QUARK MASSES

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A. Introduction

This note discusses some of the theoretical issues relevant for the determination of quark masses, which are fundamental parameters of the Standard Model of particle physics. Unlike the leptons, quarks are confined inside hadrons and are not observed as physical particles. Quark masses therefore cannot be measured directly, but must be determined indirectly through their influence on hadronic properties. Although one often speaks loosely of quark masses as one would of the mass of the electron or muon, any quantitative statement about the value of a quark mass must make careful reference to the particular theoretical framework that is used to define it. It is important to keep this *scheme dependence* in mind when using the quark mass values tabulated in the data listings.

Historically, the first determinations of quark masses were performed using quark models. The resulting masses only make sense in the limited context of a particular quark model, and cannot be related to the quark mass parameters of the Standard Model. In order to discuss quark masses at a fundamental level, definitions based on quantum field theory be used, and the purpose of this note is to discuss these definitions and the corresponding determinations of the values of the masses.

B. Mass parameters and the QCD Lagrangian

The QCD [1] Lagrangian for N_F quark flavors is

$$\mathcal{L} = \sum_{k=1}^{N_F} \overline{q}_k \left(i \not{D} - m_k \right) q_k - \tfrac{1}{4} G_{\mu\nu} G^{\mu\nu} \ , \tag{1}$$

where $\not{D} = \left(\partial_\mu - i g A_\mu \right) \gamma^\mu$ is the gauge covariant derivative, A_μ is the gluon field, $G_{\mu\nu}$ is the gluon field strength, m_k is the mass parameter of the k^{th} quark, and q_k is the quark Dirac field. After renormalization, the QCD Lagrangian Eq. (1) gives finite values for physical quantities, such as scattering amplitudes. Renormalization is a procedure that invokes a subtraction scheme to render the amplitudes finite, and requires the introduction of a dimensionful scale parameter μ. The mass parameters in the QCD Lagrangian Eq. (1) depend on the renormalization scheme used to define the theory, and also on the scale parameter μ. The most commonly used renormalization scheme for QCD perturbation theory is the $\overline{\mathrm{MS}}$ scheme.

The QCD Lagrangian has a chiral symmetry in the limit that the quark masses vanish. This symmetry is spontaneously broken by dynamical chiral symmetry breaking, and explicitly broken by the quark masses. The nonperturbative scale of dynamical chiral symmetry breaking, Λ_χ, is around $1\,\mathrm{GeV}$ [2]. It is conventional to call quarks heavy if $m > \Lambda_\chi$, so that explicit chiral symmetry breaking dominates (c, b, and t quarks are heavy), and light if $m < \Lambda_\chi$, so that spontaneous chiral symmetry breaking dominates (u, d and s quarks are light). The determination of light- and heavy-quark masses is considered separately in sections D and E below.

At high energies or short distances, nonperturbative effects, such as chiral symmetry breaking, become small and one can, in principle, determine quark masses by analyzing mass-dependent effects using QCD perturbation theory. Such computations are conventionally performed using the $\overline{\mathrm{MS}}$ scheme at a scale $\mu \gg \Lambda_\chi$, and give the $\overline{\mathrm{MS}}$ "running" mass $\overline{m}(\mu)$. We use the $\overline{\mathrm{MS}}$ scheme when reporting quark masses; one can readily convert these values into other schemes using perturbation theory.

The μ dependence of $\overline{m}(\mu)$ at short distances can be calculated using the renormalization group equation,

$$\mu^2 \frac{\mathrm{d}\overline{m}\,(\mu)}{\mathrm{d}\mu^2} = -\gamma(\overline{\alpha}_s\,(\mu))\;\overline{m}\,(\mu)\,, \tag{2}$$

where γ is the anomalous dimension which is now known to four-loop order in perturbation theory [3,4]. $\overline{\alpha}_s$ is the coupling constant in the $\overline{\mathrm{MS}}$ scheme. Defining the expansion coefficients γ_r by

$$\gamma\,(\overline{\alpha}_s) \equiv \sum_{r=1}^{\infty} \gamma_r \left(\frac{\overline{\alpha}_s}{4\pi}\right)^r ,$$

the first four coefficients are given by

$$\gamma_1 = 4,$$

$$\gamma_2 = \frac{202}{3} - \frac{20N_L}{9},$$

$$\gamma_3 = 1249 + \left(-\frac{2216}{27} - \frac{160}{3}\zeta\left(3\right)\right) N_L - \frac{140}{81}N_L^2,$$

$$\begin{aligned}\gamma_4 = {} & \frac{4603055}{162} + \frac{135680}{27}\zeta\left(3\right) - 8800\zeta\left(5\right) \\ & + \left(-\frac{91723}{27} - \frac{34192}{9}\zeta\left(3\right) + 880\zeta\left(4\right) + \frac{18400}{9}\zeta\left(5\right)\right) N_L \\ & + \left(\frac{5242}{243} + \frac{800}{9}\zeta\left(3\right) - \frac{160}{3}\zeta\left(4\right)\right) N_L^2 \\ & + \left(-\frac{332}{243} + \frac{64}{27}\zeta\left(3\right)\right) N_L^3,\end{aligned}$$

where N_L is the number of active light quark flavors at the scale μ, i.e. flavors with masses $< \mu$, and ζ is the Riemann zeta function ($\zeta(3) \simeq 1.2020569$, $\zeta(4) \simeq 1.0823232$, and $\zeta(5) \simeq 1.0369278$).

