B^0–$\overline{B}^0$ MIXING

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There are two neutral B^0–$\overline{B}^0$ meson systems, B^0_d–$\overline{B}^0_d$ and B^0_s–$\overline{B}^0_s$ (generically denoted B^0_q–$\overline{B}^0_q$, $q = s, d$), which exhibit particle-antiparticle mixing [1]. This mixing phenomenon is described in Ref. 2. In the following, we adopt the notation introduced in Ref. 2, and assume CPT conservation throughout. In each system, the light (L) and heavy (H) mass eigenstates,

$$|B_{\rm L,H}\rangle = p|B^0_q\rangle \pm q|\overline{B}^0_q\rangle\,, \tag{1}$$

have a mass difference $\Delta m_q = m_{\rm H} - m_{\rm L} > 0$, and a total decay width difference $\Delta\Gamma_q = \Gamma_{\rm L} - \Gamma_{\rm H}$. In the absence of CP violation in the mixing, $|q/p| = 1$, these differences are given by $\Delta m_q = 2|M_{12}|$ and $|\Delta\Gamma_q| = 2|\Gamma_{12}|$, where M_{12} and Γ_{12} are the off-diagonal elements of the mass and decay matrices [2]. The evolution of a pure $|B^0_q\rangle$ or $|\overline{B}^0_q\rangle$ state at $t = 0$ is given by

$$|B^0_q(t)\rangle = g_+(t)\,|B^0_q\rangle + \frac{q}{p}\,g_-(t)\,|\overline{B}^0_q\rangle\,, \tag{2}$$

$$|\overline{B}^0_q(t)\rangle = g_+(t)\,|\overline{B}^0_q\rangle + \frac{p}{q}g_-(t)\,|B^0_q\rangle\,, \tag{3}$$

which means that the flavor states remain unchanged (+) or oscillate into each other (−) with time-dependent probabilities proportional to

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}\,t\right) \pm \cos(\Delta m_q\, t)\right]\,, \tag{4}$$

where $\Gamma_q = (\Gamma_{\rm H} + \Gamma_{\rm L})/2$. In the absence of CP violation, the time-integrated mixing probability $\int |g_-(t)|^2\,dt/(\int |g_-(t)|^2\,dt + \int |g_+(t)|^2\,dt)$ is given by

$$\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2+1)}\,, \qquad \text{where} \qquad x_q = \frac{\Delta m_q}{\Gamma_q}\,, \qquad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}\,. \tag{5}$$

Standard Model predictions and phenomenology

In the Standard Model, the transitions $B^0_q \to \overline{B}^0_q$ and $\overline{B}^0_q \to B^0_q$ are due to the weak interaction. They are described, at the lowest order, by box diagrams involving two W bosons and two

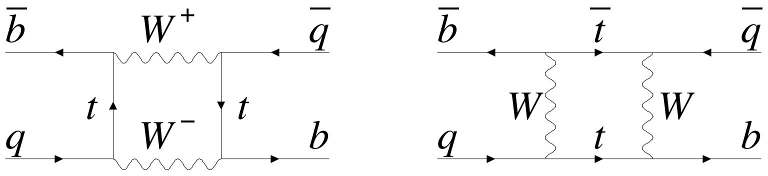

Figure 1: Dominant box diagrams for the $B_q^0 \to \overline{B}_q^0$ transitions ($q = d$ or s). Similar diagrams exist where one or both t quarks are replaced with c or u quarks.

up-type quarks (see Fig. 1), as is the case for K^0–$\overline{K}^0$ mixing. However, the long range interactions arising from intermediate virtual states are negligible for the neutral B meson systems, because the large B mass is off the region of hadronic resonances. The calculation of the dispersive and absorptive parts of the box diagrams yields the following predictions for the off-diagonal element of the mass and decay matrices [3],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} \, S_0(m_t^2/m_W^2) \, (V_{tq}^* V_{tb})^2 \,, \tag{6}$$

$$\begin{aligned} \Gamma_{12} = {} & \frac{G_F^2 m_b^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi} \\ & \times \left[(V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb} \, \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) \right. \\ & \left. + \, (V_{cq}^* V_{cb})^2 \, \mathcal{O}\left(\frac{m_c^4}{m_b^4}\right) \right] , \end{aligned} \tag{7}$$

