D^0–$\overline{D}^0$ MIXING

Revised January 2006 by D. Asner (Carleton University)

Standard Model contributions to D^0–$\overline{D}^0$ mixing are strongly suppressed by CKM and GIM factors. Thus the observation of D^0–$\overline{D}^0$ mixing might be evidence for physics beyond the Standard Model. See Burdman and Shipsey [1] for a review of D^0–$\overline{D}^0$ mixing, Ref. [2] for a compilation of mixing predictions, and Ref. [3] for later predictions.

Formalism: The time evolution of the D^0–$\overline{D}^0$ system is described by the Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}, \tag{1}$$

where the $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance requires that $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of these matrices describe the dispersive and absorptive parts of D^0–$\overline{D}^0$ mixing.

The two eigenstates D_1 and D_2 of the effective Hamiltonian matrix $(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma})$ are given by

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle\,. \tag{2}$$

The corresponding eigenvalues are

$$\lambda_{1,2} \equiv m_{1,2} - \tfrac{i}{2}\Gamma_{1,2} = \left(M - \tfrac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \tfrac{i}{2}\Gamma_{12}\right)\,, \tag{3}$$

where m_1 and Γ_1 are the mass and width of the D_1, *etc.*, and

$$\left|\frac{q}{p}\right|^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\,. \tag{4}$$

We define reduced mixing amplitudes x and y by

$$x \equiv 2M_{12}/\Gamma = (m_1 - m_2)/\Gamma = \Delta m/\Gamma \tag{5}$$

and

$$y \equiv \Gamma_{12}/\Gamma = (\Gamma_1 - \Gamma_2)/2\Gamma = \Delta\Gamma/2\Gamma\,, \tag{6}$$

where $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$. The mixing rate, R_M, is approximately $(x^2 + y^2)/2$. In Eq. (5) and Eq. (6), the middle relation holds only in the limit of CP conservation, in which case the subscripts 1 and 2 denote the CP-even and CP-odd eigenstates.

The parameters x and y are measured in several ways. The most precise constraints are obtained using the time-dependence of D decays. Since D^0–$\overline{D}^0$ mixing is a small effect, the identification tag of the initial particle as a D^0 or a $\overline{D}^0$ must be extremely accurate. The usual tag is the charge of the distinctive slow pion in the decay sequence $D^{*+} \to D^0\pi^+$ or $D^{*-} \to \overline{D}^0\pi^-$. In current experiments, the probability of mistagging is about 0.1%. Another tag of comparable accuracy is identification of one of the D's produced from $\psi(3770) \to D^0\overline{D}^0$. Time-dependent analyses are not possible at symmetric charm threshold facilities (the D^0 and $\overline{D}^0$ do not travel far enough). However, the quantum coherent $D^0\overline{D}^0$ $C = -1$ state provides time-integrated sensitivity [4, 5].

Time-Dependent Analyses: We extend the formalism of this *Review*'s note on "B^0–$\overline{B}^0$ Mixing" [6]. In addition to the "right-sign" instantaneous decay amplitudes $\overline{A}_f \equiv \langle f|H|\overline{D}^0\rangle$ and $A_{\overline{f}} \equiv \langle \overline{f}|H|D^0\rangle$ for CP conjugate final states f and $\overline{f}$, we include the "wrong-sign" amplitudes $\overline{A}_{\overline{f}} \equiv \langle \overline{f}|H|\overline{D}^0\rangle$ and $A_f \equiv \langle f|H|D^0\rangle$.

It is usual to normalize the wrong-sign decay distributions to the integrated rate of right-sign decays and to express time in units of the precisely measured D^0 mean lifetime, $\overline{\tau}_{D^0} = 1/\Gamma = 2/(\Gamma_1 + \Gamma_2)$. Starting from a pure $|D^0\rangle$ or $|\overline{D}^0\rangle$ state at $t = 0$, the time-dependent rates of production of the wrong-sign final states relative to the integrated right-sign states are then

$$r(t) = \frac{\left|\langle f|H|D^0(t)\rangle\right|^2}{\left|\overline{A}_f\right|^2} = \left|\frac{q}{p}\right|^2 \left|g_+(t)\,\chi_f^{-1} + g_-(t)\right|^2 \qquad (7)$$

and

$$\overline{r}(t) = \frac{\left|\langle \overline{f}|H|\overline{D}^0(t)\rangle\right|^2}{\left|A_{\overline{f}}\right|^2} = \left|\frac{p}{q}\right|^2 \left|g_+(t)\,\chi_{\overline{f}} + g_-(t)\right|^2 , \qquad (8)$$

where

$$\chi_f \equiv q\overline{A}_f/pA_f\,, \quad \chi_{\bar{f}} \equiv q\overline{A}_{\bar{f}}/pA_{\bar{f}}\,, \qquad (9)$$

and

$$g_{\pm}(t) = \frac{1}{2}\left(e^{-iz_1 t} \pm e^{-iz_2 t}\right)\,, \quad z_{1,2} = \frac{\lambda_{1,2}}{\Gamma}\,. \tag{10}$$

Note that a change in the convention for the relative phase of D^0 and $\overline{D}^0$ would cancel between q/p and $\overline{A}_f/A_f$ and leave χ_f invariant.