C. Lattice Gauge Theory

The use of the lattice simulations for *ab initio* determinations of the fundamental parameters of QCD, including the coupling constant and quark masses (except for the top-quark mass) is a very active area of research, with the current emphasis being on the reduction and control of the systematic uncertainties. We now briefly review some of the features of lattice QCD. In this approach space-time is approximated by a finite, discrete *lattice* of points and multi-local correlation functions are computed by the numerical evaluation of the corresponding functional integrals. To determine quark masses, one computes a convenient and appropriate set of physical quantities (frequently chosen to be a set of hadronic masses) using lattice QCD for a variety of input values of the quark masses. The true (physical) values of the quark masses are those which correctly reproduce the set of physical quantities being used for calibration.

The values of the quark masses obtained directly in lattice simulations are bare quark masses, with the lattice spacing a (i.e. the distance between neighboring points of the lattice) as the ultraviolet cut-off. In order for the lattice results to be useful in phenomenology, it is therefore necessary to relate the bare quark masses in a lattice formulation of QCD to renormalized masses in some standard renormalization scheme such as $\overline{\mathrm{MS}}$. Provided that both the ultraviolet cut-off a^{-1} and the renormalization scale are much greater than Λ_{QCD}, the bare and renormalized masses can be related in perturbation theory (this is frequently facilitated by the use of chiral Ward identities). However, the coefficients in lattice perturbation theory are often found to be large, and our ignorance of higher order terms is generally a significant source of systematic uncertainty. Increasingly, non-perturbative renormalization is used to calculate the relation between the bare and renormalized masses, circumventing the need for lattice perturbation theory.

The precision with which quark masses can be determined in lattice simulations is limited by the available computing resources. There are a number of sources of systematic uncertainty and there has been considerable progress in recent years in reducing these. In general, the main source of uncertainty arises from the difficulty of performing simulations with three flavours of sea quarks which are sufficiently light for chiral perturbation theory (see section D) to be valid. In the past the computations were performed without including sea quarks at all (this is the so-called *quenched approximation*). Current simulations are generally unquenched, but m_u and m_d are larger than their physical values and the results are extrapolated, using chiral perturbation theory where possible, to the physical point. Reducing the uncertainty in this *chiral extrapolation* is the principal challenge in improving the precision in the determination of physical quantities from lattice simulations.

In addition one has to consider the uncertainties due to the fact that the lattice spacing is non-zero (lattice artefacts) and that the volume is not infinite. The former are studied by observing the stability of the results as a is varied or by using "improved" formulations of lattice QCD. By varying the volume of the lattice one checks that finite-volume effects are indeed small.

D. Light quarks

For light quarks, one can use the techniques of chiral perturbation theory [5,6,7] to extract quark mass ratios. The mass term for light quarks in the QCD Lagrangian is

$$\overline{\Psi} M \Psi = \overline{\Psi}_L M \Psi_R + \overline{\Psi}_R M^\dagger \Psi_L, \tag{3}$$

where M is the light quark mass matrix M,

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \tag{4}$$

and $\Psi = (u, d, s)$. The mass term is the only term in the QCD Lagrangian that mixes left- and right-handed quarks. In the limit $M \to 0$, there is an independent $SU(3) \times U(1)$ flavor symmetry for the left- and right-handed quarks. The vector $U(1)$ symmetry is baryon number; the axial $U(1)$ symmetry of the classical theory is broken in the quantum theory due to the anomaly. The remaining $G_\chi = \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ chiral symmetry of the QCD Lagrangian is spontaneously broken to $SU(3)_V$, which, in the limit $M \to 0$, leads to eight massless Goldstone bosons, the π's, K's, and η.

The symmetry G_χ is only an approximate symmetry, since it is explicitly broken by the quark mass matrix M. The Goldstone bosons acquire masses which can be computed in a systematic expansion in M, in terms of low-energy constants, which are unknown nonperturbative parameters of the theory, and are not fixed by the symmetries. One treats the quark mass matrix M as an external field that transforms under G_χ as $M \to LMR^\dagger$, where $\Psi_L \to L\Psi_L$ and $\Psi_R \to R\Psi_R$ are the $SU(3)_L$ and $SU(3)_R$ transformations, and writes down the most general Lagrangian invariant under G_χ. Then one sets M to its given constant value Eq. (4), which implements the symmetry breaking. To first order in M one finds that [8]

$$\begin{aligned} m_{\pi^0}^2 &= B\,(m_u + m_d)\ , \\ m_{\pi^\pm}^2 &= B\,(m_u + m_d) + \Delta_{\mathrm{em}}\ , \end{aligned}$$

$$
\begin{aligned}
m_{K^0}^2 = m_{\overline{K}^0}^2 =& B\left(m_d+m_s\right) \ , \\
m_{K^\pm}^2 =& B\left(m_u+m_s\right)+\Delta_{\rm em} \ , \\
m_\eta^2 =& \frac{1}{3}B\left(m_u+m_d+4m_s\right) \ ,
\end{aligned}
\tag{5}
$$

