
Γ_{24}	$\pi^+ \pi^0 \pi^0 \gamma$	[*a*,*b*]	$(7.6 \, ^{+5.6}_{-3.0}) \times 10^{-6}$	
Γ_{25}	$\pi^+ \pi^+ \pi^- \gamma$	[*a*,*b*]	$(1.04 \pm 0.31) \times 10^{-4}$	
Γ_{26}	$\pi^+ \gamma \gamma$	[*b*]	$(1.10 \pm 0.32) \times 10^{-6}$	
Γ_{27}	$\pi^+ 3\gamma$	[*b*]	$< 1.0 \times 10^{-4}$	CL=90%

Leptonic modes with $\ell\overline{\ell}$ pairs

Γ_{28}	$e^+ \nu_e \nu \overline{\nu}$		$< 6 \times 10^{-5}$	CL=90%
Γ_{29}	$\mu^+ \nu_\mu \nu \overline{\nu}$		$< 6.0 \times 10^{-6}$	CL=90%
Γ_{30}	$e^+ \nu_e e^+ e^-$		$(2.48 \pm 0.20) \times 10^{-8}$	
Γ_{31}	$\mu^+ \nu_\mu e^+ e^-$		$(7.06 \pm 0.31) \times 10^{-8}$	
Γ_{32}	$e^+ \nu_e \mu^+ \mu^-$		$(1.7 \pm 0.5) \times 10^{-8}$	
Γ_{33}	$\mu^+ \nu_\mu \mu^+ \mu^-$		$< 4.1 \times 10^{-7}$	CL=90%

Lepton Family number (*LF*), Lepton number (*L*), $\Delta S = \Delta Q$ (*SQ*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes

Γ_{34}	$\pi^+ \pi^+ e^- \overline{\nu}_e$	*SQ*		$< 1.2 \times 10^{-8}$	CL=90%
Γ_{35}	$\pi^+ \pi^+ \mu^- \overline{\nu}_\mu$	*SQ*		$< 3.0 \times 10^{-6}$	CL=95%
Γ_{36}	$\pi^+ e^+ e^-$	*S1*		$(2.88 \pm 0.13) \times 10^{-7}$	
Γ_{37}	$\pi^+ \mu^+ \mu^-$	*S1*		$(8.1 \pm 1.4) \times 10^{-8}$	S=2.7
Γ_{38}	$\pi^+ \nu \overline{\nu}$	*S1*		$(1.5 {}^{+1.3}_{-0.9}) \times 10^{-10}$	
Γ_{39}	$\pi^+ \pi^0 \nu \overline{\nu}$	*S1*		$< 4.3 \times 10^{-5}$	CL=90%
Γ_{40}	$\mu^- \nu e^+ e^+$	*LF*		$< 2.0 \times 10^{-8}$	CL=90%
Γ_{41}	$\mu^+ \nu_e$	*LF*	[*e*]	$< 4 \times 10^{-3}$	CL=90%
Γ_{42}	$\pi^+ \mu^+ e^-$	*LF*		$< 1.3 \times 10^{-11}$	CL=90%
Γ_{43}	$\pi^+ \mu^- e^+$	*LF*		$< 5.2 \times 10^{-10}$	CL=90%
Γ_{44}	$\pi^- \mu^+ e^+$	*L*		$< 5.0 \times 10^{-10}$	CL=90%
Γ_{45}	$\pi^- e^+ e^+$	*L*		$< 6.4 \times 10^{-10}$	CL=90%
Γ_{46}	$\pi^- \mu^+ \mu^+$	*L*	[*e*]	$< 3.0 \times 10^{-9}$	CL=90%
Γ_{47}	$\mu^+ \overline{\nu}_e$	*L*	[*e*]	$< 3.3 \times 10^{-3}$	CL=90%
Γ_{48}	$\pi^0 e^+ \overline{\nu}_e$	*L*		$< 3 \times 10^{-3}$	CL=90%
Γ_{49}	$\pi^+ \gamma$		[*f*]	$< 2.3 \times 10^{-9}$	CL=90%

[*a*] Most of this radiative mode, the low-momentum γ part, is also included in the parent mode listed without γ's.

[*b*] See the Particle Listings below for the energy limits used in this measurement.

[*c*] Structure-dependent part.

[*d*] Direct-emission branching fraction.

[*e*] Derived from an analysis of neutrino-oscillation experiments.

[*f*] Violates angular-momentum conservation.

CONSTRAINED FIT INFORMATION

An overall fit to the mean life, a decay rate, and 12 branching ratios uses 26 measurements and one constraint to determine 8 parameters. The overall fit has a $\chi^2 = 30.0$ for 19 degrees of freedom.

The following *off-diagonal* array elements are the correlation coefficients $\left\langle\delta p_i \delta p_j\right\rangle/(\delta p_i{\cdot}\delta p_j)$, in percent, from the fit to parameters p_i, including the branching fractions, $x_i \equiv \Gamma_i/\Gamma_{\text{total}}$. The fit constrains the x_i whose labels appear in this array to sum to one.

x_3	−52						
x_4	−50	78					
x_5	−3	6	5				
x_9	−52	−36	−36	−2			
x_{10}	−8	−3	−3	0	−8		
x_{11}	−10	−6	−5	0	−9	4	
Γ	3	2	2	0	3	−1	−33
	x_2	x_3	x_4	x_5	x_9	x_{10}	x_{11}

	Mode	Rate (10^8 s^{-1})	Scale factor
Γ_2	$\mu^+\nu_\mu$	0.5122 ±0.0015	1.4
Γ_3	$\pi^0 e^+\nu_e$ Called K^+_{e3}.	0.0402 ±0.0006	1.3
Γ_4	$\pi^0\mu^+\nu_\mu$ Called $K^+_{\mu3}$.	0.0268 ±0.0005	1.2
Γ_5	$\pi^0\pi^0 e^+\nu_e$	$(1.74\ {}^{+0.35}_{-0.30})\times 10^{-5}$	
Γ_9	$\pi^+\pi^0$	0.1689 ±0.0010	1.2
Γ_{10}	$\pi^+\pi^0\pi^0$	0.01419±0.00020	1.1
Γ_{11}	$\pi^+\pi^+\pi^-$	0.04513±0.00024	

$K^\pm$ DECAY RATES

$\Gamma(\mu^+\nu_\mu)$ Γ_2

VALUE (10^6 s^{-1})	*DOCUMENT ID*	*TECN*	*CHG*
51.22±0.15 OUR FIT Error includes scale factor of 1.4.			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
51.2 ±0.8	FORD 67	CNTR	±

$\Gamma(\pi^+\pi^+\pi^-)$ Γ_{11}

VALUE (10^6 s^{-1})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
4.513±0.024 OUR FIT				
4.511±0.024		[6] FORD 70	ASPK	
• • • We do not use the following data for averages, fits, limits, etc. • • •				
4.529±0.032	3.2M	[6] FORD 70	ASPK	
4.496±0.030		[6] FORD 67	CNTR	±

[6] First FORD 70 value is second FORD 70 combined with FORD 67.

$(\Gamma(K^+) - \Gamma(K^-)) / \Gamma(K)$

$K^\pm \to \mu^\pm \nu_\mu$ RATE DIFFERENCE/AVERAGE

Test of *CPT* conservation.

VALUE (%)	*DOCUMENT ID*		*TECN*
−0.54±0.41	FORD	67	CNTR

$K^\pm \to \pi^\pm \pi^+ \pi^-$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.08±0.12		[7] FORD	70	ASPK	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−0.02±0.16		[8] SMITH	73	ASPK	±
0.10±0.14	3.2M	[7] FORD	70	ASPK	
−0.50±0.90		FLETCHER	67	OSPK	
−0.04±0.21		[7] FORD	67	CNTR	

[7] First FORD 70 value is second FORD 70 combined with FORD 67.

[8] SMITH 73 value of $K^\pm \to \pi^\pm \pi^+ \pi^-$ rate difference is derived from SMITH 73 value of $K^\pm \to \pi^\pm 2\pi^0$ rate difference.

$K^\pm \to \pi^\pm \pi^0 \pi^0$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.0 ±0.6 OUR AVERAGE					
0.08±0.58		SMITH	73	ASPK	±
−1.1 ±1.8	1802	HERZO	69	OSPK	

$K^\pm \to \pi^\pm \pi^0$ RATE DIFFERENCE/AVERAGE

Test of *CPT* conservation.

VALUE (%)	*DOCUMENT ID*		*TECN*
0.8±1.2	HERZO	69	OSPK

$K^\pm \to \pi^\pm \pi^0 \gamma$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.9±3.3 OUR AVERAGE						
0.8±5.8	2461	SMITH	76	WIRE	±	E_π 55–90 MeV
1.0±4.0	4000	ABRAMS	73B	ASPK	±	E_π 51–100 MeV

K^+ BRANCHING RATIOS

Leptonic and semileptonic modes

$\Gamma(e^+ \nu_e)/\Gamma(\mu^+ \nu_\mu)$ Γ_1/Γ_2

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
2.45±0.11 OUR AVERAGE					
2.51±0.15	404	HEINTZE	76	SPEC	+
2.37±0.17	534	HEARD	75B	SPEC	+
2.42±0.42	112	CLARK	72	OSPK	+

$\Gamma(\mu^+\nu_\mu)/\Gamma_{\text{total}}$ Γ_2/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
63.44±0.14 OUR FIT Error includes scale factor of 1.2.						
63.60±0.16 OUR AVERAGE						
63.66±0.09±0.15	865k	[9] AMBROSINO	06A	KLOE	+	
63.24±0.44	62k	CHIANG	72	OSPK	+	1.84 GeV/c K^+

[9] Fully inclusive. Used tagged kaons from ϕ decays.

$\Gamma(\pi^0 e^+\nu_e)/\Gamma_{\text{total}}$ Γ_3/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
4.98±0.07 OUR FIT Error includes scale factor of 1.3.						
4.86±0.10	3516	CHIANG	72	OSPK	+	1.84 GeV/c K^+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
4.7 ±0.3	429	SHAKLEE	64	HLBC	+	
5.0 ±0.5		ROE	61	HLBC	+	

$\Gamma(\pi^0 e^+\nu_e)/\Gamma(\mu^+\nu_\mu)$ Γ_3/Γ_2

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.0784±0.0012 OUR FIT Error includes scale factor of 1.3.					
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.069 ±0.006	350	ZELLER	69	ASPK	+
0.0775±0.0033	960	BOTTERILL	68C	ASPK	+
0.069 ±0.006	561	GARLAND	68	OSPK	+
0.0791±0.0054	295	[10] AUERBACH	67	OSPK	+

[10] AUERBACH 67 changed from 0.0797 ± 0.0054. See comment with ratio $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\mu^+\nu_\mu)$. The value 0.0785 ± 0.0025 given in AUERBACH 67 is an average of AUERBACH 67 $\Gamma(\pi^0 e^+\nu_e)/\Gamma(\mu^+\nu_\mu)$ and CESTER 66 $\Gamma(\pi^0 e^+\nu_e)/\left[\Gamma(\mu^+\nu_\mu) + \Gamma(\pi^+\pi^0)\right]$.

$\Gamma(\pi^0 e^+\nu_e)/\left[\Gamma(\mu^+\nu_\mu)+\Gamma(\pi^+\pi^0)\right]$ $\Gamma_3/(\Gamma_2+\Gamma_9)$

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
5.90±0.09 OUR FIT Error includes scale factor of 1.3.					
6.02±0.15 OUR AVERAGE					
6.16±0.22	5110	ESCHSTRUTH	68	OSPK	+
5.89±0.21	1679	CESTER	66	OSPK	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
5.92±0.65		[11] WEISSENBE...	76	SPEC	+

[11] Value calculated from WEISSENBERG 76 $(\pi^0 e\nu)$, $(\mu\nu)$, and $(\pi\pi^0)$ values to eliminate dependence on our 1974 $(\pi 2\pi^0)$ and $(\pi\pi^+\pi^-)$ fractions.

$\Gamma(\pi^0 e^+\nu_e)/\left[\Gamma(\pi^0\mu^+\nu_\mu)+\Gamma(\pi^+\pi^0)+\Gamma(\pi^+\pi^0\pi^0)\right]$ $\Gamma_3/(\Gamma_4+\Gamma_9+\Gamma_{10})$

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.1914±0.0029 OUR FIT Error includes scale factor of 1.3.					
0.1962±0.0008±0.0035	71k	SHER	03	B865	+

$\Gamma(\pi^0 e^+ \nu_e)/\Gamma(\pi^+ \pi^0)$ Γ_3/Γ_9

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
0.238±0.004 OUR FIT Error includes scale factor of 1.3.					
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.221±0.012	786	[12] LUCAS 73B	HBC	−	Dalitz pairs only

[12] LUCAS 73B gives N(K_{e3}) = 786 ± 3.1%, N(2π) = 3564 ± 3.1%. We divide.

$\Gamma(\pi^0 e^+ \nu_e)/\Gamma(\pi^+ \pi^+ \pi^-)$ Γ_3/Γ_{11}

VALUE	EVTS	DOCUMENT ID	TECN	CHG
0.890±0.014 OUR FIT Error includes scale factor of 1.3.				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.867±0.027	2768	BARMIN 87	XEBC	+
0.856±0.040	2827	BRAUN 75	HLBC	+
0.850±0.019	4385	[13] HAIDT 71	HLBC	+
0.846±0.021	4385	[13] EICHTEN 68	HLBC	+
0.94 ±0.09	854	BELLOTTI 67B	HLBC	
0.90 ±0.06	230	BORREANI 64	HBC	+

[13] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy in $\Gamma(\pi^0 \mu^+ \nu)/\Gamma(\pi^0 e^+ \nu)$ with more precise results.

$\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma_{\text{total}}$ Γ_4/Γ

VALUE (units 10^{-2})	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
3.32±0.06 OUR FIT Error includes scale factor of 1.2.					
3.33±0.16	2345	CHIANG 72	OSPK	+	1.84 GeV/c K^+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
2.8 ±0.4		[14] TAYLOR 59	EMUL	+	

[14] Earlier experiments not averaged.

$\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma(\mu^+ \nu_\mu)$ Γ_4/Γ_2

VALUE	EVTS	DOCUMENT ID	TECN	CHG
0.0524±0.0010 OUR FIT Error includes scale factor of 1.2.				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.054 ±0.009	240	ZELLER 69	ASPK	+
0.0480±0.0037	424	[15] GARLAND 68	OSPK	+
0.0486±0.0040	307	[16] AUERBACH 67	OSPK	+

[15] GARLAND 68 changed from 0.055 ± 0.004 in agreement with μ-spectrum calculation of GAILLARD 70 appendix B. L.G.Pondrom, (private communication 73).

