SUPERSYMMETRY, PART I (THEORY)

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I.1. Introduction: Supersymmetry (SUSY) is a generalization of the space-time symmetries of quantum field theory that transforms fermions into bosons and vice versa. The existence of such a non-trivial extension of the Poincaré symmetry of ordinary quantum field theory was initially surprising, and its form is highly constrained by theoretical principles [1]. Supersymmetry also provides a framework for the unification of particle physics and gravity [2–5], which is governed by the Planck energy scale, $M_{\rm P} \approx 10^{19}$ GeV (where the gravitational interactions become comparable in magnitude to the gauge interactions). In particular, it is possible that supersymmetry will ultimately explain the origin of the large hierarchy of energy scales from the W and Z masses to the Planck scale [6–9]. This is the so-called *gauge hierarchy*. The stability of the gauge hierarchy in the presence of radiative quantum corrections is not possible to maintain in the Standard Model, but can be maintained in supersymmetric theories.

If supersymmetry were an exact symmetry of nature, then particles and their superpartners (which differ in spin by half a unit) would be degenerate in mass. Since superpartners have not (yet) been observed, supersymmetry must be a broken symmetry. Nevertheless, the stability of the gauge hierarchy can still be maintained if the supersymmetry breaking is *soft* [10] and the corresponding supersymmetry-breaking mass parameters are no larger than a few TeV. (In this context, soft supersymmetry-breaking terms are non-supersymmetric terms in the Lagrangian that are either linear, quadratic or cubic in the fields, with some restrictions elucidated in Ref. [10]. The impact of such terms becomes negligible at energy scales much larger than the size of the supersymmetry-breaking masses.) The most interesting theories of this type are theories of "low-energy" (or "weak-scale") supersymmetry, where the effective scale of supersymmetry breaking is tied to the scale of electroweak symmetry breaking [6–9]. The latter is characterized by the Standard Model Higgs vacuum expectation value, $v = 246$ GeV.

Although there are no unambiguous experimental results (at present) that require the existence of new physics at the TeV-scale, expectations of the latter are primarily based on three theoretical arguments. First, a *natural* explanation (*i.e.*, one that is stable with respect to quantum corrections) of the gauge hierarchy demands new physics at the TeV-scale [9]. Second, the unification of the three gauge couplings at a very high energy close to the Planck scale does not occur in the Standard Model. However, unification can be achieved with the addition of new physics that can modify the way gauge couplings run above the electroweak scale. A simple example of successful unification arises in the minimal supersymmetric extension of the Standard Model, where supersymmetric masses lie below a few TeV [11]. Third, the existence of dark matter which makes up approximately one quarter of the energy density of the universe, cannot be explained within the Standard Model of particle physics [12]. It is tempting to attribute the dark matter to the existence of a neutral stable thermal relic (*i.e.*, a particle that was in thermal equilibrium with all other fundamental particles in the early universe at temperatures above the particle mass). Remarkably, the existence of such a particle could yield the observed density of dark matter if its mass and interaction rate were governed by new physics associated with the TeV-scale. The lightest supersymmetric particle is a promising (although not the unique) candidate for the dark matter [13].

Low-energy supersymmetry has traditionally been motivated by the three theoretical arguments just presented. More recently, some theorists [14,15] have argued that the explanation for the gauge hierarchy could lie elsewhere, in which case the effective TeV-scale theory would appear to be highly *unnatural*. Nevertheless, even without the naturalness argument, supersymmetry is expected to be a necessary ingredient of the ultimate theory at the Planck scale that unifies gravity with the other fundamental forces. Moreover, one can imagine that

Table 1: The fields of the MSSM and their SU(3)×SU(2)×U(1) quantum numbers are listed. Only one generation of quarks and leptons is exhibited. For each lepton, quark and Higgs super-multiplet, there is a corresponding anti-particle multiplet of charge-conjugated fermions and their associated scalar partners.

Field Content of the MSSM					
Super-Multiplets	Boson Fields	Fermionic Partners	SU(3)	SU(2)	U(1)
gluon/gluino	g	$\widetilde{g}$	8	0	0
gauge/	$W^{\pm}$, W^0	$\widetilde{W}^{\pm}$, $\widetilde{W}^0$	1	3	0
gaugino	B	$\widetilde{B}$	1	1	0
slepton/	$(\widetilde{\nu}, \widetilde{e}^-)_L$	$(\nu, e^-)_L$	1	2	−1
lepton	$\tilde{e}^-_R$	e^-_R	1	1	−2
squark/	$(\widetilde{u}_L, \widetilde{d}_L)$	$(u, d)_L$	3	2	1/3
quark	$\widetilde{u}_R$	u_R	3	1	4/3
	$\widetilde{d}_R$	d_R	3	1	−2/3
Higgs/	$(H^0_d\,,\, H^-_d)$	$(\widetilde{H}^0_d, \widetilde{H}^-_d)$	1	2	−1
higgsino	$(H^+_u\,,\, H^0_u)$	$(\widetilde{H}^+_u, \widetilde{H}^0_u)$	1	2	1

some remnant of supersymmetry does survive down to the TeV-scale. For example, in models of *split-supersymmetry* [15,16], some fraction of the supersymmetric spectrum remains light enough (with masses near the TeV scale) to provide successful gauge coupling unification and a viable dark matter candidate. If experimentation at future colliders uncovers evidence for (any remnant of) supersymmetry at low-energies, this would have a profound effect on the study of TeV-scale physics, and the development of a more fundamental theory of mass and symmetry-breaking phenomena in particle physics.

I.2. Structure of the MSSM: The minimal supersymmetric extension of the Standard Model (MSSM) consists of taking the fields of the two-Higgs-doublet extension of the Standard Model and adding the corresponding supersymmetric partners [4,17]. The corresponding field content of the MSSM and their gauge quantum numbers are shown in Table 1. The electric charge $Q = T_3 + \frac{1}{2}Y$ is determined in terms of the third component of the weak isospin (T_3) and the U(1) hypercharge (Y).

The gauge super-multiplets consist of the gluons and their *gluino* fermionic superpartners and the SU(2)×U(1) gauge bosons and their *gaugino* fermionic superpartners. The Higgs multiplets consist of two complex doublets of Higgs fields, their *higgsino* fermionic superpartners and the corresponding antiparticle fields. The matter super-multiplets consist of three generations of left-handed and right-handed quarks and lepton fields, their scalar superpartners (squark and slepton fields) and the corresponding antiparticle fields.

The enlarged Higgs sector of the MSSM constitutes the minimal structure needed to guarantee the cancellation of anomalies from the introduction of the higgsino superpartners. Moreover, without a second Higgs doublet, one cannot generate mass for both "up"-type and "down"-type quarks (and charged leptons) in a way consistent with the supersymmetry [18–20]. The (renormalizable) MSSM Lagrangian is then constructed by including all possible interaction terms (of dimension four or less) that satisfy the spacetime supersymmetry algebra, SU(3)×SU(2)×U(1) gauge invariance and B–L conservation (B =baryon number and L =lepton number). Finally, the most general soft-supersymmetry-breaking terms are added [10]. To generate nonzero neutrino masses, extra structure is needed as discussed in section I.8.

I.2.1. Constraints on supersymmetric parameters: If supersymmetry is associated with the origin of the electroweak scale, then the mass parameters introduced by the soft-supersymmetry-breaking must be generally of order 1 TeV or below [21] (although models have been proposed in which some supersymmetric particle masses can be larger, in the range of 1–10 TeV [22]) . Some lower bounds on these parameters exist due to the absence of supersymmetric-particle production at current accelerators [23]. Additional constraints arise from limits on the contributions of virtual supersymmetric particle exchange to a variety of Standard Model processes [24,25].

For example, the Standard Model global fit to precision electroweak data is quite good [26]. If all supersymmetric particle masses are significantly heavier than m_Z (in practice, masses greater than 300 GeV are sufficient [27]) , then the effects of the supersymmetric particles decouple in loop-corrections to electroweak observables [28]. In this case, the Standard Model global fit to precision data and the corresponding MSSM fit yield similar results. On the other hand, regions of parameter space with light supersymmetric particle masses (just above the present day experimental limits) can in some cases generate significant one-loop corrections, resulting in a slight improvement or worsening of the overall global fit to the electroweak data depending on the choice of the MSSM parameters [29]. Thus, the precision electroweak data provide some constraints on the magnitude of the soft-supersymmetry-breaking terms.

