THE MUON ANOMALOUS MAGNETIC MOMENT

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The Dirac equation predicts a muon magnetic moment, $\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{S}$, with gyromagnetic ratio $g_\mu = 2$. Quantum loop effects lead to a small calculable deviation from $g_\mu = 2$, parameterized by the anomalous magnetic moment

$$a_\mu \equiv \frac{g_\mu - 2}{2} \ . \tag{1}$$

That quantity can be accurately measured and, within the Standard Model (SM) framework, precisely predicted. Hence, comparison of experiment and theory tests the SM at its quantum loop level. A deviation in $a_\mu^{\rm exp}$ from the SM expectation would signal effects of new physics, with current sensitivity reaching up to mass scales of $\mathcal{O}$(TeV) [1, 2].

The recently completed experiment E821 at Brookhaven National Lab (BNL) studied the precession of μ^+ and μ^- in a constant external magnetic field as they circulated in a confining storage ring. It found [3]

$$\begin{aligned} a_{\mu^+}^{\rm exp} &= 11\,659\,203(6)(5) \times 10^{-10} \ , \\ a_{\mu^-}^{\rm exp} &= 11\,659\,214(8)(3) \times 10^{-10} \ , \end{aligned} \tag{2}$$

where the first errors are statistical and the second systematic. Assuming CPT invariance and taking into account correlations between systematic errors, one finds for their average [3]

$$a_\mu^{\rm exp} = 11\,659\,208.0(5.4)(3.3) \times 10^{-10} \ . \tag{3}$$

These results represent about a factor of 14 improvement over the classic CERN experiments of the 1970's [4].

The SM prediction for $a_\mu^{\rm SM}$ is generally divided into three parts (see Fig. 1 for representative Feynman diagrams)

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm Had} \ . \tag{4}$$

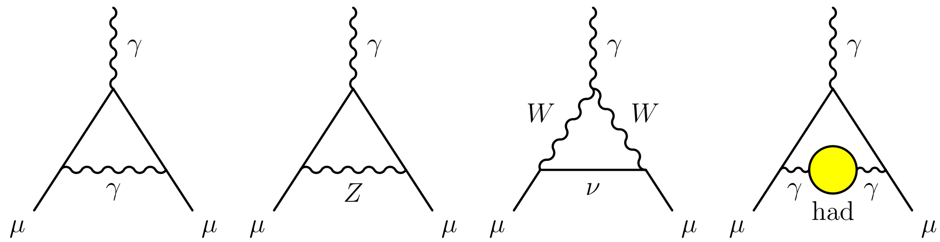

Figure 1: Representative diagrams contributing to $a_\mu^{\rm SM}$. From left to right: first order QED (Schwinger term), lowest-order weak, lowest-order hadronic.

The QED part includes all photonic and leptonic (e, μ, τ) loops starting with the classic $\alpha/2\pi$ Schwinger contribution. It has now been computed through 4 loops and estimated at the 5-loop level [5]

$$a_\mu^{\rm QED} = \frac{\alpha}{2\pi} + 0.76585741(3)\left(\frac{\alpha}{\pi}\right)^2 + 24.0505096(4)\left(\frac{\alpha}{\pi}\right)^3 + 131.01(1)\left(\frac{\alpha}{\pi}\right)^4 + 663(20)\left(\frac{\alpha}{\pi}\right)^5 + \cdots \tag{5}$$

Employing $\alpha^{-1} = 137.0359988(5)$, determined [5] from the electron a_e measurement, leads to

$$a_\mu^{\rm QED} = 116\,584\,719.0(0.1)(0.4) \times 10^{-11} \ , \tag{6}$$

where the errors result from uncertainties in the coefficients of Eq.(5) and in α (see the reviews in [2] and [6]) . Although the uncertainty in α is already very small, an experiment underway at Harvard aims to reduce the error on a_e from which it is derived by a factor of 15 [7].

