REVIEW OF CHARM DALITZ PLOT ANALYSES

Written January 2006 by D. Asner (Carleton University)

For references given here in the form SMITH 05, see the references at the end of the D^+, D^0, and D_s^+ Listings.

The formalism of Dalitz-Plot analysis is reviewed in the preceeding note. Table 1 lists reported analyses of D mesons. In the following, we discuss a number of subjects of current interest: (1) $D^0 \to K_S^0\pi^+\pi^-$; (2) $D \to \pi\pi\pi$: a $\sigma(500)$ or $f_0(600)$; (3) $D^+ \to K^-\pi^+\pi^+$: a $\kappa(800)$? (4) the $f_0(980)$, $f_0(1370)$ and $f_0(1500)$; (5) doubly Cabibbo-suppressed decays; and (6) CP violation.

$\boldsymbol{D^0 \to K_S^0\pi^+\pi^-}$: Several experiments have analyzed $D^0 \to K_S^0\pi^+\pi^-$ decay (see Table 1). The most precise results are from CLEO (BABAR and Belle, discussed below, have not yet evaluated systematic uncertainties). The CLEO analysis included ten resonances: $K_S^0\rho^0$, $K_S^0\omega$, $K_S^0 f_0(980)$, $K_S^0 f_2(1270)$, $K_S^0 f_0(1370)$, $K^*(892)^-\pi^+$, $K_0^*(1430)^-\pi^+$, $K_2^*(1430)^-\pi^+$, $K^*(1680)^-\pi^+$, and the doubly Cabibbo-suppressed mode $K^*(892)^+\pi^-$. CLEO found a much smaller nonresonant contribution than did the earliest experiments.

The source of the nonresonant component found in the early experiments has been attributed to the broad scalar resonances, the $K_0^*(1430)^-$ and $f_0(1370)$, found in the later, larger data samples. The observation of a small but significant nonresonant component in the largest data samples suggests the presence of additional broad scalar resonances, the $\kappa(800)$ and $\sigma(500)$. The CLEO analysis could accommodate the $\sigma(500)$ in lieu of the nonresonant component, but found no evidence for the $\kappa(800)$.

The ten quasi-two-body intermediate states in the CLEO analysis include both CP-even and CP-odd eigenstates and one doubly Cabibbo-suppressed channel. A time-dependent analysis of the Dalitz plot allows simultaneous determination of the strong transition amplitudes and phases and the mixing parameters x and y without phase or sign ambiguity. Using 9 fb^{-1}, CLEO obtained $(-4.5 < x < 9.3)\%$ and $(-6.4 < y < 3.6)\%$ [1].

Table 1: Reported Dalitz plot analyses.

Decay	Experiment(s)
$D^0 \to K^0_S\pi^+\pi^-$	Mark II [a], Mark III [b], E691 [c], E687 [d,e], ARGUS [f], CLEO [g], Belle [10,11], BABAR [12,13]
$D^0 \to K^-\pi^+\pi^0$	Mark III [b], E687 [e], E691 [c], CLEO [h]
$D^0 \to \overline{K}^0K^+\pi^-$	BABAR [14]
$D^0 \to K^0K^-\pi^+$	BABAR [14]
$D^0 \to K^0_S\eta\pi^0$	CLEO [i]
$D^0 \to \pi^+\pi^-\pi^0$	CLEO [j]
$D^0 \to K^0_SK^+K^-$	BABAR [k]
$D^0 \to K^-K^+K^-\pi^+$	FOCUS [l]
$D^0 \to K^-K^+\pi^-\pi^+$	FOCUS [m]
$D^+ \to K^-\pi^+\pi^+$	Mark III [b], E687 [e], E691 [c], E791 [n]
$D^+ \to \overline{K}^0\pi^+\pi^0$	Mark III [b]
$D^+ \to \pi^+\pi^+\pi^-$	E687 [o], E791 [p], FOCUS [5] [q]
$D^+ \to K^+K^-\pi^+$	FOCUS [15], E687 [r], BABAR [s]
$D^+ \to K^+\pi^+\pi^-$	E791 [t], FOCUS [u]
$D_s^+ \to K^+K^-\pi^+$	E687 [r], FOCUS [15]
$D_s^+ \to \pi^+\pi^+\pi^-$	E687 [o], E791 [v], FOCUS [5]
$D_s^+ \to K^+\pi^+\pi^-$	FOCUS [u]