There are a number of other low-energy measurements that are especially sensitive to the effects of new physics through virtual loops. For example, the virtual exchange of supersymmetric particles can contribute to the muon anomalous magnetic moment, $a_\mu \equiv \frac{1}{2}(g-2)_\mu$, and to the inclusive decay rate for $b \to s\gamma$. The most recent theoretical analysis of $(g-2)_\mu$ finds a small deviation (less than three standard deviations) of the theoretical prediction from the experimentally observed value [30]. The theoretical prediction for $\Gamma(b \to s\gamma)$ agrees quite well (within the error bars) to the experimental observation [31]. In both cases, supersymmetric corrections could have generated an observable shift from the Standard Model prediction in some regions of the MSSM parameter space [31–33]. The absence of a significant deviation places interesting constraints on the low-energy supersymmetry parameters.

I.2.2. R-Parity and the lightest supersymmetric particle: As a consequence of B–L invariance, the MSSM possesses a multiplicative R-parity invariance, where $R = (-1)^{3(B-L)+2S}$ for a particle of spin S [34]. Note that this implies that all the ordinary Standard Model particles have even R parity, whereas the corresponding supersymmetric partners have odd R parity. The conservation of R parity in scattering and decay processes has a crucial impact on supersymmetric phenomenology. For example, starting from an initial state involving ordinary (R-even) particles, it follows that supersymmetric particles must be produced in pairs. In general, these particles are highly unstable and decay into lighter states. However, R-parity invariance also implies that the lightest supersymmetric particle (LSP) is absolutely stable, and must eventually be produced at the end of a decay chain initiated by the decay of a heavy unstable supersymmetric particle.

In order to be consistent with cosmological constraints, a stable LSP is almost certainly electrically and color neutral [35]. (There are some model circumstances in which a colored gluino LSP is allowed [36], but we do not consider this possibility further here.) Consequently, the LSP in an R-parity-conserving theory is weakly interacting with ordinary matter, *i.e.*, it behaves like a stable heavy neutrino and will escape collider detectors without being directly observed. Thus, the canonical signature for conventional R-parity-conserving supersymmetric theories is missing (transverse) energy, due to the escape of the LSP. Moreover, the LSP is a prime candidate for "cold dark matter" [13], an important component of the non-baryonic dark matter that is required in many models of cosmology and galaxy formation [37]. Further aspects of dark matter can be found in Ref. [38].

I.2.3. The goldstino and gravitino: In the MSSM, supersymmetry breaking is accomplished by including the most general renormalizable soft-supersymmetry-breaking terms consistent with the SU(3)×SU(2)×U(1) gauge symmetry and R-parity invariance. These terms parameterize our ignorance of the fundamental mechanism of supersymmetry breaking. If supersymmetry breaking occurs spontaneously, then a massless Goldstone fermion called the *goldstino* ($\widetilde{G}$) must exist. The goldstino would then be the LSP and could play an important role in supersymmetric phenomenology [39]. However, the goldstino is a physical degree of freedom only in models of spontaneously-broken global supersymmetry. If supersymmetry is a local symmetry, then the theory must incorporate gravity; the resulting theory is called supergravity [40]. In models of spontaneously-broken supergravity, the goldstino is "absorbed" by the *gravitino* ($\widetilde{g}_{3/2}$), the spin-3/2 partner of the graviton [41]. By this super-Higgs mechanism, the goldstino is removed from the physical spectrum and the gravitino acquires a mass ($m_{3/2}$).

I.2.4. Hidden sectors and the structure of supersymmetry breaking: It is very difficult (perhaps impossible) to construct a realistic model of spontaneously-broken low-energy supersymmetry where the supersymmetry breaking arises solely as a consequence of the interactions of the particles of the MSSM. A more viable scheme posits a theory consisting of at least two distinct sectors: a "hidden" sector consisting of particles that are completely neutral with respect to the Standard Model gauge group, and a "visible" sector consisting of the particles of the MSSM. There are no renormalizable tree-level interactions between particles of the visible and hidden sectors. Supersymmetry breaking is assumed to occur in the hidden sector, and to then be transmitted to the MSSM by some mechanism. Two theoretical scenarios have been examined in detail: gravity-mediated and gauge-mediated supersymmetry breaking.

Supergravity models provide a natural mechanism for transmitting the supersymmetry breaking of the hidden sector to the particle spectrum of the MSSM. In models of *gravity-mediated* supersymmetry breaking, gravity is the messenger of supersymmetry breaking [42–44]. More precisely, supersymmetry breaking is mediated by effects of gravitational strength (suppressed by an inverse power of the Planck mass). In this scenario, the gravitino mass is of order the electroweak-symmetry-breaking scale, while its couplings are roughly gravitational in strength [2,45]. Such a gravitino would play no role in supersymmetric phenomenology at colliders.

In *gauge-mediated* supersymmetry breaking, supersymmetry breaking is transmitted to the MSSM via gauge forces. A typical structure of such models involves a hidden sector where supersymmetry is broken, a "messenger sector" consisting of particles (messengers) with SU(3)×SU(2)×U(1) quantum numbers, and the visible sector consisting of the fields of the MSSM [46,47]. The direct coupling of the messengers to the hidden sector generates a supersymmetry-breaking spectrum in the messenger sector. Finally, supersymmetry breaking is transmitted to the MSSM via the virtual exchange of the messengers. If this approach is extended to incorporate gravitational phenomena, then supergravity effects will also contribute to supersymmetry breaking. However, in models of gauge-mediated supersymmetry breaking, one usually chooses the model parameters in such a way that the virtual exchange of the messengers dominates the effects of the direct gravitational interactions between the hidden and visible sectors. In this scenario, the gravitino mass is typically in the eV to keV range, and is therefore the LSP. The helicity $\pm\frac{1}{2}$ components of $\widetilde{g}_{3/2}$ behave approximately like the goldstino; its coupling to the particles of the MSSM is significantly stronger than a coupling of gravitational strength.

I.2.5. Supersymmetry and extra dimensions: During the last few years, new approaches to supersymmetry breaking have been proposed, based on theories in which the number of space dimensions is greater than three. This is not a new idea—consistent superstring theories are formulated in ten spacetime dimensions, and the associated M-theory is based in eleven spacetime dimensions [48]. Nevertheless, in all approaches considered above, the string scale and the inverse size of the extra dimensions are assumed to be at or near the Planck scale, below which an effective four spacetime dimensional broken supersymmetric field theory emerges. More recently, a number of supersymmetry-breaking mechanisms have been proposed that are inherently extra-dimensional [49]. The size of the extra dimensions can be significantly larger than $M_{\rm P}^{-1}$: in some cases of order $({\rm TeV})^{-1}$ or even larger [50,51]. For example, in one approach, the fields of the MSSM live on some brane (a lower-dimensional manifold embedded in a higher dimensional spacetime), while the sector of the theory that breaks supersymmetry lives on a second separated brane. Two examples of this approach are anomaly-mediated supersymmetry breaking of Ref. [52] and gaugino-mediated supersymmetry breaking of Ref. [53]; in both cases supersymmetry-breaking is transmitted through fields that live in the bulk (the higher dimensional space between the two branes). This setup has some features in common with both gravity-mediated and gauge-mediated supersymmetry breaking (*e.g.*, a hidden and visible sector and messengers).

Alternatively, one can consider a higher dimensional theory that is compactified to four spacetime dimensions. In this approach, supersymmetry is broken by boundary conditions on the compactified space that distinguish between fermions and bosons. This is the so-called Scherk-Schwarz mechanism [54]. The phenomenology of such models can be strikingly different from that of the usual MSSM [55]. All these extra-dimensional ideas clearly deserve further investigation, although they will not be discussed further here.

I.2.6. Split-supersymmetry: If supersymmetry is not connected with the origin of the electroweak scale, string theory suggests that supersymmetry still plays a significant role in Planck-scale physics. However, it may still be possible that some remnant of the superparticle spectrum survives down to the TeV-scale or below. This is the idea of split-supersymmetry [15], in which supersymmetric scalar partners of the quarks and leptons are significantly heavier (perhaps by many orders of magnitude) than 1 TeV, whereas the fermionic partners of the gauge and Higgs bosons have masses of order 1 TeV or below (presumably protected by some chiral symmetry). With the exception of a single light neutral scalar whose properties are indistinguishable from those of the Standard Model Higgs boson, all other Higgs bosons are also taken to be very heavy.