where G_F is the Fermi constant, m_W the W boson mass, and m_i the mass of quark i; m_{B_q}, f_{B_q} and B_{B_q} are the B_q^0 mass, weak decay constant and bag parameter, respectively. The known function $S_0(x_t)$ can be approximated very well by $0.784\, x_t^{0.76}$ [4], and V_{ij} are the elements of the CKM matrix [5]. The QCD corrections η_B and η'_B are of order unity. The only non-negligible contributions to M_{12} are from box diagrams involving two top quarks. The phases of M_{12} and Γ_{12} satisfy

$$\phi_M - \phi_\Gamma = \pi + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) , \tag{8}$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the K^0–$\overline{K}^0$ system, the heavy state is expected to have a smaller decay width than that of the light state: $\Gamma_{\rm H} < \Gamma_{\rm L}$. Hence, $\Delta\Gamma = \Gamma_{\rm L} - \Gamma_{\rm H}$ is expected to be positive in the Standard Model.

Furthermore, the quantity

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2}\frac{m_b^2}{m_W^2}\frac{1}{S_0(m_t^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right) \tag{9}$$

is small, and a power expansion of $|q/p|^2$ yields

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right|\sin(\phi_M - \phi_\Gamma) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right). \tag{10}$$

Therefore, considering both Eqs. (8) and (9), the CP-violating parameter

$$1 - \left|\frac{q}{p}\right|^2 \simeq \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \tag{11}$$

is expected to be very small: $\sim \mathcal{O}(10^{-3})$ for the B^0_d–$\overline{B}{}^0_d$ system and $\lesssim \mathcal{O}(10^{-4})$ for the B^0_s–$\overline{B}{}^0_s$ system [6].

In the approximation of negligible CP violation in mixing, the ratio $\Delta\Gamma_q/\Delta m_q$ is equal to the small quantity $|\Gamma_{12}/M_{12}|$ of Eq. (9); it is hence independent of CKM matrix elements, *i.e.*, the same for the B^0_d–$\overline{B}{}^0_d$ and B^0_s–$\overline{B}{}^0_s$ systems. It can be calculated with lattice QCD techniques; typical results are $\sim 5 \times 10^{-3}$ with quoted uncertainties of $\sim 30\%$. Given the current experimental knowledge on the mixing parameter x_q (obtained from published results only),

$$\begin{cases} x_d = 0.776 \pm 0.008 & (B^0_d\text{–}\overline{B}{}^0_d \text{ system}) \\ x_s > 19.9 \text{ at } 95\% \text{ CL} & (B^0_s\text{–}\overline{B}{}^0_s \text{ system}) \end{cases}, \tag{12}$$

the Standard Model thus predicts that $\Delta\Gamma_d/\Gamma_d$ is very small (below 1%), but $\Delta\Gamma_s/\Gamma_s$ considerably larger ($\sim 10\%$). These width differences are caused by the existence of final states to which both the B^0_q and $\overline{B}{}^0_q$ mesons can decay. Such decays involve $b \to c\bar{c}q$ quark-level transitions, which are Cabibbo-suppressed if $q = d$ and Cabibbo-allowed if $q = s$.

Experimental issues and methods for oscillation analyses

Time-integrated measurements of B^0–$\overline{B}{}^0$ mixing were published for the first time in 1987 by UA1 [7] and ARGUS [8], and since then by many other experiments. These measurements are typically based on counting same-sign and opposite-sign lepton pairs from the semileptonic decay of the produced $b\overline{b}$ pairs. Such analyses cannot easily separate the contributions from the different b-hadron species, therefore, the clean environment of $\Upsilon(4S)$ machines (where only B^0_d and charged B_u mesons are produced) is in principle best suited to measure χ_d.

However, better sensitivity is obtained from time-dependent analyses aiming at the direct measurement of the oscillation frequencies Δm_d and Δm_s, from the proper time distributions of B^0_d or B^0_s candidates identified through their decay in (mostly) flavor-specific modes, and suitably tagged as mixed or unmixed. This is particularly true for the B^0_s–$\overline{B}^0_s$ system, where the large value of x_s implies maximal mixing, *i.e.*, $\chi_s \simeq 1/2$. In such analyses, the B^0_d or B^0_s mesons are either fully reconstructed, partially reconstructed from a charm meson, selected from a lepton with the characteristics of a $b \to \ell^-$ decay, or selected from a reconstructed displaced vertex. At high-energy colliders (LEP, SLC, Tevatron), the proper time $t = \frac{m_B}{p} L$ is measured from the distance L between the production vertex and the B decay vertex, and from an estimate of the B momentum p. At asymmetric B factories (KEKB, PEP-II), producing $e^+e^- \to \Upsilon(4S) \to B^0_d\overline{B}^0_d$ events with a boost $\beta\gamma$ (= 0.425, 0.55), the proper time difference between the two B candidates is estimated as $\Delta t \simeq \frac{\Delta z}{\beta\gamma c}$, where Δz is the spatial separation between the two B decay vertices along the boost direction. In all cases, the good resolution needed on the vertex positions is obtained with silicon detectors.