See the end of the D^+, D^0 and D_s^+ Listings for these references: [a]SCHINDLER 81, [b]ADLER 87, [c]ANJOS 93, [d]FRABETTI 92B, [e]FRABETTI 94G, [f]ALBRECHT 93D, [g]MURAMATSU 02, [h]KOPP 01, [i]RUBIN 04, [j]CRONIN-HENNESSY 05, [k]AUBERT 05B, [l]LINK 03G, [m]LINK 05C, [n]AITALA 02, [o]FRABETTI 97D, [p]AITALA 01B, [q]LINK 04, [r]FRABETTI 95B, [s]AUBERT 05A, [t]AITALA 97C, [u]LINK 04F, [v]AITALA 01A.

The decay $D^0 \to K^0_S\pi^+\pi^-$, important for the study of the CKM angle γ/ϕ_3 [6], is under study by Belle [10,11] and BABAR [12,13]. The CLEO model does not provide a good description of the higher-statistics BABAR and Belle data samples. An improved description is obtained in two ways: First, by adding more Breit-Wigner resonances, including two $\pi\pi$ resonances with arbitrary mass and width, denoted as σ_1

and σ_2. Second, following the methodology of FOCUS [LINK 04], by applying a K-matrix model to the $\pi\pi$ S-wave [12].

Charm Dalitz-plot analyses might also prove useful for calibrating tools used to study B decays: specifically, to extract α from $B^0 \to \pi^+\pi^-\pi^0$, β from $b \to s$ penguin decays (*e.g.*, $B^0 \to \overline{K}^0 K^+K^-$), and γ from $B^\pm \to DK^\pm$ followed by $D^0 \to \pi^+\pi^-\pi^0$ or $K^0_S K^+K^-$ or $K^+K^-\pi^0$, in addition to the well-studied $D^0 \to K^0_S\pi^+\pi^-$[2, 3].

$D \to \pi\pi\pi$: a $\sigma(500)$ or $f_0(600)$: The decay $D^+ \to \pi^+\pi^+\pi^-$ has been studied by the E687, E791 and FOCUS experiments (see Table 1). The E687 analysis considered the modes $\rho(770)^0\pi^+$, $f_0(980)\pi^+$, $f_2(1270)\pi^+$, and a nonresonant component. E791 included, in addition, $f_0(1370)\pi^+$ and $\rho(1450)^0\pi^+$. Both analyses found a very large fraction ($\sim$ 50%) for the nonresonant component, perhaps indicating a broad scalar contribution. E791 found the nonresonant amplitude to be consistent with zero if a broad scalar resonance was included. FOCUS analyzed its data using both the Breit-Wigner formalism and the K-matrix formalism for the $\pi^+\pi^-$ S-wave, following a 5-pole, 5-resonance model of Anisovich and Sarantsev [16]. The Breit-Wigner analysis included $\rho(770)^0, f_0(980), f_2(1270)^0, f_0(1500), \sigma(500)$, and a nonresonant component. The K-matrix formalism, with Breit-Wigner forms for the $\rho(770)$ and $f_2(1270)$, also describe the FOCUS data well. None of these analyses has modeled the dynamics of the $\pi^+\pi^+$ interaction. Consideration of the $I = 2$ S- and D-wave phase shifts, also measured in $\pi^+p \to \pi^+\pi^+n$ [18], could affect the $\pi^+\pi^-$ S-wave result.

Using the E791 data, Bediaga and Miranda [19] found additional evidence that the low-mass $\pi^+\pi^-$ feature is resonant by examining the phase of the $\pi^+\pi^-$ amplitude in the vicinity of the reported $\sigma(500)$ mass. The phase variation with invariant mass is consistent with a resonant interpretation.

Table 1 gives the parameters of the $\sigma(500)$ determined in charm Dalitz-plot analyses. A consistent relative phase between the $\sigma(500)$ and $\rho(770)$ resonances is observed.