The supersymmetry-breaking required to produce such a scenario would destabilize the gauge hierarchy. In particular, split-supersymmetry cannot provide a natural explanation for the existence of the light Standard Model–like Higgs boson whose mass lies orders below the the mass scale of the heavy scalars. Nevertheless, models of split-supersymmetry can account for the dark matter (which is assumed to be the LSP) and gauge coupling unification. Thus, there is some motivation for pursuing the phenomenology of such approaches [16]. One notable difference from the usual MSSM phenomenology is the existence of a long-lived gluino [56].

I.3. Parameters of the MSSM: The parameters of the MSSM are conveniently described by considering separately the supersymmetry-conserving sector and the supersymmetry-breaking sector. A careful discussion of the conventions used in defining the tree-level MSSM parameters can be found in Ref. [57]. (Additional fields and parameters must be introduced if one wishes to account for non-zero neutrino masses. We shall not pursue this here; see section I.8 for a discussion of supersymmetric approaches that incorporate neutrino masses.) For simplicity, consider first the case of one generation of quarks, leptons, and their scalar superpartners.

I.3.1. The supersymmetric-conserving parameters: The parameters of the supersymmetry-conserving sector consist of: (i) gauge couplings: g_s, g, and g', corresponding to the Standard Model gauge group SU(3)×SU(2)×U(1) respectively; (ii) a supersymmetry-conserving higgsino mass parameter μ; and (iii) Higgs-fermion Yukawa coupling constants: λ_u, λ_d, and λ_e (corresponding to the coupling of one generation of left and right-handed quarks and leptons and their superpartners to the Higgs bosons and higgsinos). Because there is no right-handed neutrino (and its superpartner) in the MSSM as defined here, one cannot introduce a Yukawa coupling λ_ν.

I.3.2. The supersymmetric-breaking parameters: The supersymmetry-breaking sector contains the following set of parameters: (i) gaugino Majorana masses M_3, M_2, and M_1 associated with the SU(3), SU(2), and U(1) subgroups of the Standard Model; (ii) five scalar squared-mass parameters for the squarks and sleptons, $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, and $M^2_{\widetilde{E}}$ [corresponding to the five electroweak gauge multiplets, *i.e.*, superpartners of $(u, d)_L$, u^c_L, d^c_L, $(\nu,\ e^-)_L$, and e^c_L, where the superscript c indicates a charge-conjugated fermion]; and (iii) Higgs-squark-squark and Higgs-slepton-slepton trilinear interaction terms, with coefficients $\lambda_u A_U$, $\lambda_d A_D$, and $\lambda_e A_E$ (which define the so-called "A-parameters"). It is traditional to factor out the Yukawa couplings in the definition of the A-parameters (originally motivated by a simple class of gravity-mediated supersymmetry-breaking models [2,4]) . If the A-parameters defined in this way are parametrically of the same order (or smaller) as compared to other supersymmetry-breaking mass parameters, then only the A-parameters of the third generation will be phenomenologically relevant.

Finally, we add: (iv) three scalar squared-mass parameters—two of which (m_1^2 and m_2^2) contribute to the diagonal Higgs squared-masses, given by $m_1^2 + |\mu|^2$ and $m_2^2 + |\mu|^2$, and a third which contributes to the off-diagonal Higgs squared-mass term, $m_{12}^2 \equiv B\mu$ (which defines the "B-parameter"). The breaking of the electroweak symmetry SU(2)×U(1) to U(1)$_{\rm EM}$ is only possible after introducing the supersymmetry-breaking Higgs squared-mass parameters. Minimizing the resulting Higgs scalar potential, these three squared-mass parameters can be re-expressed in terms of the two Higgs vacuum expectation values, v_d and v_u (also called v_1 and v_2, respectively, in the literature), and one physical Higgs mass. Here, v_d [v_u] is the vacuum expectation value of the neutral component of the Higgs field H_d [H_u] that couples exclusively to down-type (up-type) quarks and leptons. Note that $v_d^2 + v_u^2 = 4m_W^2/g^2 = (246\ \text{GeV})^2$ is fixed by the W mass and the gauge coupling, whereas the ratio

$$\tan\beta = v_u/v_d \tag{1}$$

is a free parameter of the model. By convention, the Higgs field phases are chosen such that $0 \leq \beta \leq \pi/2$.

I.3.3. MSSM-124: The total number of degrees of freedom of the MSSM is quite large, primarily due to the parameters of the soft-supersymmetry-breaking sector. In particular, in the case of three generations of quarks, leptons, and their superpartners, $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, and $M^2_{\widetilde{E}}$ are hermitian 3×3 matrices, and A_U, A_D and A_E are complex 3×3 matrices. In addition, M_1, M_2, M_3, B, and μ are in general complex. Finally, as in the Standard Model, the Higgs-fermion Yukawa couplings, λ_f ($f = u$, d, and e), are complex 3×3 matrices that are related to the quark and lepton mass matrices via: $M_f = \lambda_f v_f/\sqrt{2}$, where $v_e \equiv v_d$ (with v_u and v_d as defined above). However, not all these parameters are physical. Some of the MSSM parameters can be eliminated by expressing interaction eigenstates in terms of the mass eigenstates, with an appropriate redefinition of the MSSM fields to remove unphysical degrees of freedom. The analysis of Ref. [58] shows that the MSSM possesses 124 independent parameters. Of these, 18 parameters correspond to Standard Model parameters (including the QCD vacuum angle $\theta_{\rm QCD}$), one corresponds to a Higgs sector parameter (the analogue of the Standard Model Higgs mass), and 105 are genuinely new parameters of the model.

The latter include: five real parameters and three CP-violating phases in the gaugino/higgsino sector, 21 squark and slepton masses, 36 real mixing angles to define the squark and slepton mass eigenstates, and 40 CP-violating phases that can appear in squark and slepton interactions. The most general R-parity-conserving minimal supersymmetric extension of the Standard Model (without additional theoretical assumptions) will be denoted henceforth as MSSM-124 [59].

I.4. The supersymmetric-particle sector: Consider the sector of supersymmetric particles (*sparticles*) in the MSSM. The supersymmetric partners of the gauge and Higgs bosons are fermions, whose names are obtained by appending "ino" at the end of the corresponding Standard Model particle name. The gluino is the color octet Majorana fermion partner of the gluon with mass $M_{\widetilde{g}} = |M_3|$. The supersymmetric partners of the electroweak gauge and Higgs bosons (the gauginos and higgsinos) can mix. As a result, the physical states of definite mass are model-dependent linear combinations of the charged and neutral gauginos and higgsinos, called *charginos* and *neutralinos*, respectively. Like the gluino, the neutralinos are also Majorana fermions, which provide for some distinctive phenomenological signatures [60,61].

I.4.1. The charginos and neutralinos: The mixing of the charged gauginos ($\widetilde{W}^{\pm}$) and charged higgsinos (H_u^+ and H_d^-) is described (at tree-level) by a 2×2 complex mass matrix [62–64]:

$$M_C \equiv \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g v_u \\ \frac{1}{\sqrt{2}} g v_d & \mu \end{pmatrix} . \tag{2}$$

To determine the physical chargino states and their masses, one must perform a singular value decomposition [65] of the complex matrix M_C:

$$U^* M_C V^{-1} = \mathrm{diag}(M_{\widetilde{\chi}_1^+} \, , \, M_{\widetilde{\chi}_2^+}) \, , \tag{3}$$

where U and V are unitary matrices and the right hand side of Eq. (3) is the diagonal matrix of (non-negative) chargino masses. The physical chargino states are denoted by $\widetilde{\chi}_1^{\pm}$ and $\widetilde{\chi}_2^{\pm}$. These are linear combinations of the charged gaugino and higgsino states determined by the matrix elements of U and V [62–64]. The chargino masses correspond to the *singular values* [65] of M_C, *i.e.*, the positive square roots of the eigenvalues of $M_C^\dagger M_C$:

$$M^2_{\widetilde{\chi}_1^+,\widetilde{\chi}_2^+} = \tfrac{1}{2}\Bigg\{|\mu|^2 + |M_2|^2 + 2m_W^2 \mp \Bigg[\left(|\mu|^2 + |M_2|^2 + 2m_W^2\right)^2 - 4|\mu|^2|M_2|^2 - 4m_W^4 \sin^2 2\beta + 8m_W^2 \sin 2\beta \,\mathrm{Re}(\mu M_2)\Bigg]^{1/2}\Bigg\}, \quad (4)$$

where the states are ordered such that $M_{\widetilde{\chi}_1^+} \leq M_{\widetilde{\chi}_2^+}$. It is often convenient to choose a convention where $\tan\beta$ and M_2 are real and positive. Note that the relative phase of M_2 and μ is meaningful. (If CP-violating effects are neglected, then μ can be chosen real but may be either positive or negative.) The sign of μ is convention-dependent; the reader is warned that both sign conventions appear in the literature. The sign convention for μ in Eq. (2) is used by the LEP collaborations [23] in their plots of exclusion contours in the M_2 vs. μ plane derived from the non-observation of $e^+e^- \to \widetilde{\chi}_1^+\widetilde{\chi}_1^-$.

