# LIGHT QUARKS ($u$, $d$, $s$)

OMITTED FROM SUMMARY TABLE

## $u$-QUARK MASS

The $u$-, $d$-, and $s$-quark masses are estimates of so-called "current-quark masses," in a mass- independent subtraction scheme such as $\overline{\text{MS}}$. The ratios $m_u/m_d$ and $m_s/m_d$ are extracted from pion and kaon masses using chiral symmetry. The estimates of $d$ and $u$ masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the $u$ quark could be essentially massless. The $s$-quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **1.5 to 3.0 OUR EVALUATION** | | | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| $2.7 \pm 0.4$ | [1] JAMIN | 06 | THEO | $\overline{\text{MS}}$ scheme |
| $2.8 \pm 0.2$ | [2] NARISON | 06 | THEO | $\overline{\text{MS}}$ scheme |
| $1.7 \pm 0.3$ | [3] AUBIN | 04A | LATT | $\overline{\text{MS}}$ scheme |
| $2.9 \pm 0.6$ | [4] JAMIN | 02 | THEO | $\overline{\text{MS}}$ scheme |
| $2.3 \pm 0.4$ | [5] NARISON | 99 | THEO | $\overline{\text{MS}}$ scheme |
| $3.9 \pm 1.1$ | [6] JAMIN | 95 | THEO | $\overline{\text{MS}}$ scheme |
| $3.0 \pm 0.7$ | [7] NARISON | 95C | THEO | $\overline{\text{MS}}$ scheme |

[1] JAMIN 06 determine $m_u$(2 GeV) by combining the value of $m_s$ obtained from the spectral function for the scalar $K\pi$ form factor with other determinations of the quark mass ratios.

[2] NARISON 06 uses sum rules for $e^+e^- \to$ hadrons to order $\alpha_s^3$ to determine $m_s$ combined with other determinations of the quark mass ratios.

[3] AUBIN 04A employ a partially quenched lattice calculation of the pseudoscalar meson masses.

[4] JAMIN 02 first calculates the strange quark mass from QCD sum rules using the scalar channel, and then combines with the quark mass ratios obtained from chiral perturbation theory to obtain $m_u$.

[5] NARISON 99 uses sum rules to order $\alpha_s^3$ for $\phi$ meson decays to get $m_s$, and finds $m_u$ by combining with sum rule estimates of $m_u + m_d$ and Dashen's formula.

[6] JAMIN 95 uses QCD sum rules at next-to-leading order. We have rescaled $m_u(1\,\text{GeV}) = 5.3 \pm 1.5$ to $\mu = 2$ GeV.

[7] For NARISON 95C, we have rescaled $m_u(1\,\text{GeV}) = 4 \pm 1$ to $\mu = 2$ GeV.

## *d*-QUARK MASS

See the comment for the $u$ quark above.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **3 to 7 OUR EVALUATION** | | | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 4.8±0.5 | [8] JAMIN | 06 | THEO | $\overline{\text{MS}}$ scheme |
| 5.1±0.4 | [9] NARISON | 06 | THEO | $\overline{\text{MS}}$ scheme |
| 3.9±0.5 | [10] AUBIN | 04A | LATT | $\overline{\text{MS}}$ scheme |
| 5.2±0.9 | [11] JAMIN | 02 | THEO | $\overline{\text{MS}}$ scheme |
| 6.4±1.1 | [12] NARISON | 99 | THEO | $\overline{\text{MS}}$ scheme |
| 7.0±1.1 | [13] JAMIN | 95 | THEO | $\overline{\text{MS}}$ scheme |
| 7.4±0.7 | [14] NARISON | 95C | THEO | $\overline{\text{MS}}$ scheme |

[8] JAMIN 06 determine $m_d$(2 GeV) by combining the value of $m_s$ obtained from the spectral function for the scalar $K\pi$ form factor with other determinations of the quark mass ratios.

[9] NARISON 06 uses sum rules for $e^+e^- \to$ hadrons to order $\alpha_s^3$ to determine $m_s$ combined with other determinations of the quark mass ratios.

[10] AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses, and one-loop perturbative renormalization constant.

[11] JAMIN 02 first calculates the strange quark mass from QCD sum rules using the scalar channel, and then combines with the quark mass ratios obtained from chiral perturbation theory to obtain $m_d$.