The average statistical significance $\mathcal{S}$ of a B^0_d or B^0_s oscillation signal can be approximated as [9]

$$\mathcal{S} \approx \sqrt{N/2}\, f_{\rm sig}\, (1-2\eta)\, e^{-(\Delta m\, \sigma_t)^2/2}\,, \tag{13}$$

where N is the number of selected and tagged candidates, $f_{\rm sig}$ is the fraction of signal in that sample, η is the total mistag probability, and σ_t is the resolution on proper time (or proper time difference). The quantity $\mathcal{S}$ decreases very quickly as Δm increases; this dependence is controlled by σ_t, which is therefore a critical parameter for Δm_s analyses. At high-energy colliders, the proper time resolution $\sigma_t \sim \frac{m_B}{\langle p \rangle}\sigma_L \oplus t\frac{\sigma_p}{p}$ includes a constant contribution due to the decay length resolution σ_L (typically 0.05–0.3 ps), and a term due to the relative momentum resolution σ_p/p (typically 10–20% for partially reconstructed decays), which increases with proper time. At B factories, the boost of the B mesons is estimated from the known beam energies, and the term due to the spatial resolution dominates (typically 1–1.5 ps because of the much smaller B boost).

In order to tag a B candidate as mixed or unmixed, it is necessary to determine its flavor both in the initial state and in the final state. The initial and final state mistag probabilities, η_i and η_f, degrade $\mathcal{S}$ by a total factor $(1-2\eta) = (1-2\eta_i)(1-2\eta_f)$. In lepton-based analyses, the final state is tagged by the charge of the lepton from $b \to \ell^-$ decays; the largest contribution to η_f is then due to $\overline{b} \to \overline{c} \to \ell^-$ decays. Alternatively, the charge of a reconstructed charm meson (D^{*-} from B_d^0 or D_s^- from B_s^0), or that of a kaon hypothesized to come from a $b \to c \to s$ decay [10], can be used. For fully inclusive analyses based on topological vertexing, final state tagging techniques include jet charge [11] and charge dipole [12,13] methods.

At high-energy colliders, the methods to tag the initial state (*i.e.*, the state at production), can be divided into two groups: the ones that tag the initial charge of the $\overline{b}$ quark contained in the B candidate itself (same-side tag), and the ones that tag the initial charge of the other b quark produced in the event (opposite-side tag). On the same side, the charge of a track from the primary vertex is correlated with the production state of the B if that track is a decay product of a B^{**} state or the first particle in the fragmentation chain [14,15]. Jet- and vertex-charge techniques work on both sides and on the opposite side, respectively. Finally, the charge of a lepton from $b \to \ell^-$ or of a kaon from $b \to c \to s$ can be used as opposite side tags, keeping in mind that their performance is degraded due to integrated mixing. At SLC, the beam polarization produced a sizeable forward-backward asymmetry in the $Z \to b\overline{b}$ decays, and provided another very interesting and effective initial state tag based on the polar angle of the B candidate [12]. Initial state tags have also been combined to reach $\eta_i \sim 26\%$ at

LEP [15,16], or even 22% at SLD [12] with full efficiency. In the case $\eta_f = 0$, this corresponds to an effective tagging efficiency $Q = \epsilon D^2 = \epsilon(1-2\eta)^2$, where ϵ is the tagging efficiency, in the range $23 - 31\%$. The equivalent figure achieved by CDF during Tevatron Run I was $\sim 3.5\%$ [17] reflecting the fact that tagging is more difficult at hadron colliders. The current CDF and DØ analyses of Tevatron Run II data reach $\epsilon D^2 = (1.5 \pm 0.1)\%$ [18] and $(2.5 \pm 0.2)\%$ [19] for opposite-side tagging, while same-side kaon tagging (for B_s^0 oscillation analyses) is contributing an additional $(3.4 \pm 1.0)\%$ at CDF [18].

At B factories, the flavor of a B_d^0 meson at production cannot be determined, since the two neutral B mesons produced in a $\Upsilon(4S)$ decay evolve in a coherent P-wave state where they keep opposite flavors at any time. However, as soon as one of them decays, the other follows a time-evolution given by Eqs. (2) or (3), where t is replaced with Δt (which will take negative values half of the time). Hence, the "initial state" tag of a B can be taken as the final state tag of the other B. Effective tagging efficiencies Q of 30% are achieved by BABAR and Belle [20], using different techniques including $b \to \ell^-$ and $b \to c \to s$ tags. It is worth noting that, in this case, mixing of the other B (*i.e.*, the coherent mixing occurring before the first B decay) does not contribute to the mistag probability.