We expand $r(t)$ and $\overline{r}(t)$ to second order in time for modes where the ratio of decay amplitudes $R_D = |A_f/\overline{A}_f|^2$ is very small.

Semileptonic decays: In semileptonic D decays, $A_f = \overline{A}_{\overline{f}} = 0$ in the Standard Model. Then in the limit of weak mixing, where $|ix + y| \ll 1$, $r(t)$ is given by

$$r(t) = |g_-(t)|^2 \left|\frac{q}{p}\right|^2 \approx \frac{e^{-t}}{4}(x^2+y^2)\,t^2 \left|\frac{q}{p}\right|^2\,. \tag{11}$$

For $\overline{r}(t)$ one replaces q/p here with p/q. In the limit of CP conservation, $r(t) = \overline{r}(t)$, and the time-integrated mixing rate relative to the time-integrated right-sign decay rate is

$$R_M = \int_0^\infty r(t)dt = \left|\frac{q}{p}\right|^2 \frac{x^2+y^2}{2+x^2-y^2} \approx \frac{1}{2}(x^2+y^2)\,. \tag{12}$$

Table 1 summarizes results from semileptonic decays.

Table 1: Results for R_M in D^0 semileptonic decays.

Year	Exper.	Final state(s)	R_M (90 (95)% C.L.)
2005	Belle[a]	$K^{(*)+}e^-\overline{\nu}_e$	$< 1.0 \times 10^{-3}$
2005	CLEO[b]	$K^{(*)+}e^-\overline{\nu}_e$	$< 7.8 \times 10^{-3}$
2004	BABAR[c]	$K^{(*)+}e^-\overline{\nu}_e$	$< 4.2(4.6) \times 10^{-3}$
2002	FOCUS [7]	$K^+\mu^-\overline{\nu}_\mu$	$< 1.01(1.31) \times 10^{-3}$
1996	E791[d]	$K^+\ell^-\overline{\nu}_\ell$	$< 5.0 \times 10^{-3}$

See the end of the D^0 listings for these references: [a]BITENC 05, [b]CAWLFIELD 05, [c]AUBERT 04, [d]AITALA 96C.

Wrong-sign decays to hadronic non-CP eigenstates: Consider the final state $f = K^+\pi^-$, where A_f is doubly Cabibbo-suppressed. The ratio of decay amplitudes is

$$\frac{A_f}{\overline{A}_f} = -\sqrt{R_D}\, e^{-i\delta}, \qquad \left|\frac{A_f}{\overline{A}_f}\right| \sim O(\tan^2\theta_c)\,, \tag{13}$$

where R_D is the doubly Cabibbo-suppressed (DCS) decay rate relative to the Cabibbo-favored (CF) rate, the minus sign originates from the sign of V_{us} relative to V_{cd}, and δ is the phase difference between DCS and CF processes not attributed to the first-order electroweak spectator diagram.

We characterize the violation of CP in the mixing amplitude, the decay amplitude, and the interference between mixing and decay, by real-valued parameters A_M, A_D, and ϕ. We adopt a parametrization similar to that of Nir [8] and CLEO [GODANG 00] and express these quantities in a way that is convenient to describe the three types of CP violation:

$$\left|\frac{q}{p}\right| = 1 + A_M\,, \tag{14}$$

$$\chi_f^{-1} \equiv \frac{pA_f}{q\overline{A}_f} = \frac{-\sqrt{R_D}(1+A_D)}{(1+A_M)}\; e^{-i(\delta+\phi)}\,, \tag{15}$$

$$\chi_{\overline{f}} \equiv \frac{q\overline{A}_{\overline{f}}}{pA_{\overline{f}}} = \frac{-\sqrt{R_D}(1+A_M)}{(1+A_D)}\; e^{-i(\delta-\phi)}\,. \tag{16}$$