The mixing of the neutral gauginos ($\widetilde{B}$ and $\widetilde{W}^0$) and neutral higgsinos ($\widetilde{H}_d^0$ and $\widetilde{H}_u^0$) is described (at tree-level) by a 4×4 complex symmetric mass matrix [62,63,66,67]:

$$M_N \equiv \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g'v_d & \frac{1}{2}g'v_u \\ 0 & M_2 & \frac{1}{2}gv_d & -\frac{1}{2}gv_u \\ -\frac{1}{2}g'v_d & \frac{1}{2}gv_d & 0 & -\mu \\ \frac{1}{2}g'v_u & -\frac{1}{2}gv_u & -\mu & 0 \end{pmatrix}. \quad (5)$$

To determine the physical neutralino states and their masses, one must perform a Takagi factorization [65,68] of the complex symmetric matrix M_N:

$$W^T M_N W = \mathrm{diag}(M_{\widetilde{\chi}_1^0}\,,\, M_{\widetilde{\chi}_2^0}\,,\, M_{\widetilde{\chi}_3^0}\,,\, M_{\widetilde{\chi}_4^0})\,, \quad (6)$$

where W is a unitary matrix and the right hand side of Eq. (6) is the diagonal matrix of (non-negative) neutralino masses. The physical neutralino states are denoted by $\widetilde{\chi}_i^0$ ($i = 1, \ldots 4$), where the states are ordered such that $M_{\widetilde{\chi}_1^0} \leq M_{\widetilde{\chi}_2^0} \leq M_{\widetilde{\chi}_3^0} \leq M_{\widetilde{\chi}_4^0}$. The $\widetilde{\chi}_i^0$ are the linear combinations of the neutral gaugino and higgsino states determined by the matrix elements of W (in Ref. [62], $W = N^{-1}$). The neutralino masses correspond to the singular values of M_N (*i.e.*, the positive square roots of the eigenvalues of $M_N^\dagger M_N$). Exact formulae for these masses can be found in Ref. [66,69].

If a chargino or neutralino state approximates a particular gaugino or higgsino state, it is convenient to employ the corresponding nomenclature. Specifically, if M_1 and M_2 are small compared to m_Z and $|\mu|$, then the lightest neutralino $\widetilde{\chi}_1^0$ would be nearly a pure *photino*, $\widetilde{\gamma}$, the supersymmetric partner of the photon. If M_1 and m_Z are small compared to M_2 and $|\mu|$, then the lightest neutralino would be nearly a pure *bino*, $\widetilde{B}$, the supersymmetric partner of the weak hypercharge gauge boson. If M_2 and m_Z are small compared to M_1 and $|\mu|$, then the lightest chargino pair and neutralino would constitute a triplet of roughly mass-degenerate pure *winos*, $\widetilde{W}^{\pm}$, and $\widetilde{W}_3^0$, the supersymmetric partners of the weak SU(2) gauge bosons. Finally, if $|\mu|$ and m_Z are small compared to M_1 and M_2, then the lightest neutralino would be nearly a pure *higgsino*. Each of the above cases leads to a strikingly different phenomenology.

I.4.2. The squarks, sleptons and sneutrinos: The supersymmetric partners of the quarks and leptons are spin-zero bosons: the *squarks*, charged *sleptons*, and *sneutrinos*. For a given fermion f, there are two supersymmetric partners, $\widetilde{f}_L$ and $\widetilde{f}_R$, which are scalar partners of the corresponding left- and right-handed fermion. (There is no $\widetilde{\nu}_R$ in the MSSM.) However, in general, $\widetilde{f}_L$ and $\widetilde{f}_R$ are not mass-eigenstates, since there is $\widetilde{f}_L$–$\widetilde{f}_R$ mixing. For three generations of squarks, one must in general diagonalize 6×6 matrices corresponding to the basis $(\widetilde{q}_{iL}, \widetilde{q}_{iR})$, where $i = 1, 2, 3$ are the generation labels. For simplicity, only the one-generation case is illustrated in detail below (using the notation of the third family). In this case, the tree-level squark squared-mass matrix is given by [70]

$$M_F^2 = \begin{pmatrix} M_{\widetilde{Q}}^2 + m_q^2 + L_q & m_q X_q^* \\ m_q X_q & M_{\widetilde{R}}^2 + m_q^2 + R_q \end{pmatrix} , \qquad (7)$$

where

$$X_q \equiv A_q - \mu^* (\cot\beta)^{2T_{3q}} , \qquad (8)$$

and $T_{3q} = \frac{1}{2}$ $[-\frac{1}{2}]$ for $q = t$ $[b]$. The diagonal squared-masses are governed by soft-supersymmetry breaking squared-masses $M^2_{\widetilde{Q}}$ and $M^2_{\widetilde{R}} \equiv M^2_{\widetilde{U}}$ $[M^2_{\widetilde{D}}]$ for $q = t$ $[b]$, the corresponding quark masses m_t $[m_b]$, and electroweak correction terms:

$$L_q \equiv (T_{3q} - e_q \sin^2\theta_W) m_Z^2 \cos 2\beta \,,$$

$$R_q \equiv e_q \sin^2\theta_W \, m_Z^2 \cos 2\beta \,, \tag{9}$$

where $e_q = \frac{2}{3}$ $[-\frac{1}{3}]$ for $q = t$ $[b]$. The off-diagonal squared squark masses are proportional to the corresponding quark masses and depend on $\tan\beta$ [Eq. (1)], the soft-supersymmetry-breaking A-parameters and the higgsino mass parameter μ. The signs of the A and μ parameters are convention-dependent; other choices appear frequently in the literature. Due to the appearance of the *quark* mass in the off-diagonal element of the squark squared-mass matrix, one expects the $\widetilde{q}_L$–$\widetilde{q}_R$ mixing to be small, with the possible exception of the third-generation, where mixing can be enhanced by factors of m_t and $m_b \tan\beta$.

In the case of third generation $\widetilde{q}_L$–$\widetilde{q}_R$ mixing, the mass eigenstates (usually denoted by $\widetilde{q}_1$ and $\widetilde{q}_2$, with $m_{\widetilde{q}_1} < m_{\widetilde{q}_2}$) are determined by diagonalizing the 2×2 matrix M_F^2 given by Eq. (7). The corresponding squared-masses and mixing angle are given by [70]:

$$m^2_{\tilde{q}_{1,2}} = \frac{1}{2}\left[\mathrm{Tr}\ M_F^2 \pm \sqrt{(\mathrm{Tr}\ M_F^2)^2 - 4\det\ M_F^2}\right] \,,$$

$$\sin 2\theta_{\tilde{q}} = \frac{2m_q|X_q|}{m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}} \,. \tag{10}$$

The one-generation results above also apply to the charged sleptons, with the obvious substitutions: $q \to \tau$ with $T_{3\tau} = -\frac{1}{2}$ and $e_\tau = -1$, and the replacement of the supersymmetry-breaking parameters: $M^2_{\widetilde{Q}} \to M^2_{\widetilde{L}}$, $M^2_{\widetilde{D}} \to M^2_{\widetilde{E}}$ and $A_q \to A_\tau$. For the neutral sleptons, $\widetilde{\nu}_R$ does not exist in the MSSM, so $\widetilde{\nu}_L$ is a mass-eigenstate.