[12] NARISON 99 uses sum rules to order $\alpha_s^3$ for $\phi$ meson decays to get $m_s$, and finds $m_d$ by combining with sum rule estimates of $m_u+m_d$ and Dashen's formula.

[13] JAMIN 95 uses QCD sum rules at next-to-leading order. We have rescaled $m_d$(1 GeV) $= 9.4 \pm 1.5$ to $\mu = 2$ GeV.

[14] For NARISON 95C, we have rescaled $m_d$(1 GeV) $= 10 \pm 1$ to $\mu = 2$ GeV.

## $\overline{m} = (m_u+m_d)/2$

See the comments for the $u$ quark above.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **2.5 to 5.5 OUR EVALUATION** | | | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 4.08 ±0.25 ±0.42 | [15] GOCKELER | 06 | LATT | $\overline{\text{MS}}$ scheme |
| 4.7 ±0.2 ±0.3 | [16] GOCKELER | 06A | LATT | $\overline{\text{MS}}$ scheme |

| | | | | | | |
|---|---|---|---|---|---|---|
| 3.95 | $\pm 0.3$ | | [17] NARISON | 06 | THEO | $\overline{\text{MS}}$ scheme |
| 2.8 | $\pm 0.3$ | | [18] AUBIN | 04 | LATT | $\overline{\text{MS}}$ scheme |
| 4.29 | $\pm 0.14$ | $\pm 0.65$ | [19] AOKI | 03 | LATT | $\overline{\text{MS}}$ scheme |
| $3.223^{+0.046}_{-0.069}$ | | | [20] AOKI | 03B | LATT | $\overline{\text{MS}}$ scheme |
| 4.4 | $\pm 0.1$ | $\pm 0.4$ | [21] BECIREVIC | 03 | LATT | $\overline{\text{MS}}$ scheme |
| 4.1 | $\pm 0.3$ | $\pm 1.0$ | [22] CHIU | 03 | LATT | $\overline{\text{MS}}$ scheme |
| $3.45^{+0.14}_{-0.20}$ | | | [23] ALIKHAN | 02 | LATT | $\overline{\text{MS}}$ scheme |
| 5.3 | $\pm 0.3$ | | [24] CHIU | 02 | LATT | $\overline{\text{MS}}$ scheme |
| 3.9 | $\pm 0.6$ | | [25] MALTMAN | 02 | THEO | $\overline{\text{MS}}$ scheme |
| 3.9 | $\pm 0.6$ | | [26] MALTMAN | 01 | THEO | $\overline{\text{MS}}$ scheme |
| 4.57 | $\pm 0.18$ | | [27] AOKI | 00 | LATT | $\overline{\text{MS}}$ scheme |
| 4.4 | $\pm 2$ | | [28] GOCKELER | 00 | LATT | $\overline{\text{MS}}$ scheme |
| 4.23 | $\pm 0.29$ | | [29] AOKI | 99 | LATT | $\overline{\text{MS}}$ scheme |
| $\geq 2.1$ | | | [30] STEELE | 99 | THEO | $\overline{\text{MS}}$ scheme |
| 4.5 | $\pm 0.4$ | | [31] BECIREVIC | 98 | LATT | $\overline{\text{MS}}$ scheme |
| 4.6 | $\pm 1.2$ | | [32] DOSCH | 98 | THEO | $\overline{\text{MS}}$ scheme |
| 4.7 | $\pm 0.9$ | | [33] PRADES | 98 | THEO | $\overline{\text{MS}}$ scheme |
| 2.7 | $\pm 0.2$ | | [34] EICKER | 97 | LATT | $\overline{\text{MS}}$ scheme |
| 3.6 | $\pm 0.6$ | | [35] GOUGH | 97 | LATT | $\overline{\text{MS}}$ scheme |
| 3.4 | $\pm 0.4$ | $\pm 0.3$ | [36] GUPTA | 97 | LATT | $\overline{\text{MS}}$ scheme |
| $>3.8$ | | | [37] LELLOUCH | 97 | THEO | $\overline{\text{MS}}$ scheme |
| 4.5 | $\pm 1.0$ | | [38] BIJNENS | 95 | THEO | $\overline{\text{MS}}$ scheme |

[15] GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization, to obtain $\overline{m}(2\text{ GeV}) = 4.08 \pm 0.25 \pm 0.19 \pm 0.23$ MeV, where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.