In the absence of experimental observation of a decay-width difference, oscillation analyses typically neglect $\Delta\Gamma$ in Eq. (4), and describe the data with the physics functions $\Gamma e^{-\Gamma t}(1 \pm \cos(\Delta m t))/2$ (high-energy colliders) or $\Gamma e^{-\Gamma|\Delta t|}(1 \pm \cos(\Delta m \Delta t))/4$ (asymmetric $\Upsilon(4S)$ machines). As can be seen from Eq. (4), a non-zero value of $\Delta\Gamma$ would effectively reduce the oscillation amplitude with a small time-dependent factor that would be very difficult to distinguish from time resolution effects. Measurements of Δm_d are usually extracted from the data using a maximum likelihood fit. To extract information useful for the interpretation of B_s^0 oscillation searches and for the combination of their results, a method [9] is followed in which a B_s^0 oscillation amplitude $\mathcal{A}$ is measured as a function of a fixed test value of Δm_s, using a maximum likelihood fit based on the functions $\Gamma_s e^{-\Gamma_s t}(1 \pm \mathcal{A}\cos(\Delta m_s t))/2$. To a good approximation, the statistical uncertainty on $\mathcal{A}$ is Gaussian and equal to $1/\mathcal{S}$ from Eq. (13). If Δm_s is equal to its true value, one expects $\mathcal{A} = 1$ within the total uncertainty $\sigma_{\mathcal{A}}$; in case a signal is seen, its observed (or expected) significance will be defined as $\mathcal{A}/\sigma_{\mathcal{A}}$ (or $1/\sigma_{\mathcal{A}}$). However, if Δm_s is (far) below its true value, a measurement consistent with $\mathcal{A} = 0$ is expected. A value of Δm_s can be excluded at 95% CL if $\mathcal{A} + 1.645\,\sigma_{\mathcal{A}} \leq 1$ (since the integral of a normal distribution from $-\infty$ to 1.645 is equal to 0.95). Because of the proper time resolution, the quantity $\sigma_{\mathcal{A}}(\Delta m_s)$ is a steadily increasing function of Δm_s. We define the sensitivity for 95% CL exclusion of Δm_s values (or for a $3\,\sigma$ or $5\,\sigma$ observation of B_s^0 oscillations) as the value of Δm_s for which $1/\sigma_{\mathcal{A}} = 1.645$ (or $1/\sigma_{\mathcal{A}} = 3$ or 5).

B_d^0 *mixing studies*

Many B_d^0–$\overline{B}_d^0$ oscillations analyses have been published [21] by the ALEPH [22], BABAR [23], Belle [24], CDF [14], DELPHI [13,25], L3 [26], and OPAL [27] collaborations. Although a variety of different techniques have been used, the individual Δm_d results obtained at high-energy colliders have remarkably similar precision. Their average is compatible with the recent and more precise measurements from asymmetric B factories. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or b-hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the b-hadron lifetimes and fractions published in this *Review*. Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fragmentation fractions, lifetimes, branching ratios of b hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, tagging, background description). Combining all published measurements [13,14,22–27] and accounting for all identified correlations yields $\Delta m_d = 0.507 \pm 0.003(\text{stat}) \pm 0.003(\text{syst})\ \text{ps}^{-1}$ [28], a result now dominated by the B factories.

On the other hand, ARGUS and CLEO have published time-integrated measurements [29–31], which average to $\chi_d = 0.182 \pm 0.015$. Following Ref. 31, the width difference $\Delta\Gamma_d$ could in principle be extracted from the measured value of Γ_d and the above averages for Δm_d and χ_d (see Eq. (5)), provided that $\Delta\Gamma_d$ has a negligible impact on the Δm_d measurements. However, direct time-dependent studies published by DELPHI [13] and BABAR [32] yield stronger constraints, which can be combined to yield $\mathrm{sign}(\mathrm{Re}\lambda_{\mathrm{CP}})\Delta\Gamma_{\mathrm{d}}/\Gamma_{\mathrm{d}} = 0.009 \pm 0.037$ [28].