with two unknown constants B and $\Delta_{\rm em}$, the electromagnetic mass difference. From Eq. (5), one can determine the quark mass ratios [8]

$$
\begin{aligned}
\frac{m_u}{m_d} =& \frac{2m_{\pi^0}^2-m_{\pi^+}^2+m_{K^+}^2-m_{K^0}^2}{m_{K^0}^2-m_{K^+}^2+m_{\pi^+}^2} = 0.56 \ , \\
\frac{m_s}{m_d} =& \frac{m_{K^0}^2+m_{K^+}^2-m_{\pi^+}^2}{m_{K^0}^2+m_{\pi^+}^2-m_{K^+}^2} = 20.1 \ ,
\end{aligned}
\tag{6}
$$

to lowest order in chiral perturbation theory, with an error which will be estimated below. Since the mass ratios extracted using chiral perturbation theory use the symmetry transformation property of M under the chiral symmetry G_χ, it is important to use a renormalization scheme for QCD that does not change this transformation law. Any mass independent subtraction scheme such as $\overline{\rm MS}$ is suitable. The ratios of quark masses are scale independent in such a scheme, and Eq. (6) can be taken to be the ratio of $\overline{\rm MS}$ masses. Chiral perturbation theory cannot determine the overall scale of the quark masses, since it uses only the symmetry properties of M, and any multiple of M has the same G_χ transformation law as M.

Chiral perturbation theory is a systematic expansion in powers of the light quark masses. The typical expansion parameter is $m_K^2/\Lambda_\chi^2 \sim 0.25$ if one uses $SU(3)$ chiral symmetry, and $m_\pi^2/\Lambda_\chi^2 \sim 0.02$ if one uses $SU(2)$ chiral symmetry. Electromagnetic effects at the few percent level also break $SU(2)$ and $SU(3)$ symmetry. The mass formulæ Eq. (5) were derived using $SU(3)$ chiral symmetry, and are expected to have a 25% uncertainty due to second order corrections.

There is a subtlety which arises when one tries to determine quark mass ratios at second order in chiral perturbation theory. The second order quark mass term [9]

$$
\left(M^\dagger\right)^{-1}\det M^\dagger \tag{7}
$$

(which can be generated by instantons) transforms in the same way under G_χ as M. Chiral perturbation theory cannot distinguish between M and $\left(M^\dagger\right)^{-1}\det M^\dagger$; one can make the replacement $M \to M(\lambda) = M + \lambda M\left(M^\dagger M\right)^{-1}\det M^\dagger$ in the chiral Lagrangian,

$$\begin{aligned} M(\lambda) &= \mathrm{diag}\left(m_u(\lambda)\,,\ \ m_d(\lambda)\,,\ \ m_s(\lambda)\right) \\ &= \mathrm{diag}\left(m_u + \lambda m_d m_s\,,\ \ m_d + \lambda m_u m_s\,,\ \ m_s + \lambda m_u m_d\right), \end{aligned} \tag{8}$$

and leave all observables unchanged.

The combination

$$\left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2}\left(\frac{m_s}{m_d}\right)^2 = 1 \tag{9}$$

where

$$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}, \qquad \hat{m} = \frac{1}{2}\left(m_u + m_d\right),$$

is insensitive to the transformation in Eq. (8). Eq. (9) gives an ellipse in the $m_u/m_d - m_s/m_d$ plane. The ellipse is well-determined by chiral perturbation theory, but the exact location on the ellipse, and the absolute normalization of the quark masses, has larger uncertainties. Q is determined to be in the range 21–25 from $\eta \to 3\pi$ decay and the electromagnetic contribution to the K^+–K^0 and π^+–π^0 mass differences [10].

It is particularly important to determine the quark mass ratio m_u/m_d, since there is no strong CP problem if $m_u = 0$. The chiral symmetry G_χ of the QCD Lagrangian is not enhanced even if $m_u = 0$. [The possible additional axial u-quark number symmetry is anomalous. The only additional symmetry when $m_u = 0$ is CP.] As a result $m_u = 0$ is not a special value for chiral perturbation theory.

The absolute normalization of the quark masses can be determined by using methods that go beyond chiral perturbation theory, such as spectral function sum rules [11,12] for hadronic correlation functions or lattice simulations.

Sum Rules: Sum rule methods have been extensively used to determine quark masses and for illustration we briefly discuss here their application to hadronic τ decays [13]. Other applications involve very similar techniques.

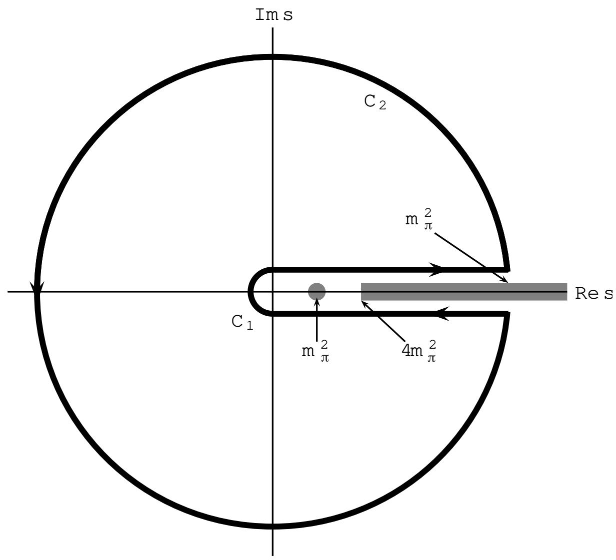

Figure 1: The analytic structure of $\Pi(s)$ in the complex s-plane. The contours C_1 and C_2 are the integration contours discussed in the text.