Loop contributions involving heavy $W^\pm, Z$ or Higgs particles are collectively labeled as $a_\mu^{\rm EW}$. They are suppressed by at least a factor of $\frac{\alpha}{\pi}\frac{m_\mu^2}{m_W^2} \simeq 4 \times 10^{-9}$. At 1-loop order [8]

$$a_\mu^{\rm EW}[\text{1-loop}] = \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left[\frac{5}{3} + \frac{1}{3}\left(1 - 4\sin^2\theta_{\rm W}\right)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_W^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right] \ , \tag{7}$$

$$= 194.8 \times 10^{-11} \ , \qquad \text{for} \ \ \sin^2\theta_{\rm W} \equiv 1 - \frac{M_W^2}{M_Z^2} \simeq 0.223 \ . \quad (8)$$

Two-loop corrections are relatively large and negative [9]

$$a_\mu^{\rm EW}[2-\text{loop}] = -40.7(1.0)(1.8) \times 10^{-11} \ , \qquad (9)$$

where the errors stem from quark triangle loops and the assumed Higgs mass range $m_H = 150^{+100}_{-40}$ GeV. The 3-loop leading logarithms are negligible [9,10], $\mathcal{O}(10^{-12})$, implying in total

$$a_\mu^{\rm EW} = 154(1)(2) \times 10^{-11} \ . \qquad (10)$$

Hadronic (quark and gluon) loop contributions to $a_\mu^{\rm SM}$ give rise to its main theoretical uncertainties. At present, those effects are not calculable from first principles, but such an approach may become possible as lattice QCD matures. Instead, one currently relies on a dispersion relation approach to evaluate the lowest-order (*i.e.*, $\mathcal{O}(\alpha^2)$) hadronic vacuum polarization contribution $a_\mu^{\rm Had}[LO]$ from corresponding cross section measurements [11]

$$a_\mu^{\rm Had}[LO] = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} R^{(0)}(s) \ , \qquad (11)$$

where $K(s)$ is a QED kernel function [12], and where $R^{(0)}(s)$ denotes the ratio of the bare* cross section for e^+e^- annihilation into hadrons to the pointlike muon-pair cross section at center-of-mass energy $\sqrt{s}$. The function $K(s) \sim 1/s$ in Eq. (11) gives a strong weight to the low-energy part of the integral. Hence, $a_\mu^{\rm Had}[LO]$ is dominated by the $\rho(770)$ resonance.

Currently, the available $\sigma(e^+e^- \to \text{hadrons})$ data give a leading order hadronic vacuum polarization (representative) contribution of [13]

* The bare cross section is defined as the measured cross section corrected for initial-state radiation, electron-vertex loop contributions and vacuum-polarization effects in the photon propagator. However, QED effects in the hadron vertex and final state, as photon radiation, must be included.

$$a_\mu^{\rm Had}[LO] = 6\,963(62)(36) \times 10^{-11} \ , \qquad (12)$$

where the errors correspond to experimental, dominated by systematic uncertainties, and QED radiative corrections to the data.

Alternatively, one can use precise vector spectral functions from $\tau \to \nu_\tau +$ hadrons decays [14] that can be related to isovector $e^+e^- \to$ hadrons cross sections by isospin rotation. When isospin-violating corrections (from QED and $m_d - m_u \neq 0$) are applied, one finds [13]

$$a_\mu^{\rm Had}[LO] = 7\,110(50)(8)(28) \times 10^{-11} \ (\tau) \ , \qquad (13)$$

where the errors are statistical and systematic, and where the last error is an estimate for the uncertainty in the isospin-breaking corrections. The discrepancy between the e^+e^- and τ-based determinations of $a_\mu^{\rm Had}[LO]$ is currently unexplained. It may be indicative of problems with one or both data sets. It may also suggest the need for additional isospin-violating corrections to the τ data. Preliminary new low-energy e^+e^- and τ data may help to resolve this discrepancy and should reduce the hadronic uncertainty.

Higher order, $\mathcal{O}(\alpha^3)$, hadronic contributions are obtained from the same $e^+e^- \to$ hadrons data [14–16] along with model-dependent estimates of the hadronic light-by-light scattering contribution motivated by large-N_C QCD [17]. Following [2], one finds

$$a_\mu^{\rm Had}[NLO] = 22(35) \times 10^{-11} \ , \qquad (14)$$

where the error is dominated by hadronic light-by-light uncertainties.