Assuming $\Delta\Gamma_d = 0$ and no CP violation in mixing, and using the measured B^0_d lifetime of $1.530 \pm 0.009\ \mathrm{ps}^{-1}$, the Δm_d and χ_d results are combined to yield the world average

$$\Delta m_d = 0.507 \pm 0.005\ \mathrm{ps}^{-1} \tag{14}$$

or, equivalently,

$$\chi_d = 0.188 \pm 0.003\,. \tag{15}$$

Evidence for CP violation in B^0_d mixing has been searched for, both with flavor-specific and inclusive B^0_d decays, in samples where the initial flavor state is tagged. In the case of semileptonic (or other flavor-specific) decays, where the final state tag is also available, the following asymmetry [2]

$$\mathcal{A}_{\mathrm{SL}} = \frac{N(\overline{B}^0_d(t) \to \ell^+\nu_\ell X) - N(B^0_d(t) \to \ell^-\overline{\nu}_\ell X)}{N(\overline{B}^0_d(t) \to \ell^+\nu_\ell X) + N(B^0_d(t) \to \ell^-\overline{\nu}_\ell X)} \simeq 1 - |q/p|^2_d \tag{16}$$

has been measured, either in time-integrated analyses at CLEO [31,33], CDF [34] and DØ [35], or in time-dependent analyses at LEP [36–38], BABAR [32,39] and Belle [40]. In the inclusive case, also investigated at LEP [37,38,41], no final state tag is used, and the asymmetry [42]

$$\begin{aligned} &\frac{N(B^0_d(t) \to \mathrm{all}) - N(\overline{B}^0_d(t) \to \mathrm{all})}{N(B^0_d(t) \to \mathrm{all}) + N(\overline{B}^0_d(t) \to \mathrm{all})} \\ &\simeq \mathcal{A}_{\mathrm{SL}}\left[\frac{x_d}{2}\sin(\Delta m_d\, t) - \sin^2\left(\frac{\Delta m_d\, t}{2}\right)\right] \end{aligned} \tag{17}$$

must be measured as a function of the proper time to extract information on CP violation. In all cases, asymmetries compatible with zero have been found, with a precision limited by the available statistics. A simple average of all published results for the B^0_d meson [31–33,36,38,39,41] yields $\mathcal{A}_{\mathrm{SL}} = -0.005 \pm 0.012$, or $|q/p|_d = 1.0026 \pm 0.0059$, a result which does not yet constrain the Standard Model.

The Δm_d result of Eq. (14) provides an estimate of $2|M_{12}|$, and can be used, together with Eq. (6), to extract the magnitude of the CKM matrix element V_{td} within the Standard Model [43]. The main experimental uncertainties on the resulting estimate of $|V_{td}|$ come from m_t and Δm_d; however, the extraction is at present completely dominated by the uncertainty on the hadronic matrix element $f_{B_d}\sqrt{B_{B_d}} = 244 \pm 26$ MeV obtained from lattice QCD calculations [44].

B_s^0 mixing studies

B_s^0–$\overline{B}_s^0$ oscillations have been the subject of many studies from ALEPH [45], DELPHI [13,16,46], OPAL [47], SLD [12,48, 49], CDF [18,50] and DØ [19,51]. The most sensitive analyses at LEP appear to be the ones based on inclusive lepton samples. Because of their better proper time resolution, the small data samples analyzed inclusively at SLD, as well as the fully reconstructed B_s decays at LEP and at the Tevatron, are also very useful to explore the high Δm_s region.

All results are limited by the available statistics. They can easily be combined, since all experiments provide measurements of the B_s^0 oscillation amplitude. All published results [12,13,16,45–48,50] are averaged [28] to yield the combined amplitudes $\mathcal{A}$ shown in Fig. 2 (top) as a function of Δm_s. The individual results have been adjusted to common physics inputs, and all known correlations have been accounted for; the sensitivities of the inclusive analyses, which depend directly through Eq. (13) on the assumed fraction f_s of B_s^0 mesons in an unbiased sample of weakly-decaying b hadrons, have also been rescaled to a common average of $f_s = 0.102 \pm 0.009$. The combined sensitivity for 95% CL exclusion of Δm_s values is found to be 18.2 ps^{-1}. All values of Δm_s below 14.4 ps^{-1} are excluded at 95% CL, which we express as

$$\Delta m_s > 14.4 \ \text{ps}^{-1} \quad \text{at 95\% CL}. \tag{18}$$

The values between 14.4 and 21.8 ps^{-1} cannot be excluded, because the data is compatible with a signal in this region.