[16] GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization.

[17] NARISON 06 uses sum rules for $e^+e^- \to$ hadrons to order $\alpha_s^3$ to determine $m_s$ combined with other determinations of the quark mass ratios.

[18] AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.

[19] AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory.

[20] AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the $\mathcal{O}(a)$ improved Wilson action.

[21] BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses $\mathcal{O}(a)$ improved Wilson action and nonperturbative renormalization.

[22] CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.

[23] ALIKHAN 02 uses lattice simulation of the meson and baryon masses with two dynamical flavors and degenerate light quarks.

[24] CHIU 02 extracts the average light quark mass from quenched lattice simulations using quenched chiral perturbation theory.

[25] MALTMAN 02 uses finite energy sum rules in the $ud$ and $us$ pseudoscalar channels. Other mass values are also obtained by similar methods.

[26] MALTMAN 01 uses Borel transformed and finite energy sum rules.

[27] AOKI 00 obtain the light quark masses from a quenched lattice simulation of the meson and baryon spectrum with the Wilson quark action.

[28] GOCKELER 00 obtained from a quenched lattice computation of the pseudoscalar meson masses using $\mathcal{O}(a)$ improved Wilson fermions and nonperturbative renormalization.

[29] AOKI 99 obtain the light quark masses from a quenched lattice simulation of the meson spectrum with the staggered quark action employing the regularization independent scheme.

[30] STEELE 99 obtain a bound on the light quark masses by applying the Holder inequality to a sum rule. We have converted their bound of $(m_u+m_d)/2 \geq 3$ MeV at $\mu$=1 GeV to $\mu$=2 GeV.

[31] BECIREVIC 98 compute the quark mass using the Alpha action in the quenched approximation. The conversion from the regularization independent scheme to the $\overline{\text{MS}}$ scheme is at NNLO.

[32] DOSCH 98 use sum rule determinations of the quark condensate and chiral perturbation theory to obtain $9.4 \leq (m_u+m_d)(1\,\text{GeV}) \leq 15.7$ MeV. We have converted to result to $\mu$=2 GeV.

[33] PRADES 98 uses finite energy sum rules for the axial current correlator.

[34] EICKER 97 use lattice gauge computations with two dynamical light flavors.

[35] GOUGH 97 use lattice gauge computations in the quenched approximation. Correcting for quenching gives $2.1 < \overline{m} < 3.5$ MeV at $\mu$=2 GeV.

[36] GUPTA 97 use Lattice Monte Carlo computations in the quenched approximation. The value for two light dynamic flavors at $\mu = 2$ GeV is $2.7 \pm 0.3 \pm 0.3$ MeV.

[37] LELLOUCH 97 obtain lower bounds on quark masses using hadronic spectral functions.

[38] BIJNENS 95 determines $m_u+m_d$ (1 GeV) $= 12 \pm 2.5$ MeV using finite energy sum rules. We have rescaled this to 2 GeV.

## *s*-QUARK MASS

See the comment for the $u$ quark above.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35.