[16] AUERBACH 67 changed from 0.0602 ± 0.0046 by erratum which brings the μ-spectrum calculation into agreement with GAILLARD 70 appendix B.

$\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma(\pi^0 e^+ \nu_e)$ Γ_4/Γ_3

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
0.668±0.008 OUR FIT					
0.670±0.008 OUR AVERAGE					
0.671±0.007±0.008	24k	HORIE 01	SPEC		
0.670±0.014		[17] HEINTZE 77	SPEC	+	
0.667±0.017	5601	BOTTERILL 68B	ASPK	+	

• • • We do not use the following data for averages, fits, limits, etc. • • •

0.608 ± 0.014	1585	[18] BRAUN 75	HLBC	+	
0.705 ± 0.063	554	[19] LUCAS 73B	HBC	−	Dalitz pairs only
0.698 ± 0.025	3480	[20] CHIANG 72	OSPK	+	1.84 GeV/c K^+
0.596 ± 0.025		[21] HAIDT 71	HLBC	+	
0.604 ± 0.022	1398	[21] EICHTEN 68	HLBC		
0.703 ± 0.056	1509	CALLAHAN 66B	HLBC		

[17] HEINTZE 77 value from fit to λ_0. Assumes μ-e universality.

[18] BRAUN 75 value is from form factor fit. Assumes μ-e universality.

[19] LUCAS 73B gives N($K_{\mu 3}$) = 554 ± 7.6%, N(K_{e3}) = 786 ± 3.1%. We divide.

[20] CHIANG 72 $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\pi^0 e^+\nu_e)$ is statistically independent of CHIANG 72 $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma_{\rm total}$ and $\Gamma(\pi^0 e^+\nu_e)/\Gamma_{\rm total}$.

[21] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy with more precise results.

$\left[\Gamma(\pi^0\mu^+\nu_\mu)+\Gamma(\pi^+\pi^0)\right]/\Gamma_{\rm total}$ $\quad(\Gamma_4+\Gamma_9)/\Gamma$

We combine these two modes for experiments measuring them in xenon bubble chamber because of difficulties of separating them there.

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
24.24±0.11 OUR FIT	Error includes scale factor of 1.1.			
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$25.4\ \pm0.9$	886	SHAKLEE 64	HLBC	+
$23.4\ \pm1.1$		ROE 61	HLBC	+

$\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\pi^+\pi^+\pi^-)$ $\quad\Gamma_4/\Gamma_{11}$

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.594±0.011 OUR FIT	Error includes scale factor of 1.2.				
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.503 ± 0.019	1505	[22] HAIDT 71	HLBC	+	
0.510 ± 0.017	1505	[22] EICHTEN 68	HLBC	+	
$0.63\ \pm0.07$	2845	[23] BISI 65B	BC	+	HBC+HLBC

[22] HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy in $\Gamma(\pi^0\mu^+\nu)/\Gamma(\pi^0 e^+\nu)$ with more precise results.

[23] Error enlarged for background problems. See GAILLARD 70.

$\Gamma(\pi^0\pi^0 e^+\nu_e)/\Gamma_{\rm total}$ $\quad\Gamma_5/\Gamma$

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
2.2 ±0.4 OUR FIT				
2.54±0.89	10	BARMIN 88B	HLBC	+

$\Gamma(\pi^0\pi^0 e^+\nu_e)/\Gamma(\pi^0 e^+\nu_e)$ $\quad\Gamma_5/\Gamma_3$

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
$4.3^{+0.9}_{-0.7}$ OUR FIT				
$4.1^{+1.0}_{-0.7}$ OUR AVERAGE				
$4.2^{+1.0}_{-0.9}$	25	BOLOTOV 86B	CALO	−
$3.8^{+5.0}_{-1.2}$	2	LJUNG 73	HLBC	+

$\Gamma(\pi^+\pi^-e^+\nu_e)/\Gamma(\pi^+\pi^+\pi^-)$ Γ_6/Γ_{11}

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
7.31±0.16 OUR AVERAGE					
7.35±0.01±0.19	388k	[24] PISLAK	01	B865	
7.21±0.32	30k	ROSSELET	77	SPEC	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
7.36±0.68	500	BOURQUIN	71	ASPK	
7.0 ±0.9	106	SCHWEINB...	71	HLBC	+
5.83±0.63	269	ELY	69	HLBC	+

[24] PISLAK 01 reports $\Gamma(\pi^+\pi^-e^+\nu_e)/\Gamma_{\text{total}} = (4.109 \pm 0.008 \pm 0.110) \times 10^{-5}$ using the PDG 00 value $\Gamma(\pi^+\pi^+\pi^-)/\Gamma_{\text{total}} = (5.59 \pm 0.05) \times 10^{-2}$. We divide by the PDG value and unfold its error from the systematic error. PISLAK 03 gives additional details on the branching ratio measurement and gives improved errors on the *S*-wave π-π scattering length: $a_0^0 = 0.216 \pm 0.013(\text{stat.}) \pm 0.002(\text{syst.}) \pm 0.002(\text{theor.})$.

$\Gamma(\pi^+\pi^-\mu^+\nu_\mu)/\Gamma_{\text{total}}$ Γ_7/Γ

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$0.77^{+0.54}_{-0.50}$	1	CLINE	65	FBC	+

$\Gamma(\pi^+\pi^-\mu^+\nu_\mu)/\Gamma(\pi^+\pi^+\pi^-)$ Γ_7/Γ_{11}

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
2.57±1.55	7	BISI	67	DBC	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
∼ 2.5	1	GREINER	64	EMUL	+

$\Gamma(\pi^0\pi^0\pi^0e^+\nu_e)/\Gamma_{\text{total}}$ Γ_8/Γ

VALUE (units 10^{-6})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<3.5	90	0	BOLOTOV	88	SPEC	−
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<9	90	0	BARMIN	92	XEBC	+

Hadronic modes

$\Gamma(\pi^+\pi^0)/\Gamma_{\text{total}}$ Γ_9/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
20.92±0.12 OUR FIT	Error includes scale factor of 1.1.					
21.18±0.28	16k	CHIANG	72	OSPK	+	1.84 GeV/c K^+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
21.0 ±0.6		CALLAHAN	65	HLBC		See $\Gamma(\pi^+\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$

$\Gamma(\pi^+\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$ Γ_9/Γ_{11}

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
3.742±0.032 OUR FIT	Error includes scale factor of 1.1.				
• • • We do not use the following data for averages, fits, limits, etc. • • •					
3.96 ±0.15	1045	CALLAHAN	66	FBC	+

$\Gamma(\pi^+\pi^0)/\Gamma(\mu^+\nu_\mu)$ Γ_9/Γ_2

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.3297±0.0024 OUR FIT Error includes scale factor of 1.1.						
0.3325±0.0032 OUR AVERAGE						
0.3329±0.0047±0.0010	45k	USHER	92	SPEC	+	$p\overline{p}$ at rest
0.3355±0.0057		[25] WEISSENBE...	76	SPEC	+	
0.3277±0.0065	4517	[26] AUERBACH	67	OSPK	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.328 ±0.005	25k	[25] WEISSENBE...	74	STRC	+	
0.305 ±0.018	1600	ZELLER	69	ASPK	+	

[25] WEISSENBERG 76 revises WEISSENBERG 74.

[26] AUERBACH 67 changed from 0.3253 ± 0.0065. See comment with ratio $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\mu^+\nu_\mu)$.

$\Gamma(\pi^+\pi^0\pi^0)/\Gamma_{\text{total}}$ Γ_{10}/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
1.757±0.024 OUR FIT Error includes scale factor of 1.1.						
1.775±0.028 OUR AVERAGE Error includes scale factor of 1.2.						
1.763±0.013±0.022		ALOISIO	04A	KLOE	±	
1.84 ±0.06	1307	CHIANG	72	OSPK	+	1.84 GeV/*c* K^+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
1.53 ±0.11	198	[27] PANDOULAS	70	EMUL	+	
1.8 ±0.2	108	SHAKLEE	64	HLBC	+	
1.7 ±0.2		ROE	61	HLBC	+	
1.5 ±0.2		[28] TAYLOR	59	EMUL	+	

[27] Includes events of TAYLOR 59.

[28] Earlier experiments not averaged.

$\Gamma(\pi^+\pi^0\pi^0)/\Gamma(\pi^+\pi^0)$ Γ_{10}/Γ_9

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.0840±0.0013 OUR FIT Error includes scale factor of 1.1.						
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.081 ±0.005	574	[29] LUCAS	73B	HBC	−	Dalitz pairs only

[29] LUCAS 73B gives $N(\pi 2\pi^0) = 574 \pm 5.9\%$, $N(2\pi) = 3564 \pm 3.1\%$. We quote $0.5N(\pi 2\pi^0)/N(2\pi)$ where 0.5 is because only Dalitz pair π^0's were used.

$\Gamma(\pi^+\pi^0\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$ Γ_{10}/Γ_{11}

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.314±0.005 OUR FIT Error includes scale factor of 1.1.						
0.303±0.009	2027	BISI	65	BC	+	HBC+HLBC
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.393±0.099	17	YOUNG	65	EMUL	+	

$\Gamma(\pi^+\pi^+\pi^-)/\Gamma_{total}$ Γ_{11}/Γ

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
5.590±0.031 OUR FIT	Error includes scale factor of 1.1.					
• • • We do not use the following data for averages, fits, limits, etc. • • •						
5.56 ±0.20	2330	[30] CHIANG	72	OSPK	+	1.84 GeV/c K^+
5.34 ±0.21	693	[31] PANDOULAS	70	EMUL	+	
5.71 ±0.15		DEMARCO	65	HBC		
6.0 ±0.4	44	YOUNG	65	EMUL	+	
5.54 ±0.12	2332	CALLAHAN	64	HLBC	+	
5.1 ±0.2	540	SHAKLEE	64	HLBC	+	
5.7 ±0.3		ROE	61	HLBC	+	

[30] Value is not independent of CHIANG 72 $\Gamma(\mu^+\nu_\mu)/\Gamma_{total}$, $\Gamma(\pi^+\pi^0)/\Gamma_{total}$, $\Gamma(\pi^+\pi^0\pi^0)/\Gamma_{total}$, $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma_{total}$, and $\Gamma(\pi^0 e^+\nu_e)/\Gamma_{total}$.

[31] Includes events of TAYLOR 59.

Leptonic and semileptonic modes with photons

$\Gamma(\mu^+\nu_\mu\gamma)/\Gamma_{total}$ Γ_{12}/Γ

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
6.2±0.8 OUR AVERAGE						
6.6±1.5		[32,33] DEMIDOV	90	XEBC		P(μ) <231.5 MeV/c
6.0±0.9		BARMIN	88	HLBC	+	P(μ) <231.5 MeV/c
• • • We do not use the following data for averages, fits, limits, etc. • • •						
3.5±0.8		[33,34] DEMIDOV	90	XEBC		E(γ) > 20 MeV
3.2±0.5	57	[35] BARMIN	88	HLBC	+	$E(\gamma)$ >20 MeV
5.4±0.3		[36] AKIBA	85	SPEC		P(μ) <231.5 MeV/c

[32] P(μ) cut given in DEMIDOV 90 paper, 235.1 MeV/c, is a misprint according to authors (private communication).

[33] DEMIDOV 90 quotes only inner bremsstrahlung (IB) part.

[34] Not independent of above DEMIDOV 90 value. Cuts differ.

[35] Not independent of above BARMIN 88 value. Cuts differ.

[36] Assumes μ-e universality and uses constraints from $K \rightarrow e\nu\gamma$.

$\Gamma(\mu^+\nu_\mu\gamma(\mathrm{SD}^+))/\Gamma_{total}$ Γ_{13}/Γ

Structure-dependent part with $+\gamma$ helicity (SD$^+$ term). See the "Note on $\pi^\pm \rightarrow \ell^\pm\nu\gamma$ and $K^\pm \rightarrow \ell^\pm\nu\gamma$ Form Factors" in the $\pi^\pm$ section of the Particle Data Listings above.

VALUE (units 10^{-5})	*CL%*	*DOCUMENT ID*		*TECN*
<3.0	90	AKIBA	85	SPEC

$\Gamma(\mu^+\nu_\mu\gamma(\mathrm{SD^+INT}))/\Gamma_{total}$ Γ_{14}/Γ

Interference term between internal Bremsstrahlung and SD$^+$ term. See the "Note on $\pi^\pm \rightarrow \ell^\pm\nu\gamma$ and $K^\pm \rightarrow \ell^\pm\nu\gamma$ Form Factors" in the $\pi^\pm$ section of the Particle Data Listings above.

VALUE (units 10^{-5})	*CL%*	*DOCUMENT ID*		*TECN*
<2.7	90	AKIBA	85	SPEC

$\Gamma(\mu^+\nu_\mu\gamma(\mathrm{SD}^- + \mathrm{SD}^-\mathrm{INT}))/\Gamma_{\mathrm{total}}$ Γ_{15}/Γ

Sum of structure-dependent part with $-\gamma$ helicity (SD$^-$ term) and interference term between internal Bremsstrahlung and SD$^-$ term. See the "Note on $\pi^\pm \to \ell^\pm\nu\gamma$ and $K^\pm \to \ell^\pm\nu\gamma$ Form Factors" in the $\pi^\pm$ section of the Particle Data Listings above.

VALUE (units 10^{-4})	*CL%*	*DOCUMENT ID*		*TECN*
<2.6	90	[37] AKIBA	85	SPEC

[37] Assumes μ-e universality and uses constraints from $K \to e\nu\gamma$.

$\Gamma(e^+\nu_e\gamma(\mathrm{SD}^+))/\Gamma_{\mathrm{total}}$ Γ_{16}/Γ

Structure-dependent part with $+\gamma$ helicity (SD$^+$ term). See the "Note on $\pi^\pm \to \ell^\pm\nu\gamma$ and $K^\pm \to \ell^\pm\nu\gamma$ Form Factors" in the $\pi^\pm$ section of the Particle Data Listings above.