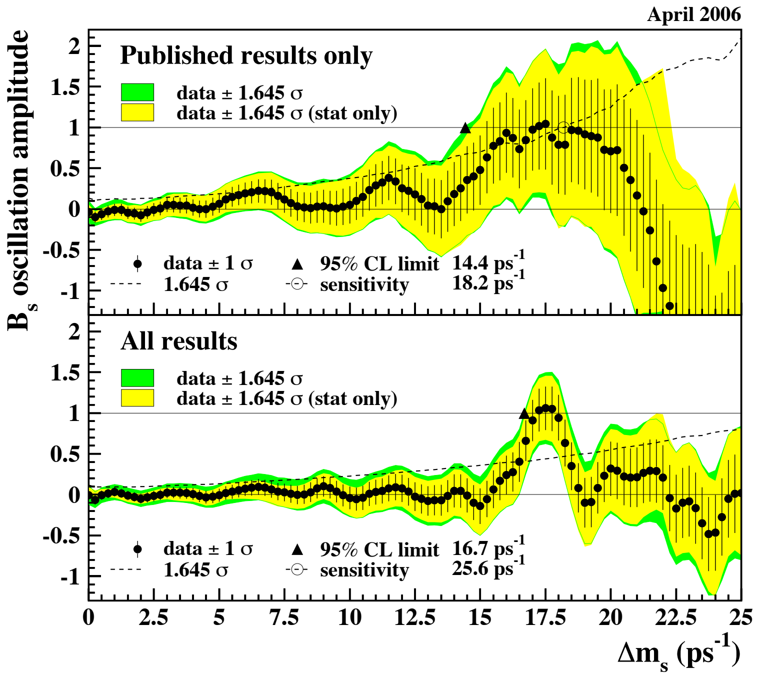

Figure 2: Combined measurements of the B_s^0 oscillation amplitude as a function of Δm_s, based on published results only (top) or on all published and unpublished results (bottom) available at the end of April 2006. The measurements are dominated by statistical uncertainties. Neighboring points are statistically correlated. See full-color version on color pages at end of book.

However, the largest deviation from $\mathcal{A} = 0$ in this range is a $1.9\,\sigma$ effect only, so no signal can be claimed.

The above average does not include the very recent results from Tevatron Run II, based on 1 fb^{-1} of data. In a paper submitted for publication [19], DØ reports the first direct two-sided bound established by a single experiment of $17 < \Delta m_s < 21$ ps^{-1} (90% CL) and a most probable value of 19 ps^{-1} with an observed (expected) significance of $2.5\,\sigma$ $(0.9\,\sigma)$. A preliminary and subsequent analysis from CDF [18] is more sensitive and leads to the first direct evidence of B_s^0 oscillations and the following measurement:

$$\Delta m_s = 17.33^{+0.42}_{-0.21}(\text{stat}) \pm 0.07(\text{syst})~~\text{ps}^{-1}\,. \qquad (19)$$

Both the observed significance and the expected significance of this signal are equal to $3.1\,\sigma$. The CDF collaboration is quoting a 0.5% probability that their data would fluctuate to produce, at any value of Δm_s, a fake signal as significant as the observed one, corresponding to a $2.6\,\sigma$ effect. Both DØ and CDF quote their Δm_s results assuming that they see the oscillation signal.

Including all unpublished analyses [18,19,49] in the average leads to the combined amplitude spectrum of Fig. 2 (bottom), which is dominated by the new CDF result, and where a consolidated signal is seen with a significance of $4.0\,\sigma$. A preliminary world average is

$$\Delta m_s = 17.4\,^{+0.3}_{-0.2}\ \text{ps}^{-1}\,. \tag{20}$$

The information on $|V_{ts}|$ obtained, in the framework of the Standard Model, from the combined amplitude spectrum, is hampered by the hadronic uncertainty, as in the B^0_d case. However, several uncertainties cancel in the frequency ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}}\,\xi^2 \left|\frac{V_{ts}}{V_{td}}\right|^2 , \tag{21}$$

where $\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}}) = 1.210\,^{+0.047}_{-0.035}$ is an SU(3) flavor-symmetry breaking factor obtained from lattice QCD calculations [44]. Using the averages of Eqs. (14) and (20), one can extract

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.208 \pm 0.002(\text{exp})\,^{+0.008}_{-0.006}(\text{lattice})\,, \tag{22}$$

in good agreement with (but more precise than) the recent result obtained by the Belle collaboration based on the observation of the $b \to d\gamma$ transition [52]. The CKM matrix can be constrained using experimental results on observables such as Δm_d, Δm_s, $|V_{ub}/V_{cb}|$, ϵ_K, and $\sin(2\beta)$ together with theoretical inputs and unitarity conditions [43,53,54]. The constraint from our knowledge on the ratio $\Delta m_s/\Delta m_d$ is presently more effective in limiting the position of the apex of the CKM unitarity triangle than the one obtained from the Δm_d measurements alone, due to the reduced hadronic uncertainty in Eq. (21). We also note that the measured value of $\mathit{\Delta m_s}$ is consistent with the Standard Model prediction obtained from CKM fits where no experimental information on $\mathit{\Delta m_s}$ is used, *e.g.* $21.2 \pm 3.2\ \text{ps}^{-1}$ [53] or $16.5\,^{+10.5}_{-3.4}\ \text{ps}^{-1}$ [54].