Table 2: Parameters of the $\sigma(500)$ resonance. The amplitude and phase are relative to the $\rho(770)$.

Experiment	E791[a]	CLEO[b]	FOCUS [5]
Decay mode	$D^+ \to \pi^+\pi^+\pi^-$	$D^0 \to K^0_S\pi^+\pi^-$	$D^+ \to \pi^+\pi^+\pi^-$
Amplitude	$1.17 \pm 0.13 \pm 0.06$	0.57 ± 0.13	—
Phase (°)	$205.7 \pm 8.0 \pm 5.2$	214 ± 11	200 ± 31
m (MeV/c^2)	$478^{+24}_{-23} \pm 17$	513 ± 32	443 ± 27
Γ (MeV/c^2)	$324^{+42}_{-40} \pm 21$	335 ± 67	443 ± 80

See the end of the D^+ and D^0 listings for these references: [a]AITALA 01B, [b]MURAMATSU 02.

CLEO has studied $D^0 \to \pi^+\pi^-\pi^0$ (see Table 1). Only the three $\rho(770)\pi$ resonant contributions are observed. No evidence is found for any $\pi\pi$ S-wave, either with the Breit-Wigner or with a K-matrix parametrization, using the 4-pole, 2-resonance model of Au, Morgan, and Pennington [17].

$D^+ \to K^-\pi^+\pi^+$: a κ(800)?: Evidence for a broad $K\pi$ scalar resonance has been found by E791 in $D^+ \to K^-\pi^+\pi^+$ (see Table 1). Fitting the Dalitz plot with $\overline{K}^*(892)^0\pi^+$, $\overline{K}^*_0(1430)^0\pi^+$, $\overline{K}^*_2(1430)^0\pi^+$, and $\overline{K}^*(1680)^0\pi^+$, plus a constant nonresonant component, E791 found results consistent with earlier analyses by E691 and E687, with a nonresonant fit fraction of over 90%. With more events than the other experiments, E791 was then led to include an extra low-mass S-wave $\overline{K}\pi$ resonance to account for the poor fit already seen by earlier experiments. A $\kappa(800)$ with mass $797 \pm 19 \pm 43$ MeV and width $410 \pm 43 \pm 87$ MeV much improved the fit. The $\kappa(800)$ became the dominant resonance and the nonresonant fit fraction was reduced from $90.9 \pm 2.6\%$ to $13.0 \pm 5.8 \pm 4.4\%$.

In addition, E791 modeled the $K\pi$ S-wave phase variation as a function of $K\pi$ mass with only the $K^*_0(1430)$ resonance and a nonresonant component following a parametrization of LASS [20]. It was necessary to relax the unitarity constraint to describe the data [21]. The $K\pi$ S-wave phase behavior in this model was consistent with the model that included the κ resonance.

Finally, E791 performed a model-independent partial-wave analysis [AITALA 05] of the S-wave component of the $K\pi$ system, finding the amplitude and phase from the $K\pi$ threshold up to 1.72 GeV. No assumptions were made regarding dependence on invariant mass, but the analysis did use the relatively well-understood P- and D-waves, described by the $K^*(892)$ and $K^*(1680)$ and by the $K_2(1430)$, respectively. The results were similar to those obtained by AITALA 02, which parametrized the S-wave with κ and $K_0(1430)$ Breit-Wigner forms and a constant complex non-resonant term. As with the $\sigma(500)$, the $K^-\pi^+$ S-wave result could be affected by including dynamics of the $I = 2$ $\pi^+\pi^+$ interaction; however in AITALA 05, the $I = 2$ elastic amplitude was found to be negligible compared to the κ.

CLEO allowed scalar $K\pi$ resonances in fits to $D^0 \to K^-\pi^+\pi^0$ and $D^0 \to K^0_S\pi^+\pi^-$ (see Table 1), and observed a significant contribution from only the $K_0^*(1430)$ [22]. BABAR fit $D^0 \to K^0K^-\pi^+$ with both positively charged and neutral $\overline{K}^*(892)$, $\overline{K}^*_0(1430)$, $\overline{K}^*_2(1430)$, and $\overline{K}^*(1680)$ resonances, as well as the $a_0(980)^-$, $a_0(1450)^-$, and $a_2(1310)^-$ resonances, and a nonresonant component [14]. BABAR also fit $D^0 \to \overline{K}^0K^+\pi^-$ with the same resonances except for the $a_2(1310)^-$. In both cases, a good fit was obtained without including the κ.

