

## 12. *CP* VIOLATION IN MESON DECAYS

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The *CP* transformation combines charge conjugation *C* with parity *P*. Under *C*, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, *e.g.*,  $Q \rightarrow -Q$  for electromagnetic charge. Under *P*, the handedness of space is reversed,  $\vec{x} \rightarrow -\vec{x}$ . Thus, for example, a left-handed electron  $e_L^-$  is transformed under *CP* into a right-handed positron,  $e_R^+$ .

If *CP* were an exact symmetry, the laws of Nature would be the same for matter and for antimatter. We observe that most phenomena are *C*- and *P*-symmetric, and therefore, also *CP*-symmetric. In particular, these symmetries are respected by the gravitational, electromagnetic, and strong interactions. The weak interactions, on the other hand, violate *C* and *P* in the strongest possible way. For example, the charged *W* bosons couple to left-handed electrons,  $e_L^-$ , and to their *CP*-conjugate right-handed positrons,  $e_R^+$ , but to neither their *C*-conjugate left-handed positrons,  $e_L^+$ , nor their *P*-conjugate right-handed electrons,  $e_R^-$ . While weak interactions violate *C* and *P* separately, *CP* is still preserved in most weak interaction processes. The *CP* symmetry is, however, violated in certain rare processes, as discovered in neutral *K* decays in 1964 [1], and observed in recent years in *B* decays. A  $K_L$  meson decays more often to  $\pi^- e^+ \bar{\nu}_e$  than to  $\pi^+ e^- \nu_e$ , thus allowing electrons and positrons to be unambiguously distinguished, but the decay-rate asymmetry is only at the 0.003 level. The *CP*-violating effects observed in *B* decays are larger: the *CP* asymmetry in  $B^0/\bar{B}^0$  meson decays to *CP* eigenstates like  $J/\psi K_S$  is about 0.70 [2,3]. These effects are related to  $K^0 - \bar{K}^0$  and  $B^0 - \bar{B}^0$  mixing, but *CP* violation arising solely from decay amplitudes has also been observed, first in  $K \rightarrow \pi\pi$  decays [4–6] and more recently in various neutral [7,8] and charged [9,10] *B* decays. *CP* violation has not yet been observed in *D* or  $B_s$  meson decays, or in the lepton sector.

In addition to parity and to continuous Lorentz transformations, there is one other spacetime operation that could be a symmetry of the interactions: time reversal *T*,  $t \rightarrow -t$ . Violations of *T* symmetry have been observed in neutral *K* decays [11], and are expected as a corollary of *CP* violation if the combined *CPT* transformation is a fundamental symmetry of Nature [12]. All observations indicate that *CPT* is indeed a symmetry of Nature. Furthermore, one cannot build a Lorentz-invariant quantum field theory with a Hermitian Hamiltonian that violates *CPT*. (At several points in our discussion, we avoid assumptions about *CPT*, in order to identify cases where evidence for *CP* violation relies on assumptions about *CPT*.)

Within the Standard Model, *CP* symmetry is broken by complex phases in the Yukawa couplings (that is, the couplings of the Higgs scalar to quarks). When all manipulations to remove unphysical phases in this model are exhausted, one finds that there is a single *CP*-violating parameter [13]. In the basis of mass eigenstates, this single phase appears in the  $3 \times 3$  unitary matrix that gives the *W*-boson couplings to an up-type antiquark and a down-type quark. (If the Standard Model is supplemented with Majorana mass terms for the neutrinos, the analogous mixing matrix for leptons has three *CP*-violating phases.) The beautifully consistent and economical Standard-Model description of *CP* violation in terms of Yukawa couplings, known as the Kobayashi-Maskawa (KM) mechanism [13],

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agrees with all measurements to date. Furthermore, one can fit the data allowing new physics contributions to loop processes to compete with, or even dominate over, the Standard Model ones [14,15]. Such an analysis provides a model-independent proof that the KM phase is different from zero, and that the matrix of three-generation quark mixing is the dominant source of *CP* violation in meson decays.