Adding Eqs. (6), (10), (12), and (14) gives the representative e^+e^- data-based SM prediction (which includes recent changes in the QED and hadronic light by light contributions)

$$a_\mu^{\rm SM} = 116\,591\,858(72)(35)(3) \times 10^{-11} \ . \qquad (15)$$

The difference between experiment and theory

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 22(10) \times 10^{-10} \ , \tag{16}$$

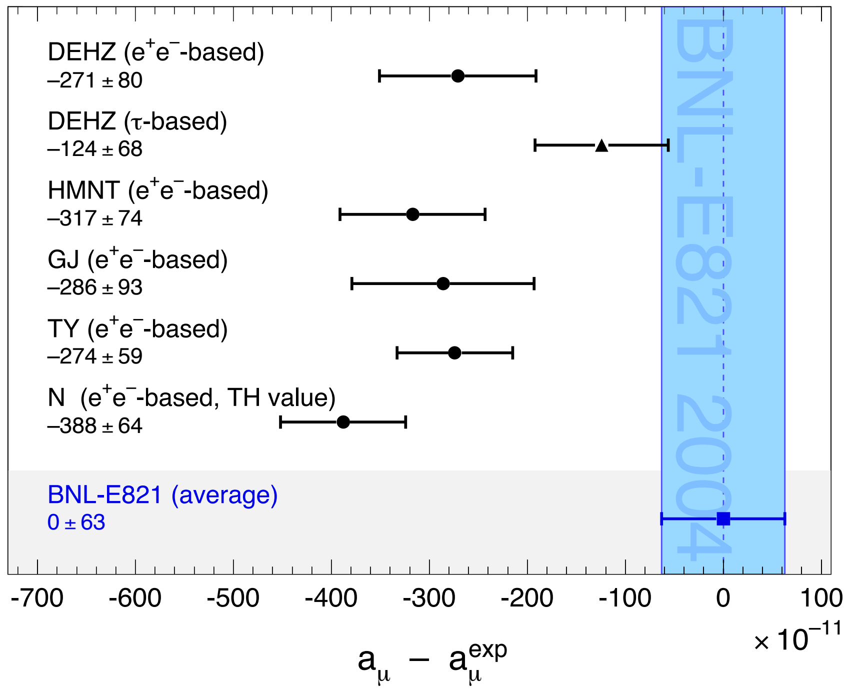

Figure 2: Compilation of recently published results for a_μ (in units of 10^{-11}), subtracted by the central value of the experimental average (3). The shaded band indicates the experimental error. The SM predictions are taken from: DEHZ [13], HMNT [16], GJ [18], TY [19], N [20]. Note that the quoted errors do not include the uncertainty on the subtracted experimental value. To obtain for each theory calculation a result equivalent to Eq. (16), one has to add the errors from theory and experiment in quadrature. See full-color version on color pages at end of book.

(with all errors combined in quadrature) represents an interesting but not compelling discrepancy of 2.2 times the estimated 1σ error. Using the recent estimates for the hadronic contribution compiled in Fig. 2, this discrepancy can exhibit up to 3σ. Those larger discrepancies arise in part because the published results illustrated there have not been updated to include more recent evaluations of the QED [5] and hadronic light-by-light [2,17] contributions. Switching to τ data reduces the discrepancy by about a factor of 3, assuming the isospin-violating corrections are under control within the estimated uncertainties.

An alternate interpretation is that Δa_μ may be a new physics signal with supersymmetric particle loops as the leading candidate explanation. Such a scenario is quite natural, since generically, supersymmetric models predict [1] an additional contribution to $a_\mu^{\rm SM}$

$$a_\mu^{\rm SUSY} \simeq \pm\, 130 \times 10^{-11} \cdot \left(\frac{100\ {\rm GeV}}{m_{\rm SUSY}}\right)^2 \tan\beta\ , \qquad (17)$$

where $m_{\rm SUSY}$ is a representative supersymmetric mass scale, and $\tan\beta \simeq 3$–40 is a potential enhancement factor. Supersymmetric particles in the mass range 100–500 GeV could be the source of the deviation Δa_μ. If so, those particles could be directly observed at the next generation of high energy colliders. New physics effects [1] other than supersymmetry could also explain a non-vanishing Δa_μ.

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