\textbf{CP VIOLATION IN \( K_S \to 3\pi \)}

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The possible final states for the decay \( K^0 \to \pi^+\pi^-\pi^0 \) have isospin \( I = 0, 1, 2, \) and \( 3 \). The \( I = 0 \) and \( I = 2 \) states have \( CP = +1 \) and \( K_S \) can decay into them without violating \( CP \) symmetry, but they are expected to be strongly suppressed by centrifugal barrier effects. The \( I = 1 \) and \( I = 3 \) states, which have no centrifugal barrier, have \( CP = -1 \) so that the \( K_S \) decay to these requires \( CP \) violation.

In order to see \( CP \) violation in \( K_S \to \pi^+\pi^-\pi^0 \), it is necessary to observe the interference between \( K_S \) and \( K_L \) decay, which determines the amplitude ratio

\[
\eta_{+0} = \frac{A(K_S \to \pi^+\pi^-\pi^0)}{A(K_L \to \pi^+\pi^-\pi^0)}.
\]  

(1)

If \( \eta_{+0} \) is obtained from an integration over the whole Dalitz plot, there is no contribution from the \( I = 0 \) and \( I = 2 \) final states and a nonzero value of \( \eta_{+0} \) is entirely due to \( CP \) violation.

Only \( I = 1 \) and \( I = 3 \) states, which are \( CP = -1 \), are allowed for \( K^0 \to \pi^0\pi^0\pi^0 \) decays and the decay of \( K_S \) into \( 3\pi^0 \) is an unambiguous sign of \( CP \) violation. Similarly to \( \eta_{+0} \), \( \eta_{000} \) is defined as

\[
\eta_{000} = \frac{A(K_S \to \pi^0\pi^0\pi^0)}{A(K_L \to \pi^0\pi^0\pi^0)}.
\]  

(2)

If one assumes that \( CPT \) invariance holds and that there are no transitions to \( I = 3 \) (or to nonsymmetric \( I = 1 \) states), it can be shown that

\[
\eta_{+0} = \eta_{000} = \epsilon + i \frac{\text{Im } a_1}{\text{Re } a_1}.
\]  

(3)

With the Wu-Yang phase convention, \( a_1 \) is the weak decay amplitude for \( K^0 \) into \( I = 1 \) final states; \( \epsilon \) is determined from \( CP \) violation in \( K_L \to 2\pi \) decays. The real parts of \( \eta_{+0} \) and \( \eta_{000} \) are equal to \( \text{Re}(\epsilon) \). Since currently-known upper limits on \( |\eta_{+0}| \) and \( |\eta_{000}| \) are much larger than \( |\epsilon| \), they can be interpreted as upper limits on \( \text{Im}(\eta_{+0}) \) and \( \text{Im}(\eta_{000}) \) and so as limits on the \( CP \)-violating phase of the decay amplitude \( a_1 \).