The current level of experimental accuracy and the theoretical uncertainties involved in the interpretation of the various observations leave room, however, for additional subdominant sources of *CP* violation from new physics. Indeed, almost all extensions of the Standard Model imply that there are such additional sources. Moreover, *CP* violation is a necessary condition for baryogenesis, the process of dynamically generating the matter-antimatter asymmetry of the Universe [16]. Despite the phenomenological success of the KM mechanism, it fails (by several orders of magnitude) to accommodate the observed asymmetry [17]. This discrepancy strongly suggests that Nature provides additional sources of *CP* violation beyond the KM mechanism. (Recent evidence for neutrino masses implies that *CP* can be violated also in the lepton sector. This situation makes leptogenesis [18], a scenario where *CP*-violating phases in the Yukawa couplings of the neutrinos play a crucial role in the generation of the baryon asymmetry, a very attractive possibility.) The expectation of new sources motivates the large ongoing experimental effort to find deviations from the predictions of the KM mechanism.

*CP* violation can be experimentally searched for in a variety of processes, such as meson decays, electric dipole moments of neutrons, electrons and nuclei, and neutrino oscillations. Meson decays probe flavor-changing *CP* violation. The search for electric dipole moments may find (or constrain) sources of *CP* violation that, unlike the KM phase, are not related to flavor-changing couplings. Future searches for *CP* violation in neutrino oscillations might provide further input on leptogenesis.

The present measurements of *CP* asymmetries provide some of the strongest constraints on the weak couplings of quarks. Future measurements of *CP* violation in *K*, *D*, *B*, and *B<sub>s</sub>* meson decays will provide additional constraints on the flavor parameters of the Standard Model, and can probe new physics. In this review, we give the formalism and basic physics that are relevant to present and near future measurements of *CP* violation in meson decays.

Before going into details, we list here the independent *CP*-violating observables where a signal has been established [19,20,21]:

1. Indirect *CP* violation in  $K \rightarrow \pi\pi$  decays and in  $K \rightarrow \pi\ell\nu$  decays is given by

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3} . \quad (12.1)$$

2. Direct *CP* violation in  $K \rightarrow \pi\pi$  decays is given by

$$\mathcal{R}e(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3} . \quad (12.2)$$

3. *CP* violation in the interference of mixing and decay in the  $B \rightarrow \psi K^0$  and other related modes is given by (we use  $K^0$  throughout to denote results that combine  $K_S$  and  $K_L$  modes, but use the sign appropriate to  $K_S$ ):

$$S_{\psi K^0} = +0.673 \pm 0.023. \quad (12.3)$$

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4. *CP* violation in the interference of mixing and decay in the  $B \rightarrow \eta' K^0$  modes is given by

$$S_{\eta' K^0} = +0.60 \pm 0.07. \quad (12.4)$$

5. *CP* violation in the interference of mixing and decay in the  $B \rightarrow K^+ K^- K_S$  mode is given by

$$S_{(K^+ K^- K_0)_+} = -0.74^{+0.12}_{-0.10}. \quad (12.5)$$

6. *CP* violation in the interference of mixing and decay in the  $B \rightarrow \pi^+ \pi^-$  mode is given by

$$S_{\pi^+ \pi^-} = -0.61 \pm 0.08. \quad (12.6)$$

7. *CP* violation in the interference of mixing and decay in the  $B \rightarrow \psi \pi^0$  mode is given by

$$S_{\psi \pi^0} = -0.94 \pm 0.29. \quad (12.7)$$

8. *CP* violation in the interference of mixing and decay in the  $B \rightarrow D^+ D^-$  mode is given by

$$S_{D^+ D^-} = -0.87 \pm 0.26. \quad (12.8)$$

9. *CP* violation in the interference of mixing and decay in the  $B \rightarrow D^{*+} D^{*-}$  mode is given by

$$S_{D^{*+} D^{*-}} = -0.70 \pm 0.16. \quad (12.9)$$

10. Direct *CP* violation in the  $\bar{B}^0 \rightarrow K^- \pi^+$  mode is given by

$$\mathcal{A}_{K^\mp \pi^\pm} = -0.098 \pm 0.013. \quad (12.10)$$

11. Direct *CP* violation in the  $B^0 \rightarrow \eta K^{*0}$  mode is given by

$$\mathcal{A}_{\eta K^{*0}} = +0.19 \pm 0.05. \quad (12.11)$$

12. Direct *CP* violation in the  $B^- \rightarrow K^- \rho^0$  mode is given by

$$\mathcal{A}_{\rho^0 K^\mp} = +0.37 \pm 0.10. \quad (12.12)$$

In addition, there is evidence for *CP* violation in neutral  $B$  decays into final  $D^* D$  and  $D^* \pi$  and various additional  $(c\bar{c})K_S$  modes.