The experimentally measured quantity is R_τ,

$$\frac{\mathrm{d}R_\tau}{\mathrm{d}s} = \frac{\mathrm{d}\Gamma/\mathrm{d}s\left(\tau^- \to \text{hadrons} + \nu_\tau(\gamma)\right)}{\Gamma\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau(\gamma)\right)} \tag{10}$$

the hadronic invariant mass spectrum in semihadronic τ decay, normalized to the leptonic τ decay rate. It is useful to define q as the total momentum of the hadronic final state, so $s = q^2$ is the hadronic invariant mass. The total hadronic τ decay rate R_τ is then given by integrating $\mathrm{d}R_\tau/\mathrm{d}s$ over the kinematically allowed range $0 \leq s \leq M_\tau^2$.

R_τ can be written as

$$\begin{aligned} R_\tau = & 12\pi \int_0^{M_\tau^2} \frac{\mathrm{d}s}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2 \\ & \times \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\,\Pi^T(s) + \mathrm{Im}\,\Pi^L(s)\right] \end{aligned} \tag{11}$$

where $s = q^2$, and the hadronic spectral functions $\Pi^{L,T}$ are defined from the time-ordered correlation function of two weak currents is the time-ordered correlator of the weak interaction current ($j^\mu(x)$ and $j^\nu(0)$) by

$$\Pi^{\mu\nu}(q) = i \int \mathrm{d}^4x \; e^{iq\cdot x} \langle 0| T\left(j^\mu(x) j^\nu(0)^\dagger\right)|0\rangle \,, \tag{12}$$

$$\Pi^{\mu\nu}(q) = \left(-g^{\mu\nu} + q^\mu q^\nu\right) \Pi^T(s) + q^\mu q^\nu \Pi^L(s), \tag{13}$$

and the decomposition Eq. (13) is the most general possible structure consistent with Lorentz invariance.

By the optical theorem, the imaginary part of $\Pi^{\mu\nu}$ is proportional to the total cross-section for the current to produce all possible states. A detailed analysis including the phase space factors leads to Eq. (11). The spectral functions $\Pi^{L,T}(s)$ are analytic in the complex s plane, with singularities along the real axis. There is an isolated pole at $s = m_\pi^2$, and single- and multi-particle singularities for $s \geq 4m_\pi^2$, the two-particle threshold. The discontinuity along the real axis is $\Pi^{L,T}(s+i0^+) - \Pi^{L,T}(s-i0^+) = 2i\mathrm{Im}\ \Pi^{L,T}(s)$. As a result, Eq. (11) can be rewritten with the replacement $\mathrm{Im}\ \Pi^{L,T}(s) \to -i\Pi^{L,T}(s)/2$, and the integration being over the contour C_1. Finally, the contour C_1 can be deformed to C_2 without crossing any singularities, and so leaving the integral unchanged. One can derive a series of sum rules analogous to Eq. (11) by weighting the differential τ hadronic decay rate by different powers of the hadronic invariant mass,

$$R_\tau^{kl} = \int_0^{M_\tau^2} \mathrm{d}s \; \left(1 - \frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{\mathrm{d}R_\tau}{\mathrm{d}s} \tag{14}$$

where $\mathrm{d}R_\tau/\mathrm{d}s$ is the hadronic invariant mass distribution in τ decay normalized to the leptonic decay rate. This leads to the final form of the sum rule(s),

$$\begin{aligned} R_\tau^{kl} = &- 6\pi i \int_{C_2} \frac{\mathrm{d}s}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^{2+k} \left(\frac{s}{M_\tau^2}\right)^l \\ &\times \left[\left(1 + 2\frac{s}{M_\tau^2}\right)\Pi^T(s) + \Pi^L(s)\right]. \end{aligned} \tag{15}$$

The manipulations so far are completely rigorous and exact, relying only on the general analytic structure of quantum field theory. The left-hand side of the sum rule Eq. (15) is obtained from experiment. The right hand-side can be computed for s far away from any physical cuts using the operator product expansion (OPE) for the time-ordered product of currents in Eq. (12), and QCD perturbation theory. The OPE is an expansion for the time-ordered product Eq. (12) in a series of local operators, and is an expansion about the $q \to \infty$ limit. It gives $\Pi(s)$ as an expansion in powers of $\alpha_s(s)$ and $\Lambda^2_{\rm QCD}/s$, and is valid when s is far (in units of $\Lambda^2_{\rm QCD}$) from any singularities in the complex s-plane.