In general, $\chi_{\overline{f}}$ and χ_f^{-1} are independent complex numbers. To leading order,

$$r(t) = e^{-t} \times \Big[R_D(1+A_D)^2 + \sqrt{R_D}(1+A_M)(1+A_D)y'_{-}t + \frac{(1+A_M)^2 R_M}{2}\, t^2\Big] \tag{17}$$

and

$$\overline{r}(t) = e^{-t} \times \Big[\frac{R_D}{(1+A_D)^2} + \frac{\sqrt{R_D}}{(1+A_D)(1+A_M)} y'_{+} t + \frac{R_M}{2(1+A_M)^2}\, t^2\Big]. \tag{18}$$

Here

$$y'_{\pm} \equiv y' \cos\phi \pm x' \sin\phi = y \cos(\delta \mp \phi) - x \sin(\delta \mp \phi) \quad (19)$$

$$y' \equiv y \cos\delta - x \sin\delta, \quad x' \equiv x \cos\delta + y \sin\delta\,, \quad (20)$$

and R_M is the mixing rate relative to the time-integrated right-sign rate.

The three terms in Eq. (17) and Eq. (18) probe the three fundamental types of CP violation. In the limit of CP conservation, A_M, A_D, and ϕ are all zero, and then

$$r(t) = \overline{r}(t) = e^{-t}\left(R_D + \sqrt{R_D}\, y' t + \frac{1}{2} R_M\, t^2\right)\,, \quad (21)$$

and the time-integrated wrong-sign rate relative to the integrated right-sign rate is

$$R = \int_0^\infty r(t)\, dt = R_D + \sqrt{R_D}\, y' + R_M\,. \quad (22)$$

The ratio R is the most readily accessible experimental quantity. Table 2 gives recent measurements of R in $D^0 \to K^+\pi^-$ decay. The average of these results, $R = (0.376 \pm 0.009)\,\%$, is about two standard deviations from the average of earlier, less precise results, $R = (0.81 \pm 0.23)\,\%$, which we have omitted.

Table 2: Results for R in $D^0 \to K^+\pi^-$.

Year	Exper.	Technique	$R(\times 10^{-3})$	$A_D(\%)$
2006	Belle[a]	$e^+e^- \to \Upsilon(4S)$	$3.77 \pm 0.08 \pm 0.05$	—
2005	FOCUS[b]	γ BeO	$4.29 \pm 0.63 \pm 0.28$	$18.0 \pm 14.0 \pm 4.1$
2003	BABAR[c]	$e^+e^- \to \Upsilon(4S)$	$3.57 \pm 0.22 \pm 0.27$	$9.5 \pm 6.1 \pm 8.3$
2000	CLEO[d]	$e^+e^- \to \Upsilon(4S)$	$3.32^{+0.63}_{-0.65} \pm 0.40$	$2^{+19}_{-20} \pm 1$

See the end of the D^0 listings for these references: [a]ZHANG 06, [b]LINK 05, [c]AUBERT 03Z, [d]GODANG 00.

The contributions to R—allowing for CP violation—can be extracted by fitting the $D^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^-\pi^+$ decay rates. Table 2 gives the constraints on A_D with $x' = y' = 0$. Table 3 summarizes the results for y' and $x'^2/2$. Figure 1 shows the two-dimensional allowed regions. No meaningful constraints on A_M and ϕ have been reported.

Table 3: Results from studies of the time dependence $r(t)$.

Year	Exper.	y' (95% C.L.)	$x'^2/2$ (95% C.L.)
2006	Belle[a]	$-2.8 < y' < 2.1$ %	< 0.036 %
2005	FOCUS[b]	$-11.2 < y' < 6.7$ %	< 0.40 %
2003	BABAR[c]	$-5.6 < y' < 3.9$ %	< 0.11 %
2000	CLEO[d]	$-5.8 < y' < 1.0$ %	< 0.041 %

See the end of the D^0 listings for these references: [a]ZHANG 06, [b]LINK 05, [c]AUBERT 03Z, [d]GODANG 00.

Extraction of the amplitudes x and y from the results in Table 3 requires knowledge of the relative strong phase δ, a subject of theoretical discussion [4,9–11]. In most cases, it appears difficult for theory to accommodate $\delta > 25°$, although the judicious placement of a $K\pi$ resonance could allow δ to be as large as 40°.

A quantum interference effect that provides useful sensitivity to δ arises in the decay chain $\psi(3770) \to D^0\overline{D}^0 \to (f_{cp})(K^+\pi^-)$, where f_{cp} denotes a CP eigenstate from D^0 decay, such as K^+K^-[1, 16]. Here, the amplitude triangle relation

$$\sqrt{2}\, A(D_\pm \to K^-\pi^+) = A(D^0 \to K^-\pi^+) \pm A(\overline{D}^0 \to K^-\pi^+), \tag{23}$$

where $D_\pm$ denotes a CP eigenstate, implies that

$$\cos\delta = \frac{B(D_+ \to K^-\pi^+) - B(D_- \to K^-\pi^+)}{2\sqrt{R_D}\, B(D^0 \to K^-\pi^+)} , \tag{24}$$

neglecting CP violation and exploiting $R_D \ll \sqrt{R_D}$.