Information on $\Delta\Gamma_s$ can be obtained by studying the proper time distribution of untagged B_s^0 samples [55]. In the case of an inclusive B_s^0 selection [56], or a semileptonic (or flavour-specific) B_s^0 decay selection [16,57,58], both the short- and long-lived components are present, and the proper time distribution is a superposition of two exponentials with decay constants $\Gamma_{\mathrm{L,H}} = \Gamma_s \pm \Delta\Gamma_s/2$. In principle, this provides sensitivity to both Γ_s and $(\Delta\Gamma_s/\Gamma_s)^2$. Ignoring $\Delta\Gamma_s$ and fitting for a single exponential leads to an estimate of Γ_s with a relative bias proportional to $(\Delta\Gamma_s/\Gamma_s)^2$. An alternative approach, which is directly sensitive to first order in $\Delta\Gamma_s/\Gamma_s$, is to determine the lifetime of B_s^0 candidates decaying to CP eigenstates; measurements exist for $B_s^0 \to J/\psi\phi$ [59,60] and $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ [61], which are mostly CP-even states [62]. However, in the case of $B_s^0 \to J/\psi\phi$ this technique has now been replaced by more sensitive time-dependent angular analyses that allow the simultaneous extraction of $\Delta\Gamma_s/\Gamma_s$ and the CP-even and CP-odd amplitudes [63]. An estimate of $\Delta\Gamma_s/\Gamma_s$ has also been obtained directly from a measurement of the $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ branching ratio [61], under the assumption that these decays account for all the CP-even final states (however, no systematic uncertainty due to this assumption is given, so the average quoted below will not include this estimate).

Applying the combination procedure of Ref. 28 (including the constraint from the flavour-specific lifetime measurements) on the published results [16,57,59,61,63] yields

$$\Delta\Gamma_s/\Gamma_s = +0.31\,^{+0.11}_{-0.13} \qquad \text{and} \qquad 1/\Gamma_s = 1.398\,^{+0.049}_{-0.050}\ \text{ps}\,, \tag{23}$$

or equivalently

$$1/\Gamma_{\mathrm{L}} = 1.21 \pm 0.09\ \text{ps} \qquad \text{and} \qquad 1/\Gamma_{\mathrm{H}} = 1.66\,^{+0.11}_{-0.12}\ \text{ps}\,. \tag{24}$$

This result can be compared with the theoretical prediction $\Delta\Gamma_s/\Gamma_s = +0.12 \pm 0.05$ [64] within the Standard Model.

Average b-hadron mixing probability and b-hadron production fractions in Z decays and at high energy

Mixing measurements can significantly improve our knowledge on the fractions f_u, f_d, f_s and f_{baryon}, defined as the fractions of B_u, B_d^0, B_s^0 and b-baryon in an unbiased sample of weakly decaying b hadrons produced in high-energy collisions. Indeed, time-integrated mixing analyses performed with lepton pairs from $b\overline{b}$ events at high energy measure the quantity

$$\overline{\chi} = f_d' \chi_d + f_s' \chi_s \,, \tag{25}$$

where f_d' and f_s' are the fractions of B_d^0 and B_s^0 hadrons in a sample of semileptonic b-hadron decays. Assuming that all b hadrons have the same semileptonic decay width implies $f_q' = f_q/(\Gamma_q \tau_b)$ $(q = s, d)$, where τ_b is the average b-hadron lifetime. Hence $\overline{\chi}$ measurements, together with the χ_d average of Eq. (15) and the very good approximation $\chi_s = 1/2$ (in fact $\chi_s > 0.4988$ at 95% CL from Eqs. (5), (18) and (23)), provide constraints on the fractions f_d and f_s.