The OPE gives $\Pi(s)$ as a series in α_s, quark masses, and various non-perturbative vacuum matrix element. By computing $\Pi(s)$ theoretically, and comparing with the experimental values of R_τ^{kl}, one determines various parameters such as α_s and the quark masses. The theoretical uncertainties in using Eq. (15) arise from neglected higher order corrections (both perturbative and non-perturbative), and because the OPE is no longer valid near the real axis, where Π has singularities. The contribution of neglected higher order corrections can be estimated as for any other perturbative computation. The error due to the failure of the OPE is more difficult to estimate. In Eq. (15), the OPE fails on the endpoints of C_2 that touch the real axis at $s = M_\tau^2$. The weight factor $(1 - s/M_\tau^2)$ in Eq. (15) vanishes at this point, so the importance of the endpoint can be reduced by choosing larger values of k.

Lattice Gauge Theory: Lattice simulations allow for detailed studies of the behaviour of hadronic masses and matrix elements as functions of the quark masses. Moreover, the quark masses do not have to take their physical values, but can be varied freely and chiral perturbation theory applies also for unphysical masses, provided that they are sufficiently light. From such recent studies of pseudoscalar masses and decay constants, the relevant higher-order couplings in the chiral Lagrangian have been estimated, strongly suggesting that $m_u \neq 0$ [14,15,16]. In order to make this evidence conclusive, the lattice systematic errors must be reduced; in particular the range of light quark masses should be increased and the validity of chiral perturbation theory for this range established.

In recent years there have been a number of unquenched determinations of the masses of the light quarks using a variety of formulations of lattice QCD (see, for example, the set of results in refs. [17,18,19,20,21,22,23,24]) . Some of the simulations have been performed with two flavours of sea quarks and some with three flavours. The lattice systematic uncertainties in these determinations are different (e.g. due to the different lattice formulations of QCD, the use of perturbative and non-perturbative renormalization and the different chiral and continuum extrapolations). Taking these into consideration, we give below our current estimates for the quark masses determined from lattice simulations.

In current lattice simulations it is the combination $(m_u + m_d)/2$ which can be determined. In the evaluation of m_s one gets a result which is about 20–25% larger if the ϕ-meson is used as input rather than the K-meson. This is evidence that the errors due to quenching are significant. It is reassuring that this difference is eliminated or reduced significantly in the cited unquenched studies.

The quark masses for light quarks discussed so far are often referred to as current quark masses. Nonrelativistic quark models use constituent quark masses, which are of order 350 MeV for the u and d quarks. Constituent quark masses model the effects of dynamical chiral symmetry breaking, and are not related to the quark mass parameters m_k of the QCD Lagrangian Eq. (1). Constituent masses are only defined in the context of a particular hadronic model.

E. Heavy quarks

The masses and decay rates of hadrons containing a single heavy quark, such as the B and D mesons can be determined using the heavy quark effective theory (HQET) [25]. The theoretical calculations involve radiative corrections computed in perturbation theory with an expansion in $\alpha_s(m_Q)$ and non-perturbative corrections with an expansion in powers of $\Lambda_{\rm QCD}/m_Q$. Due to the asymptotic nature of the QCD perturbation series, the two kinds of corrections are intimately related; an example of this are renormalon effects in the perturbative expansion which are associated with non-perturbative corrections.

Systems containing two heavy quarks such as the Υ or J/Ψ are treated using NRQCD [26]. The typical momentum and energy transfers in these systems are $\alpha_s m_Q$, and $\alpha_s^2 m_Q$, respectively, so these bound states are sensitive to scales much smaller than m_Q. However, smeared observables, such as the cross-section for $e^+e^- \to \overline{b}b$ averaged over some range of s that includes several bound state energy levels, are better behaved and only sensitive to scales near m_Q. For this reason, most determinations of the b quark mass using perturbative calculations compare smeared observables with experiment [27,28,29].

Lattice simulations of QCD requires the quark mass to be much smaller than a^{-1}, where a is the lattice spacing, in order to avoid large errors due to the granularity of the lattice. Since computing resources limit a^{-1} in current simulations to be typically in the range 1.5 – 2.5 GeV, this is not possible for the b-quark and is marginal for the c-quark. For this reason, particularly for the b-quark, simulations are performed using effective theories, including HQET and NRQCD. Using effective theories, m_b is obtained from what is essentially a computation of the difference of $M_{H_b} - m_b$, where M_{H_b} is the mass of a hadron H_b containing a b-quark. The relative error on m_b is therefore much smaller than that for $M_{H_b} - m_b$, and this is the reason for the small errors quoted in section F. The principal systematic errors are the matching of the effective theories to QCD and the presence of power divergences in a^{-1} in the $1/m_b$ corrections which have to be subtracted numerically. The use of HQET or NRQCD is less precise for the charm quark, and in this case *improved* formulations of QCD, in which the errors to the finite lattice spacing are formally reduced are being used (see in particular refs. [30,31]) .