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### 12.1. Formalism

The phenomenology of *CP* violation is superficially different in *K*, *D*, *B*, and *B<sub>s</sub>* decays. This is primarily because each of these systems is governed by a different balance between decay rates, oscillations, and lifetime splitting. However, the underlying mechanisms of *CP* violation are identical for all pseudoscalar mesons.

In this section, we present a general formalism for, and classification of, *CP* violation in the decay of a pseudoscalar meson *M* that might be a charged or neutral *K*, *D*, *B*, or *B<sub>s</sub>* meson. Subsequent sections describe the *CP*-violating phenomenology, approximations, and alternative formalisms that are specific to each system.

**12.1.1. Charged- and neutral-meson decays:** We define decay amplitudes of *M* (which could be charged or neutral) and its *CP* conjugate  $\overline{M}$  to a multi-particle final state *f* and its *CP* conjugate  $\overline{f}$  as

$$\begin{aligned} A_f &= \langle f | \mathcal{H} | M \rangle \quad , \quad \overline{A}_f = \langle f | \mathcal{H} | \overline{M} \rangle \quad , \\ A_{\overline{f}} &= \langle \overline{f} | \mathcal{H} | M \rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f} | \mathcal{H} | \overline{M} \rangle \quad , \end{aligned} \quad (12.13)$$

where  $\mathcal{H}$  is the Hamiltonian governing weak interactions. The action of *CP* on these states introduces phases  $\xi_M$  and  $\xi_f$  that depend on their flavor content, according to

$$CP|M\rangle = e^{+i\xi_M} |\overline{M}\rangle \quad , \quad CP|f\rangle = e^{+i\xi_f} |\overline{f}\rangle \quad , \quad (12.14)$$

with

$$CP|\overline{M}\rangle = e^{-i\xi_M} |M\rangle \quad , \quad CP|\overline{f}\rangle = e^{-i\xi_f} |f\rangle \quad (12.15)$$

so that  $(CP)^2 = 1$ . The phases  $\xi_M$  and  $\xi_f$  are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If *CP* is conserved by the dynamics,  $[CP, \mathcal{H}] = 0$ , then  $A_f$  and  $\overline{A}_{\overline{f}}$  have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_M)} A_f \quad . \quad (12.16)$$

**12.1.2. Neutral-meson mixing:** A state that is initially a superposition of  $M^0$  and  $\overline{M}^0$ , say

$$|\psi(0)\rangle = a(0)|M^0\rangle + b(0)|\overline{M}^0\rangle \quad , \quad (12.17)$$

will evolve in time acquiring components that describe all possible decay final states  $\{f_1, f_2, \dots\}$ , that is,

$$|\psi(t)\rangle = a(t)|M^0\rangle + b(t)|\overline{M}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \dots \quad (12.18)$$

If we are interested in computing only the values of  $a(t)$  and  $b(t)$  (and not the values of all  $c_i(t)$ ), and if the times  $t$  in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [22]. The simplified time evolution is determined by a  $2 \times 2$  effective Hamiltonian  $\mathbf{H}$  that is not

Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as  $\mathbf{H}$ , can be written in terms of Hermitian matrices  $\mathbf{M}$  and  $\mathbf{\Gamma}$  as

$$\mathbf{H} = \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} . \quad (12.19)$$

$\mathbf{M}$  and  $\mathbf{\Gamma}$  are associated with  $(M^0, \overline{M}^0) \leftrightarrow (M^0, \overline{M}^0)$  transitions via off-shell (dispersive), and on-shell (absorptive) intermediate states, respectively. Diagonal elements of  $\mathbf{M}$  and  $\mathbf{\Gamma}$  are associated with the flavor-conserving transitions  $M^0 \rightarrow M^0$  and  $\overline{M}^0 \rightarrow \overline{M}^0$ , while off-diagonal elements are associated with flavor-changing transitions  $M^0 \leftrightarrow \overline{M}^0$ .