FOCUS has conclusively observed a $K\pi$ S-wave as a distortion of the $K^*(892)$ line-shape in semileptonic charm decays [LINK 02E, LINK 05D].

The $f_0(980)$, $f_0(1370)$ and $f_0(1500)$: The meson content of the 0^{++} nonet and the quark content of the $f_0(980)$, $a_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ mesons are current puzzles in light-meson spectroscopy [22]. Measuring branching fractions and couplings to different final states and comparing scalar-meson production rates among D^0, D^+, and D_s^+ mesons may help solve these puzzles.

For example: A large contribution of $f_0(980)$ to $D^0 \to K^0_SK^+K^-$ was reported by ARGUS [ALBRECHT 87E] and by BABAR [14]. This is inconsistent with the smaller contribution of $f_0(980)$ observed in $D^0 \to K^0_S\pi^+\pi^-$ by CLEO.

The explanation is that $D^0 \rightarrow K^0_S K^+ K^-$ has a large contribution from $a_0(980)^0 \rightarrow K^+K^-$. Therefore CLEO studied $D^0 \rightarrow K^0_S \eta\pi^0$ [RUBIN 04], where the dominant contribution is from $K^0_S\, a_0(980)^0$, $a_0(980)^0 \rightarrow \eta\pi^0$, and there can be no $f_0(980)$. A more recent BABAR analysis of $D^0 \rightarrow K^0_s K^+ K^-$ found a large amount of $a_0(980) \rightarrow K\overline{K}$ and little $f_0(980)$ [AUBERT 05B].

The proximity of the $K\overline{K}$ threshold requires either a coupled-channel Breit-Wigner function [23] or a Flatte parametrization [24] of the $f_0(980)$. The width of the $f_0(980)$ is poorly known. E791 and FOCUS [LINK 05C] [5] used a coupled-channel Breit-Wigner function to describe the $f_0(980)$ in $D^+_s \rightarrow \pi^+\pi^+\pi^-$. BESII studied the $f_0(980)$ in $J/\psi \rightarrow \phi\pi^+\pi^-$ and ϕK^+K^- [25]. The values found for the couplings to the $\pi\pi$ and $K\overline{K}$ channels, $g_{\pi\pi}$ and g_{KK}, were not consistent. Results such as these are desirable for input to the analysis of $D^+_s \rightarrow K^+K^-\pi^+$ [15], which includes both the $f_0(980)$ and $a_0(980)$.

The quark content of the $f_0(1370)$ and $f_0(1500)$ can perhaps be inferred from how they populate various Dalitz plots. Results so far are confusing. The E791 analysis of $D^+ \rightarrow \pi^+\pi^+\pi^-$ [AITALA 01B] found some $f_0(1370)$ but no $f_0(1500)$, while the FOCUS analysis [5] of this mode found little $f_0(1370)$. In $D^+_s \rightarrow \pi^+\pi^+\pi^-$, E687 and FOCUS [5] found no $f_0(1370)$, but did find a resonance with parameters similar to the $f_0(1500)$, whereas E791 found a $\pi^+\pi^-$ resonance with mass $1434 \pm 18 \pm 9$ MeV and width $172 \pm 32 \pm 6$ MeV, consistent with neither the $f_0(1370)$ or $f_0(1500)$. BABAR [AUBERT 05B] in $D^0 \rightarrow \overline{K}^0 K^+K^-$ found neither the $f_0(1370)$ nor the $f_0(1500)$, but did observe a K^+K^- resonance consistent with the values from E791 given above, while CLEO has observed the $f_0(1370)$ in $D^0 \rightarrow K^0_S\pi^+\pi^-$. The FOCUS analysis that used the K-matrix formalism for the $\pi\pi$ S-wave observed significant couplings to five T-matrix poles— $f_0(980)$, $f_0(1300)$, $f_0(1200-1600)$, $f_0(1500)$, $f_0(1750)$— in both $D^+ \rightarrow \pi^+\pi^-\pi^+$ and $D^+_s \rightarrow \pi^+\pi^-\pi^+$. Again, the quark content of each pole might be inferred from the coupling to various Dalitz plots.