For an observable particle such as the electron, the position of the pole in the propagator is the definition of the particle mass. In QCD this definition of the quark mass is known as the pole mass. It is known that the on-shell quark propagator has no infrared divergences in perturbation theory [32,33], so this provides a perturbative definition of the quark mass. The pole mass cannot be used to arbitrarily high accuracy because of nonperturbative infrared effects in QCD. The full quark propagator has no pole because the quarks are confined, so that the pole mass cannot be defined outside of perturbation theory. The relation between the pole mass m_Q and the $\overline{\rm MS}$ mass $\overline{m}_Q$ is known to three loops [34,35,36]

$$\begin{aligned} m_Q = \overline{m}_Q(\overline{m}_Q)\Bigg\{1 + \frac{4\overline{\alpha}_s(\overline{m}_Q)}{3\pi} \\ + \left[-1.0414\sum_k\left(1 - \frac{4}{3}\frac{\overline{m}_{Q_k}}{\overline{m}_Q}\right) + 13.4434\right]\left[\frac{\overline{\alpha}_s(\overline{m}_Q)}{\pi}\right]^2 \\ + \left[0.6527N_L^2 - 26.655N_L + 190.595\right]\left[\frac{\overline{\alpha}_s(\overline{m}_Q)}{\pi}\right]^3\Bigg\}, \end{aligned} \tag{16}$$

where $\overline{\alpha}_s(\mu)$ is the strong interaction coupling constants in the $\overline{\rm MS}$ scheme, and the sum over k extends over the N_L flavors Q_k lighter than Q. The complete mass dependence of the α_s^2 term can be found in [34]; the mass dependence of the α_s^3 term is not known. For the b-quark, Eq. (16) reads

$$m_b = \overline{m}_b\left(\overline{m}_b\right)\left[1 + 0.09 + 0.05 + 0.03\right], \tag{17}$$

where the contributions from the different orders in α_s are shown explicitly. The two and three loop corrections are comparable in size and have the same sign as the one loop term. This is a signal of the asymptotic nature of the perturbation series [there is a renormalon in the pole mass]. Such a badly behaved perturbation expansion can be avoided by directly extracting the $\overline{\rm MS}$ mass from data without extracting the pole mass as an intermediate step.

F. Numerical values and caveats

The quark masses in the particle data listings have been obtained by using a wide variety of methods. Each method involves its own set of approximations and errors. In most cases, the errors are a best guess at the size of neglected higher-order corrections or other uncertainties. The expansion parameters for some of the approximations are not very small (for example, they are $m_K^2/\Lambda_\chi^2 \sim 0.25$ for the chiral expansion and $\Lambda_{\rm QCD}/m_b \sim 0.1$ for the heavy-quark expansion), so an unexpectedly large coefficient in a neglected higher-order term could significantly alter the results. It is also important to note that the quark mass values can be significantly different in the different schemes.

The heavy quark masses obtained using HQET, QCD sum rules, or lattice gauge theory are consistent with each other if they are all converted into the same scheme and scale. We have specified all masses in the $\overline{\rm MS}$ scheme. For light quarks, the renormalization scale has been chosen to be $\mu = 2\,\mathrm{GeV}$. The light quark masses at $1\,\mathrm{GeV}$ are significantly different from those at $2\,\mathrm{GeV}$, $\overline{m}(1\,\mathrm{GeV})/\overline{m}(2\,\mathrm{GeV}) \sim 1.35$. It is conventional to choose the renormalization scale equal to the quark mass for a heavy quark, so we have quoted $\overline{m}_Q(\mu)$ at $\mu = \overline{m}_Q$ for the c and b quarks. Recent analyses of inclusive B meson decays have shown that recently proposed mass definitions lead to a better behaved perturbation series than for the $\overline{\rm MS}$ mass, and hence to more accurate mass values. We have chosen to also give values for one of these, the b quark mass in the 1S-scheme [37,38]. Other schemes that have been proposed are the PS-scheme [39] and the kinetic scheme [40].

If necessary, we have converted values in the original papers to our chosen scheme using two-loop formulæ. It is important to realized that our conversions introduce significant additional errors. In converting to the $\overline{\rm MS}$ b-quark mass, for example, the three-loop conversions from the 1S and pole masses give values about 40 MeV and 135 MeV lower than the two-loop conversions. The uncertainty in $\alpha_s(M_Z) = 0.1187(20)$ gives an uncertainty of ± 20 MeV and ± 35 MeV respectively in the same conversions. We have not added these additional errors when we do our conversions.

A summary of the quark masses using lattice and continuum methods is given below. The mass values quoted in the listings combine values extracted using both methods; here we present the separate results.

From the spread of results and taking into account the different treatment of systematic errors in each of the unquenched lattice simulations (with 2 and 3 flavours of sea quarks), we find the current results for the light quark masses renormalized at a scale of 2 GeV:

$$\frac{1}{2}\left(\overline{m}_u + \overline{m}_d\right)\big|_{\mu=2\,\mathrm{GeV}} = (3.8 \pm 0.8)\,\mathrm{MeV} \qquad \text{[Lattice only]}$$

and

$$\overline{m}_s\big|_{\mu=2\,\mathrm{GeV}} = (95 \pm 20)\,\mathrm{MeV} \qquad \text{[Lattice only]}\,.$$

It is to be expected that the recent progress in reducing the systematic uncertainties in unquenched simulations, including in the evaluation of the renormalization constants and in the control of the chiral extrapolation, will continue so that errors quoted above for the best results will decrease significantly.

