# THE MUON ANOMALOUS MAGNETIC MOMENT

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The Dirac equation predicts a muon magnetic moment, $\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{S}$, with gyromagnetic ratio $g_\mu = 2$. Quantum loop effects lead to a small calculable deviation from $g_\mu = 2$, parameterized by the anomalous magnetic moment

$$a_\mu \equiv \frac{g_\mu - 2}{2} \ . \tag{1}$$

That quantity can be accurately measured and, within the Standard Model (SM) framework, precisely predicted. Hence, comparison of experiment and theory tests the SM at its quantum loop level. A deviation in $a_\mu^{\rm exp}$ from the SM expectation would signal effects of new physics, with current sensitivity reaching up to mass scales of $\mathcal{O}$(TeV) [1,2]. For a recent and very thorough muon $g-2$ review, see Ref. 3.

The E821 experiment at Brookhaven National Lab (BNL) studied the precession of $\mu^+$ and $\mu^-$ in a constant external magnetic field as they circulated in a confining storage ring. It found [4]

$$\begin{aligned} a_{\mu^+}^{\rm exp} &= 11\,659\,203(6)(5) \times 10^{-10}\,, \\ a_{\mu^-}^{\rm exp} &= 11\,659\,214(8)(3) \times 10^{-10}\,, \end{aligned} \tag{2}$$

where the first errors are statistical and the second systematic. Assuming CPT invariance and taking into account correlations between systematic errors, one finds for their average [4]

$$a_\mu^{\rm exp} = 11\,659\,208.0(5.4)(3.3) \times 10^{-10} \ . \tag{3}$$

These results represent about a factor of 14 improvement over the classic CERN experiments of the 1970's [5].

The SM prediction for $a_\mu^{\rm SM}$ is generally divided into three parts (see Fig. 1 for representative Feynman diagrams)

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm Had} \ . \tag{4}$$

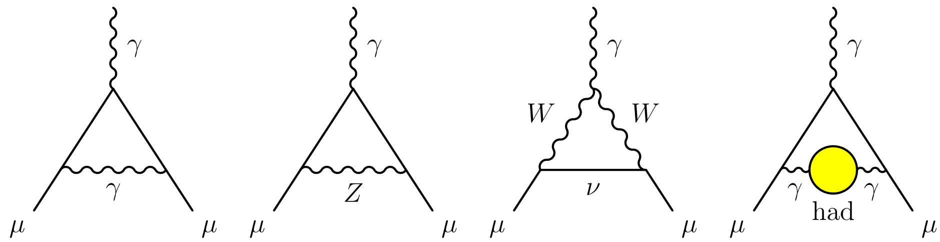


**Figure 1:** Representative diagrams contributing to $a_\mu^{\rm SM}$. From left to right: first order QED (Schwinger term), lowest-order weak, lowest-order hadronic.

The QED part includes all photonic and leptonic $(e, \mu, \tau)$ loops starting with the classic $\alpha/2\pi$ Schwinger contribution. It has been computed through 4 loops and estimated at the 5-loop level [6]

$$a_\mu^{\rm QED} = \frac{\alpha}{2\pi} + 0.765857410(27)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050964(87)\left(\frac{\alpha}{\pi}\right)^3 + 130.8055(80)\left(\frac{\alpha}{\pi}\right)^4 + 663(20)\left(\frac{\alpha}{\pi}\right)^5 + \cdots \tag{5}$$

Employing $\alpha^{-1} = 137.035999070(98)$, determined [6,7] from the electron $a_e$ measurement, leads to

$$a_\mu^{\rm QED} = 116\,584\,718.10(0.16) \times 10^{-11}\,, \tag{6}$$

where the error results from uncertainties in the coefficients of Eq. (5) and in $\alpha$.

Loop contributions involving heavy $W^\pm, Z$ or Higgs particles are collectively labeled as $a_\mu^{\rm EW}$. They are suppressed by at least a factor of $\frac{\alpha}{\pi}\frac{m_\mu^2}{m_W^2} \simeq 4 \times 10^{-9}$. At 1-loop order [8]

$$\begin{aligned} a_\mu^{\rm EW}[\text{1-loop}] &= \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left[\frac{5}{3} + \frac{1}{3}\left(1 - 4\sin^2\theta_{\rm W}\right)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_W^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right], \\ &= 194.8 \times 10^{-11}\,, \end{aligned} \tag{7}$$