It is noteworthy that the S-wave observed in B Dalitz-plot analyses appears to be different than that observed in D-meson decays.

Doubly Cabibbo-Suppressed Decays: There are two classes of multibody doubly Cabibbo-suppressed (DCS) decays of D mesons. The first consists of those in which the DCS and corresponding Cabbibo-favored (CF) decays populate distinct Dalitz plots: the pairs $D^0 \to K^+\pi^-\pi^0$ and $D^0 \to K^-\pi^+\pi^0$, or $D^+ \to K^+\pi^+\pi^-$ and $D^+ \to K^-\pi^+\pi^+$, are examples. CLEO [BRANDENBURG 01] and Belle [TIAN 05] have reported $\Gamma(D^0 \to K^+\pi^-\pi^0)/\Gamma(D^0 \to K^-\pi^+\pi^0) = (0.43^{+0.11}_{-0.10} \pm 0.07)\%$ and $(0.229 \pm 0.015^{+0.013}_{-0.009})\%$, respectively. E791 and FOCUS have reported $\Gamma(D^+ \to K^+\pi^-\pi^+)/\Gamma(D^+ \to K^-\pi^+\pi^+) = (0.77 \pm 0.17 \pm 0.08)\%$ and $(0.65 \pm 0.08 \pm 0.04)\%$, respectively.

The second class consists of decays in which the DCS and CF modes populate the same Dalitz plot; for example, $D^0 \to K^{*-}\pi^+$ and $D^0 \to K^{*+}\pi^-$ both contribute to $D^0 \to K^0_S\pi^+\pi^-$. In this case, the potential for interference of DCS and CF amplitudes increases the sensitivity to the DCS amplitude. CLEO found the relative amplitude and phase to be $(7.1 \pm 1.3^{+2.6}_{-0.6}{}^{+2.6}_{-0.6})\%$ and $(189 \pm 10 \pm 3^{+15}_{-5})^\circ$, corresponding to $\Gamma(D^0 \to K^*(892)^+\pi^-)/\Gamma(D^0 \to K^*(892)^-\pi^+) = (0.5 \pm 0.2^{+0.5}_{-0.1}{}^{+0.4}_{-0.1})\%$. In addition to $D^0 \to K^*(892)^+\pi^-$, Belle [10,11] and
BABAR [12,13] have found evidence for $D^0 \to K_0(1430)^+\pi^-$ and $K_2(1430)^+\pi^-$, and Belle has also found evidence for $K^*(1680)^+\pi^-$.

CP Violation: In the limit of CP conservation, charge conjugate decays will have the same Dalitz-plot distribution. The $D^{*\pm}$ tag enables the discrimination between D^0 and $\overline{D}^0$. The integrated CP violation across the Dalitz plot is determined in two ways. The first uses

$$\mathcal{A}_{CP} = \int \left(\frac{|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2} \right) dm^2_{ab}\, dm^2_{bc} \Bigg/ \int dm^2_{ab}\, dm^2_{bc}\,, \quad (1)$$

where $\mathcal{M}$ and $\overline{\mathcal{M}}$ are the D^0 and $\overline{D}^0$ Dalitz-plot amplitudes. The second uses the asymmetry in the efficiency-corrected D^0 and $\overline{D}^0$ yields,

$$\mathcal{A}_{CP} = \frac{N_{D^0} - N_{\overline{D}^0}}{N_{D^0} + N_{\overline{D}^0}} \, . \tag{2}$$

These expressions are less sensitive to CP violation than are the individual resonant submodes [ASNER 04A]. Table 3 lists the results for CP violation. No evidence of CP violation has been observed in D-meson decays.

Table 3: Dalitz-plot-integrated CP violation. Measurements computing $\mathcal{A}_{CP}$ with Eq. (2) rather than Eq. (1) are denoted †.