VALUE (units 10^{-5})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<7.1	90	MACEK	70	OSPK	+	P(e) 234–247

$\Gamma(e^+\nu_e\gamma(\mathrm{SD}^+))/\Gamma(e^+\nu_e)$ Γ_{16}/Γ_1

Structure-dependent part with $+\gamma$ helicity (SD$^+$ term). See the "Note on $\pi^\pm \to \ell^\pm\nu\gamma$ and $K^\pm \to \ell^\pm\nu\gamma$ Form Factors" in the $\pi^\pm$ section of the Particle Data Listings above.

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$1.05^{+0.25}_{-0.30}$	56	[38] HEARD	75	SPEC	+	P(e) 236–247

[38] This value is included in the first HEINTZE 79 value in the section on $\Gamma(e^+\nu_e\gamma(\mathrm{SD}^+))/\Gamma(\mu^+\nu_\mu)$ above.

$\Gamma(e^+\nu_e\gamma(\mathrm{SD}^+))/\Gamma(\mu^+\nu_\mu)$ Γ_{16}/Γ_2

Structure-dependent part with $+\gamma$ helicity (SD$^+$ term). See the "Note on $\pi^\pm \to \ell^\pm\nu\gamma$ and $K^\pm \to \ell^\pm\nu\gamma$ Form Factors" in the $\pi^\pm$ section of the Particle Data Listings above.

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
2.40±0.36	107	[39] HEINTZE	79	SPEC	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
2.33±0.42	51	[39] HEINTZE	79	SPEC	+

[39] First HEINTZE 79 result is second combined with HEARD 75 result from section $\Gamma(e^+\nu_e\gamma(\mathrm{SD}^+))/\Gamma(e^+\nu_e)$ below.

$\Gamma(e^+\nu_e\gamma(\mathrm{SD}^-))/\Gamma_{\mathrm{total}}$ Γ_{17}/Γ

Structure-dependent part with $-\gamma$ helicity (SD$^-$ term). See the "Note on $\pi^\pm \to \ell^\pm\nu\gamma$ and $K^\pm \to \ell^\pm\nu\gamma$ Form Factors" in the $\pi^\pm$ section of the Particle Data Listings above.

VALUE (units 10^{-4})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*
<1.6	90	[40] HEINTZE	79	SPEC	+

[40] Implies (axial vector/vector) amplitude ratio outside range from -1.8 to -0.54.

$\Gamma(\pi^0 e^+ \nu_e \gamma)/\Gamma(\pi^0 e^+ \nu_e)$ Γ_{18}/Γ_3

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.54±0.04 OUR AVERAGE Error includes scale factor of 1.1.						
0.46±0.08	82	[41] BARMIN	91	XEBC		$E(\gamma) > 10$ MeV, $0.6 < \cos\theta_{e\gamma} < 0.9$
0.56±0.04	192	[42] BOLOTOV	86B	CALO	−	$E(\gamma) > 10$ MeV
• • • We do not use the following data for averages, fits, limits, etc. • • •						
1.51±0.25	82	[41] BARMIN	91	XEBC		$E(\gamma) > 10$ MeV, $\cos\theta_{e\gamma} < 0.98$
0.48±0.20	16	[43] LJUNG	73	HLBC	+	$E(\gamma) > 30$ MeV
$0.22^{+0.15}_{-0.10}$		[43] LJUNG	73	HLBC	+	$E(\gamma) > 30$ MeV
0.76±0.28	13	[44] ROMANO	71	HLBC		$E(\gamma) > 10$ MeV
0.53±0.22		[44] ROMANO	71	HLBC	+	$E(\gamma) > 30$ MeV

[41] BARMIN 91 quotes branching ratio $\Gamma(K \to e\pi^0\nu\gamma)/\Gamma_{\text{all}}$. The measured normalization is $[\Gamma(K \to e\pi^0\nu) + \Gamma(K \to \pi^+\pi^+\pi^-)]$. For comparison with other experiments we used $\Gamma(K \to e\pi^0\nu)/\Gamma_{\text{all}} = 0.0482$ to calculate the values quoted here.

[42] $\cos\theta(e\gamma)$ between 0.6 and 0.9.

[43] First LJUNG 73 value is for $\cos\theta(e\gamma) < 0.9$, second value is for $\cos\theta(e\gamma)$ between 0.6 and 0.9 for comparison with ROMANO 71.

[44] Both ROMANO 71 values are for $\cos\theta(e\gamma)$ between 0.6 and 0.9. Second value is for comparison with second LJUNG 73 value. We use lowest $E(\gamma)$ cut for Summary Table value. See ROMANO 71 for E_γ dependence.

$\Gamma(\pi^0 e^+ \nu_e \gamma(\text{SD}))/\Gamma_{\text{total}}$ Γ_{19}/Γ

Structure-dependent part.

VALUE (units 10^{-5})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*
<5.3	90	BOLOTOV	86B	CALO	−

$\Gamma(\pi^0 \mu^+ \nu_\mu \gamma)/\Gamma_{\text{total}}$ Γ_{20}/Γ

VALUE (units 10^{-5})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
2.4±0.5±0.6		125	SHIMIZU	06	K470	+	$E_\gamma > 30$ MeV; $\Theta_{\mu\gamma} > 20°$
• • • We do not use the following data for averages, fits, limits, etc. • • •							
<6.1	90	0	LJUNG	73	HLBC	+	$E(\gamma) > 30$ MeV

$\Gamma(\pi^0 \pi^0 e^+ \nu_e \gamma)/\Gamma_{\text{total}}$ Γ_{21}/Γ

VALUE (units 10^{-6})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<5	90	0	BARMIN	92	XEBC	+	$E_\gamma > 10$ MeV

Hadronic modes with photons

$\Gamma(\pi^+ \pi^0 \gamma)/\Gamma_{\text{total}}$ Γ_{22}/Γ

VALUE (units 10^{-4})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
2.75±0.15 OUR AVERAGE							
2.71±0.45		140	BOLOTOV	87	WIRE	−	$T\pi^-$ 55–90 MeV
2.87±0.32		2461	SMITH	76	WIRE	±	$T\pi^\pm$ 55–90 MeV
2.71±0.19		2100	ABRAMS	72	ASPK	±	$T\pi^+$ 55–90 MeV

• • • We do not use the following data for averages, fits, limits, etc. • • •

$1.5\ ^{+1.1}_{-0.6}$			[45] LJUNG	73	HLBC	+	Tπ^+ 55–80 MeV
$2.6\ ^{+1.5}_{-1.1}$			[45] LJUNG	73	HLBC	+	Tπ^+ 55–90 MeV
$6.8\ ^{+3.7}_{-2.1}$		17	[45] LJUNG	73	HLBC	+	Tπ^+ 55–102 MeV
2.4 ± 0.8		24	EDWARDS	72	OSPK		Tπ^+ 58–90 MeV
<1.0		0	[46] MALTSEV	70	HLBC	+	Tπ^+ <55 MeV
<1.9	90	0	EMMERSON	69	OSPK		Tπ^+ 55–80 MeV
2.2 ± 0.7		18	CLINE	64	FBC	+	Tπ^+ 55–80 MeV

[45] The LJUNG 73 values are not independent.

[46] MALTSEV 70 selects low π^+ energy to enhance direct emission contribution.

$\Gamma(\pi^+\pi^0\gamma(\text{DE}))/\Gamma_{\text{total}}$ Γ_{23}/Γ

Direct emission (DE) part of $\Gamma(\pi^+\pi^0\gamma)/\Gamma_{\text{total}}$, assuming that interference (INT) component is zero.

VALUE (units 10^{-6})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
4.4±0.7 OUR AVERAGE						
$3.7 \pm 3.9 \pm 1.0$	930	UVAROV	06	ISTR	−	Tπ^- 55–90 MeV
$3.2 \pm 1.3 \pm 1.0$	4k	[47] ALIEV	03	K470	+	Tπ^+ 55–90 MeV
$4.7 \pm 0.8 \pm 0.3$	20k	[48] ADLER	00C	B787	+	Tπ^+ 55–90 MeV

• • • We do not use the following data for averages, fits, limits, etc. • • •

$6.1 \pm 2.5 \pm 1.9$	4k	[47] ALIEV	03	K470	+	Tπ^+ full range
$20.5 \pm 4.6\ ^{+3.9}_{-2.3}$		BOLOTOV	87	WIRE	−	Tπ^- 55–90 MeV
$15.6 \pm 3.5 \pm 5.0$		ABRAMS	72	ASPK	±	T$\pi^\pm$ 55–90 MeV

[47] ALIEV 03 "Tπ^+ full range" result is extrapolated from their Tπ >35 MeV measurement. They calculate the "Tπ^+ 55–90 MeV" result for comparison with other experiments. They measure the INT component to be $(-0.58^{+0.91}_{-0.83})\%$ of the inner bremsstrahlung (IB) component. The DE component is measured assuming INT=0.

[48] ADLER 00C measures the INT component to be $(-0.4 \pm 1.6)\%$ of the inner bremsstrahlung (IB) component. The DE component is measured assuming INT=0.

$\Gamma(\pi^+\pi^0\pi^0\gamma)/\Gamma(\pi^+\pi^0\pi^0)$ Γ_{24}/Γ_{10}

VALUE (units 10^{-4})	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
$\mathbf{4.3^{+3.2}_{-1.7}}$	BOLOTOV	85	SPEC	−	$E(\gamma) > 10$ MeV

$\Gamma(\pi^+\pi^+\pi^-\gamma)/\Gamma_{\text{total}}$ Γ_{25}/Γ

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
1.04±0.31 OUR AVERAGE						
1.10 ± 0.48	7	BARMIN	89	XEBC		$E(\gamma) > 5$ MeV
1.0 ± 0.4		STAMER	65	EMUL	+	$E(\gamma) > 11$ MeV

$\Gamma(\pi^+\gamma\gamma)/\Gamma_{\rm total}$ Γ_{26}/Γ

VALUE (units 10^{-7})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
11± 3 ±1		31	[49] KITCHING	97	B787		

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (units 10^{-7})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
< 0.083	90		[50] ARTAMONOV	05	B949	+	P_{π^+} >213 MeV/c
< 10	90	0	ATIYA	90B	B787		$T\pi$ 117–127 MeV
< 84	90	0	ASANO	82	CNTR	+	$T\pi$ 117–127 MeV
−420±520		0	ABRAMS	77	SPEC	+	$T\pi$ <92 MeV
< 350	90	0	LJUNG	73	HLBC	+	6–102, 114–127 MeV
< 500	90	0	KLEMS	71	OSPK	+	$T\pi$ <117 MeV
−100±600			CHEN	68	OSPK	+	$T\pi$ 60–90 MeV

[49] KITCHING 97 is extrapolated from their model-independent branching fraction $(6.0 \pm 1.5 \pm 0.7) \times 10^{-7}$ for 100 MeV/c<P_{π^+} < 180 MeV/c using Chiral Perturbation Theory.

[50] ARTAMONOV 05 limit assumes ChPT with $\hat{c}$= 1.8 with unitarity corrections. With $\hat{c}$= 1.6 and no unitarity corrections they obtain < 2.3×10^{-8} at 90% CL. This partial branching ratio is predicted to be 6.10×10^{-9} and 0.49×10^{-9} for the cases with and without unitarity correction.

$\Gamma(\pi^+ 3\gamma)/\Gamma_{\rm total}$ Γ_{27}/Γ

Values given here assume a phase space pion energy spectrum.

VALUE (units 10^{-4})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<1.0	90	ASANO	82	CNTR	+	$T(\pi)$ 117–127 MeV

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (units 10^{-4})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<3.0	90	KLEMS	71	OSPK	+	$T(\pi)$ >117 MeV

Leptonic modes with $\ell\overline{\ell}$ pairs

$\Gamma(e^+\nu_e\nu\overline{\nu})/\Gamma(e^+\nu_e)$ Γ_{28}/Γ_1

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<3.8	90	0	HEINTZE	79	SPEC	+

$\Gamma(\mu^+\nu_\mu\nu\overline{\nu})/\Gamma_{\rm total}$ Γ_{29}/Γ

VALUE (units 10^{-6})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
<6.0	90	0	[51] PANG	73	CNTR	+

[51] PANG 73 assumes μ spectrum from ν-ν interaction of BARDIN 70.

$\Gamma(e^+\nu_e e^+ e^-)/\Gamma_{\rm total}$ Γ_{30}/Γ

VALUE (units 10^{-8})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
2.48± 0.14±0.14	410	POBLAGUEV	02	B865	+	m_{ee} >150 MeV

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE (units 10^{-8})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
20 ±20	4	DIAMANT-...	76	SPEC	+	$m_{e^+e^-}$ >140 MeV

$\Gamma(\mu^+\nu_\mu e^+e^-)/\Gamma_{\text{total}}$ Γ_{31}/Γ

VALUE (units 10^{-8})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
7.06± 0.16±0.26	2.7k	POBLAGUEV 02	B865	+	m_{ee} >145 MeV
• • • We do not use the following data for averages, fits, limits, etc. • • •					
100 ±30	14	DIAMANT-... 76	SPEC	+	$m_{e^+e^-}$ >140 MeV

$\Gamma(e^+\nu_e\mu^+\mu^-)/\Gamma_{\text{total}}$ Γ_{32}/Γ

VALUE (units 10^{-8})	*CL%*	*DOCUMENT ID*	*TECN*
1.72±0.45		MA 06	B865
• • • We do not use the following data for averages, fits, limits, etc. • • •			
<50	90	ADLER 98	B787

$\Gamma(\mu^+\nu_\mu\mu^+\mu^-)/\Gamma_{\text{total}}$ Γ_{33}/Γ

VALUE (units 10^{-7})	*CL%*	*DOCUMENT ID*	*TECN*	*CHG*
<4.1	90	ATIYA 89	B787	+

Lepton Family number (*LF*), Lepton number (*L*), $\Delta S = \Delta Q$ (*SQ*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes

$\Gamma(\pi^+\pi^+e^-\overline{\nu}_e)/\Gamma_{\text{total}}$ Γ_{34}/Γ

Test of $\Delta S = \Delta Q$ rule.

VALUE (units 10^{-7})	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
• • • We do not use the following data for averages, fits, limits, etc. • • •					
< 9.0	95	0	SCHWEINB... 71	HLBC	+
< 6.9	95	0	ELY 69	HLBC	+
<20.	95		BIRGE 65	FBC	+

$\Gamma(\pi^+\pi^+e^-\overline{\nu}_e)/\Gamma(\pi^+\pi^-e^+\nu_e)$ Γ_{34}/Γ_6

Test of $\Delta S = \Delta Q$ rule.