for $\sin^2\theta_{\rm W} \equiv 1 - M_W^2/M_Z^2 \simeq 0.223$, and where $G_\mu \simeq 1.166 \times 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant. Two-loop corrections are relatively large and negative [9]

$$a_\mu^{\rm EW}[\text{2-loop}] = -40.7(1.0)(1.8) \times 10^{-11}\,, \tag{8}$$

where the errors stem from quark triangle loops and the assumed Higgs mass range between 100 and 500 GeV. The 3-loop leading logarithms are negligible [9,10], $\mathcal{O}(10^{-12})$, implying in total

$$a_\mu^{\mathrm{EW}} = 154(1)(2) \times 10^{-11} \quad . \tag{9}$$

Hadronic (quark and gluon) loop contributions to $a_\mu^{\mathrm{SM}}$ give rise to its main theoretical uncertainties. At present, those effects are not calculable from first principles, but such an approach, at least partially, may become possible as lattice QCD matures. Instead, one currently relies on a dispersion relation approach to evaluate the lowest-order (*i.e.*, $\mathcal{O}(\alpha^2)$) hadronic vacuum polarization contribution $a_\mu^{\mathrm{Had}}[\mathrm{LO}]$ from corresponding cross section measurements [11]

$$a_\mu^{\mathrm{Had}}[\mathrm{LO}] = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds\, \frac{K(s)}{s} R^{(0)}(s)\,, \tag{10}$$

where $K(s)$ is a QED kernel function [12], and where $R^{(0)}(s)$ denotes the ratio of the bare[1] cross section for $e^+e^-$ annihilation into hadrons to the pointlike muon-pair cross section at center-of-mass energy $\sqrt{s}$. The function $K(s) \sim 1/s$ in Eq. (10) gives a strong weight to the low-energy part of the integral. Hence, $a_\mu^{\mathrm{Had}}[\mathrm{LO}]$ is dominated by the $\rho(770)$ resonance.

Currently, the available $\sigma(e^+e^- \to \text{hadrons})$ data give a leading-order hadronic vacuum polarization (representative) contribution of [13]

$$a_\mu^{\mathrm{Had}}[\mathrm{LO}] = 6\,894(42)(18) \times 10^{-11}\,, \tag{11}$$

where the first error is experimental (dominated by systematic uncertainties), and the second due to QED radiative corrections to the data.

Alternatively, one can use precise vector spectral functions from $\tau \to \nu_\tau + \text{hadrons}$ decays [14] that can be related to

[1] The bare cross section is defined as the measured cross section corrected for initial-state radiation, electron-vertex loop contributions and vacuum-polarization effects in the photon propagator. However, QED effects in the hadron vertex and final state, such as photon radiation, are included.

isovector $e^+e^- \to$ hadrons cross sections by isospin rotation. When isospin-violating corrections (from QED and $m_d - m_u \neq 0$) are applied, one finds [15,16]

$$a_\mu^{\rm Had}[\rm LO] = 7\,103(50)(7)(28) \times 10^{-11}\ (\tau)\,, \qquad (12)$$

where the errors are statistical and systematic, and where the last error is an estimate for the uncertainty in the isospin-breaking corrections. The discrepancy between the $e^+e^-$ and $\tau$-based determinations of $a_\mu^{\rm Had}[\rm LO]$ is currently unexplained. It may be indicative of problems with one or both data sets. It may also suggest the need for additional isospin-violating corrections to the $\tau$ data. Forthcoming low-energy $e^+e^-$ and $\tau$ data may help to resolve this discrepancy and should reduce the hadronic uncertainty.

Higher order, $\mathcal{O}(\alpha^3)$, hadronic contributions are obtained from dispersion relations using the same $e^+e^- \to$ hadrons data [13,14,17], giving $a_\mu^{\rm Had,Disp}[\rm NLO] = (-98 \pm 1) \times 10^{-11}$, along with model-dependent estimates of the hadronic light-by-light scattering contribution, $a_\mu^{\rm Had,LBL}[\rm NLO]$, motivated by large-$N_C$ QCD [18–23]. [2] Following [2], one finds for the sum of the two terms

$$a_\mu^{\rm Had}[\rm NLO] = 22(35) \times 10^{-11}\,, \qquad (13)$$

where the error is dominated by hadronic light-by-light uncertainties.