| *VALUE* (MeV) | | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|---|
| **95 ±25 OUR EVALUATION** | | | | | |
| 105 ± 6 ± 7 | [39] | CHETYRKIN | 06 | THEO | $\overline{\text{MS}}$ scheme |
| 111 ± 6 ±10 | [40] | GOCKELER | 06 | LATT | $\overline{\text{MS}}$ scheme |
| 119 ± 5 ± 8 | [41] | GOCKELER | 06A | LATT | $\overline{\text{MS}}$ scheme |
| 92 ± 9 | [42] | JAMIN | 06 | THEO | $\overline{\text{MS}}$ scheme |
| 104 ±15 | [43] | NARISON | 06 | THEO | $\overline{\text{MS}}$ scheme |
| $\geq 71 \pm 4$, $\leq 151 \pm 14$ | [44] | NARISON | 06 | THEO | $\overline{\text{MS}}$ scheme |
| $96\ ^{+\ 5}_{-\ 3}\ ^{+16}_{-18}$ | [45] | BAIKOV | 05 | THEO | $\overline{\text{MS}}$ scheme |
| 81 ±22 | [46] | GAMIZ | 05 | THEO | $\overline{\text{MS}}$ scheme |
| 125 ±28 | [47] | GORBUNOV | 05 | THEO | $\overline{\text{MS}}$ scheme |
| 93 ±32 | [48] | NARISON | 05 | THEO | $\overline{\text{MS}}$ scheme |
| 76 ± 8 | [49] | AUBIN | 04 | LATT | $\overline{\text{MS}}$ scheme |
| 116 ± 6 ± 0.65 | [50] | AOKI | 03 | LATT | $\overline{\text{MS}}$ scheme |
| $84.5^{+12}_{-\ 1.7}$ | [51] | AOKI | 03B | LATT | $\overline{\text{MS}}$ scheme |
| 106 ± 2 ± 8 | [52] | BECIREVIC | 03 | LATT | $\overline{\text{MS}}$ scheme |
| 92 ± 9 ±16 | [53] | CHIU | 03 | LATT | $\overline{\text{MS}}$ scheme |
| 117 ±17 | [54] | GAMIZ | 03 | THEO | $\overline{\text{MS}}$ scheme |

| | | | | | |
|---|---|---|---|---|---|
| 103 | $\pm 17$ | [55] GAMIZ | 03 | THEO | $\overline{\text{MS}}$ scheme |
| 88 | $^{+\ 3}_{-\ 6}$ | [56] ALIKHAN | 02 | LATT | $\overline{\text{MS}}$ scheme |
| 115 | $\pm\ 8$ | [57] CHIU | 02 | LATT | $\overline{\text{MS}}$ scheme |
| 99 | $\pm 16$ | [58] JAMIN | 02 | THEO | $\overline{\text{MS}}$ scheme |
| 100 | $\pm 12$ | [59] MALTMAN | 02 | THEO | $\overline{\text{MS}}$ scheme |
| 116 | $^{+20}_{-25}$ | [60] CHEN | 01B | THEO | $\overline{\text{MS}}$ scheme |
| 125 | $\pm 27$ | [61] KOERNER | 01 | THEO | $\overline{\text{MS}}$ scheme |
| 130 | $\pm 15$ | [62] AOKI | 00 | LATT | $\overline{\text{MS}}$ scheme |
| 97 | $\pm\ 4$ | [63] GARDEN | 00 | LATT | $\overline{\text{MS}}$ scheme |
| 105 | $\pm\ 4$ | [64] GOCKELER | 00 | LATT | $\overline{\text{MS}}$ scheme |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | |
|---|---|---|---|---|---|
| 118 | $\pm 14$ | [65] AOKI | 99 | LATT | $\overline{\text{MS}}$ scheme |
| 170 | $^{+44}_{-55}$ | [66] BARATE | 99R | ALEP | $\overline{\text{MS}}$ scheme |
| 115 | $\pm\ 8$ | [67] MALTMAN | 99 | THEO | $\overline{\text{MS}}$ scheme |
| 129 | $\pm 24$ | [68] NARISON | 99 | THEO | $\overline{\text{MS}}$ scheme |
| 114 | $\pm 23$ | [69] PICH | 99 | THEO | $\overline{\text{MS}}$ scheme |
| 111 | $\pm 12$ | [70] BECIREVIC | 98 | LATT | $\overline{\text{MS}}$ scheme |
| 148 | $\pm 48$ | [71] CHETYRKIN | 98 | THEO | $\overline{\text{MS}}$ scheme |
| 103 | $\pm 10$ | [72] CUCCHIERI | 98 | LATT | $\overline{\text{MS}}$ scheme |
| 115 | $\pm 19$ | [73] DOMINGUEZ | 98 | THEO | $\overline{\text{MS}}$ scheme |
| 152.4 | $\pm 14.1$ | [74] CHETYRKIN | 97 | THEO | $\overline{\text{MS}}$ scheme |
| $\geq 89$ | | [75] COLANGELO | 97 | THEO | $\overline{\text{MS}}$ scheme |
| 140 | $\pm 20$ | [76] EICKER | 97 | LATT | $\overline{\text{MS}}$ scheme |
| 95 | $\pm 16$ | [77] GOUGH | 97 | LATT | $\overline{\text{MS}}$ scheme |
| 100 | $\pm 21\ \ \pm 10$ | [78] GUPTA | 97 | LATT | $\overline{\text{MS}}$ scheme |
| $>100$ | | [79] LELLOUCH | 97 | THEO | $\overline{\text{MS}}$ scheme |
| 140 | $\pm 24$ | [80] JAMIN | 95 | THEO | $\overline{\text{MS}}$ scheme |

[39] CHETYRKIN 06 use QCD sum rules in the pseudoscalar channel to order $\alpha_s^4$.