The LEP experiments have measured $f_s \times \mathrm{BR}(B_s^0 \to D_s^- \ell^+ \nu_\ell X)$ [65], $\mathrm{BR}(b \to \mathit{\Lambda}_b^0) \times \mathrm{BR}(\mathit{\Lambda}_b^0 \to \mathit{\Lambda}_c^+ \ell^- \overline{\nu}_\ell X)$ [66], and $\mathrm{BR}(b \to \mathit{\Xi}_b^-) \times \mathrm{BR}(\mathit{\Xi}_b^- \to \mathit{\Xi}^- \ell^- \overline{\nu}_\ell X)$ [67] from partially reconstructed final states, including a lepton, f_{baryon} from protons identified in b events [68], and the production rate of charged b hadrons [69]. The b-hadron fractions measured at CDF with electron-charm final states [70] are at slight discrepancy with the ones measured at LEP. Furthermore the values of $\overline{\chi}$ measured at LEP, 0.1259 ± 0.0042 [71], and at CDF, 0.152 ± 0.013 [72], show a $1.9\,\sigma$ deviation with respect to each other. This may be a hint that the fractions at the Tevatron might be different from the ones in Z decays. Combining [28] all the available information under the constraints $f_u = f_d$ and $f_u + f_d + f_s + f_{\mathrm{baryon}} = 1$ yields the two set of averages shown in Table 1. The second set, obtained using both LEP and Tevatron results, has larger errors than the first set, obtained using LEP results only, because we have applied scale factors as advocated by the PDG for the treatment of marginally consistent data.

Table 1: $\overline{\chi}$ and b-hadron fractions (see text).

	in Z decays	at high energy
$\overline{\chi}$	0.1259 ± 0.0042	0.1283 ± 0.0076
$f_u = f_d$	0.399 ± 0.010	0.398 ± 0.012
f_s	0.102 ± 0.009	0.103 ± 0.014
f_{baryon}	0.100 ± 0.017	0.100 ± 0.020

Summary and prospects

B^0–$\overline{B}^0$ mixing has been and still is a field of intense study. The mass difference in the B^0_d–$\overline{B}^0_d$ system is now very precisely known (with an experimental error of 0.9%) but, despite an impressive theoretical effort, the hadronic uncertainty keeps limiting the precision of the extracted estimate of $|V_{td}|$ within the Standard Model (SM). On the other hand measurements of $\Delta\Gamma_d$ and of CP violation in B^0_d–$\overline{B}^0_d$ mixing are consistent with zero, with an uncertainty still large compared to the SM predictions. Impressive new B^0_s results are becoming available from Run II of the Tevatron: preliminary direct evidence for B^0_s–$\overline{B}^0_s$ oscillations has been reported, with a frequency in agreement with the SM. New time-dependent angular analyses of $B^0_s \to J/\psi\phi$ decays at CDF and DØ have improved our knowledge of $\Delta\Gamma_s/\Gamma_s$ to an absolute uncertainty of $\sim 10\%$, of the same size as the central value of the SM prediction. The data clearly prefer $\Gamma_{\rm L} > \Gamma_{\rm H}$ as predicted in the SM.

Improved results on B^0_s–$\overline{B}^0_s$ mixing are still to come from the Tevatron, with very promising prospects in the next couple of years, both for Δm_s and $\Delta\Gamma_s$. With a few fb^{-1} of data, the CDF and DØ collaborations will have the potential to confirm their Δm_s signals and make $> 5\,\sigma$ observations of B^0_s oscillations. Further studies with $B^0_s \to J/\psi\phi$ decays will not only improve on $\Delta\Gamma_s$, but perhaps also allow a very first investigation of the CP-violating phase ϕ_s induced by B^0_s–$\overline{B}^0_s$ mixing, about which nothing is known experimentally at present. However, the SM value of ϕ_s is very small ($\phi_s = -2\beta_s$ where $\beta_s \equiv \arg(-V_{ts}V^*_{tb}/(V_{cs}V^*_{cb}))$ is about one degree), and a full search for new physics effects in this observable will require much larger statistics. These will become available at CERN's Large Hadron Collider scheduled to start operation in 2007, where the LHCb collaboration expects to be able to measure ϕ_s down to the SM value after several years of operations [73].

B mixing may not have delivered all its secrets yet, because it is one of the phenomena where new physics might still reveal itself (although a dominant contribution is becoming unlikely). Theoretical calculations in lattice QCD have become more reliable, and further progress in reducing hadronic uncertainties is expected. In the long term, a stringent check of the consistency, within the SM, of the B^0_d and B^0_s mixing amplitudes (magnitudes and phases) with all other measured flavour-physics observables (including CP asymmetries in B decays) will be possible, leading to further limits on new physics or, better, new physics discovery.

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