In the case of three generations, the supersymmetry-breaking scalar squared-masses $[M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$ and $M^2_{\widetilde{E}}]$ and the A-parameters that parameterize the Higgs couplings to up and down-type squarks and charged sleptons (henceforth denoted by A_U, A_D and A_E, respectively) are now 3×3 matrices as noted in Section I.3. The diagonalization of the 6×6 squark mass matrices yields $\widetilde{f}_{iL}$–$\widetilde{f}_{jR}$ mixing (for $i \neq j$). In practice, since the $\widetilde{f}_L$–$\widetilde{f}_R$ mixing is appreciable only for the third generation, this additional complication can usually be neglected.

Radiative loop corrections will modify all tree-level results for masses quoted in this section. These corrections must be included in any precision study of supersymmetric phenomenology [71]. Beyond tree-level, the definition of the supersymmetric parameters becomes convention-dependent. For example, one can define physical couplings or running couplings, which differ beyond tree-level. This provides a challenge to any effort that attempts to extract supersymmetric parameters from data. The supersymmetric parameter analysis (SPA) project proposes a set of conventions [72] based on a consistent set of conventions and input parameters. dimensional reduction scheme for the regularization of higher-order loop corrections in supersymmetric theories recently advocated in Ultimately, these efforts will facilitate the reconstruction of the fundamental supersymmetric theory (and its breaking mechanism) from high precision studies of supersymmetric phenomena at future colliders.

I.5. The Higgs sector of the MSSM: Next, consider the MSSM Higgs sector [19,20,73]. Despite the large number of potential CP-violating phases among the MSSM-124 parameters, the tree-level MSSM Higgs sector is automatically CP-conserving. That is, unphysical phases can be absorbed into the definition of the Higgs fields such that $\tan\beta$ is a real parameter (conventionally chosen to be positive). Moreover, the physical neutral Higgs scalars are CP eigenstates. The model contains five physical Higgs particles: a charged Higgs boson pair ($H^{\pm}$), two CP-even neutral Higgs bosons (denoted by h^0 and H^0 where $m_h \leq m_H$), and one CP-odd neutral Higgs boson (A^0).

I.5.1 The Tree-level MSSM Higgs sector: The properties of the Higgs sector are determined by the Higgs potential, which is made up of quadratic terms [whose squared-mass coefficients were mentioned above Eq. (1)] and quartic interaction terms whose coefficients are dimensionless couplings. The quartic interaction terms are manifestly supersymmetric at tree-level (and are modified by supersymmetry-breaking effects only at the loop level). In general, the quartic couplings arise from two sources: (i) the supersymmetric generalization of the scalar potential (the so-called "F-terms"), and (ii) interaction terms related by supersymmetry to the coupling of the scalar fields and the gauge fields, whose coefficients are proportional to the corresponding gauge couplings (the so-called "D-terms"). In the MSSM, F-term contributions to the quartic couplings are absent (although such terms may be present in extensions of the MSSM, *e.g.*, models with Higgs singlets). As a result, the strengths of the MSSM quartic Higgs interactions are fixed in terms of the gauge couplings. Due to the resulting constraint on the form of the two-Higgs-doublet scalar potential, all the tree-level MSSM Higgs-sector parameters depend only on two quantities: $\tan\beta$ [defined in Eq. (1)] and one Higgs mass (usually taken to be m_A). From these two quantities, one can predict the values of the remaining Higgs boson masses, an angle α (which measures the component of the original $Y = \pm 1$ Higgs doublet states in the physical CP-even neutral scalars), and the Higgs boson self-couplings.

I.5.2 The radiatively-corrected MSSM Higgs sector: When radiative corrections are incorporated, additional parameters of the supersymmetric model enter via virtual loops. The impact of these corrections can be significant [74]. For example, the tree-level MSSM-124 prediction for the upper bound of the lightest CP-even Higgs mass, $m_h \leq m_Z|\cos 2\beta| \leq m_Z$ [19,20], can be substantially modified when radiative corrections are included. The qualitative behavior of these radiative corrections can be most easily seen in the large top-squark mass limit, where in addition, both the splitting of the two diagonal entries and the two off-diagonal entries of the top-squark squared-mass matrix [Eq. (7)] are small in comparison to the average of the two top-squark squared-masses, $M_{\rm S}^2 \equiv \frac{1}{2}(M^2_{\widetilde{t}_1} + M^2_{\widetilde{t}_2})$. In this case (assuming $m_A > m_Z$), the predicted upper bound for m_h

(which reaches its maximum at large $\tan\beta$) is approximately given by

$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left\{\ln\left(M_{\rm S}^2/m_t^2\right) + \frac{X_t^2}{M_{\rm S}^2}\left(1 - \frac{X_t^2}{12M_{\rm S}^2}\right)\right\}, \quad (11)$$

where $X_t \equiv A_t - \mu\cot\beta$ is the top-squark mixing factor [see Eq. (7)]. A more complete treatment of the radiative corrections [75] shows that Eq. (11) somewhat overestimates the true upper bound of m_h. These more refined computations, which incorporate renormalization group improvement and the leading two-loop contributions, yield $m_h \lesssim 135$ GeV (with an accuracy of a few GeV) for $m_t = 175$ GeV and $M_S \lesssim 2$ TeV [75]. This Higgs mass upper bound can be relaxed somewhat in non-minimal extensions of the MSSM, as noted in Section I.9.

In addition, one-loop radiative corrections can introduce CP-violating effects in the Higgs sector, which depend on some of the CP-violating phases among the MSSM-124 parameters [76]. Although these effects are more model-dependent, they can have a non-trivial impact on the Higgs searches at future colliders. A summary of the current MSSM Higgs mass limits can be found in Ref. [77].

I.6. Restricting the MSSM parameter freedom: In Sections I.4 and I.5 we surveyed the parameters that comprise the MSSM-124. However in its most general form, the MSSM-124 is not a phenomenologically-viable theory over most of its parameter space. This conclusion follows from the observation that a generic point in the MSSM-124 parameter space exhibits: (i) no conservation of the separate lepton numbers L_e, L_μ, and L_τ; (ii) unsuppressed FCNC's; and (iii) new sources of CP violation that are inconsistent with the experimental bounds.

For example, the MSSM contains many new sources of CP violation [78]. In particular, some combinations of the complex phases of the gaugino-mass parameters, the A parameters, and μ must be less than of order 10^{-2}—10^{-3} (for a supersymmetry-breaking scale of 100 GeV) to avoid generating electric dipole moments for the neutron, electron, and atoms in conflict with observed data [79–81]. The non-observation of FCNC's [24,25] places additional strong constraints on the off-diagonal matrix elements of the squark and slepton soft-supersymmetry-breaking squared masses and A-parameters (see Section I.3.3). As a result of the phenomenological deficiencies listed above, almost the entire MSSM-124 parameter space is ruled out! This theory is viable only at very special "exceptional" regions of the full parameter space.

The MSSM-124 is also theoretically incomplete since it provides no explanation for the origin of the supersymmetry-breaking parameters (and in particular, why these parameters should conform to the exceptional points of the parameter space mentioned above). Moreover, there is no understanding of the choice of parameters that leads to the breaking of the electroweak symmetry. What is needed ultimately is a fundamental theory of supersymmetry breaking, which would provide a rationale for some set of soft-supersymmetry breaking terms that would be consistent with the phenomenological constraints referred to above. Presumably, the number of independent parameters characterizing such a theory would be considerably less than 124.

I.6.1. Bottom-up approach for constraining the parameters of the MSSM: In the absence of a fundamental theory of supersymmetry breaking, there are two general approaches for reducing the parameter freedom of MSSM-124. In the low-energy approach, an attempt is made to elucidate the nature of the exceptional points in the MSSM-124 parameter space that are phenomenologically viable. Consider the following two possible choices. First, one can assume that $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, $M^2_{\widetilde{E}}$, and A_U, A_D, A_E are generation-independent (horizontal universality [7,58,82]) . Alternatively, one can simply require that all the aforementioned matrices are flavor diagonal in a basis where the quark and lepton mass matrices are diagonal (flavor alignment [83]) . In either case, L_e, L_μ, and L_τ are separately conserved, while tree-level FCNC's are automatically absent. In both cases, the number of free parameters characterizing the MSSM is substantially less than 124. Both scenarios are phenomenologically viable, although there is no strong theoretical basis for either scenario.