The strong phase δ might also be determined by constructing amplitude quadrangles from a complete set of branching fraction measurements of the other DCS D decays to two pseudoscalars [12]. This analysis would have to assume that the amplitudes from both $\Delta I = 1$ and $\Delta I = 0$ that populate the total $I = 1/2$ $K\pi$ state have the same strong phase relative to the amplitude that populates the total $I = 3/2$ $K\pi$ state.

The Dalitz-plot analyses of DCS D decays to a pseudoscalar and a vector allow the measurement of the relative strong phase

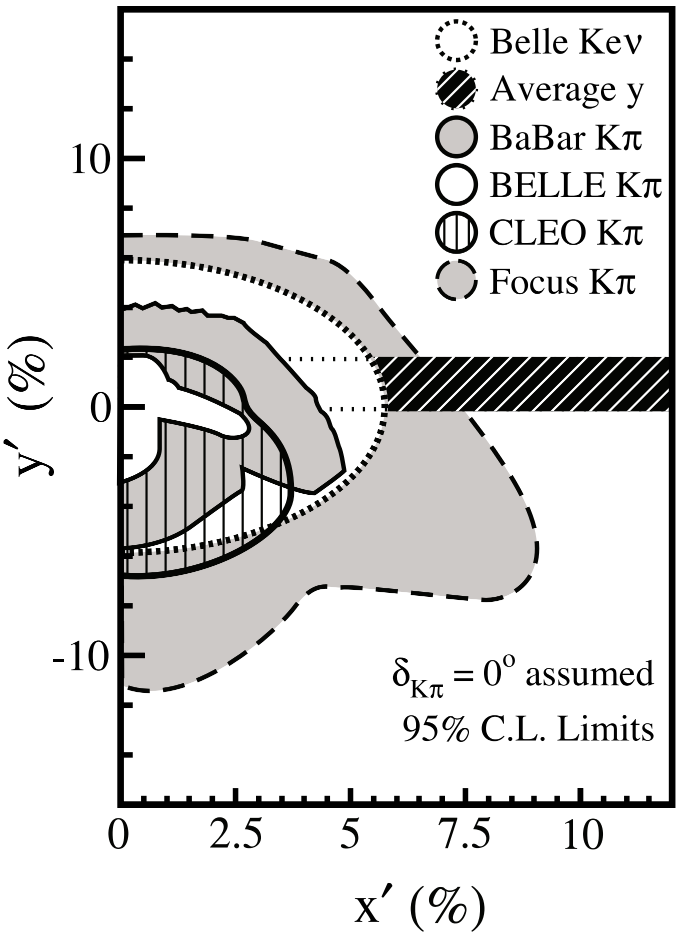

Figure 1: Allowed regions in the $x'y'$ plane. The allowed region for y is the average of the results from E791[a], FOCUS[b], CLEO[c], BABAR[d], and Belle[e]. Also shown is the limit from $D^0 \to K^{(*)}\ell\nu$ from Belle[f] and limits from $D \to K\pi$ from CLEO[g], BABAR[h], Belle[i] and FOCUS[j]. The CLEO, BABAR and Belle results allow CP violation in the decay and mixing amplitudes, and in the interference between these two processes. The FOCUS result does not allow CP violation. We assume $\delta = 0$ to place the y results. A non-zero δ would rotate the $D^0 \to CP$ eigenstates confidence region clockwise about the origin by δ. All results are consistent with the absence of mixing. See the end of the D^0 listings for these references: [a]AITALA 99E, [b]LINK 00, [c]CSORNA 02, [d]AUBERT 03P, [e]ABE 02I, [f]BITENC 05, [g]GODANG 00, [h]AUBERT 03Z, [i]ZHANG 06, [j]LINK 05. See full-color version on color pages at end of book.

between some amplitudes, providing additional constraints to the amplitude quadrangle [13] and thus the determination of the strong phase difference between the relevant DCS and CF amplitudes. In $D^0 \to K_S^0\pi^+\pi^-$, the DCS and CF decay amplitudes populate the same Dalitz plot, which allows direct measurement of the relative strong phase. CLEO has measured the relative phase between $D^0 \to K^*(892)^+\pi^-$ and $D^0 \to K^*(892)^-\pi^+$ to be $(189 \pm 10 \pm 3^{+15}_{-5})^\circ$ [MURAMATSU 02], consistent with the 180° expected from Cabibbo factors and a small strong phase.