Experiment	Decay mode	$\mathcal{A}_{CP}(\%)$
BABAR [a]	$D^+ \to K^+K^-\pi^+$	$1.4 \pm 1.0 \pm 0.8$
Belle [b†]	$D^0 \to K^+\pi^-\pi^0$	-0.6 ± 5.3
Belle [b†]	$D^0 \to K^+\pi^-\pi^+\pi^-$	-1.8 ± 4.4
CLEO [c]	$D^0 \to K^-\pi^+\pi^0$	-3.1 ± 8.6
CLEO [d†]	$D^0 \to K^+\pi^-\pi^0$	$+9^{+22}_{-25}$
CLEO [e]	$D^0 \to K^0_S\pi^+\pi^-$	$-0.9 \pm 2.1^{+1.0+1.3}_{-4.3-3.7}$
CLEO [f]	$D^0 \to \pi^+\pi^-\pi^0$	$+1^{+9}_{-7} \pm 9$

See the end of the D^+ and D^0 Listings for these references: [a]AUBERT 05A, [b]TIAN 05, [c]KOPP 01, [d]BRANDENBURG 01, [e]ASNER 04A, [f]CRONIN-HENNESSY 05.

The possibility of interference between CP–conserving and CP–violating amplitudes provides a more sensitive probe of CP violation. The constraints on the square of the CP–violating amplitude obtained in the resonant submodes of $D^0 \to K^0_S\pi^+\pi^-$ range from 3.5×10^{-4} to 28.4×10^{-4} at 95% confidence level [ASNER 04A].

References

1. See the note on "D^0–$\overline{D}^0$ Mixing" in this *Review.*
2. See the note on "The CKM Quark Mixing Matrix" in this *Review.*
3. See the note on "CP Violation in Meson Decays" in this *Review.*

4. Dalitz plot analysis of the wrong sign rate $D^0 \to K^+\pi^-\pi^0$ [BRANDENBURG 01] and the time dependence of Dalitz plot analysis of $D^0 \to K^0_S\pi^+\pi^-$ [ASNER 05] are two candidate processes.
5. S. Malvezzi, AIP Conf. Proc. **688**, 276 (2004) [Nucl. Phys. Proc. Suppl. **126**, 220 (2004)].
6. A. Giri *et al.*, Phys. Rev. **D68**, 054018 (2003).
7. A. Bondar and A. Poluektov, `hep-ph/0510246`.
8. J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics*, New York: John Wiley & Sons (1952).
9. F. von Hippel and C. Quigg, Phys. Rev. **D5**, 624, (1972).
10. A. Poluektov *et al.* (Belle Collab.), Phys. Rev. D **70**, 072003 (2004).
11. K. Abe *et al.* Belle Collaboration, `hep-ex/0411049`.
12. B. Aubert *et al.* (BABAR Collab.), Phys. Rev. Lett. **95**, 121802 (2005).
13. B. Aubert *et al.* (BABAR Collab.), `hep-ex/0507101`.
14. B. Aubert *et al.* (BABAR Collab.), `hep-ex/0207089`; contributed to the 31st International Conference on High Energy Physics (ICHEP 2002).
15. S. Malvezzi, AIP Conf. Proc. **549**, 569 (2002).
16. V. V. Anisovich and A. V. Sarantsev, Eur. Phys. J. A **16**, 229 (2003).
17. K. L. Au *et al.*, Phys. Rev. D **35**, 1633 (1987).
18. W. Hoogland *et al.*, Nucl. Phys. **B69**, 266 (1974).
19. I. Bediaga (E791 Collab.), AIP Conf. Proc. **688**, 252 (2004).
20. D. Aston *et al.* (LASS Collab.), Nucl. Phys. **B296**, 493 (1988).
21. C. Gobel (Fermilab E791 Collab.), AIP Conf. Proc. **688**, 266 (2004).
22. See the "Note on Scalar Mesons" in this *Review.*
23. T. A. Armstrong *et al.* (WA76 Collab.), Z. Phys. **C51**, 351 (1991).
24. S. M. Flatte, Phys. Lett. **B63**, 224 (1976).
25. M. Ablikim *et al.* (BES Collab.), Phys. Lett. B **607**, 243 (2005).