I.6.2. Top-down approach for constraining the parameters of the MSSM: In the high-energy approach, one imposes a particular structure on the soft-supersymmetry-breaking terms at a common high-energy scale (such as the Planck scale, $M_{\rm P}$). Using the renormalization group equations, one can then derive the low-energy MSSM parameters relevant for collider physics. The initial conditions (at the appropriate high-energy scale) for the renormalization group equations depend on the mechanism by which supersymmetry breaking is communicated to the effective low energy theory. Examples of this scenario are provided by models of gravity-mediated and gauge-mediated supersymmetry breaking (see Section I.2). One bonus of such an approach is that one of the diagonal Higgs squared-mass parameters is typically driven negative by renormalization group evolution [84]. Thus, electroweak symmetry breaking is generated radiatively, and the resulting electroweak symmetry-breaking scale is intimately tied to the scale of low-energy supersymmetry breaking.

One prediction of the high-energy approach that arises in most grand unified supergravity models and gauge-mediated supersymmetry-breaking models is the unification of the (tree-level) gaugino mass parameters at some high-energy scale $M_{\rm X}$:

$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}\,. \qquad (12)$$

Consequently, the effective low-energy gaugino mass parameters (at the electroweak scale) are related:

$$M_3 = (g_s^2/g^2)M_2\;, \qquad M_1 = (5g'^2/3g^2)M_2 \simeq 0.5M_2\;. \quad (13)$$

In this case, the chargino and neutralino masses and mixing angles depend only on three unknown parameters: the gluino mass, μ, and $\tan\beta$. If in addition $|\mu| \gg M_1 \gtrsim m_Z$, then the lightest neutralino is nearly a pure bino, an assumption often made in supersymmetric particle searches at colliders.

I.6.3. Anomaly-mediated supersymmetry-breaking: In some supergravity models, tree-level masses for the gauginos are absent. The gaugino mass parameters arise at one-loop and do not satisfy Eq. (13). In this case, one finds a model-independent contribution to the gaugino mass whose origin can be traced to the super-conformal (super-Weyl) anomaly, which is common to all supergravity models [52]. This approach is called *anomaly-mediated* supersymmetry breaking (AMSB). Eq. (13) is then replaced (in the one-loop approximation) by:

$$M_i \simeq \frac{b_i g_i^2}{16\pi^2} m_{3/2}\,, \tag{14}$$

where $m_{3/2}$ is the gravitino mass (assumed to be of order 1 TeV), and b_i are the coefficients of the MSSM gauge beta-functions corresponding to the corresponding U(1), SU(2) and SU(3) gauge groups: $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$. Eq. (14) yields $M_1 \simeq 2.8M_2$ and $M_3 \simeq -8.3M_2$, which implies that the lightest chargino pair and neutralino comprise a nearly mass-degenerate triplet of winos, $\widetilde{W}^{\pm}$, $\widetilde{W}^0$ (*c.f.* Table 1), over most of the MSSM parameter space . (For example, if $|\mu| \gg m_Z$, then Eq. (14) implies that $M_{\widetilde{\chi}_1^{\pm}} \simeq M_{\widetilde{\chi}_1^0} \simeq M_2$ [85].) The corresponding supersymmetric phenomenology differs significantly from the standard phenomenology based on Eq. (13), and is explored in detail in Ref. [86]. Anomaly-mediated supersymmetry breaking also generates (approximate) flavor-diagonal squark and slepton mass matrices. However, this yields negative squared-mass contributions for the sleptons in the MSSM. This fatal flaw may be possible to cure in approaches beyond the minimal supersymmetric model [87]. Alternatively, one may conclude that anomaly-mediation is not the sole source of supersymmetry-breaking in the slepton sector.

I.7. The constrained MSSMs: mSUGRA, GMSB, and SGUTs: One way to guarantee the absence of significant FCNC's mediated by virtual supersymmetric-particle exchange is to posit that the diagonal soft-supersymmetry-breaking scalar squared-masses are universal at some energy scale.

I.7.1. The minimal supergravity (mSUGRA) model: In the *minimal* supergravity (mSUGRA) framework [2–4], the soft-supersymmetry-breaking parameters at the Planck scale take a particularly simple form in which the scalar squared-masses and the A-parameters are flavor-diagonal and universal [43]:

$$M^2_{\widetilde{Q}}(M_{\rm P}) = M^2_{\widetilde{U}}(M_{\rm P}) = M^2_{\widetilde{D}}(M_{\rm P}) = m_0^2\mathbf{1}\,,$$

$$
\begin{aligned}
&M^2_{\widetilde{L}}(M_{\rm P}) = M^2_{\widetilde{E}}(M_{\rm P}) = m_0^2\mathbf{1}\,, \\
&m_1^2(M_{\rm P}) = m_2^2(M_{\rm P}) = m_0^2\,, \\
&A_U(M_{\rm P}) = A_D(M_{\rm P}) = A_E(M_{\rm P}) = A_0\mathbf{1}\,,
\end{aligned}
\tag{15}
$$

where $\mathbf{1}$ is a 3×3 identity matrix in generation space. Renormalization group evolution is then used to derive the values of the supersymmetric parameters at the low-energy (electroweak) scale. For example, to compute squark masses, one must use the *low-energy* values for $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$ and $M^2_{\widetilde{D}}$ in Eq. (7). Through the renormalization group running with boundary conditions specified in Eq. (13) and Eq. (15), one can show that the low-energy values of $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$ and $M^2_{\widetilde{D}}$ depend primarily on m_0^2 and $m^2_{1/2}$. A number of useful approximate analytic expressions for superpartner masses in terms of the mSUGRA parameters can be found in Ref. [88].

Clearly, in the mSUGRA approach, the MSSM-124 parameter freedom has been significantly reduced. Typical mSUGRA models give low-energy values for the scalar mass parameters that satisfy $M_{\widetilde{L}} \approx M_{\widetilde{E}} < M_{\widetilde{Q}} \approx M_{\widetilde{U}} \approx M_{\widetilde{D}}$, with the squark mass parameters somewhere between a factor of 1–3 larger than the slepton mass parameters (*e.g.*, see Ref. [88]) . More precisely, the low-energy values of the squark mass parameters of the first two generations are roughly degenerate, while $M_{\widetilde{Q}_3}$ and $M_{\widetilde{U}_3}$ are typically reduced by a factor of 1–3 from the values of the first and second generation squark mass parameters, because of renormalization effects due to the heavy top-quark mass.

As a result, one typically finds that four flavors of squarks (with two squark eigenstates per flavor) and $\widetilde{b}_R$ are nearly mass-degenerate. The $\widetilde{b}_L$ mass and the diagonal $\widetilde{t}_L$ and $\widetilde{t}_R$ masses are reduced compared to the common squark mass of the first two generations. In addition, there are six flavors of nearly mass-degenerate sleptons (with two slepton eigenstates per flavor for the charged sleptons and one per flavor for the sneutrinos); the sleptons are expected to be somewhat lighter than the mass-degenerate squarks. Finally, third generation squark masses and tau-slepton masses are sensitive to the strength of the respective

$\widetilde{f}_L$–$\widetilde{f}_R$ mixing, as discussed below Eq. (7). If $\tan\beta \gg 1$, then the pattern of third generation squark masses is somewhat altered, as discussed in Ref. [89].

In mSUGRA models, the LSP is typically the lightest neutralino, $\widetilde{\chi}^0_1$, which is dominated by its bino component. In particular, one can reject those mSUGRA parameter regimes in which the LSP is a chargino or the $\widetilde{\tau}_1$ (the lightest scalar superpartner of the τ-lepton). In general, if one imposes the constraints of supersymmetric particle searches and those of cosmology (say, by requiring the LSP to be a suitable dark matter candidate), one obtains significant restrictions to the mSUGRA parameter space [90].

One can count the number of independent parameters in the mSUGRA framework. In addition to 18 Standard Model parameters (excluding the Higgs mass), one must specify m_0, $m_{1/2}$, A_0, and Planck-scale values for μ and B-parameters (denoted by μ_0 and B_0). In principle, A_0, B_0, and μ_0 can be complex, although in the mSUGRA approach, these parameters are taken (arbitrarily) to be real. As previously noted, renormalization group evolution is used to compute the low-energy values of the mSUGRA parameters, which then fixes all the parameters of the low-energy MSSM. In particular, the two Higgs vacuum expectation values (or equivalently, m_Z and $\tan\beta$) can be expressed as a function of the Planck-scale supergravity parameters. The simplest procedure is to remove μ_0 and B_0 in favor of m_Z and $\tan\beta$ [the sign of μ_0, denoted sgn(μ_0) below, is not fixed in this process]. In this case, the MSSM spectrum and its interaction strengths are determined by five parameters:

$$m_0\,,\; A_0\,,\; m_{1/2}\,,\; \tan\beta\,,\; \text{and}\;\; \mathrm{sgn}(\mu_0)\,, \tag{16}$$

in addition to the 18 parameters of the Standard Model. However, the mSUGRA approach is probably too simplistic. Theoretical considerations suggest that the universality of Planck-scale soft-supersymmetry-breaking parameters is not generic [91]. In particular, it is easy to write down effective operators at the Planck scale that do not respect flavor universality, and it is difficult to find a theoretical principle that would forbid them.