There are several results for R measured in multibody final states with nonzero strangeness. Here R, defined in Eq. (22), becomes an average over the Dalitz space, weighted by experimental efficiencies and acceptance. Table 4 summarizes the results.

Table 4: Results for R in $D^0 \to K^{(*)+}\pi^-(n\pi)$.

Year	Exper.	D^0 final state	$R(\%)$
2005	Belle[a]	$K^+\pi^-\pi^+\pi^-$	$0.320 \pm 0.019^{+0.018}_{-0.013}$
2005	Belle[a]	$K^+\pi^-\pi^0$	$0.229 \pm 0.017^{+0.013}_{-0.009}$
2002	CLEO[b]	$K^{*+}\pi^-$	$0.5 \pm 0.2^{+0.6}_{-0.1}$
2001	CLEO[c]	$K^+\pi^-\pi^+\pi^-$	$0.41^{+0.12}_{-0.11} \pm 0.04$
2001	CLEO[d]	$K^+\pi^-\pi^0$	$0.43^{+0.11}_{-0.10} \pm 0.07$
1998	E791[e]	$K^+\pi^-\pi^+\pi^-$	$0.68^{+0.34}_{-0.33} \pm 0.07$

See the end of the D^0 listings for these references: [a]TIAN 05, [b]MURAMATSU 02, [c]DYTMAN 01, [d]BRANDENBURG 01, [e]AITALA 98.

For multibody final states, Eqs. (13)–(22) apply to one point in the Dalitz space. Although x and y do not vary across the space, knowledge of the resonant substructure is needed to extrapolate the strong phase difference δ from point to point. Both the sign and magnitude of x and y may be measured using the time-dependent resonant substructure of multibody D^0 decays. CLEO has performed a time-dependent Dalitz-plot analysis of $D^0 \to K_S^0\pi^+\pi^-$, and reports $(-4.5 < x < 9.3)\%$ and

$(-6.4 < y < 3.6)\%$ at the 95% confidence level, without phase or sign ambiguity [ASNER 05], as shown in Figure 2.

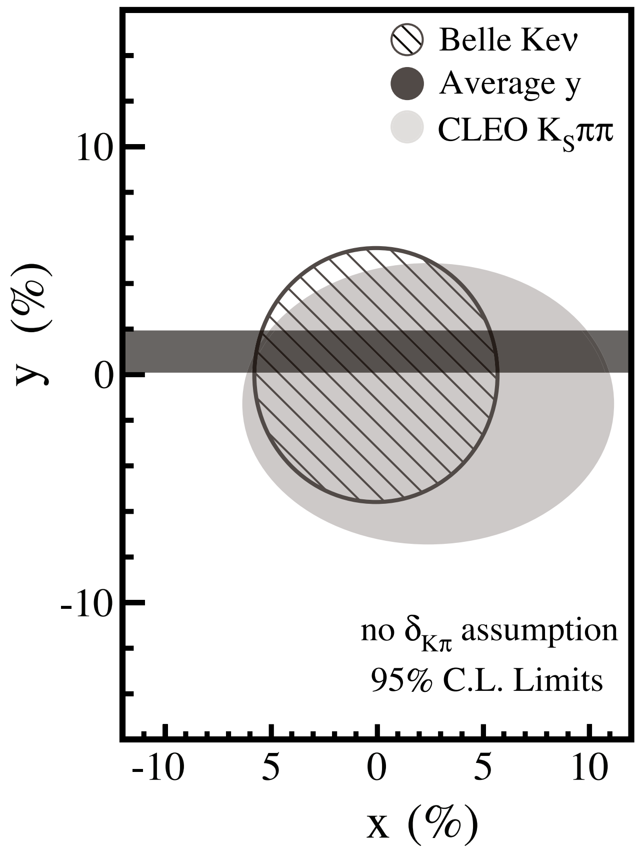

Figure 2: Allowed regions in the xy plane. No assumption is made regarding δ. The allowed region for y is the average of the results from E791[a], FOCUS[b], CLEO[c], BABAR[d], and Belle[e]. Also shown is the limit from $D^0 \to K^{(*)}\ell\nu$ from Belle[f]. The CLEO experiment has constrained x and y with the time-dependent Dalitz-plot analysis of $D^0 \to K^0_S\pi^+\pi^-$[g]. All results are consistent with the absence of mixing. See the end of the D^0 listings for these references: [a]AITALA 99E, [b]LINK 00, [c]CSORNA 02, [d]AUBERT 03P, [e]ABE 02I, [f]BITENC 05, [g]ASNER 05.