[40] GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization, to obtain $\overline{m}_s(2\text{ GeV}) = 111 \pm 6 \pm 4 \pm 6$ MeV, where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.

[41] GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization.

[42] JAMIN 06 determine $\overline{m}_s(2\text{ GeV})$ from the spectral function for the scalar $K\pi$ form factor.

[43] NARISON 06 uses sum rules for $e^+e^- \to$ hadrons to order $\alpha_s^3$.

[44] NARISON 06 obtains the quoted range from positivity of the spectral functions.

[45] BAIKOV 05 determines $\overline{m}_s(M_\tau) = 100^{+5+17}_{-3-19}$ from sum rules using the strange spectral function in $\tau$ decay. The computations were done to order $\alpha_s^3$, with an estimate of the $\alpha_s^4$ terms. We have converted the result to $\mu = 2$ GeV.

[46] GAMIZ 05 determines $\overline{m}_s(2\text{ GeV})$ from sum rules using the strange spectral function in $\tau$ decay. The computations were done to order $\alpha_s^2$, with an estimate of the $\alpha_s^3$ terms.

[47] GORBUNOV 05 use hadronic tau decays to N$^3$LO, including power corrections.

[48] NARISON 05 determines $\overline{m}_s(2\text{ GeV})$ from sum rules using the strange spectral function in $\tau$ decay. The computations were done to order $\alpha_s^3$.

[49] AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.

[50] AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory. Determines $m_s$=113.8 ± 2.3 $^{+5.8}_{-2.9}$ using $K$ mass as input and $m_s$=142.3 ± 5.8 $^{+22}_{-0}$ using $\phi$ mass as input. We have performed a weighted average of these values.

[51] AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the $\mathcal{O}(a)$ improved Wilson action.

[52] BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses $\mathcal{O}(a)$ improved Wilson action and nonperturbative renormalization. They also quote $\overline{m}/m_s$=24.3 ± 0.2 ± 0.6.

[53] CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.

[54] GAMIZ 03 determines $m_s$ from SU(3) breaking in the $\tau$ hadronic width. The value of $V_{us}$ is chosen to satisfy CKM unitarity.

[55] GAMIZ 03 determines $m_s$ from SU(3) breaking in the $\tau$ hadronic width. The value of $V_{us}$ is taken from the PDG.

[56] ALIKHAN 02 uses lattice simulation of the meson and baryon masses with two dynamical flavors and degenerate light quarks. The above value uses the $K$-meson mass to determine $m_s$. If the $\phi$ meson is used, the number changes to $90^{+5}_{-10}$.

[57] CHIU 02 extracts the strange quark mass from quenched lattice simulations using quenched chiral perturbation theory.

[58] JAMIN 02 calculates the strange quark mass from QCD sum rules using the scalar channel.

[59] MALTMAN 02 uses finite energy sum rules in the $ud$ and $us$ pseudoscalar channels. Other mass values are also obtained by similar methods.

[60] CHEN 01B uses an analysis of the hadronic spectral function in $\tau$ decay.

[61] KOERNER 01 obtain the $s$ quark mass of $m_s(m_\tau) = 130 \pm 27(\text{exp}) \pm 9(\text{thy})$ MeV from an analysis of Cabibbo suppressed $\tau$ decays. We have converted this to $\mu$ = 2 GeV.

[62] AOKI 00 obtain the light quark masses from a quenched lattice simulation of the meson and baryon spectrum with the Wilson quark action. We have averaged their results of $m_s$= 115.6 ± 2.3 and $m_s$= 143.7 ± 5.8 obtained using $m_K$ and $m_\phi$, respectively, to normalize the spectrum.

[63] GARDEN 00 use a quenched lattice computation of the hadron spectrum.