The continuum determinations of the u and d quark masses is

$$\overline{m}_u\Big|_{\mu=2\ \mathrm{GeV}} = 3 \pm 1\,\mathrm{MeV} \quad \text{[Excluding lattice]}.$$

$$\overline{m}_d\Big|_{\mu=2\ \mathrm{GeV}} = 6.0 \pm 1.5\,\mathrm{MeV} \quad \text{[Excluding lattice]}.$$

The absolute values of the u and d quark masses are difficult to determine directly, since they are small compared to typical hadronic scales, and their effects are comparable in size to isospin violating electromagnetic corrections. Most u and d quark mass values are obtained by determining the ratios to m_s using chiral perturbation theory, and then extracting a value for m_s. The average u, d mass is

$$\frac{1}{2}\left(\overline{m}_u + \overline{m}_d\right)\Big|_{\mu=2\ \mathrm{GeV}} = 4.4 \pm 1.5\,\mathrm{MeV} \quad \text{[Excluding lattice]}.$$

The continuum extractions for the s-quark give

$$\overline{m}_s\Big|_{\mu=2\ \mathrm{GeV}} = 103 \pm 20\,\mathrm{MeV} \quad \text{[Excluding lattice]}$$

where we have excluded older values, many of which have been superceded by more recent determinations by the same authors, in the fit.

There have been recent advances in computing the perturbative corrections to the two-point correlation functions used in the sum rule extractions for the s-quark mass. The results are now known to order α_s^2, with an estimate of the order α_s^3 terms. These NNLO corrections are large, and reduce the value for $\overline{m}_s$, so that newer determinations give smaller values for $\overline{m}_s$. The full α_s^3 correction to strangeness-changing current correlator in τ decays has been recently computed [41].

The continuum determinations of the c-quark mass is

$$\overline{m}_c\,(\overline{m}_c) = 1.24 \pm 0.09\,\text{GeV} \quad \text{[Excluding lattice]}.$$

Recent determinations include at least two-loop corrections, and give values consistent with this range.

There are still rather few lattice determinations of m_c, as the charm quark is too light for comfortable use of HQET, and yet heavy enough that one must be careful about lattice artifacts. The published results are from quenched simulations (see for example refs. [42,43,44]) and as the best result we take

$$\overline{m}_c(\overline{m}_c) = (1.30 \pm 0.03 \pm 0.20)\,\text{GeV} \qquad \text{[Lattice only]},$$

where the second error of 15% is our estimate of possible quenching effects. Recent preliminary unquenched results lie in the above range, and we expect that in future editions the best result will be dominated by determinations from unquenched simulations.

There has been much recent work on the b-quark mass, which has led to greatly improved accuracy in the quark mass values. The main progress has been both theoretical, due to using better quark mass definitions and the inclusion of higher order terms in the $1/m_b$ expansion, and experimental, due to the precision B-factory data. For the value from continuum extractions, we find

$$\overline{m}_b\,(\overline{m}_b) = 4.20 \pm 0.07\,\text{GeV} \quad \text{[Excluding lattice]},$$

$$\overline{m}_b^{1\text{S}} = 4.70 \pm 0.07\,\text{GeV} \quad \text{[Excluding lattice]},$$

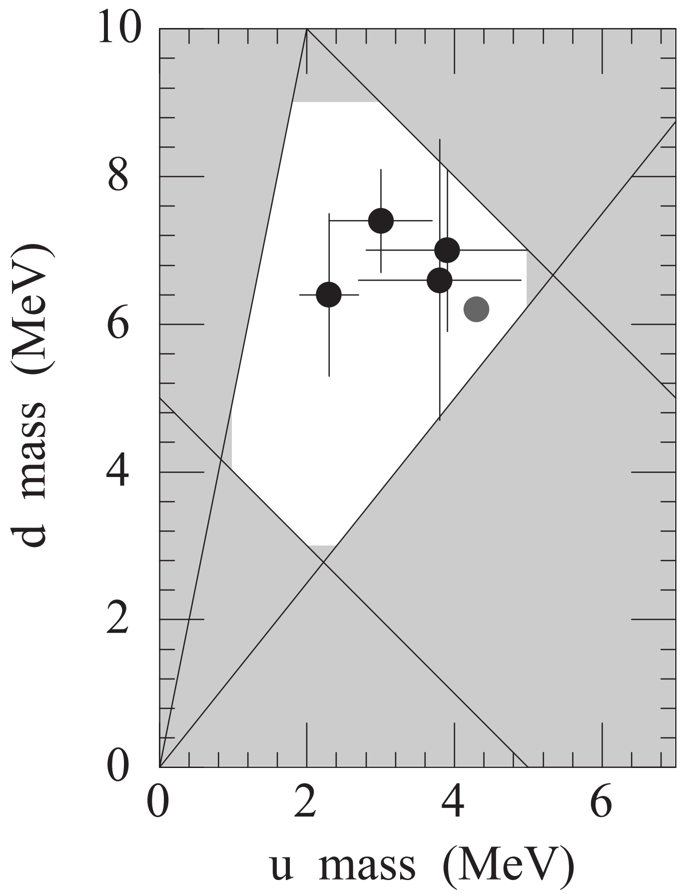

Figure 2: The allowed region (shown in white) for up quark and down quark masses. This region was determined in part from papers reporting values for m_u and m_d (data points shown) and in part from analysis of the allowed ranges of other mass parameters (see Fig. 3). The parameter $(m_u + m_d)/2$ yields the two downward-sloping lines, while m_u/m_d yields the two rising lines originating at (0,0). The grey point is from a paper giving no error bars.