VALUE (units 10^{-4})	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*
< 3	90	3	[52] BLOCH 76	SPEC
• • • We do not use the following data for averages, fits, limits, etc. • • •				
<130.	95	0	BOURQUIN 71	ASPK

[52] BLOCH 76 quotes 3.6×10^{-4} at CL = 95%, we convert.

$\Gamma(\pi^+\pi^+\mu^-\overline{\nu}_\mu)/\Gamma_{\text{total}}$ Γ_{35}/Γ

Test of $\Delta S = \Delta Q$ rule.

VALUE (units 10^{-6})	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
<3.0	95	0	BIRGE 65	FBC	+

$\Gamma(\pi^+e^+e^-)/\Gamma_{\text{total}}$ Γ_{36}/Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by combined first-order weak and electromagnetic interactions.

VALUE (units 10^{-7})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
2.88±0.13 OUR AVERAGE				
2.94±0.05±0.14	10300	[53] APPEL 99	SPEC	+
2.75±0.23±0.13	500	[54] ALLIEGRO 92	SPEC	+
2.7 ±0.5	41	[55] BLOCH 75	SPEC	+

[53] APPEL 99 establishes vector nature of this decay and determines form factor $f(Z)$= $f_0(1+\delta Z)$, $Z=M^2_{ee}/m^2_K$, δ=2.14 ± 0.13 ± 0.15.

[54] ALLIEGRO 92 assumes a vector interaction with a form factor given by $\lambda = 0.105 \pm 0.035 \pm 0.015$ and a correlation coefficient of −0.82.

[55] BLOCH 75 assumes a vector interaction.

$\Gamma(\pi^+\mu^+\mu^-)/\Gamma_{total}$ Γ_{37}/Γ

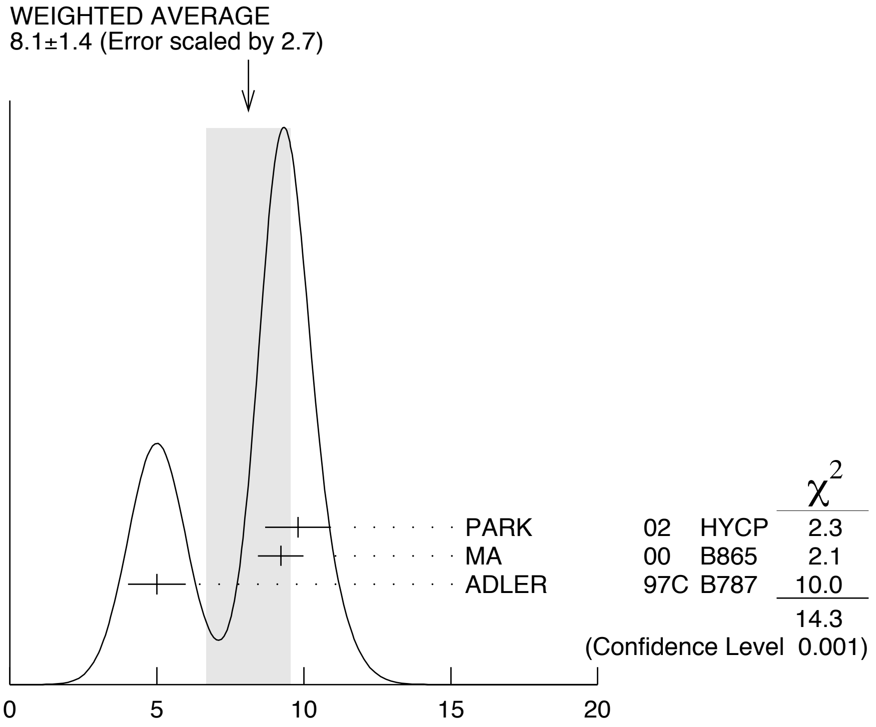
Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions.

VALUE (units 10^{-8})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
8.1 ±1.4 OUR AVERAGE Error includes scale factor of 2.7. See the ideogram below.						
9.8 ±1.0 ±0.5		110	[56] PARK	02	HYCP	±
9.22±0.60±0.49		402	[57] MA	00	B865	+
5.0 ±0.4 ±0.9		207	[58] ADLER	97C	B787	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
9.7 ±1.2 ±0.4		65	PARK	02	HYCP	+
10.0 ±1.9 ±0.7		35	PARK	02	HYCP	−
<23	90		ATIYA	89	B787	+

[56] PARK 02 "±" result comes from combining $K^+ \rightarrow \pi^+\mu^+\mu^-$ and $K^- \rightarrow \pi^-\mu^+\mu^-$, assuming *CP* is conserved.

[57] MA 00 establishes vector nature of this decay and determines form factor $f(Z)$= $f_0(1+\delta Z)$, $Z=M^2_{\mu\mu}/m^2_K$, $\delta=2.45^{+1.30}_{-0.95}$. .

[58] ADLER 97C gives systematic error 0.7×10^{-8} and theoretical uncertainty 0.6×10^{-8}, which we combine in quadrature to obtain our second error.



$\Gamma(\pi^+\mu^+\mu^-)/\Gamma_{total}$ Γ_{37}/Γ

$\Gamma(\pi^+\nu\overline{\nu})/\Gamma_{\text{total}}$ Γ_{38}/Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions. Branching ratio values are extrapolated from the momentum or energy regions shown in the comments assuming Standard Model phase space except for those labeled "Scalar" or "Tensor" to indicate the assumed non-Standard-Model interaction.

VALUE (units 10^{-9})	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
$\mathbf{0.147^{+0.130}_{-0.089}}$		3	[59] ANISIMOVSK...04	B949	+	$211<P_\pi<229$ MeV/c
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$<$ 2.2	90		[60] ADLER 04	B787	+	$140<P_\pi<195$ MeV/c
$0.157^{+0.175}_{-0.082}$		2	ADLER 02	B787		$P_\pi>211$ MeV/c
$<$ 4.2	90	1	ADLER 02C	B787		$140<P_\pi<195$ MeV/c
$<$ 4.7	90		ADLER 02C	B787		Scalar
$<$ 2.5	90		ADLER 02C	B787		Tensor
$0.15^{+0.34}_{-0.12}$		1	ADLER 00	B787		In ADLER 02
$0.42^{+0.97}_{-0.35}$		1	ADLER 97	B787		
$<$ 2.4	90		ADLER 96	B787		
$<$ 7.5	90		ATIYA 93	B787	+	$T(\pi)$ 115–127 MeV
$<$ 5.2	90		[61] ATIYA 93	B787	+	
$<$ 17	90	0	ATIYA 93B	B787	+	$T(\pi)$ 60–100 MeV
$<$ 34	90		ATIYA 90	B787	+	
$<$140	90		ASANO 81B	CNTR	+	$T(\pi)$ 116–127 MeV

[59] Value obtained combining the previous E787 result ADLER 02 with 2 evts and the present E949 with 1 evt. The additional event has a signal-to-background ratio 0.9.

[60] Value obtained combining the previous result ADLER 02C with 1 event and the present result with 0 events to obtain an expected background 1.22 ± 0.24 evts and 1 evt observed.

[61] Combining ATIYA 93 and ATIYA 93B results. Superseded by ADLER 96.

$\Gamma(\pi^+\pi^0\nu\overline{\nu})/\Gamma_{\text{total}}$ Γ_{39}/Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by higher-order electroweak interactions.

VALUE (units 10^{-5})	*CL%*	*DOCUMENT ID*	*TECN*
<4.3	90	[62] ADLER 01	SPEC

[62] Search region defined by 90 MeV/$c<P_{\pi^+}<188$ MeV/c and 135 MeV$<E_{\pi^0}<180$ MeV.

$\Gamma(\mu^-\nu e^+e^+)/\Gamma(\pi^+\pi^-e^+\nu_e)$ Γ_{40}/Γ_6

Test of lepton family number conservation.

VALUE (units 10^{-3})	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
<0.5	90	0	[63] DIAMANT-... 76	SPEC	+

[63] DIAMANT-BERGER 76 quotes this result times our 1975 $\pi^+\pi^-e\nu$ BR ratio.

$\Gamma(\mu^+\nu_e)/\Gamma_{\text{total}}$ Γ_{41}/Γ

Forbidden by lepton family number conservation.

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
<0.004	90	0	[64] LYONS	81	HLBC	0	200 GeV K^+ narrow band ν beam
• • • We do not use the following data for averages, fits, limits, etc. • • •							
<0.012	90		[64] COOPER	82	HLBC		Wideband ν beam

[64] COOPER 82 and LYONS 81 limits on ν_e observation are here interpreted as limits on lepton family number violation in the absence of mixing.

$\Gamma(\pi^+\mu^+e^-)/\Gamma_{\text{total}}$ Γ_{42}/Γ

Test of lepton family number conservation.

VALUE (units 10^{-10})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*
<0.13	90	[65] SHER	05	RVUE	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
<0.21	90	SHER	05	E865	+
<0.39	90	APPEL	00	B865	+
<2.1	90	LEE	90	SPEC	+

[65] This result combines SHER 05 1998 data, APPEL 00 1996 data, and data from BERGMAN 97 and PISLAK 97 theses, all from BNL-E865, with LEE 90 BNL-E777 data.

$\Gamma(\pi^+\mu^-e^+)/\Gamma_{\text{total}}$ Γ_{43}/Γ

Test of lepton family number conservation.

VALUE (units 10^{-10})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
< 5.2	90	0	APPEL	00B	B865	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<70	90	0	[66] DIAMANT-...	76	SPEC	+

[66] Measurement actually applies to the sum of the $\pi^+\mu^-e^+$ and $\pi^-\mu^+e^+$ modes.

$\Gamma(\pi^-\mu^+e^+)/\Gamma_{\text{total}}$ Γ_{44}/Γ

Test of total lepton number conservation.

VALUE (units 10^{-10})	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
< 5.0	90	0	APPEL	00B	B865	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<70	90	0	[67] DIAMANT-...	76	SPEC	+

[67] Measurement actually applies to the sum of the $\pi^+\mu^-e^+$ and $\pi^-\mu^+e^+$ modes.

$\Gamma(\pi^-e^+e^+)/\Gamma_{\text{total}}$ Γ_{45}/Γ

Test of total lepton number conservation.

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
$<6.4 \times 10^{-10}$	90	0	APPEL	00B	B865	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$<9.2 \times 10^{-9}$	90	0	DIAMANT-...	76	SPEC	+
$<1.5 \times 10^{-5}$			CHANG	68	HBC	−

$\Gamma(\pi^- \mu^+ \mu^+)/\Gamma_{\text{total}}$ Γ_{46}/Γ

Forbidden by total lepton number conservation.

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
$\mathbf{<3.0 \times 10^{-9}}$	90	0	APPEL 00B	B865	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$<1.5 \times 10^{-4}$	90		[68] LITTENBERG 92	HBC	

[68] LITTENBERG 92 is from retroactive data analysis of CHANG 68 bubble chamber data.

$\Gamma(\mu^+ \overline{\nu}_e)/\Gamma_{\text{total}}$ Γ_{47}/Γ

Forbidden by total lepton number conservation.

VALUE (units 10^{-3})	*CL%*	*DOCUMENT ID*	*TECN*	*COMMENT*
<3.3	90	[69] COOPER 82	HLBC	Wideband ν beam

[69] COOPER 82 limit on $\overline{\nu}_e$ observation is here interpreted as a limit on lepton number violation in the absence of mixing.

$\Gamma(\pi^0 e^+ \overline{\nu}_e)/\Gamma_{\text{total}}$ Γ_{48}/Γ

Forbidden by total lepton number conservation.

VALUE	*CL%*	*DOCUMENT ID*	*TECN*	*COMMENT*
<0.003	90	[70] COOPER 82	HLBC	Wideband ν beam

[70] COOPER 82 limit on $\overline{\nu}_e$ observation is here interpreted as a limit on lepton number violation in the absence of mixing.

$\Gamma(\pi^+ \gamma)/\Gamma_{\text{total}}$ Γ_{49}/Γ

Violates angular momentum conservation and gauge invariance. Current interest in this decay is as a search for non-commutative space-time effects as discussed in ARTAMONOV 05 and for exotic physics such as a vacuum expectation value of a new vector field, non-local Superstring effects, or departures from Lorentz invariance, as discussed in ADLER 02B.

VALUE (units 10^{-9})	*CL%*	*DOCUMENT ID*	*TECN*	*CHG*
< 2.3	90	ARTAMONOV 05	B949	+
• • • We do not use the following data for averages, fits, limits, etc. • • •				
< 360	90	ADLER 02B	B787	+
<1400	90	ASANO 82	CNTR	+
<4000	90	[71] KLEMS 71	OSPK	+

[71] Test of model of Selleri, Nuovo Cimento **60A** 291 (1969).

K^+ LONGITUDINAL POLARIZATION OF EMITTED μ^+

VALUE	*CL%*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
<−0.990	90	[72] AOKI 94	SPEC	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
<−0.990	90	IMAZATO 92	SPEC	+	Repl. by AOKI 94
−0.970±0.047		[73] YAMANAKA 86	SPEC	+	
−1.0 ±0.1		[73] CUTTS 69	SPRK	+	
−0.96 ±0.12		[73] COOMBES 57	CNTR	+	

[72] AOKI 94 measures $\xi P_\mu = -0.9996 \pm 0.0030 \pm 0.0048$. The above limit is obtained by summing the statistical and systematic errors in quadrature, normalizing to the physically significant region ($|\xi P_\mu| < 1$) and assuming that ξ=1, its maximum value.

[73] Assumes ξ=1.

DALITZ PLOT PARAMETERS FOR $K \rightarrow 3\pi$ DECAYS

Revised 1999 by T.G. Trippe (LBNL).

The Dalitz plot distribution for $K^\pm \rightarrow \pi^\pm\pi^\pm\pi^\mp$, $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$, and $K^0_L \rightarrow \pi^+\pi^-\pi^0$ can be parameterized by a series expansion such as that introduced by Weinberg [1]. We use the form

$$\begin{aligned}\left|M\right|^2 \propto 1 &+ g\frac{(s_3 - s_0)}{m^2_{\pi^+}} + h\left[\frac{s_3 - s_0}{m^2_{\pi^+}}\right]^2 \\ &+ j\ \frac{(s_2 - s_1)}{m^2_{\pi^+}} + k\left[\frac{s_2 - s_1}{m^2_{\pi^+}}\right]^2 \\ &+ f\ \frac{(s_2 - s_1)}{m^2_{\pi^+}}\frac{(s_3 - s_0)}{m^2_{\pi^+}} + \cdots\ , \qquad (1)\end{aligned}$$

where $m^2_{\pi^+}$ has been introduced to make the coefficients g, h, j, and k dimensionless, and

$$\begin{aligned}s_i &= (P_K - P_i)^2 = (m_K - m_i)^2 - 2m_K T_i\ ,\ i = 1,2,3, \\ s_0 &= \frac{1}{3}\sum_i s_i = \frac{1}{3}(m^2_K + m^2_1 + m^2_2 + m^2_3)\quad .\end{aligned}$$

Here the P_i are four-vectors, m_i and T_i are the mass and kinetic energy of the i^{th} pion, and the index 3 is used for the odd pion.