Decays to CP Eigenstates: When the final state f is a CP eigenstate, there is no distinction between f and $\overline{f}$, and then $A_f = A_{\overline{f}}$ and $\overline{A}_{\overline{f}} = \overline{A}_f$. We denote final states with CP eigenvalues ± 1 by $f_\pm$. In analogy with Eqs. (7)–(8), the decay rates to CP eigenstates are then

$$\begin{aligned} r_\pm(t) = & \frac{\left|\langle f_\pm|H|D^0(t)\rangle\right|^2}{\left|\overline{A}_\pm\right|^2} \\ & = \frac{1}{4}\left| h_\pm(t)\left(\frac{A_\pm}{\overline{A}_\pm} \pm \frac{q}{p}\right) + h_\mp(t)\left(\frac{A_\pm}{\overline{A}_\pm} \mp \frac{q}{p}\right)\right|^2, \\ & \propto \frac{1}{|p|^2}\left|h_\pm(t) + \eta_\pm h_\mp(t)\right|^2, \end{aligned} \tag{25}$$

and

$$\overline{r}_\pm(t) = \frac{\left|\langle f_\pm|H|\overline{D}^0(t)\rangle\right|^2}{\left|A_\pm\right|^2} \propto \frac{1}{|q|^2}\left|h_\pm(t) - \eta_\pm h_\mp(t)\right|^2, \tag{26}$$

where

$$h_\pm(t) = g_+(t) \pm g_-(t) = e^{-iz_\pm t}\,, \tag{27}$$

and

$$\eta_\pm \equiv \frac{pA_\pm \mp q\overline{A}_\pm}{pA_\pm \pm q\overline{A}_\pm} = \frac{1 \mp \chi_\pm}{1 \pm \chi_\pm}\,. \tag{28}$$

The variable $\eta_\pm$ describes CP violation; it can receive contributions from each of the three fundamental types of CP violation.

The quantity y may be measured by comparing the rate for decays to non-CP eigenstates such as $D^0 \to K^-\pi^+$ with decays to CP eigenstates such as $D^0 \to K^+K^-$ [11]. A positive y would make K^+K^- decays appear to have a shorter lifetime than $K^-\pi^+$ decays. The decay rate for a D^0 into a CP eigenstate is not described by a single exponential in the presence of CP violation.

In the limit of weak mixing, where $|ix + y| \ll 1$, and small CP violation, where $|A_M|$, $|A_D|$, and $|\sin\phi| \ll 1$, the time dependence of decays to CP eigenstates is proportional to a single exponential:

$$r_\pm(t) \propto \exp\left(-[1 \pm \left|\frac{p}{q}\right|(y\cos\phi - x\sin\phi)]t\right), \tag{29}$$

$$\overline{r}_\pm(t) \propto \exp\left(-[1 \pm \left|\frac{q}{p}\right|(y\cos\phi + x\sin\phi)]t\right), \tag{30}$$

$$r_\pm(t) + \overline{r}_\pm(t) \propto e^{-(1\pm y_{CP})t}. \tag{31}$$

Here

$$\begin{aligned} y_{CP} = y\cos\phi &\left[\frac{1}{2}\left(\left|\frac{p}{q}\right| + \left|\frac{q}{p}\right|\right) + \frac{A_{\text{prod}}}{2}\left(\left|\frac{p}{q}\right| - \left|\frac{q}{p}\right|\right)\right] \\ - x\sin\phi &\left[\frac{1}{2}\left(\left|\frac{p}{q}\right| - \left|\frac{q}{p}\right|\right) + \frac{A_{\text{prod}}}{2}\left(\left|\frac{p}{q}\right| + \left|\frac{q}{p}\right|\right)\right], \end{aligned} \tag{32}$$

and

$$A_{\text{prod}} \equiv \frac{N(D^0) - N(\overline{D}^0)}{N(D^0) + N(\overline{D}^0)} \tag{33}$$

is defined as the production asymmetry of the D^0 and $\overline{D}^0$.

The possibility of CP violation has been considered in the limit of weak mixing and small CP violation. In this limit there is no sensitivity to CP violation in direct decay. Belle [14] and BABAR [AUBERT 03P] have measure A_Γ, where

$$A_\Gamma \equiv \frac{r_\pm(t) - \overline{r}_\pm(t)}{r_\pm(t) - \overline{r}_\pm(t)} \approx A_M\, y\cos\phi - x\sin\phi,$$

allowing CP violation in interference and mixing.

In the limit of CP conservation, $A_\pm = \pm\overline{A}_\pm$, $\eta_\pm = 0$, $y = y_{CP}$, and

$$r_\pm(t)\left|\overline{A}_\pm\right|^2 = \overline{r}_\pm(t)\left|A_\pm\right|^2 \propto e^{-(1\pm y_{CP})t}\,. \tag{34}$$

All measurements of y and A_Γ are relative to the $D^0 \to K^-\pi^+$ decay rate. Table 5 summarizes the current status of measurements. The average of the six y_{CP} measurements is $0.90 \pm 0.42\,\%$.