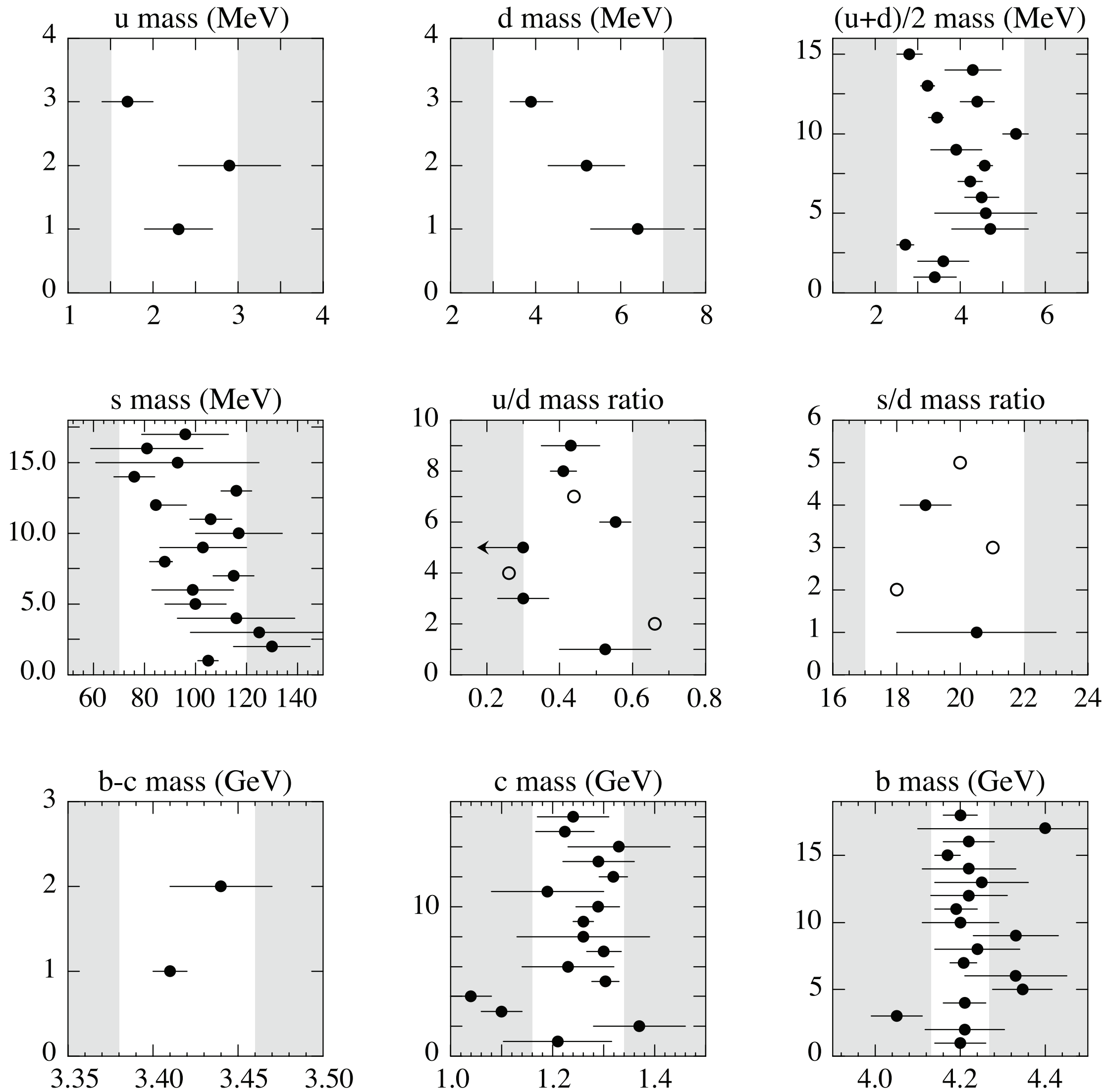

Figure 3. The values of each quark mass parameter taken from the Data Listings. Points from papers reporting no error bars are colored grey. Arrows indicate limits reported. The grey regions indicate values excluded by our evaluations; some regions were determined in part though examination of Fig. 2.

As the current best lattice results for $\overline{m}_b$ we take:

$$\overline{m}_b(\overline{m}_b) = (4.2 \pm 0.1 \pm 0.1)\,\mathrm{GeV} \qquad [\text{Lattice only}]\,,$$

where the first error is a rounding of the lattice errors quotes by authors using the static heavy quark and the second is a (conservative) 10% on $m_B - \overline{m}_b$ due to the fact that the HQET simulations [45,46,47] were performed with two flavours of sea quark and because $O(\Lambda^2_{\mathrm{QCD}}/m_b)$ corrections have not been included up to now. The value of $\overline{m}_b$ obtained using NRQCD [48] with 3 flavours of light sea quark is consistent with that given above, but with the matching to QCD currently performed only to one-loop order in perturbation theory.

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