The coefficient g is a measure of the slope in the variable s_3 (or T_3) of the Dalitz plot, while h and k measure the quadratic dependence on s_3 and $(s_2 - s_1)$, respectively. The coefficient j is related to the asymmetry of the plot and must be zero if CP

invariance holds. Note also that if CP is good, g, h, and k must be the same for $K^+ \to \pi^+\pi^+\pi^-$ as for $K^- \to \pi^-\pi^-\pi^+$.

Since different experiments use different forms for $\left|M\right|^2$, in order to compare the experiments we have converted to g, h, j, and k whatever coefficients have been measured. Where such conversions have been done, the measured coefficient a_y, a_t, a_u, or a_v is given in the comment at the right. For definitions of these coefficients, details of this conversion, and discussion of the data, see the April 1982 version of this note [2].

References

1. S. Weinberg, Phys. Rev. Lett. **4**, 87 (1960).
2. Particle Data Group, Phys. Lett. **111B**, 69 (1982).

ENERGY DEPENDENCE OF $K^\pm$ DALITZ PLOT

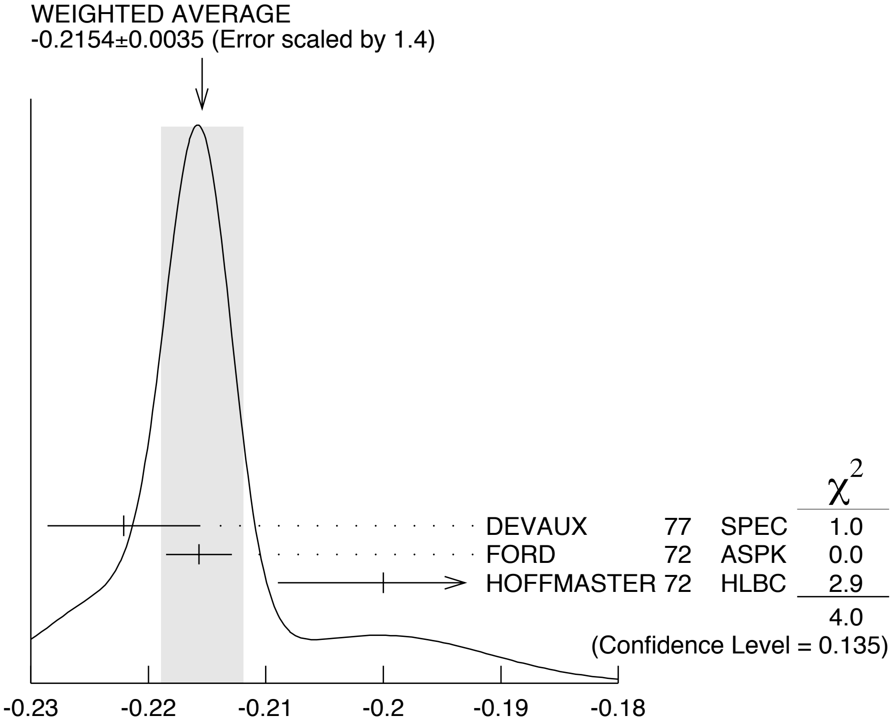
$\left|\text{matrix element}\right|^2 = 1 + gu + hu^2 + kv^2$
where $u = (s_3 - s_0) / m_\pi^2$ and $v = (s_2 - s_1) / m_\pi^2$

LINEAR COEFFICIENT g_+ FOR $K^+ \to \pi^+\pi^+\pi^-$

Some experiments use Dalitz variables x and y. In the comments we give a_y = coefficient of y term. See note above on "Dalitz Plot Parameters for $K \to 3\pi$ Decays." For discussion of the conversion of a_y to g, see the earlier version of the same note in the *Review* published in Physics Letters **111B** 70 (1982).

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
−0.2154±0.0035 OUR AVERAGE	Error includes scale factor of 1.4. See the ideogram below.					
−0.2221±0.0065	225k	DEVAUX	77	SPEC	+	a_y=.2814 ± .0082
−0.2157±0.0028	750k	FORD	72	ASPK	+	a_y=.2734 ± .0035
−0.200 ±0.009	39819	[74] HOFFMASTER	72	HLBC	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •						
−0.196 ±0.012	17898	[75] GRAUMAN	70	HLBC	+	a_y=0.228 ± 0.030
−0.218 ±0.016	9994	[76] BUTLER	68	HBC	+	a_y=0.277 ± 0.020
−0.22 ±0.024	5428	[76,77] ZINCHENKO	67	HBC	+	a_y=0.28 ± 0.03

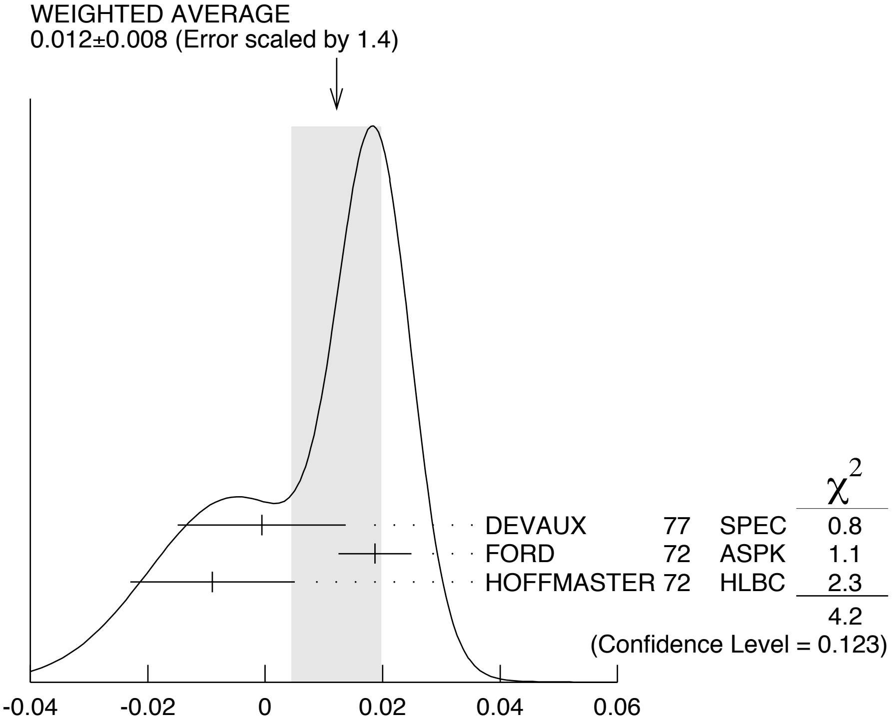
[74] HOFFMASTER 72 includes GRAUMAN 70 data.
[75] Emulsion data added — all events included by HOFFMASTER 72.
[76] Experiments with large errors not included in average.
[77] Also includes DBC events.



Linear energy dependence for $K^+ \rightarrow \pi^+\pi^+\pi^-$

QUADRATIC COEFFICIENT *h* FOR $K^+ \rightarrow \pi^+\pi^+\pi^-$

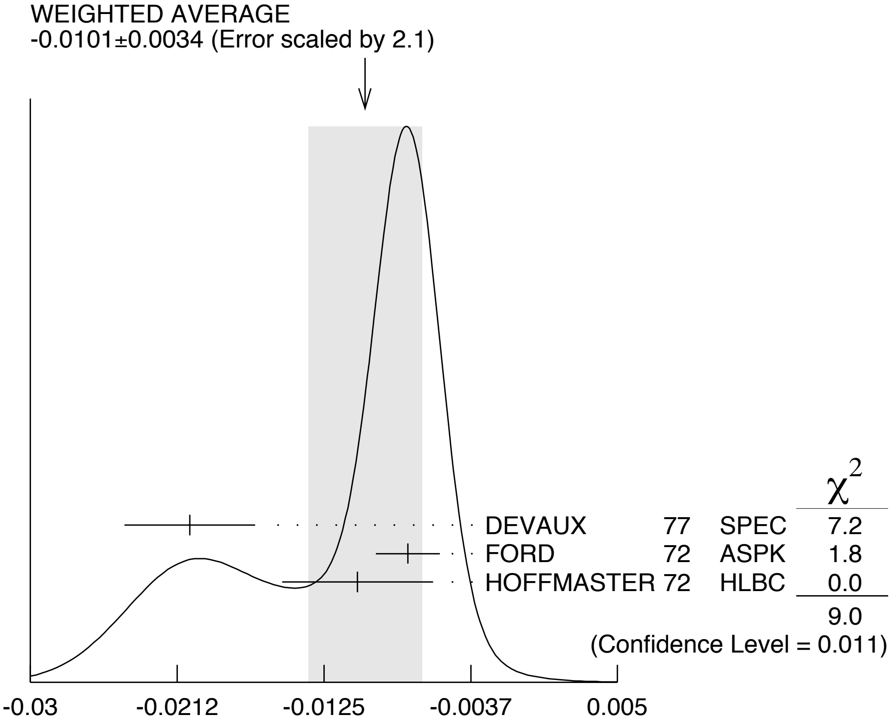
VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.012 ±0.008 OUR AVERAGE	Error includes scale factor of 1.4. See the ideogram below.				
−0.0006±0.0143	225k	DEVAUX	77	SPEC	+
0.0187±0.0062	750k	FORD	72	ASPK	+
−0.009 ±0.014	39819	HOFFMASTER	72	HLBC	+



Quadratic coefficient h for $K^+ \rightarrow \pi^+\pi^+\pi^-$

QUADRATIC COEFFICIENT k FOR $K^+ \rightarrow \pi^+\pi^+\pi^-$

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
$-$0.0101$\pm$0.0034 OUR AVERAGE	Error includes scale factor of 2.1. See the ideogram below.				
-0.0205 ± 0.0039	225k	DEVAUX	77	SPEC	+
-0.0075 ± 0.0019	750k	FORD	72	ASPK	+
-0.0105 ± 0.0045	39819	HOFFMASTER	72	HLBC	+



Quadratic coefficient k for $K^+ \to \pi^+\pi^+\pi^-$

LINEAR COEFFICIENT g_- FOR $K^- \to \pi^-\pi^-\pi^+$

Some experiments use Dalitz variables x and y. In the comments we give a_y = coefficient of y term. See note above on "Dalitz Plot Parameters for $K \to 3\pi$ Decays." For discussion of the conversion of a_y to g, see the earlier version of the same note in the *Review* published in Physics Letters **111B** 70 (1982).

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
−0.217 ±0.007 OUR AVERAGE		Error includes scale factor of 2.5.				
−0.2186±0.0028	750k	FORD	72	ASPK	−	a_y=.2770 ± .0035
−0.193 ±0.010	50919	MAST	69	HBC	−	a_y=0.244 ± 0.013
• • • We do not use the following data for averages, fits, limits, etc. • • •						
−0.199 ±0.008	81k	[78] LUCAS	73	HBC	−	a_y=0.252 ± 0.011
−0.190 ±0.023	5778	[79,80] MOSCOSO	68	HBC	−	a_y=0.242 ± 0.029
−0.220 ±0.035	1347	[81] FERRO-LUZZI	61	HBC	−	a_y=0.28 ± 0.045

[78] Quadratic dependence is required by K_L^0 experiments. For comparison we average only those $K^\pm$ experiments which quote quadratic fit values.

[79] Experiments with large errors not included in average.

[80] Also includes DBC events.

[81] No radiative corrections included.

QUADRATIC COEFFICIENT h FOR $K^- \to \pi^-\pi^-\pi^+$

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.010 ±0.006 OUR AVERAGE					
0.0125±0.0062	750k	FORD	72	ASPK	−
−0.001 ±0.012	50919	MAST	69	HBC	−

QUADRATIC COEFFICIENT k FOR $K^- \to \pi^-\pi^-\pi^+$

VALUE	EVTS	DOCUMENT ID	TECN	CHG
−0.0084±0.0019 OUR AVERAGE				
−0.0083±0.0019	750k	FORD 72	ASPK	−
−0.014 ±0.012	50919	MAST 69	HBC	−

$(g_+ - g_-) / (g_+ + g_-)$ FOR $K^\pm \to \pi^\pm\pi^+\pi^-$

A nonzero value for this quantity indicates *CP* violation.

VALUE (units 10^{-4})	EVTS	DOCUMENT ID	TECN
1.5± 2.9 OUR AVERAGE			
1.7± 2.1±2.0	1.7G	[82] BATLEY 06	NA48
−70.0±53	3.2M	FORD 70	ASPK

[82] This measurement neglects any possible charge asymmetry in higher order slope parameters h or k.

LINEAR COEFFICIENT g FOR $K^\pm \to \pi^\pm\pi^0\pi^0$

Unless otherwise stated, all experiments include terms quadratic in $(s_3 - s_0) / m^2_{\pi^+}$. See note above on "Dalitz Plot Parameters for $K \to 3\pi$ Decays."

See BATUSOV 98 for a discussion of the discrepancy between their result and others, especially BOLOTOV 86. At this time we have no way to resolve the discrepancy so we depend on the large scale factor as a warning.

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
0.626 ±0.007 OUR AVERAGE					
0.6259±0.0043±0.0093	493k	AKOPDZHAN..05B	TNF	±	
0.627 ±0.004 ±0.010	252k	[83,84] AJINENKO 03B	ISTR	−	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.736 ±0.014 ±0.012	33k	BATUSOV 98	SPEC	+	
0.582 ±0.021	43k	BOLOTOV 86	CALO	−	
0.670 ±0.054	3263	BRAUN 76B	HLBC	+	
0.630 ±0.038	5635	SHEAFF 75	HLBC	+	
0.510 ±0.060	27k	SMITH 75	WIRE	+	
0.67 ±0.06	1365	AUBERT 72	HLBC	+	
0.544 ±0.048	4048	DAVISON 69	HLBC	+	Also emulsion

[83] Measured using in-flight decays of the 25 GeV negative secondary beam.