The eigenvectors of  $\mathbf{H}$  have well-defined masses and decay widths. To specify the components of the strong interaction eigenstates,  $M^0$  and  $\overline{M}^0$ , in the light ( $M_L$ ) and heavy ( $M_H$ ) mass eigenstates, we introduce three complex parameters:  $p$ ,  $q$ , and, for the case that both  $CP$  and  $CPT$  are violated in mixing,  $z$ :

$$\begin{aligned} |M_L\rangle &\propto p\sqrt{1-z}|M^0\rangle + q\sqrt{1+z}|\overline{M}^0\rangle \\ |M_H\rangle &\propto p\sqrt{1+z}|M^0\rangle - q\sqrt{1-z}|\overline{M}^0\rangle , \end{aligned} \quad (12.20)$$

with the normalization  $|q|^2 + |p|^2 = 1$  when  $z = 0$ . (Another possible choice, which is in standard usage for  $K$  mesons, defines the mass eigenstates according to their lifetimes:  $K_S$  for the short-lived and  $K_L$  for the long-lived state. The  $K_L$  is experimentally found to be the heavier state.)

The real and imaginary parts of the eigenvalues  $\omega_{L,H}$  corresponding to  $|M_{L,H}\rangle$  represent their masses and decay widths, respectively. The mass and width splittings are

$$\begin{aligned} \Delta m &\equiv m_H - m_L = \mathcal{R}e(\omega_H - \omega_L) \quad , \\ \Delta\Gamma &\equiv \Gamma_H - \Gamma_L = -2\mathcal{I}m(\omega_H - \omega_L) . \end{aligned} \quad (12.21)$$

Note that here  $\Delta m$  is positive by definition, while the sign of  $\Delta\Gamma$  is to be experimentally determined. The sign of  $\Delta\Gamma$  has not yet been established for the  $B$  and  $B_s$  mesons, while  $\Delta\Gamma < 0$  is established for  $K$  mesons and  $\Delta\Gamma > 0$  is established for  $D$  mesons. The Standard Model predicts  $\Delta\Gamma < 0$  for  $B$  and  $B_s$  mesons (for this reason,  $\Delta\Gamma = \Gamma_L - \Gamma_H$ , which is still a signed quantity, is often used in the  $B$  and  $B_s$  literature and is the convention used in the PDG experimental summaries).

Solving the eigenvalue problem for  $\mathbf{H}$  yields

$$\left(\frac{q}{p}\right)^2 = \frac{\mathbf{M}_{12}^* - (i/2)\mathbf{\Gamma}_{12}^*}{\mathbf{M}_{12} - (i/2)\mathbf{\Gamma}_{12}} \quad (12.22)$$

and

$$z \equiv \frac{\delta m - (i/2)\delta\Gamma}{\Delta m - (i/2)\Delta\Gamma} , \quad (12.23)$$

where

$$\delta m \equiv \mathbf{M}_{11} - \mathbf{M}_{22} \quad , \quad \delta\Gamma \equiv \mathbf{\Gamma}_{11} - \mathbf{\Gamma}_{22} \quad (12.24)$$

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are the differences in effective mass and decay-rate expectation values for the strong interaction states  $M^0$  and  $\overline{M}^0$ .

If either  $CP$  or  $CPT$  is a symmetry of  $\mathbf{H}$  (independently of whether  $T$  is conserved or violated), then the values of  $\delta m$  and  $\delta\Gamma$  are both zero, and hence  $z = 0$ . We also find that

$$\omega_H - \omega_L = 2\sqrt{\left(\mathbf{M}_{12} - \frac{i}{2}\mathbf{\Gamma}_{12}\right)\left(\mathbf{M}_{12}^* - \frac{i}{2}\mathbf{\Gamma}_{12}^*\right)}. \quad (12.25)$$