Table 5: Results for y from $D^0 \to K^+K^-$ and $\pi^+\pi^-$.

Year	Exper.	D^0 final state(s)	$y_{CP}(\%)$	$A_\Gamma(\times 10^{-3})$
2003	Belle [14]	K^+K^-	$1.15 \pm 0.69 \pm 0.38$	$-2.0 \pm 6.3 \pm 3.0$
2003	BABAR[a]	$K^+K^-, \pi^+\pi^-$	$0.8 \pm 0.4^{+0.5}_{-0.4}$	$-8 \pm 6 \pm 2$
2001	CLEO[b]	$K^+K^-, \pi^+\pi^-$	$-1.1 \pm 2.5 \pm 1.4$	—
2001	Belle[c]	K^+K^-	$-0.5 \pm 1.0^{+0.7}_{-0.8}$	—
2000	FOCUS[d]	K^+K^-	$3.4 \pm 1.4 \pm 0.7$	—
1999	E791[e]	K^+K^-	$0.8 \pm 2.9 \pm 1.0$	—

See the end of the D^0 listings for these references: [a]AUBERT 03P, [b]CSORNA 02, [c]ABE 02I, [d]LINK 00, [e]AITALA 99E.

Substantial work on the integrated CP asymmetries in decays to CP eigenstates indicates that A_{CP} is consistent with zero at the few percent level [15]. The expression for the integrated CP asymmetry that includes the possibility of CP violation in mixing is

$$A_{CP} = \frac{\Gamma(D^0 \to f_\pm) - \Gamma(\overline{D}^0 \to f_\pm)}{\Gamma(D^0 \to f_\pm) + \Gamma(\overline{D}^0 \to f_\pm)} \tag{35}$$

$$= \frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} + 2\mathrm{Re}(\eta_\pm)\,. \tag{36}$$

Coherent $D^0\overline{D}^0$ Analyses: Measurements of R_D, $\cos\delta$, x, and y can be made simultaneously in a combined fit to the single-tag (ST) and double-tag (DT) yields or individually by a series of "targeted" analyses [16, 17].

The "comprehensive" analysis simultaneously measures mixing and DCS parameters by examining various ST and DT rates. Due to quantum correlations in the $C=-1$ and $C=+1$ $D^0\overline{D}^0$ pairs produced in the reactions $e^+e^- \to D^0\overline{D}^0(\pi^0)$ and $e^+e^- \to D^0\overline{D}^0\gamma(\pi^0)$, respectively, the time-integrated $D^0\overline{D}^0$ decay rates are sensitive to interference between amplitudes for indistinguishable final states. The size of this interference is governed by the relevant amplitude ratios and can include contributions from D^0-$\overline{D}^0$ mixing.

Table 6: CLEO-c results from time-integrated yields at $\psi(3770) \to D\bar{D}$.

Parameter	CLEO-c fitted value	Other results
y (Table 5)	-0.058 ± 0.066	$(0.90 \pm 0.42)\%$
$\cos \delta_{K\pi}$	1.09 ± 0.66	—
R_M (Table 1)	$(1.7 \pm 1.5) \times 10^{-3}$	$<0.1\%$ (95% C.L.)
$x^2/2$ (Table 3)	$< 0.44\%$ @(95% C.L.)	$<0.036\%$ (95% C.L.)

The following categories of final states are considered:

f or $\bar{f}$: Hadronic states accessed from either D^0 or $\overline{D}^0$ decay but that are not CP eigenstates. An example is $K^-\pi^+$, which results from Cabibbo-favored D^0 transitions or DCS $\overline{D}^0$ transitions.

ℓ^+ or ℓ^-: Semileptonic or purely leptonic final states, which, in the absence of mixing, tag unambiguously the flavor of the parent D.

S_+ or S_-: CP-even and CP-odd eigenstates, respectively.

The decay rates for $D^0\overline{D}^0$ pairs to all possible combinations of the above categories of final states are calculated in Ref. [4], for both $C = -1$ and $C = +1$, reproducing the work of Refs. [5, 10]. Such $D^0\overline{D}^0$ combinations, where both D final states are specified, are double tags. In addition, the rates for single tags, where either the D^0 or $\overline{D}^0$ is identified and the other neutral D decays generically are given in Ref. [4].