[84] They form new world averages $g_- = (0.617 \pm 0.018)$ and $g_+ = (0.684 \pm 0.033)$ which give $\Delta g_{\tau'} = 0.051 \pm 0.028$.

QUADRATIC COEFFICIENT h FOR $K^\pm \to \pi^\pm\pi^0\pi^0$

VALUE	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
0.052 ±0.008 OUR AVERAGE					
0.0551±0.0044±0.0086	493k	AKOPDZHAN..05B	TNF	±	
0.046 ±0.004 ±0.012	252k	[85] AJINENKO 03B	ISTR	−	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.128 ±0.015 ±0.024	33k	BATUSOV 98	SPEC	+	
0.037 ±0.024	43k	BOLOTOV 86	CALO	−	
0.152 ±0.082	3263	BRAUN 76B	HLBC	+	
0.041 ±0.030	5635	SHEAFF 75	HLBC	+	
0.009 ±0.040	27k	SMITH 75	WIRE	+	
−0.01 ±0.08	1365	AUBERT 72	HLBC	+	
0.026 ±0.050	4048	DAVISON 69	HLBC	+	Also emulsion

[85] Measured using in-flight decays of the 25 GeV negative secondary beam.

QUADRATIC COEFFICIENT *k* FOR $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
0.0054±0.0035 OUR AVERAGE	Error includes scale factor of 2.5.			
0.0082±0.0011±0.0014	493k	AKOPDZHAN..05B	TNF	±
0.001 ±0.001 ±0.002	252k	[86] AJINENKO 03B	ISTR	−

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
0.0197±0.0045±0.0029	33k	BATUSOV 98	SPEC	+

[86] Measured using in-flight decays of the 25 GeV negative secondary beam.

$(g_{+} - g_{-}) / (g_{+} + g_{-})$ FOR $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

A nonzero value for this quantity indicates *CP* violation.

VALUE (%)	*EVTS*	*DOCUMENT ID*	*TECN*
0.02±0.18±0.05	619k	[87] AKOPDZHAN..05	TNF

[87] Asymmetry obtained assuming that $g_{+} + g_{-} = 2 \times 0.652$ (PDG 02) and that asymmetries in *h* and *k* are zero.

ALTERNATIVE PARAMETERIZATION OF $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ DALITZ PLOT

The following functional form for the matrix element suggested by $\pi\pi$ rescattering in $K^{+} \rightarrow \pi^{+}$"$\pi^{+}\pi^{-}$"$\rightarrow \pi^{+}\pi^{0}\pi^{0}$ is used for this fit (CABIBBO 04A, CABIBBO 05): Matrix element = $M_0 + M_1$ where $M_0 = 1 + (1/2)g_0\ u + (1/2)\ h'\ u^2$ with $u = (s_3 - s_0)/(m_{\pi^+})^2$ and where M_1 takes into account the non-analytic piece due to pi pi rescattering amplitudes a_0 and a_2; The parameters g_0 and h' are related to the parameters g and h of the matrix element squared given in the previous section by the approximations $g_0 \sim g^{PDG}$ and $h' \sim h^{PDG} - (g/2)^2$.

LINEAR COEFFICIENT g_0 FOR $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
0.645±0.004±0.009	23M	[88] BATLEY 06B	NA48	±

[88] This fit is obtained with the CABIBBO 05 matrix element in the $2\pi^0$ invariant mass squared range 0.074 GeV^2 $< m^2_{2\pi^0} <$ 0.097 GeV^2, assuming $k = 0$ (no term proportional to $(s_2 - s_1)^2$) and excluding the kinematic region around the cusp ($m^2_{2\pi^0} = (2m_{\pi^+})^2 \pm 0.000525$ GeV^2). Also $\pi - \pi$ phase shifts a_0 and a_2 are measured: $(a_0 - a_2)m_{\pi^+} = 0.268 \pm 0.010 \pm 0.004 \pm 0.013$(external) and $a_2\ m_{\pi^+} = -0.041 \pm 0.022 \pm 0.014$.

QUADRATIC COEFFICIENT h' FOR $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

VALUE	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*
−0.047±0.012±0.011	23M	[89] BATLEY 06B	NA48	±

[89] This fit is obtained with the CABIBBO 05 matrix element in the $2\pi^0$ invariant mass squared range 0.074 GeV^2 $< m^2_{2\pi^0} <$ 0.097 GeV^2, assuming $k = 0$ (no term proportional to $(s_2 - s_1)^2$) and excluding the kinematic region around the cusp ($m^2_{2\pi^0} = (2m_{\pi^+})^2 \pm 0.000525$ GeV^2). Also $\pi - \pi$ phase shifts a_0 and a_2 are measured: $(a_0 - a_2)m_{\pi^+} = 0.268 \pm 0.010 \pm 0.004 \pm 0.013$(external) and $a_2\ m_{\pi^+} = -0.041 \pm 0.022 \pm 0.014$.

Citation: W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) (URL: http://pdg.lbl.gov)

$K^{\pm}_{\ell 3}$ AND $K^{0}_{\ell 3}$ FORM FACTORS

Revised June 2006 by T.G. Trippe (LBNL).

Assuming that only the vector current contributes to $K \rightarrow \pi \ell \nu$ decays, we write the matrix element as

$$M \propto f_+(t) \left[(P_K + P_\pi)_\mu \overline{\ell} \gamma_\mu (1+\gamma_5)\nu\right] + f_-(t) \left[m_\ell \overline{\ell}(1+\gamma_5)\nu\right] \ , \tag{1}$$

where P_K and P_π are the four-momenta of the K and π mesons, m_ℓ is the lepton mass, and f_+ and f_- are dimensionless form factors which can depend only on $t = (P_K - P_\pi)^2$, the square of the four-momentum transfer to the leptons. If time-reversal invariance holds, f_+ and f_- are relatively real. $K_{\mu 3}$ experiments, discussed immediately below, measure f_+ and f_-, while K_{e3} experiments, discussed further below, are sensitive only to f_+ because the small electron mass makes the f_- term negligible.

$K_{\mu 3}$ Experiments. Analyses of $K_{\mu 3}$ data frequently assume a linear dependence of f_+ and f_- on t, *i.e.*,

$$f_\pm(t) = f_\pm(0)\left[1 + \lambda_\pm (t/m^2_{\pi^+})\right] \tag{2}$$

Most $K_{\mu 3}$ data are adequately described by Eq. (2) for f_+ and a constant f_- (*i.e.*, $\lambda_- = 0$).

There are two equivalent parametrizations commonly used in these analyses:

(1) $\lambda_+, \xi(0)$ parametrization. Older analyses of $K_{\mu 3}$ data often introduce the ratio of the two form factors

$$\xi(t) = f_-(t)/f_+(t) \ . \tag{3}$$

The $K_{\mu 3}$ decay distribution is then described by the two parameters λ_+ and $\xi(0)$ (assuming time reversal invariance and $\lambda_- = 0$).

(2) $\boldsymbol{\lambda_+, \lambda_0}$ parametrization. More recent $K_{\mu 3}$ analyses have parameterized in terms of the form factors f_+ and f_0 which are associated with vector and scalar exchange, respectively, to the lepton pair. f_0 is related to f_+ and f_- by

$$f_0(t) = f_+(t) + \left[t/(m_K^2 - m_\pi^2)\right] f_-(t) \ . \tag{4}$$

Here $f_0(0)$ must equal $f_+(0)$ unless $f_-(t)$ diverges at $t = 0$. The earlier assumption that f_+ is linear in t and f_- is constant leads to f_0 linear in t:

$$f_0(t) = f_0(0) \left[1 + \lambda_0 (t/m_{\pi^+}^2)\right] \ . \tag{5}$$

With the assumption that $f_0(0) = f_+(0)$, the two parametrizations, $(\lambda_+, \xi(0))$ and (λ_+, λ_0) are equivalent as long as correlation information is retained. (λ_+, λ_0) correlations tend to be less strong than $(\lambda_+, \xi(0))$ correlations.

In this edition of the *Review* we no longer quote results in the $(\lambda_+, \xi(0))$ parameterization. We have removed many older low statistics results from the listings. See the 2004 version of this note [4] for these older results and the 1982 version [5] for additional discussion of the $K_{\mu 3}^0$ parameters, correlations, and conversion between parametrizations.

Quadratic Parameterization. More recent high statistics experiments have included a quadratic term in the expansion of $f_+(t)$,

$$f_+(t) = f_+(0) \left[1 + \lambda_+^{'} (t/m_{\pi^+}^2) + \frac{\lambda_+^{''}}{2} (t/m_{\pi^+}^2)^2\right] \tag{6}$$

If there is a non-vanishing quadratic term, then λ_+ of Eq. (2) represents the average slope, which is then different from $\lambda_+^{'}$. Our convention is to include the factor $\frac{1}{2}$ in the quadratic term and to use m_{π^+} even for K_{e3}^+ and $K_{\mu 3}^+$ decays. We have converted other's parameterizations to match our conventions, as noted in the beginning of the $K^{\pm}_{\ell 3}$ *and* $K^0_{\ell 3}$ *Form Factors* sections of the *Data Listings*.

Pole Parameterization. The pole model describes the t dependence of $f_+(t)$ and $f_0(t)$ in terms of the exchange of the lightest vector and scalar K^* mesons with masses M_v and M_s, respectively:

$$f_+(t) = f_+(0)\left[\frac{M_v^2}{M_v^2 - t}\right] \ , \qquad f_0(t) = f_0(0)\left[\frac{M_s^2}{M_s^2 - t}\right] \ . \tag{7}$$

K_{e3} Experiments. Analysis of K_{e3} data is simpler than that of $K_{\mu 3}$ because the second term of the matrix element assuming a pure vector current [Eq. (1) above] can be neglected. Here f_+ can be assumed to be linear in t, in which case the linear coefficient λ_+ of Eq. (2) is determined, or quadratic, in which case the linear coefficient $\lambda^{'}_+$ and quadratic coefficient $\lambda^{''}_+$ of Eq. (6) are determined.

If we remove the assumption of a pure vector current, then the matrix element for the decay, in addition to the terms in Eq. (1), would contain

$$\begin{aligned} &+2m_K\, f_S\, \overline{\ell}(1+\gamma_5)\nu \\ &\qquad +(2f_T/m_K)(P_K)_\lambda(P_\pi)_\mu\, \overline{\ell}\, \sigma_{\lambda\mu}(1+\gamma_5)\nu \ , \end{aligned} \tag{8}$$

where f_S is the scalar form factor, and f_T is the tensor form factor. In the case of the K_{e3} decays where the f_- term can be neglected, experiments have yielded limits on $|f_S/f_+|$ and $|f_T/f_+|$.

Fits for $K_{\ell 3}$FormFactors. For K_{e3} data we determine best values for the three parameterizations: linear (λ_+), quadratic ($\lambda^{'}_+$, $\lambda^{''}_+$) and pole (M_v). For $K_{\mu 3}$ data we determine best values for the three parameterizations: linear (λ_+, λ_0), quadratic

$(\lambda'_+, \lambda''_+,\ \lambda_0))$ and pole $(M_v,\ M_s)$. We then assume $\mu - e$ universality so that we can combine K_{e3} and $K_{\mu3}$ data and again determine best values for the three parameterizations: linear $(\lambda_+,\ \lambda_0)$, quadratic $(\lambda'_+, \lambda''_+,\ \lambda_0))$ and pole $(M_v,\ M_s)$. When there is more than one parameter, fits are done including input correlations. Simple averages suffice in the two K_{e3} cases where there is only one parameter: linear (λ_+) and pole (M_v).

Both KTeV and KLOE see an improvement in the quality of their fits relative to linear fits when a quadratic term is introduced, as well as when the pole parameterization is used. The quadratic parameterization has the disadvantage that the quadratic parameter λ''_+ is highly correlated with the linear parameter λ'_+, in the neighborhood of 95%, and that neither parameter is very well determined. The pole fit has the same number of parameters as the linear fit but yields slightly better fit probabilities so that it would be advisable for all experiments to include the pole parameterization as one of their choices [6].

The *Kaon Particle Listings* show the results with and without assuming μ-e universality. The *Meson Summary Tables* show all of the results assuming μ-e universality, but most results not assuming μ-e universality are given only in the *Listings*.

References

1. L.M. Chounet, J.M. Gaillard, and M.K. Gaillard, Phys. Reports **4C**, 199 (1972).
2. H.W. Fearing, E. Fischbach, and J. Smith, Phys. Rev. **D2**, 542 (1970).
3. N. Cabibbo and A. Maksymowicz, Phys. Lett. **9**, 352 (1964).
4. S. Eidleman *et al.*, Particle Data Group, Phys. Lett. **B592**, 1 (2004).

5. M. Roos *et al.*, Particle Data Group, Phys. Lett. **111B**, 73 (1982).
6. We thank P. Franzini (Rome U. and Frascati) for useful discussions on this point.

$K^{\pm}_{\ell 3}$ FORM FACTORS

In the form factor comments, the following symbols are used.

f_+ and f_- are form factors for the vector matrix element.

f_S and f_T refer to the scalar and tensor term.

$f_0 = f_+ + f_- \; t/(m^2_{K^+} - m^2_{\pi^0})$.

t = momentum transfer to the π.

λ_+ and λ_0 are the linear expansion coefficients of f_+ and f_0:

$f_+(t) = f_+(0)\;(1 + \lambda_+ t\;/m^2_{\pi^+})$

For quadratic expansion

$f_+(t) = f_+(0)\;(1 + \lambda'_+ t\;/m^2_{\pi^+} + \frac{\lambda''_+}{2}\; t^2/m^4_{\pi^+}\;)$

as used by KTeV. If there is a non-vanishing quadratic term, then λ_+ represents an average slope, which is then different from λ'_+.

NA48 and ISTRA quadratic expansion coefficients are converted with

${\lambda'_+}^{PDG} = \lambda_+^{NA48}$ and ${\lambda''_+}^{PDG} = 2\;{\lambda'_+}^{NA48}$

${\lambda'_+}^{PDG} = (\frac{m_{\pi^+}}{m_{\pi^0}})^2\;\lambda_+^{ISTRA}$ and

${\lambda''_+}^{PDG} = 2\;(\frac{m_{\pi^+}}{m_{\pi^0}})^4\;{\lambda'_+}^{ISTRA}$

ISTRA linear expansion coefficients are converted with

$\lambda_+^{PDG} = (\frac{m_{\pi^+}}{m_{\pi^0}})^2\;\lambda_+^{ISTRA}$ and $\lambda_0^{PDG} = (\frac{m_{\pi^+}}{m_{\pi^0}})^2\;\lambda_0^{ISTRA}$

The pole parametrization is

$$f_+(t) = f_+(0)\;(\frac{M_V^2}{M_V^2 - t}\;)$$

$$f_0(t) = f_0(0)\;(\frac{M_S^2}{M_S^2 - t}\;)$$

where M_V and M_S are the vector and scalar pole masses.