Adding Eqs. (6), (9), (11) and (13) gives the representative $e^+e^-$ data-based SM prediction

$$a_\mu^{\rm SM} = 116\,591\,788(2)(46)(35) \times 10^{-11}\,, \qquad (14)$$

where the errors are due to the electroweak, lowest-order hadronic, and higher-order hadronic contributions, respectively. The difference between experiment and theory

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 292(63)(58) \times 10^{-11}\,, \qquad (15)$$

[2] Some representative recent estimates of the hadronic light-by-light scattering contribution, $a_\mu^{\rm Had,LBL}[\rm NLO]$, that followed after the sign correction of [20], are: $120(35) \times 10^{-11}$ [2], $110(40) \times 10^{-11}$ [18], $136(25) \times 10^{-11}$ [19], $100(39) \times 10^{-11}$ [24].

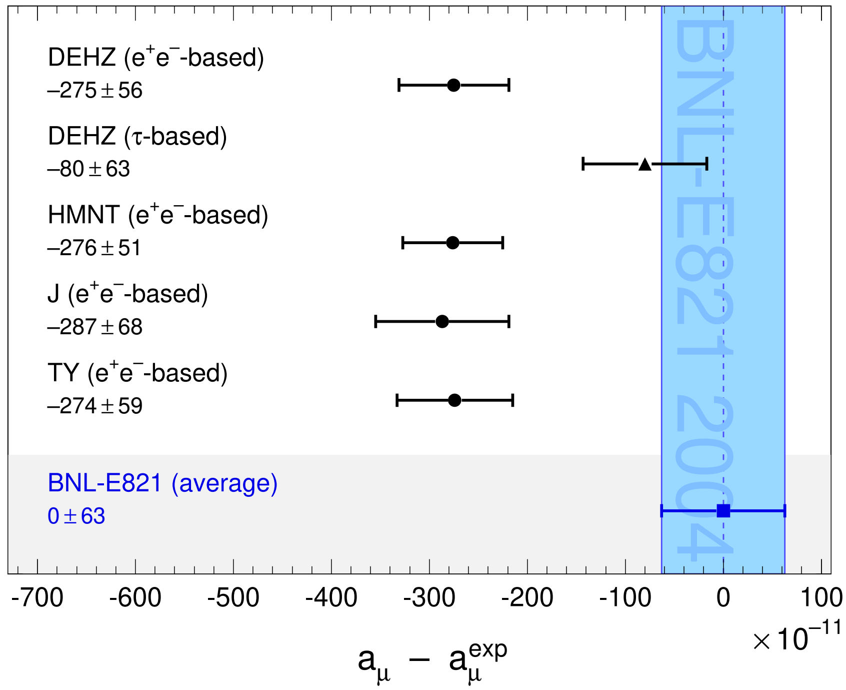


**Figure 2:** Compilation of recently published results for $a_\mu$ (in units of $10^{-11}$), subtracted by the central value of the experimental average (3). The shaded band indicates the experimental error. The SM predictions are taken from: DEHZ [15,16], TY [25], HMNT [13], J [24]. Note that the quoted errors do not include the uncertainty on the subtracted experimental value. To obtain for each theory calculation a result equivalent to Eq. (15), the errors from theory and experiment must be added in quadrature. See full-color version on color pages at end of book.

(with all errors combined in quadrature) represents an interesting but not yet conclusive discrepancy of 3.4 times the estimated $1\sigma$ error. All the recent estimates for the hadronic contribution compiled in Fig. 2 exhibit similar discrepancies. Switching to $\tau$ data reduces the discrepancy to $0.9\sigma$, assuming the isospin-violating corrections are under control within the estimated uncertainties.

An alternate interpretation is that $\Delta a_\mu$ may be a new physics signal with supersymmetric particle loops as the leading candidate explanation. Such a scenario is quite natural, since generically, supersymmetric models predict [1] an additional contribution to $a_\mu^{\rm SM}$

$$a_\mu^{\rm SUSY} \simeq \pm\, 130 \times 10^{-11} \cdot \left(\frac{100\ {\rm GeV}}{m_{\rm SUSY}}\right)^2 \tan\beta\,, \qquad (16)$$

where $m_{\rm SUSY}$ is a representative supersymmetric mass scale, and $\tan\beta \simeq 3$–$40$ is a potential enhancement factor. Supersymmetric particles in the mass range 100–500 GeV could be the source of the deviation $\Delta a_\mu$. If so, those particles could be directly observed at the next generation of high energy colliders. New physics effects [1] other than supersymmetry could also explain a non-vanishing $\Delta a_\mu$.

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