If either  $CP$  or  $T$  is a symmetry of  $\mathbf{H}$  (independently of whether  $CPT$  is conserved or violated), then  $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$  is real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_M} \Rightarrow \left|\frac{q}{p}\right| = 1, \quad (12.26)$$

where  $\xi_M$  is the arbitrary unphysical phase introduced in Eq. (12.15). If, and only if,  $CP$  is a symmetry of  $\mathbf{H}$  (independently of  $CPT$  and  $T$ ), then both of the above conditions hold, with the result that the mass eigenstates are orthogonal

$$\langle M_H | M_L \rangle = |p|^2 - |q|^2 = 0. \quad (12.27)$$

**12.1.3.  $CP$ -violating observables :** All  $CP$ -violating observables in  $M$  and  $\overline{M}$  decays to final states  $f$  and  $\overline{f}$  can be expressed in terms of phase-convention-independent combinations of  $A_f$ ,  $\overline{A}_f$ ,  $A_{\overline{f}}$ , and  $\overline{A}_{\overline{f}}$ , together with, for neutral-meson decays only,  $q/p$ .  $CP$  violation in charged-meson decays depends only on the combination  $|\overline{A}_{\overline{f}}/A_f|$ , while  $CP$  violation in neutral-meson decays is complicated by  $M^0 \leftrightarrow \overline{M}^0$  oscillations, and depends, additionally, on  $|q/p|$  and on  $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$ .

The decay rates of the two neutral  $K$  mass eigenstates,  $K_S$  and  $K_L$ , are different enough ( $\Gamma_S/\Gamma_L \sim 500$ ) that one can, in most cases, actually study their decays independently. For neutral  $D$ ,  $B$ , and  $B_s$  mesons, however, values of  $\Delta\Gamma/\Gamma$  (where  $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$ ) are relatively small, and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure  $|M^0\rangle$  or  $|\overline{M}^0\rangle$  after an elapsed proper time  $t$  as  $|M_{\text{phys}}^0(t)\rangle$  or  $|\overline{M}_{\text{phys}}^0(t)\rangle$ , respectively. Using the effective Hamiltonian approximation, but not assuming  $CPT$  is a good symmetry, we obtain

$$\begin{aligned} |M_{\text{phys}}^0(t)\rangle &= (g_+(t) + z g_-(t)) |M^0\rangle - \sqrt{1 - z^2} \frac{q}{p} g_-(t) |\overline{M}^0\rangle, \\ |\overline{M}_{\text{phys}}^0(t)\rangle &= (g_+(t) - z g_-(t)) |\overline{M}^0\rangle - \sqrt{1 - z^2} \frac{p}{q} g_-(t) |M^0\rangle, \end{aligned} \quad (12.28)$$

where

$$g_{\pm}(t) \equiv \frac{1}{2} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) \quad (12.29)$$

and  $z = 0$  if either *CPT* or *CP* is conserved.

Defining  $x \equiv \Delta m/\Gamma$  and  $y \equiv \Delta\Gamma/(2\Gamma)$ , and assuming  $z = 0$ , one obtains the following time-dependent decay rates:

$$\begin{aligned} \frac{d\Gamma[M_{\text{phys}}^0(t) \rightarrow f]/dt}{e^{-\Gamma t}\mathcal{N}_f} = & \\ & \left(|A_f|^2 + |(q/p)\bar{A}_f|^2\right) \cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\bar{A}_f|^2\right) \cos(x\Gamma t) \\ & + 2\operatorname{Re}((q/p)A_f^*\bar{A}_f) \sinh(y\Gamma t) - 2\operatorname{Im}((q/p)A_f^*\bar{A}_f) \sin(x\Gamma t), \end{aligned} \quad (12.30)$$

$$\begin{aligned} \frac{d\Gamma[\bar{M}_{\text{phys}}^0(t) \rightarrow f]/dt}{e^{-\Gamma t}\mathcal{N}_f} = & \\ & \left(|(p/q)A_f|^2 + |\bar{A}_f|^2\right) \cosh(y\Gamma t) - \left(|(p/q)A_f|^2 - |\bar{A}_f|^2\right) \cos(x\Gamma t) \\ & + 2\operatorname{Re}((p/q)A_f\bar{A}_f^*) \sinh(y\Gamma t) - 2\operatorname{Im}((p/q)A_f\bar{A}_f^*) \sin(x\Gamma t), \end{aligned} \quad (12.31)$$