The following abbreviations are used:

DP = Dalitz plot analysis.

PI = π spectrum analysis.

MU = μ spectrum analysis.

POL= μ polarization analysis.

BR = $K^{\pm}_{\mu 3}/K^{\pm}_{e3}$ branching ratio analysis.

E = positron or electron spectrum analysis.

RC = radiative corrections.

λ_+ (LINEAR ENERGY DEPENDENCE OF f_+ IN $K^{\pm}_{e3}$ DECAY)

These results are for a linear expansion only. See the next section for fits including a quadratic term. For radiative correction of the $K^{\pm}_{e3}$ Dalitz plot, see GINSBERG 67,

BECHERRAWY 70, CIRIGLIANO 02, CIRIGLIANO 04, and ANDRE 04. Results labeled OUR FIT are discussed in the review "$K^{\pm}_{\ell 3}$ and $K^{0}_{\ell 3}$ Form Factors" above. For earlier, lower statistics results, see the 2004 edition of this review, Phys. Lett. B592, 1 (2004).

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
2.96 ±0.05 OUR FIT Assuming μ-e universality					
2.96 ±0.06 OUR AVERAGE					
2.966±0.050±0.034	919k	[90] YUSHCHENKO 04B	ISTR	−	DP
2.78 ±0.26 ±0.30	41k	SHIMIZU 00	SPEC	+	DP
2.84 ±0.27 ±0.20	32k	[91] AKIMENKO 91	SPEC		PI, no RC
2.9 ±0.4	62k	[92] BOLOTOV 88	SPEC		PI, no RC
• • • We do not use the following data for averages, fits, limits, etc. • • •					
3.06 ±0.09 ±0.06	550k	[90,93] AJINENKO 03C	ISTR	−	DP
2.93 ±0.15 ±0.2	130k	[93] AJINENKO 02	SPEC		DP

[90] Rescaled to agree with our conventions as noted above.

[91] AKIMENKO 91 state that radiative corrections would raise λ_+ by 0.0013.

[92] BOLOTOV 88 state radiative corrections of GINSBERG 67 would raise λ_+ by 0.002.

[93] Superseded by YUSHCHENKO 04B.

λ_+ (LINEAR ENERGY DEPENDENCE OF f_+ IN $K^{\pm}_{\mu 3}$ DECAY)

Results labeled OUR FIT are discussed in the review "$K^{\pm}_{\ell 3}$ and $K^{0}_{\ell 3}$ Form Factors" above. For earlier, lower statistics results, see the 2004 edition of this review, Phys. Lett. B592, 1 (2004).

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
2.96±0.05 OUR FIT Assuming μ-e universality					
2.96±0.17 OUR FIT Not assuming μ-e universality					
2.96±0.14±0.10	540k	[94] YUSHCHENKO 04	ISTR	−	DP
• • • We do not use the following data for averages, fits, limits, etc. • • •					
3.21±0.45	112k	[95] AJINENKO 03	ISTR	−	DP

[94] Rescaled to agree with our conventions as noted above.

[95] Superseded by YUSHCHENKO 04.

λ_0 (LINEAR ENERGY DEPENDENCE OF f_0 IN $K^{\pm}_{\mu 3}$ DECAY)

Results labeled OUR FIT are discussed in the review "$K^{\pm}_{\ell 3}$ and $K^{0}_{\ell 3}$ Form Factors" above. For earlier, lower statistics results, see the 2004 edition of this review, Phys. Lett. B592, 1 (2004).

VALUE (units 10^{-2})	$d\lambda_0/d\lambda_+$	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1.96±0.12 OUR FIT Assuming μ-e universality. Correlation is $d\lambda_0/d\lambda_+ = -0.35$.						
1.96±0.13 OUR FIT Not assuming μ-e universality. Correlation is $d\lambda_0/d\lambda_+ = -0.35$.						
+1.96±0.12±0.06	−0.348	540k	[96] YUSHCHENKO 04	ISTR	−	DP
• • • We do not use the following data for averages, fits, limits, etc. • • •						
+2.09±0.45	−0.46	112k	[97] AJINENKO 03	ISTR	−	DP
+1.9 ±0.64		24k	[98] HORIE 01	SPEC	+	BR
+1.9 ±1.0	+0.03	55k	[99] HEINTZE 77	SPEC	+	BR

[96] Rescaled to agree with our conventions as noted above.

[97] Superseded by YUSHCHENKO 04.

[98] HORIE 01 assumes μ-e universality in $K^{+}_{\ell 3}$ decay and uses SHIMIZU 00 value λ=0.0278±0.0040 from $K^{\pm}_{e3}$ decay.

[99] HEINTZE 77 uses $\lambda_+ = 0.029 \pm 0.003$. $d\lambda_0/d\lambda_+$ estimated by us.

λ'_+ (LINEAR $K^{\pm}_{e3}$ FORM FACTOR FROM QUADRATIC FIT)

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
2.485±0.163±0.034	919k	[100,101] YUSHCHENKO 04B	ISTR	−	DP
• • • We do not use the following data for averages, fits, limits, etc. • • •					
3.07 ±0.21	550k	[100,102] AJINENKO 03C	ISTR	−	DP

[100] Rescaled to agree with our conventions as noted above.

[101] YUSHCHENKO 04B λ'_+ and λ''_+ are strongly correlated with coefficient $\rho(\lambda'_+, \lambda''_+) = -0.95$.

[102] Superseded by YUSHCHENKO 04B.

λ''_+(QUADRATIC $K^{\pm}_{e3}$ FORM FACTOR)

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.192±0.062±0.071	919k	[103,104] YUSHCHENKO 04B	ISTR	−	DP
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−0.5 ±0.7 ±1.5	550k	[103,105] AJINENKO 03C	ISTR	−	DP

[103] Rescaled to agree with our conventions as noted above.

[104] YUSHCHENKO 04B λ'_+ and λ''_+ are strongly correlated with coefficient $\rho(\lambda'_+, \lambda''_+) = -0.95$.

[105] Superseded by YUSHCHENKO 04B.

$|f_S/f_+|$ FOR $K^{\pm}_{e3}$ DECAY

Ratio of scalar to f_+ couplings.

VALUE (units 10^{-2})	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
$-0.3^{+0.8}_{-0.7}$ OUR AVERAGE						
$-0.37^{+0.66}_{-0.56}\pm0.41$		919k	YUSHCHENKO 04B	ISTR	−	λ'_+, λ''_+, f_S fit
$0.2 \pm 2.6 \pm 1.4$		41k	SHIMIZU 00	SPEC	+	λ_+, f_S, f_T fit
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$0.2^{+2.0}_{-2.2}\pm0.3$		550k	[106] AJINENKO 03C	ISTR	−	λ_+, f_S, f_T fit
$-1.9^{+2.5}_{-1.6}$		130k	[106] AJINENKO 02	SPEC		λ_+, f_S fit
$7.0 \pm 1.6 \pm 1.6$		32k	AKIMENKO 91	SPEC		λ_+, f_S, f_T, ϕ fit
0 ± 10		2827	[107] BRAUN 75	HLBC	+	
<13	90	4017	CHIANG 72	OSPK	+	
14^{+3}_{-4}		2707	[107] STEINER 71	HLBC	+	λ_+, f_S, f_T, ϕ fit
<23	90		BOTTERILL 68C	ASPK		
<18	90		BELLOTTI 67B	HLBC		
<30	95		KALMUS 67	HLBC	+	

[106] Superseded by YUSHCHENKO 04B.

[107] Statistical errors only.

$|f_T/f_+|$ FOR $K^{\pm}_{e3}$ DECAY

Ratio of tensor to f_+ couplings.

VALUE (units 10^{-2})	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
−1.2 ± 2.3 OUR AVERAGE						
−1.2 ± 2.1 ± 1.1		919k	YUSHCHENKO 04B	ISTR	−	λ'_{+}, λ''_{+}, f_T fit
1 ± 14 ± 9		41k	SHIMIZU 00	SPEC	+	λ_+, f_S, f_T fit
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$2.1^{+6.4}_{-7.5} \pm 2.6$		550k	[108] AJINENKO 03C	ISTR	−	λ_+, f_S, f_T fit
$-4.5^{+6.0}_{-5.7}$		130k	[108] AJINENKO 02	SPEC		λ_+, f_T fit
$53^{+9}_{-10} \pm 10$		32k	AKIMENKO 91	SPEC		λ_+, f_S, f_T, ϕ fit
7 ± 37		2827	[109] BRAUN 75	HLBC	+	
< 75.	90	4017	CHIANG 72	OSPK	+	
24^{+16}_{-14}		2707	[109] STEINER 71	HLBC	+	λ_+, f_S, f_T, ϕ fit
< 58.	90		BOTTERILL 68C	ASPK		
< 58.	90		BELLOTTI 67B	HLBC		
< 110.	95		KALMUS 67	HLBC	+	

[108] Superseded by YUSHCHENKO 04B.

[109] Statistical errors only.

f_S/f_+ FOR $K^{\pm}_{\mu 3}$ DECAY

Ratio of scalar to f_+ couplings.

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
0.17 ± 0.14 ± 0.54	540k	[110] YUSHCHENKO 04	ISTR	−	DP
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.4 ± 0.5 ± 0.5	112k	[111] AJINENKO 03	ISTR	−	DP

[110] The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for λ_0, ±0.0053, combined in quadrature with the systematic error ±0.0009.

[111] The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for λ_0. Superseded by YUSHCHENKO 04.

f_T/f_+ FOR $K^{\pm}_{\mu 3}$ DECAY

Ratio of tensor to f_+ couplings.

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
−0.07 ± 0.71 ± 0.20	540k	YUSHCHENKO 04	ISTR	−	DP
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−2.1 ± 2.8 ± 1.4	112k	[112] AJINENKO 03	ISTR	−	DP
2 ± 12	1585	BRAUN 75	HLBC		

[112] The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for λ_0. Superseded by YUSHCHENKO 04.

DECAY FORM FACTORS FOR $K^{\pm} \rightarrow \pi^+ \pi^- e^{\pm} \nu_e$

Given in PISLAK 01, ROSSELET 77, BEIER 73, and BASILE 71C.

DECAY FORM FACTOR FOR $K^{\pm} \rightarrow \pi^0 \pi^0 e^{\pm} \nu$

Given in BOLOTOV 86B, BARMIN 88B, and SHIMIZU 04.

$K^{\pm} \rightarrow \ell^{\pm} \nu \gamma$ FORM FACTORS

For definitions of the axial-vector F_A and vector F_V form factor, see the "Note on $\pi^{\pm} \rightarrow \ell^{\pm} \nu \gamma$ and $K^{\pm} \rightarrow \ell^{\pm} \nu \gamma$ Form Factors" in the $\pi^{\pm}$ section. In the kaon literature, often different definitions $a_K = F_A/m_K$ and $v_K = F_V/m_K$ are used.

$F_A + F_V$, SUM OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \rightarrow e \nu_e \gamma$

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*
0.148±0.010 OUR AVERAGE				
0.147±0.011	51	[113] HEINTZE	79	SPEC
$0.150^{+0.018}_{-0.023}$	56	[114] HEARD	75	SPEC

[113] HEINTZE 79 quotes absolute value of $|F_A + F_V| \sin\theta_c$. We use $\sin\theta_c = V_{us} = 0.2205$.

[114] HEARD 75 quotes absolute value of $|F_A + F_V| \sin\theta_c$. We use $\sin\theta_c = V_{us} = 0.2205$.

$F_A + F_V$, SUM OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \rightarrow \mu \nu_\mu \gamma$

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.165±0.007±0.011		2588	[115] ADLER	00B	B787	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
−1.2 to 1.1	90		DEMIDOV	90	XEBC	
< 0.23	90		[115] AKIBA	85	SPEC	

[115] Quotes absolute value. Sign not determined.

$F_A - F_V$, DIFFERENCE OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \rightarrow e \nu_e \gamma$

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*
<0.49	90	[116] HEINTZE	79	SPEC

[116] HEINTZE 79 quotes $|F_A - F_V| < \sqrt{11}\,|F_A + F_V|$.

$F_A - F_V$, DIFFERENCE OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \rightarrow \mu \nu_\mu \gamma$

VALUE	*CL%*	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
−0.24 to 0.04	90	2588	ADLER	00B	B787	+
• • • We do not use the following data for averages, fits, limits, etc. • • •						
−2.2 to 0.6	90		DEMIDOV	90	XEBC	
−2.5 to 0.3	90		AKIBA	85	SPEC	

$K^{\pm}$ CHARGE RADIUS

VALUE (fm)	*DOCUMENT ID*		*COMMENT*
0.560±0.031 OUR AVERAGE			
0.580±0.040	AMENDOLIA	86B	$K e \rightarrow K e$
0.530±0.050	DALLY	80	$K e \rightarrow K e$
• • • We do not use the following data for averages, fits, limits, etc. • • •			
0.620±0.037	BLATNIK	79	VMD + dispersion relations

CP VIOLATION TESTS IN K^+ AND K^- DECAYS

$$\Delta(K^{\pm}_{\pi\mu\mu}) = \frac{\Gamma(K^{+}_{\pi\mu\mu})-\Gamma(K^{-}_{\pi\mu\mu})}{\Gamma(K^{+}_{\pi\mu\mu})+\Gamma(K^{-}_{\pi\mu\mu})}$$

VALUE	*DOCUMENT ID*		*TECN*
−0.02±0.11±0.04	PARK	02	HYCP

T VIOLATION TESTS IN K^+ AND K^- DECAYS

P_T in $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$

T-violating muon polarization. Sensitive to new sources of *CP* violation beyond the Standard Model.

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
−1.7±2.3±1.1		[117] ABE	04F	K246	+
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−4.2±4.9±0.9	3.9M	ABE	99S	K246	+

[117] Includes three sets of data: 96-97 (ABE 99S), 98, and 99-00 totaling about three times the ABE 99S data sample. Corresponds to $P_T < 5.0 \times 10^{-3}$ at 90% CL.

P_T in $K^+ \rightarrow \mu^+ \nu_\mu \gamma$

T-violating muon polarization. Sensitive to new sources of *CP* violation beyond the Standard Model.