CLEO-c has reported results using 281 pb^{-1} of $e^+e^- \to \psi(3770)$ data [18], where the quantum coherent $D^0\overline{D}^0$ pairs are in the $C = -1$ state. The values of y, R_M, and $\cos\delta$ are determined from a combined fit to the ST (hadronic only) and DT yields. The hadronic final states included in the analysis are $K^-\pi^+$ (f), $K^+\pi^-$ ($\bar{f}$), K^-K^+ ($S+$), $\pi^+\pi^-$ (S_+), $K^0_S\pi^0\pi^0$ (S_+), and $K^0_S\pi^0$ (S_-). Both of the two flavored final states, $K^-\pi^+$ and $K^+\pi^-$, can be reached via CF or DCS transitions.

Semileptonic DT yields are also included, where one D is fully reconstructed in one of the hadronic modes listed above, and the other D is partially reconstructed, requiring that only the electron be found. When the electron is accompanied by a flavor tag ($D \to K^-\pi^+$ or $K^+\pi^-$), only the "right-sign" DT sample, where the electron and kaon charges are the same, is used. Extraction of the DCS "wrong-sign" semileptonic yield is not feasible with the current CLEO-c data sample, and the parameter $r_{K\pi}$ is constrained to the world average. Table 6 shows the results of the fit to the CLEO-c data.

References

1. G. Burdman and I. Shipsey, Ann. Rev. Nucl. and Part. Sci. **53**, 431 (2003).
2. H.N. Nelson, in *Proceedings of the 19th Intl. Symp. on Lepton and Photon Interactions at High Energy LP99*, ed. J.A. Jaros and M.E. Peskin, SLAC (1999); A.A. Petrov, `hep-ph/0311371`, contributed to Flavor Physics and CP Violation (FPCP2003), Paris, June 2003.
3. I.I. Bigi and N.G. Uraltsev, Nucl. Phys. **B592**, 92 (2001); C.K. Chua and W.S. Hou, `hep-ph/0110106`; A.F. Falk *et al.*, Phys. Rev. **D65**, 054034 (2002); S. Bianco *et al.*, Riv. Nuovo Cim. **26N7**, 1 (2003); A. F. Falk *et al.*, Phys. Rev. D **69**, 114021 (2004) E. Golowich and A. A. Petrov, Phys. Lett. B **625**, 53 (2005).
4. D. M. Asner and W. M. Sun, Phys. Rev. D **73**, 034024 (2006).
5. D. Atwood and A. A. Petrov, Phys. Rev. D **71**, 054032 (2005); Z. z. Xing, Phys. Rev. D **55**, 196 (1997); M. Goldhaber and J. L. Rosner, Phys. Rev. D **15**, 1254 (1977).
6. See the review on B^0–$\overline{B}^0$ mixing in this *Review*.
7. K. Stenson, presented at the April Meeting of the American Physical Society (APS 03), Philadelphia, Pennsylvania, April 5-8, 2003; M. Hosack, (FOCUS Collab.), Fermilab-Thesis-2002-25.
8. Y. Nir, Lectures given at 27th SLAC Summer Institute on Particle Physics: CP Violation in and Beyond the Standard Model (SSI 99), Stanford, California, 7-16 Jul 1999. Published in Trieste 1999, *Particle Physics*, pp. 165-243.
9. L. Chau and H. Cheng, Phys. Lett. **B333**, 514 (1994); T.E. Browder and S. Pakvasa, Phys. Lett. **B383**, 475 (1995); A.F. Falk, Y. Nir, and A.A. Petrov, JHEP **9912**, 19 (1999); G. Blaylock, A. Seiden, and Y. Nir, Phys. Lett. **B355**, 555 (1995).
10. M. Gronau *et al.*, Phys. Lett. B **508**, 37 (2001).
11. S. Bergmann *et al.*, Phys. Lett. **B486**, 418 (2000).
12. E. Golowich and S. Pakvasa, Phys. Lett. **B505**, 94 (2001).

13. C.W. Chiang and J.L. Rosner, Phys. Rev. **D65**, 054007 (2002).

14. K. Abe *et al.*, (Belle Collab.), hep-ex/0308034, contributed to the 21st International Symposium on Lepton and Photon Interactions at High Energies, (LP 03), Batavia, Illinois, 11-16 Aug 2003.

15. See the tabulation of A_{CP} results in the decays of D^0 and D^+ in this *Review*.

16. R. A. Briere *et al.*, (CLEO Collab.), CLNS 01-1742, (2001).

17. G. Cavoto *et al.*, *Prepared for 3rd Workshop on the Unitarity Triangle: CKM 2005, San Diego, California, 15-18 Mar 2005*, hep-ph/0603019.

18. W. Sun, for the CLEO Collaboration, *To appear in the proceedings of Particles and Nuclei International Conference (PANIC 05), Santa Fe, New Mexico, 24-28 Oct 2005*, hep-ex/0603031.