where  $\mathcal{N}_f$  is a common, time-independent, normalization factor. Decay rates to the *CP*-conjugate final state  $\bar{f}$  are obtained analogously, with  $\mathcal{N}_f = \mathcal{N}_{\bar{f}}$  and the substitutions  $A_f \rightarrow A_{\bar{f}}$  and  $\bar{A}_f \rightarrow \bar{A}_{\bar{f}}$  in Eqs. (12.30, 12.31). Terms proportional to  $|A_f|^2$  or  $|\bar{A}_f|^2$  are associated with decays that occur without any net  $M \leftrightarrow \bar{M}$  oscillation, while terms proportional to  $|(q/p)\bar{A}_f|^2$  or  $|(p/q)A_f|^2$  are associated with decays following a net oscillation. The  $\sinh(y\Gamma t)$  and  $\sin(x\Gamma t)$  terms of Eqs. (12.30, 12.31) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

When neutral pseudoscalar mesons are produced coherently in pairs from the decay of a vector resonance,  $V \rightarrow M^0\bar{M}^0$  (for example,  $\Upsilon(4S) \rightarrow B^0\bar{B}^0$  or  $\phi \rightarrow K^0\bar{K}^0$ ), the time-dependence of their subsequent decays to final states  $f_1$  and  $f_2$  has a similar form to Eqs. (12.30, 12.31):

$$\begin{aligned} \frac{d\Gamma[V_{\text{phys}}(t_1, t_2) \rightarrow f_1 f_2]/dt}{e^{-\Gamma|\Delta t|}\mathcal{N}_{f_1 f_2}} = & \\ & \left(|a_+|^2 + |a_-|^2\right) \cosh(y\Gamma\Delta t) + \left(|a_+|^2 - |a_-|^2\right) \cos(x\Gamma\Delta t) \\ & - 2\operatorname{Re}(a_+^* a_-) \sinh(y\Gamma\Delta t) + 2\operatorname{Im}(a_+^* a_-) \sin(x\Gamma\Delta t), \end{aligned} \quad (12.32)$$

where  $\Delta t \equiv t_2 - t_1$  is the difference in the production times,  $t_1$  and  $t_2$ , of  $f_1$  and  $f_2$ , respectively, and the dependence on the average decay time and on decay angles has been integrated out. The coefficients in Eq. (12.32) are determined by the amplitudes for no

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net oscillation from  $t_1 \rightarrow t_2$ ,  $\bar{A}_{f_1} A_{f_2}$ , and  $A_{f_1} \bar{A}_{f_2}$ , and for a net oscillation,  $(q/p)\bar{A}_{f_1} \bar{A}_{f_2}$  and  $(p/q)A_{f_1} A_{f_2}$ , via

$$\begin{aligned} a_+ &\equiv \bar{A}_{f_1} A_{f_2} - A_{f_1} \bar{A}_{f_2} , \\ a_- &\equiv -\sqrt{1-z^2} \left( \frac{q}{p} \bar{A}_{f_1} \bar{A}_{f_2} - \frac{p}{q} A_{f_1} A_{f_2} \right) + z (\bar{A}_{f_1} A_{f_2} + A_{f_1} \bar{A}_{f_2}) . \end{aligned} \quad (12.33)$$

Assuming *CPT* conservation,  $z = 0$ , and identifying  $\Delta t \rightarrow t$  and  $f_2 \rightarrow f$ , we find that Eqs. (12.32) and (12.33) reduce to Eq. (12.30) with  $A_{f_1} = 0$ ,  $\bar{A}_{f_1} = 1$ , or to Eq. (12.31) with  $\bar{A}_{f_1} = 0$ ,  $A_{f_1} = 1$ . Indeed, such a situation plays an important role in experiments. Final states  $f_1$  with  $A_{f_1} = 0$  or  $\bar{A}_{f_1} = 0$  are called tagging states, because they identify the decaying pseudoscalar meson as, respectively,  $\bar{M}^0$  or  $M^0$ . Before one of  $M^0$  or  $\bar{M}^0$  decays, they evolve in phase, so that there is always one  $M^0$  and one  $\bar{M}^0$  present. A tagging decay of one meson sets the clock for the time evolution of the other: it starts at  $t_1$  as purely  $M^0$  or  $\bar{M}^0$ , with time evolution that depends only on  $t_2 - t_1$ .