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
−0.64±1.85±0.10	114k	[118] ANISIMOVSK...	03	K246	+

[118] Muons stopped and polarization measured from decay to positrons.

Im(ξ) in $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ DECAY (from transverse μ pol.)

Test of *T* reversal invariance.

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
−0.006 ±0.008 OUR AVERAGE						
−0.0053±0.0071±0.0036		[119] ABE	04F	K246	+	
−0.016 ±0.025	20M	CAMPBELL	81	CNTR	+	Pol.
• • • We do not use the following data for averages, fits, limits, etc. • • •						
−0.013 ±0.016 ±0.003	3.9M	ABE	99S	CNTR	+	p_T K^+ at rest

[119] Includes three sets of data: 96-97 (ABE 99S), 98, and 99-00 totaling about three times the ABE 99S data sample. Corresponds to Im(ξ) < 0.016 at 90% CL.

$K^{\pm}$ REFERENCES

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BATLEY	06	PL B634 474	J.R. Batley *et al.*	(CERN NA48/2 Collab.)
BATLEY	06B	PL B633 173	J.R. Batley *et al.*	(CERN NA48/2 Collab.)
MA	06	PR D73 037101	H. Ma *et al.*	(BNL E865 Collab.)
SHIMIZU	06	PL B633 190	S. Shimizu *et al.*	(KEK E470 Collab.)
UVAROV	06	PAN 69 26	V.A. Uvarov *et al.*	(ISTRA+ Collab.)
AKOPDZHAN...	05	EPJ C40 343	G.A. Akopdzhanov *et al.*	(IHEP)
Also		PAN 68 948 Translated from YAF 68 986.	G.A. Akopdzhanov *et al.*	(IHEP)
AKOPDZHAN...	05B	JETPL 82 675 Translated from ZETFP 82 771.	G.A. Akopdzhanov *et al.*	(IHEP)
ARTAMONOV	05	PL B623 192	A.V. Artamonov *et al.*	(BNL E949 Collab.)
CABIBBO	05	JHEP 0503 021	N. Cabibbo, G. Isidori	(CERN, ROMAI, FRAS)
SHER	05	PR D72 012005	A. Sher *et al.*	(BNL E865 Collab.)
ABE	04F	PRL 93 131601	M. Abe *et al.*	(KEK E246 Collab.)
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ALOISIO	04A	PL B597 139	A. Aloisio *et al.*	(KLOE Collab.)
ANDRE	04	hep-ph/0406006	T. Andre	(EFI)
ANISIMOVSK...	04	PRL 93 031801	V.V. Animovsky *et al.*	(BNL E949 Collab.)
CABIBBO	04A	PRL 93 121801	N. Cabibbo	(CERN, ROMAI)
CIRIGLIANO	04	EPJ C35 53	V. Cirigliano, H. Neufeld, H. Pichl	(CIT, VALE+)
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AJINENKO	03	PAN 66 105 Translated from YAF 66 107.	I.V. Ajinenko *et al.*	(IHEP, INRM)
AJINENKO	03B	PL B567 159	I.V. Ajinenko *et al.*	(IHEP, INRM)
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ALIEV	03	PL B554 7	M.A. Aliev *et al.*	(KEK E470 Collab.)
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PISLAK	03	PR D67 072004	S. Pislak *et al.*	(BNL E865 Collab.)
SHER	03	PRL 91 261802	A. Sher *et al.*	(BNL E865 Collab.)
ADLER	02	PRL 88 041803	S. Adler *et al.*	(BNL E787 Collab.)
ADLER	02B	PR D65 052009	S. Adler *et al.*	(BNL E787 Collab.)
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AJINENKO	02	PAN 65 2064 Translated from YAF 65 2125.	I.V. Ajinenko *et al.*	(IHEP, INRM)
CIRIGLIANO	02	EPJ C23 121	V. Cirigliano *et al.*	(VIEN, VALE, MARS)
PARK	02	PRL 88 111801	H.K. Park *et al.*	(FNAL HyperCP Collab.)
PDG	02	PR D66 010001	K. Hagiwara *et al.*	
POBLAGUEV	02	PRL 89 061803	A.A. Poblaguev *et al.*	(BNL 865 Collab.)
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HORIE	01	PL B513 311	K. Horie *et al.*	(KEK E426 Collab.)
PISLAK	01	PRL 87 221801	S. Pislak *et al.*	(BNL E865 Collab.)
Also		PR D67 072004	S. Pislak *et al.*	(BNL E865 Collab.)
ADLER	00	PRL 84 3768	S. Adler *et al.*	(BNL E787 Collab.)
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APPEL	00	PRL 85 2450	R. Appel *et al.*	(BNL 865 Collab.)
Also		Thesis, Yale Univ.	D.R. Bergman	
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APPEL	00B	PRL 85 2877	R. Appel *et al.*	(BNL 865 Collab.)
MA	00	PRL 84 2580	H. Ma *et al.*	(BNL 865 Collab.)
PDG	00	EPJ C15 1	D.E. Groom *et al.*	
SHIMIZU	00	PL B495 33	S. Shimizu *et al.*	(KEK E246 Collab.)
ABE	99S	PRL 83 4253	M. Abe *et al.*	(KEK E246 Collab.)
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ADLER	98	PR D58 012003	S. Adler *et al.*	(BNL E787 Collab.)
BATUSOV	98	NP B516 3	V.Y. Batusov *et al.*	
ADLER	97	PRL 79 2204	S. Adler *et al.*	(BNL E787 Collab.)
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KOPTEV	95	JETPL 61 877 Translated from ZETFP 61 865.	V.P. Koptev *et al.*	(PNPI)

AOKI	94	PR D50 69	M. Aoki *et al.*	(INUS, KEK, TOKMS)
ATIYA	93	PRL 70 2521	M.S. Atiya *et al.*	(BNL E787 Collab.)
Also		PRL 71 305 (erratum)	M.S. Atiya *et al.*	(BNL E787 Collab.)
ATIYA	93B	PR D48 R1	M.S. Atiya *et al.*	(BNL E787 Collab.)
ALLIEGRO	92	PRL 68 278	C. Alliegro *et al.*	(BNL, FNAL, PSI+)
BARMIN	92	SJNP 55 547	V.V. Barmin *et al.*	(ITEP)
		Translated from YAF 55 976.		
IMAZATO	92	PRL 69 877	J. Imazato *et al.*	(KEK, INUS, TOKY+)
IVANOV	92	THESIS	Yu.M. Ivanov	(PNPI)
LITTENBERG	92	PRL 68 443	L.S. Littenberg, R.E. Shrock	(BNL, STON)
USHER	92	PR D45 3961	T. Usher *et al.*	(UCI)
AKIMENKO	91	PL B259 225	S.A. Akimenko *et al.*	(SERP, JINR, TBIL+)
BARMIN	91	SJNP 53 606	V.V. Barmin *et al.*	(ITEP)
		Translated from YAF 53 981.		
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ATIYA	90B	PRL 65 1188	M.S. Atiya *et al.*	(BNL E787 Collab.)
DEMIDOV	90	SJNP 52 1006	V.S. Demidov *et al.*	(ITEP)
		Translated from YAF 52 1595.		
LEE	90	PRL 64 165	A.M. Lee *et al.*	(BNL, FNAL, VILL, WASH+)
ATIYA	89	PRL 63 2177	M.S. Atiya *et al.*	(BNL E787 Collab.)
BARMIN	89	SJNP 50 421	V.V. Barmin *et al.*	(ITEP)
		Translated from YAF 50 679.		
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		Translated from YAF 47 1011.		
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		Translated from YAF 48 1719.		
BOLOTOV	88	JETPL 47 7	V.N. Bolotov *et al.*	(ASCI)
		Translated from ZETFP 47 8.		
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BARMIN	87	SJNP 45 62	V.V. Barmin *et al.*	(ITEP)
		Translated from YAF 45 97.		
BOLOTOV	87	SJNP 45 1023	V.N. Bolotov *et al.*	(INRM)
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AMENDOLIA	86B	PL B178 435	S.R. Amendolia *et al.*	(CERN NA7 Collab.)
BOLOTOV	86	SJNP 44 73	V.N. Bolotov *et al.*	(INRM)
		Translated from YAF 44 117.		
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Also		PRL 52 329	R.S. Hayano *et al.*	(TOKY, KEK)
AKIBA	85	PR D32 2911	Y. Akiba *et al.*	(TOKY, TINT, TSUK, KEK)
BOLOTOV	85	JETPL 42 481	V.N. Bolotov *et al.*	(INRM)
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ASANO	82	PL 113B 195	Y. Asano *et al.*	(KEK, TOKY, INUS, OSAK)
COOPER	82	PL 112B 97	A.M. Cooper *et al.*	(RL)
PDG	82B	PL 111B 70	M. Roos *et al.*	(HELS, CIT, CERN)
ASANO	81B	PL 107B 159	Y. Asano *et al.*	(KEK, TOKY, INUS, OSAK)
CAMPBELL	81	PRL 47 1032	M.K. Campbell *et al.*	(YALE, BNL)
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LUM	81	PR D23 2522	G.K. Lum *et al.*	(LBL, NBS+)
LYONS	81	ZPHY C10 215	L. Lyons, C. Albajar, G. Myatt	(OXF)
DALLY	80	PRL 45 232	E.B. Dally *et al.*	(UCLA+)
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HEINTZE	79	NP B149 365	J. Heintze *et al.*	(HEIDP, CERN)
ABRAMS	77	PR D15 22	R.J. Abrams *et al.*	(BNL)
DEVAUX	77	NP B126 11	B. Devaux *et al.*	(SACL, GEVA)
HEINTZE	77	PL 70B 482	J. Heintze *et al.*	(HEIDP, CERN)
ROSSELET	77	PR D15 574	L. Rosselet *et al.*	(GEVA, SACL)
BLOCH	76	PL 60B 393	P. Bloch *et al.*	(GEVA, SACL)
BRAUN	76B	LNC 17 521	H.M. Braun *et al.*	(AACH3, BARI, BELG+)
DIAMANT-...	76	PL 62B 485	A.M. Diamant-Berger *et al.*	(SACL, GEVA)
HEINTZE	76	PL 60B 302	J. Heintze *et al.*	(HEIDP)
SMITH	76	NP B109 173	K.M. Smith *et al.*	(GLAS, LIVP, OXF+)
WEISSENBE...	76	NP B115 55	A.O. Weissenberg *et al.*	(ITEP, LEBD)
BLOCH	75	PL 56B 201	P. Bloch *et al.*	(SACL, GEVA)
BRAUN	75	NP B89 210	H.M. Braun *et al.*	(AACH3, BARI, BRUX+)
CHENG	75	NP A254 381	S.C. Cheng *et al.*	(COLU, YALE)
HEARD	75	PL 55B 324	K.S. Heard *et al.*	(CERN, HEIDH)
HEARD	75B	PL 55B 327	K.S. Heard *et al.*	(CERN, HEIDH)

SHEAFF	75	PR D12 2570	M. Sheaff	(WISC)
SMITH	75	NP B91 45	K.M. Smith *et al.*	(GLAS, LIVP, OXF+)
WEISSENBE...	74	PL 48B 474	A.O. Weissenberg *et al.*	(ITEP, LEBD)
ABRAMS	73B	PRL 30 500	R.J. Abrams *et al.*	(BNL)
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BEIER	73	PRL 30 399	E.W. Beier *et al.*	(PENN)
LJUNG	73	PR D8 1307	D. Ljung, D. Cline	(WISC)
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SMITH	73	NP B60 411	K.M. Smith *et al.*	(GLAS, LIVP, OXF+)
ABRAMS	72	PRL 29 1118	R.J. Abrams *et al.*	(BNL)
AUBERT	72	NC 12A 509	B. Aubert *et al.*	(ORSAY, BRUX, EPOL)
CHIANG	72	PR D6 1254	I.H. Chiang *et al.*	(ROCH, WISC)
CLARK	72	PRL 29 1274	A.R. Clark *et al.*	(LBL)
EDWARDS	72	PR D5 2720	R.T. Edwards *et al.*	(ILL)
FORD	72	PL 38B 335	W.T. Ford *et al.*	(PRIN)
HOFFMASTER	72	NP B36 1	S. Hoffmaster *et al.*	(STEV, SETO, LEHI)
BASILE	71C	PL 36B 619	P. Basile *et al.*	(SACL, GEVA)
BOURQUIN	71	PL 36B 615	M.H. Bourquin *et al.*	(GEVA, SACL)
HAIDT	71	PR D3 10	D. Haidt	(AACH, BARI, CERN, EPOL, NIJM+)
Also		PL 29B 691	D. Haidt *et al.*	(AACH, BARI, CERN, EPOL+)
KLEMS	71	PR D4 66	J.H. Klems, R.H. Hildebrand, R. Stiening	(CHIC+)
Also		PRL 24 1086	J.H. Klems, R.H. Hildebrand, R. Stiening	(LRL+)
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OTT	71	PR D3 52	R.J. Ott, T.W. Pritchard	(LOQM)
ROMANO	71	PL 36B 525	F. Romano *et al.*	(BARI, CERN, ORSAY)
SCHWEINB...	71	PL 36B 246	W. Schweinberger	(AACH, BELG, CERN, NIJM+)
STEINER	71	PL 36B 521	H.J. Steiner	(AACH, BARI, CERN, EPOL, ORSAY+)
BARDIN	70	PL 32B 121	D.Y. Bardin, S.N. Bilenky, B.M. Pontecorvo	(JINR)
BECHERRAWY	70	PR D1 1452	T. Becherrawy	(ROCH)
FORD	70	PRL 25 1370	W.T. Ford *et al.*	(PRIN)
GAILLARD	70	CERN 70-14	J.M. Gaillard, L.M. Chounet	(CERN, ORSAY)
GRAUMAN	70	PR D1 1277	J. Grauman *et al.*	(STEV, SETO, LEHI)
Also		PRL 23 737	J.U. Grauman *et al.*	(STEV, SETO, LEHI)
MACEK	70	PR D1 1249	R.J. Macek *et al.*	(PENN)
MALTSEV	70	SJNP 10 678 Translated from YAF 10 1195.	E.I. Maltsev *et al.*	(JINR)
PANDOULAS	70	PR D2 1205	D. Pandoulas *et al.*	(STEV, SETO)
CUTTS	69	PR 184 1380	D. Cutts *et al.*	(LRL, MIT)
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DAVISON	69	PR 180 1333	D.C. Davison *et al.*	(UCR)
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