[64] GOCKELER 00 obtained from a quenched lattice computation of the pseudoscalar meson masses using $\mathcal{O}(a)$ improved Wilson fermions and nonperturbative renormalization.

[65] AOKI 99 obtain the light quark masses from a quenched lattice simulation of the meson spectrum with the Staggered quark action employing the regularization independent scheme. We have averaged their results of $m_s$=106.0 ± 7.1 and $m_s$=129 ± 12 obtained using $m_K$ and $m_\phi$, respectively, to normalize the spectrum.

[66] BARATE 99R obtain the strange quark mass from an analysis of the observed mass spectra in $\tau$ decay. We have converted their value of $m_s(m_\tau)$= 176 $^{+46}_{-57}$ MeV to $\mu$=2 GeV.

[67] MALTMAN 99 determines the strange quark mass using finite energy sum rules.

[68] NARISON 99 uses sum rules to order $\alpha_s^3$ for $\phi$ meson decays.

[69] PICH 99 obtain the $s$-quark mass from an analysis of the moments of the invariant mass distribution in $\tau$ decays.

[70] BECIREVIC 98 compute the quark mass using the Alpha action in the quenched approximation. The conversion from the regularization independent scheme to the $\overline{\text{MS}}$ scheme is at NNLO.

[71] CHETYRKIN 98 uses spectral moments of hadronic $\tau$ decays to determine $m_s$(1 GeV)=200 ± 70 MeV. We have rescaled the result to $\mu$=2 GeV.

[72] CUCCHIERI 98 obtains the quark mass using a quenched lattice computation of the hadronic spectrum.

[73] DOMINGUEZ 98 uses hadronic spectral function sum rules (to four loops, and including dimension six operators) to determine $m_s(1\,\text{GeV}) < 155 \pm 25$ MeV. We have rescaled the result to $\mu$=2 GeV.
[74] CHETYRKIN 97 obtains 205.5 $\pm$ 19.1 MeV at $\mu$=1 GeV from QCD sum rules including fourth-order QCD corrections. We have rescaled the result to 2 GeV.
[75] COLANGELO 97 is QCD sum rule computation. We have rescaled $m_s(1\,\text{GeV}) > 120$ to $\mu = 2$ GeV.
[76] EICKER 97 use lattice gauge computations with two dynamical light flavors.
[77] GOUGH 97 use lattice gauge computations in the quenched approximation. Correcting for quenching gives 54 $< m_s <$ 92 MeV at $\mu$=2 GeV.
[78] GUPTA 97 use Lattice Monte Carlo computations in the quenched approximation. The value for two light dynamical flavors at $\mu = 2$ GeV is 68 $\pm$ 12 $\pm$ 7 MeV.
[79] LELLOUCH 97 obtain lower bounds on quark masses using hadronic spectral functions.
[80] JAMIN 95 uses QCD sum rules at next-to-leading order. We have rescaled $m_s(1\,\text{GeV}) = 189 \pm 32$ to $\mu = 2$ GeV.

## LIGHT QUARK MASS RATIOS

### *u/d* MASS RATIO

| VALUE | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|
| **0.3 to 0.6 OUR EVALUATION** | | | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 0.43 $\pm$0.08 | [81] AUBIN | 04A | LATT | $\overline{\text{MS}}$ scheme |
| 0.410$\pm$0.036 | [82] NELSON | 03 | LATT | $\overline{\text{MS}}$ scheme |
| 0.44 | [83] GAO | 97 | THEO | $\overline{\text{MS}}$ scheme |
| 0.553$\pm$0.043 | [84] LEUTWYLER | 96 | THEO | Compilation |
| $<$0.3 | [85] CHOI | 92 | THEO | |
| 0.26 | [86] DONOGHUE | 92 | THEO | |
| 0.30 $\pm$0.07 | [87] DONOGHUE | 92B | THEO | |
| 0.66 | [88] GERARD | 90 | THEO | |
| 0.4 to 0.65 | [89] LEUTWYLER | 90B | THEO | |
| 0.05 to 0.78 | [90] MALTMAN | 90 | THEO | |