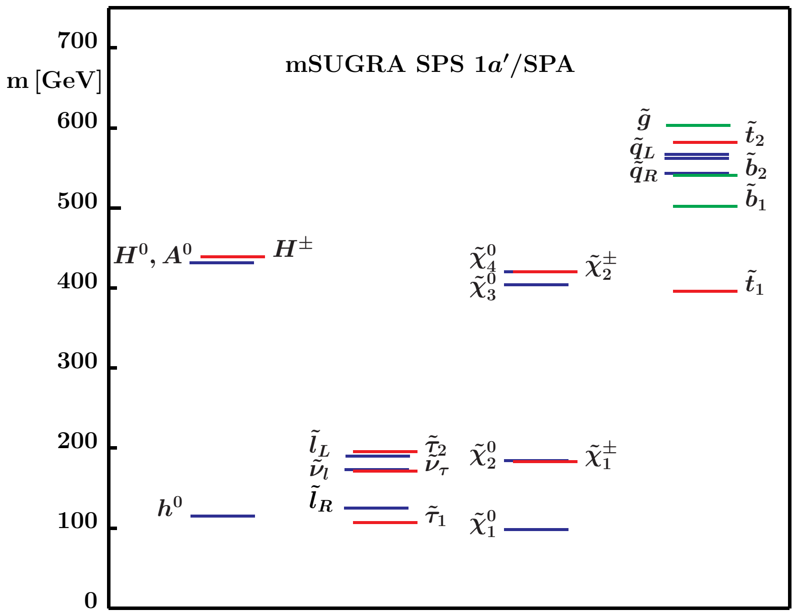

Figure 1: Mass spectrum of supersymmetric particles and Higgs bosons for the mSUGRA reference point SPS 1a′. The masses of the first and second generation squarks, sleptons and sneutrinos are denoted collectively by $\widetilde{q}$, $\widetilde{\ell}$ and $\widetilde{\nu}_\ell$, respectively. Taken from Ref. [72]. See full-color version on color pages at end of book.

In order to facilitate studies of supersymmetric phenomenology at colliders, it has been a valuable exercise to compile a set of benchmark supersymmetric parameters, from which supersymmetric spectra and couplings can be derived [92]. A compilation of benchmark mSUGRA points consistent with present data from particle physics and cosmology can be found in Ref. [93]. One particular well-studied benchmark points, the so-called SPS 1a′ reference point [72] (this is a slight modification of the SPS 1a point of Ref. [92], which incorporates the latest constraints from collider data and cosmology) has been especially useful in experimental studies of supersymmetric phenomena at future colliders. The supersymmetric particle spectrum for the SPS 1a′ reference point is exhibited in Figure 1. However, it is important to keep in mind that even within the mSUGRA framework, the resulting supersymmetric theory and its attendant phenomenology can be quite different from the SPS 1a′ reference point.

I.7.2. Gauge-mediated supersymmetry breaking: In contrast to models of gravity-mediated supersymmetry breaking, the universality of the fundamental soft-supersymmetry-breaking squark and slepton squared-mass parameters is guaranteed in gauge-mediated supersymmetry-breaking because the supersymmetry-breaking is communicated to the sector of MSSM fields via gauge interactions. In the minimal gauge-mediated supersymmetry-breaking (GMSB) approach, there is one effective mass scale, Λ, that determines all low-energy scalar and gaugino mass parameters through loop-effects (while the resulting A parameters are suppressed). In order that the resulting superpartner masses be of order 1 TeV or less, one must have $\Lambda \sim 100$ TeV. The origin of the μ and B-parameters is quite model-dependent, and lies somewhat outside the ansatz of gauge-mediated supersymmetry breaking. The simplest models of this type are even more restrictive than mSUGRA, with two fewer degrees of freedom. Benchmark reference points for GMSB models have been proposed in Ref. [92] to facilitate collider studies. However, minimal GMSB is not a fully realized model. The sector of supersymmetry-breaking dynamics can be very complex, and no complete model of gauge-mediated supersymmetry yet exists that is both simple and compelling.

It was noted in Section I.2 that the gravitino is the LSP in GMSB models. Thus, in such models, the next-to-lightest supersymmetric particle (NLSP) plays a crucial role in the phenomenology of supersymmetric particle production and decay. Note that unlike the LSP, the NLSP can be charged. In GMSB models, the most likely candidates for the NLSP are $\widetilde{\chi}_1^0$ and $\widetilde{\tau}_R^\pm$. The NLSP will decay into its superpartner plus a gravitino (*e.g.*, $\widetilde{\chi}_1^0 \to \gamma\widetilde{g}_{3/2}$, $\widetilde{\chi}_1^0 \to Z\widetilde{g}_{3/2}$ or $\widetilde{\tau}_R^\pm \to \tau^\pm\widetilde{g}_{3/2}$), with lifetimes and branching ratios that depend on the model parameters.

Different choices for the identity of the NLSP and its decay rate lead to a variety of distinctive supersymmetric phenomenologies [47,94]. For example, a long-lived $\widetilde{\chi}_1^0$-NLSP that decays outside collider detectors leads to supersymmetric decay chains with missing energy in association with leptons and/or hadronic jets (this case is indistinguishable from the canonical phenomenology of the $\widetilde{\chi}_1^0$-LSP). On the other hand, if

$\widetilde{\chi}_1^0 \to \gamma\widetilde{g}_{3/2}$ is the dominant decay mode, and the decay occurs inside the detector, then nearly *all* supersymmetric particle decay chains would contain a photon. In contrast, the case of a $\widetilde{\tau}_R^{\pm}$-NLSP would lead either to a new long-lived charged particle (*i.e.*, the $\widetilde{\tau}_R^{\pm}$) or to supersymmetric particle decay chains with τ leptons.

I.7.3. Supersymmetric grand unification: Finally, grand unification [95] can impose additional constraints on the MSSM parameters. As emphasized in Section I.1, it is striking that the SU(3)×SU(2)×U(1) gauge couplings unify in models of supersymmetric grand unified theories (SGUTs) [7,15,96,97] with (some of) the supersymmetry-breaking parameters of order 1 TeV or below. Gauge coupling unification, which takes place at an energy scale of order 10^{16} GeV, is quite robust [98]. For example, successful unification depends weakly on the details of the theory at the unification scale. In particular, given the low-energy values of the electroweak couplings $g(m_Z)$ and $g'(m_Z)$, one can predict $\alpha_s(m_Z)$ by using the MSSM renormalization group equations to extrapolate to higher energies, and by imposing the unification condition on the three gauge couplings at some high-energy scale, $M_{\rm X}$. This procedure, which fixes $M_{\rm X}$, can be successful (*i.e.*, three running couplings will meet at a single point) only for a unique value of $\alpha_s(m_Z)$. The extrapolation depends somewhat on the low-energy supersymmetric spectrum (so-called low-energy "threshold effects"), and on the SGUT spectrum (high-energy threshold effects), which can somewhat alter the evolution of couplings. Ref. [99] summarizes the comparison of data with the expectations of SGUTs, and shows that the measured value of $\alpha_s(m_Z)$ is in good agreement with the predictions of supersymmetric grand unification for a reasonable choice of supersymmetric threshold corrections.

Additional SGUT predictions arise through the unification of the Higgs-fermion Yukawa couplings (λ_f). There is some evidence that $\lambda_b = \lambda_\tau$ is consistent with observed low-energy data [100], and an intriguing possibility that $\lambda_b = \lambda_\tau = \lambda_t$ may be phenomenologically viable [89,101] in the parameter regime where $\tan\beta \simeq m_t/m_b$. Finally, grand unification imposes constraints on the soft-supersymmetry-breaking parameters. For example, gaugino-mass unification leads to the relations given by Eq. (13). Diagonal squark and slepton soft-supersymmetry-breaking scalar masses may also be unified, which is analogous to the unification of Higgs-fermion Yukawa couplings.