[81] AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses.
[82] NELSON 03 computes coefficients in the order $p^4$ chiral Lagrangian using a lattice calculation with three dynamical flavors. The ratio $m_u/m_d$ is obtained by combining this with the chiral perturbation theory computation of the meson masses to order $p^4$.
[83] GAO 97 uses electromagnetic mass splittings of light mesons.
[84] LEUTWYLER 96 uses a combined fit to $\eta \to 3\pi$ and $\psi' \to J/\psi$ ($\pi$,$\eta$) decay rates, and the electromagnetic mass differences of the $\pi$ and $K$.
[85] CHOI 92 result obtained from the decays $\psi(2S) \to J/\psi(1S)\,\pi$ and $\psi(2S) \to J/\psi(1S)\,\eta$, and a dilute instanton gas estimate of some unknown matrix elements.
[86] DONOGHUE 92 result is from a combined analysis of meson masses, $\eta \to 3\pi$ using second-order chiral perturbation theory including nonanalytic terms, and $(\psi(2S) \to J/\psi(1S)\,\pi)/(\psi(2S) \to J/\psi(1S)\,\eta)$.
[87] DONOGHUE 92B computes quark mass ratios using $(\psi(2S) \to J/\psi(1S)\,\pi)/(\psi(2S) \to J/\psi(1S)\,\eta)$, and an estimate of $L_{14}$ using Weinberg sum rules.
[88] GERARD 90 uses large $N$ and $\eta$-$\eta'$ mixing.
[89] LEUTWYLER 90B determines quark mass ratios using second-order chiral perturbation theory for the meson and baryon masses, including nonanalytic corrections. Also uses Weinberg sum rules to determine $L_7$.
[90] MALTMAN 90 uses second-order chiral perturbation theory including nonanalytic terms for the meson masses. Uses a criterion of "maximum reasonableness" that certain coefficients which are expected to be of order one are $\leq 3$.

## $s/d$ MASS RATIO

| VALUE | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|
| **17 to 22 OUR EVALUATION** | | | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 20.0 | [91] GAO | 97 | THEO | $\overline{\text{MS}}$ scheme |
| 18.9±0.8 | [92] LEUTWYLER | 96 | THEO | Compilation |
| 21 | [93] DONOGHUE | 92 | THEO | |
| 18 | [94] GERARD | 90 | THEO | |
| 18 to 23 | [95] LEUTWYLER | 90B | THEO | |

[91] GAO 97 uses electromagnetic mass splittings of light mesons.

[92] LEUTWYLER 96 uses a combined fit to $\eta \to 3\pi$ and $\psi' \to J/\psi\ (\pi,\eta)$ decay rates, and the electromagnetic mass differences of the $\pi$ and $K$.

[93] DONOGHUE 92 result is from a combined analysis of meson masses, $\eta \to 3\pi$ using second-order chiral perturbation theory including nonanalytic terms, and $(\psi(2S) \to J/\psi(1S)\pi)/(\psi(2S) \to J/\psi(1S)\eta)$.

[94] GERARD 90 uses large $N$ and $\eta$-$\eta'$ mixing.

[95] LEUTWYLER 90B determines quark mass ratios using second-order chiral perturbation theory for the meson and baryon masses, including nonanalytic corrections. Also uses Weinberg sum rules to determine $L_7$.

## $m_s/\overline{m}$ MASS RATIO

$\overline{m} \equiv (m_u + m_d)/2$

| VALUE | DOCUMENT ID | | TECN |
|---|---|---|---|
| **25 to 30 OUR EVALUATION** | | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | |
| 27.4±0.4 | [96] AUBIN | 04 | LATT |

[96] Three flavor dynamical lattice calculation of pseudoscalar meson masses.

## $Q$ MASS RATIO

$Q \equiv \sqrt{(m^2{}_s - \overline{m}^2)/(m^2{}_d - m^2{}_u)}; \quad \overline{m} \equiv (m_u + m_d)/2$

| VALUE | DOCUMENT ID | | TECN |
|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | |
| 22.8±0.4 | [97] MARTEMYAN..05 | | THEO |
| 22.7±0.8 | [98] ANISOVICH | 96 | THEO |

[97] MARTEMYANOV 05 determine $Q$ from $\eta \to 3\pi$ decay.

[98] ANISOVICH 96 find $Q$ from $\eta \to \pi^+\pi^-\pi^0$ decay using dispersion relations and chiral perturbation theory.

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