In the absence of a fundamental theory of supersymmetry breaking, further progress will require a detailed knowledge of the supersymmetric-particle spectrum in order to determine the nature of the high-energy parameters. Of course, any of the theoretical assumptions described in this section could be wrong and must eventually be tested experimentally.

I.8. Massive neutrinos in low-energy supersymmetry: With the overwhelming evidence for neutrino masses and mixing [102], it is clear that any viable supersymmetric model of fundamental particles must incorporate some form of L violation in the low-energy theory [103]. This requires an extension of the MSSM, which (as in the case of the minimal Standard Model) contains three generations of massless neutrinos. To construct a supersymmetric model with massive neutrinos, one can follow one of two different approaches.

I.8.1. The supersymmetric seesaw: In the first approach, one starts with a modified Standard Model which incorporates new structure that yields nonzero neutrino masses. Following the procedures of Sections I.2 and I.3, one then formulates the supersymmetric extension of the modified Standard Model. For example, neutrino masses can be incorporated into the Standard Model by introducing an SU(3)×SU(2)×U(1) singlet right-handed neutrino (ν_R) and a super-heavy Majorana mass (typically of order a grand unified mass) for the ν_R. In addition, one must also include a standard Yukawa coupling between the lepton doublet, the Higgs doublet and ν_R. The Higgs vacuum expectation value then induces an off-diagonal ν_L–ν_R mass of order the electroweak scale. Diagonalizing the neutrino mass matrix (in the three-generation model) yields three superheavy neutrino states and three very light neutrino states that are identified as the light neutrino states observed in nature. This is the seesaw mechanism [104]. The supersymmetric generalization of the seesaw model of neutrino masses is now easily constructed [105,106].

I.8.2. R-parity-violating supersymmetry: Another approach to incorporating massive neutrinos in supersymmetric models is to retain the minimal particle content of the MSSM but remove the assumption of R-parity invariance [107]. The most general R-parity-violating (RPV) theory involving the MSSM spectrum introduces many new parameters to both the supersymmetry-conserving and the supersymmetry-breaking sectors. Each new interaction term violates either B or L conservation. For example, consider new scalar-fermion Yukawa couplings derived from the following interactions:

$$(\lambda_L)_{pmn}\widehat{L}_p\widehat{L}_m\widehat{E}^c_n+(\lambda'_L)_{pmn}\widehat{L}_p\widehat{Q}_m\widehat{D}^c_n+(\lambda_B)_{pmn}\widehat{U}^c_p\widehat{D}^c_m\widehat{D}^c_n\,, \quad (17)$$

where p, m, and n are generation indices, and gauge group indices are suppressed. In the notation above, $\widehat{Q}$, $\widehat{U}^c$, $\widehat{D}^c$, $\widehat{L}$, and $\widehat{E}^c$ respectively represent $(u,d)_L$, u^c_L, d^c_L, $(\nu,\, e^-)_L$, and e^c_L and the corresponding superpartners. The Yukawa interactions are obtained from Eq. (17) by taking all possible combinations involving two fermions and one scalar superpartner. Note that the term in Eq. (17) proportional to λ_B violates B, while the other two terms violate L. Even if all the terms of Eq. (17) are absent, there is one more possible supersymmetric source of R-parity violation. In the notation of Eq. (17), one can add a term of the form $(\mu_L)_p\widehat{H}_u\widehat{L}_p$, where $\widehat{H}_u$ represents the $Y=1$ Higgs doublet and its higgsino superpartner. This term is the RPV generalization of the supersymmetry-conserving Higgs mass parameter μ of the MSSM, in which the $Y=-1$ Higgs/higgsino super-multiplet $\widehat{H}_d$ is replaced by the slepton/lepton super-multiplet $\widehat{L}_p$. The RPV-parameters $(\mu_L)_p$ also violate L.

Phenomenological constraints derived from data on various low-energy B- and L-violating processes can be used to establish limits on each of the coefficients $(\lambda_L)_{pmn}$, $(\lambda'_L)_{pmn}$, and $(\lambda_B)_{pmn}$ taken one at a time [107,108]. If more than one coefficient is simultaneously non-zero, then the limits are, in general, more complicated [109]. All possible RPV terms cannot be simultaneously present and unsuppressed; otherwise the proton decay rate would be many orders of magnitude larger than the present experimental bound. One way to avoid proton decay is to impose B or L invariance (either one alone would suffice). Otherwise, one must accept the requirement that certain RPV coefficients must be extremely suppressed.

One particularly interesting class of RPV models is one in which B is conserved, but L is violated. It is possible to enforce baryon number conservation, while allowing for lepton number violating interactions by imposing a discrete $\mathbf{Z_3}$ baryon *triality* symmetry on the low-energy theory [110], in place of the standard $\mathbf{Z_2}$ R-parity. Since the distinction between the Higgs and matter super-multiplets is lost in RPV models, R-parity violation permits the mixing of sleptons and Higgs bosons, the mixing of neutrinos and neutralinos, and the mixing of charged leptons and charginos, leading to more complicated mass matrices and mass eigenstates than in the MSSM.

The supersymmetric phenomenology of the RPV models exhibits features that are quite distinct from that of the MSSM [107]. The LSP is no longer stable, which implies that not all supersymmetric decay chains must yield missing-energy events at colliders. Nevertheless, the loss of the missing-energy signature is often compensated by other striking signals (which depend on which R-parity-violating parameters are dominant). For example, supersymmetric particles in RPV models can be singly produced (in contrast to R-parity-conserving models where supersymmetric particles must be produced in pairs). The phenomenology of pair-produced supersymmetric particles in RPV models can also differ significantly from expectations due to new decay chains not present in R-parity-conserving supersymmetry [107].

In RPV models with lepton number violation (these include low-energy supersymmetry models with baryon triality mentioned above), both $\Delta L=1$ and $\Delta L=2$ phenomena are allowed, leading to neutrino masses and mixing [111], neutrinoless double-beta decay [112], sneutrino-antisneutrino mixing [106,113,114], and s-channel resonant production of sneutrinos in e^+e^- collisions [115] and charged sleptons in $p\bar{p}$ and pp collisions [116]. For example, Ref. [117] demonstrates how one can fit both the solar and atmospheric neutrino data in an RPV supersymmetric model where μ_L provides the dominant source of R-parity violation.

I.9. Other non-minimal extensions of the MSSM: There are additional motivations for extending the supersymmetric model beyond the MSSM. Here we mention just a few. The μ parameter of the MSSM is a supersymmetric-*preserving* parameter; nevertheless it must be of order the supersymmetry-breaking scale to yield a consistent supersymmetric phenomenology. In the MSSM, one must devise a theoretical mechanism to guarantee that the magnitude of μ is not larger than the TeV-scale (*e.g.*, in gravity-mediated supersymmetry, the Giudice-Masiero mechanism of Ref. [118] is the most cited explanation).

In extensions of the MSSM, new compelling solutions to the so-called μ-problem are possible. For example, one can replace μ by the vacuum expectation value of a new SU(3)×SU(2)×U(1) singlet scalar field. In such a model, the Higgs sector of the MSSM is enlarged (and the corresponding fermionic higgsino superpartner is added). This is the so-called NMSSM (here, NM stands for non-minimal) [119].

Non-minimal extensions of the MSSM involving additional matter super-multiplets can also yield a less restrictive bound on the mass of the lightest Higgs boson (as compared to the MSSM Higgs mass bound quoted in Section I.5.2). For example, by imposing gauge coupling unification, the upper limit on the lightest Higgs boson mass can be as high as 200—300 GeV [120] (a similar relaxation of the Higgs mass bound has been observed in split supersymmetry [121] and in extra-dimensional scenarios [122]) . Note that these less restrictive Higgs mass upper bounds are comparable to the (experimentally determined) upper bound for the Higgs boson mass based on the Standard Model global fits to precision electroweak data [26,123].

Other MSSM extensions considered in the literature include an enlarged electroweak gauge group beyond SU(2)×U(1) [124]; and/or the addition of new, possibly exotic, matter super-multiplets (*e.g.*, a vector-like color triplet with electric charge $\frac{1}{3}e$; such states sometimes occur as low-energy remnants in E_6

grand unification models). A possible theoretical motivation for such new structures arises from the study of phenomenologically viable string theory ground states [125].

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