When  $f_1$  is a state that both  $M^0$  and  $\bar{M}^0$  can decay into, then Eq. (12.32) contains interference terms proportional to  $A_{f_1} \bar{A}_{f_1} \neq 0$  that are not present in Eqs. (12.30, 12.31). Even when  $f_1$  is dominantly produced by  $M^0$  decays rather than  $\bar{M}^0$  decays, or vice versa,  $A_{f_1} \bar{A}_{f_1}$  can be non-zero owing to doubly-CKM-suppressed decays (with amplitudes suppressed by at least two powers of  $\lambda$  relative to the dominant amplitude, in the language of Section 12.3), and these terms should be considered for precision studies of *CP* violation in coherent  $V \rightarrow M^0 \bar{M}^0$  decays [23].

**12.1.4. Classification of *CP*-violating effects :** We distinguish three types of *CP*-violating effects in meson decays:

I. *CP* violation in decay is defined by

$$|\bar{A}_{\bar{f}}/A_f| \neq 1 . \quad (12.34)$$

In charged meson decays, where mixing effects are absent, this is the only possible source of *CP* asymmetries:

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma(M^- \rightarrow f^-) - \Gamma(M^+ \rightarrow f^+)}{\Gamma(M^- \rightarrow f^-) + \Gamma(M^+ \rightarrow f^+)} = \frac{|\bar{A}_{f^-}/A_{f^+}|^2 - 1}{|\bar{A}_{f^-}/A_{f^+}|^2 + 1} . \quad (12.35)$$

II. *CP* (and *T*) violation in mixing is defined by

$$|q/p| \neq 1 . \quad (12.36)$$

In charged-current semileptonic neutral meson decays  $M, \bar{M} \rightarrow \ell^\pm X$  (taking  $|A_{\ell^+ X}| = |\bar{A}_{\ell^- X}|$  and  $A_{\ell^- X} = \bar{A}_{\ell^+ X} = 0$ , as is the case in the Standard Model, to lowest order in  $G_F$ , and in most of its reasonable extensions), this is the only



source of *CP* violation, and can be measured via the asymmetry of “wrong-sign” decays induced by oscillations:

$$\begin{aligned} \mathcal{A}_{\text{SL}}(t) &\equiv \frac{d\Gamma/dt[\overline{M}_{\text{phys}}^0(t) \rightarrow \ell^+ X] - d\Gamma/dt[M_{\text{phys}}^0(t) \rightarrow \ell^- X]}{d\Gamma/dt[\overline{M}_{\text{phys}}^0(t) \rightarrow \ell^+ X] + d\Gamma/dt[M_{\text{phys}}^0(t) \rightarrow \ell^- X]} \\ &= \frac{1 - |q/p|^4}{1 + |q/p|^4}. \end{aligned} \quad (12.37)$$

Note that this asymmetry of time-dependent decay rates is actually time-independent.

III. *CP* violation in interference between a decay without mixing,  $M^0 \rightarrow f$ , and a decay with mixing,  $M^0 \rightarrow \overline{M}^0 \rightarrow f$  (such an effect occurs only in decays to final states that are common to  $M^0$  and  $\overline{M}^0$ , including all *CP* eigenstates), is defined by

$$\text{Im}(\lambda_f) \neq 0, \quad (12.38)$$

with

$$\lambda_f \equiv \frac{q}{p} \frac{\overline{A}_f}{A_f}. \quad (12.39)$$

This form of *CP* violation can be observed, for example, using the asymmetry of neutral meson decays into final *CP* eigenstates  $f_{CP}$

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{M}_{\text{phys}}^0(t) \rightarrow f_{CP}] - d\Gamma/dt[M_{\text{phys}}^0(t) \rightarrow f_{CP}]}{d\Gamma/dt[\overline{M}_{\text{phys}}^0(t) \rightarrow f_{CP}] + d\Gamma/dt[M_{\text{phys}}^0(t) \rightarrow f_{CP}]} . \quad (12.40)$$

If  $\Delta\Gamma = 0$  and  $|q/p| = 1$ , as expected to a good approximation for *B* mesons, but not for *K* mesons, then  $\mathcal{A}_{f_{CP}}$  has a particularly simple form (see Eq. (12.74), below). If, in addition, the decay amplitudes fulfill  $|\overline{A}_{f_{CP}}| = |A_{f_{CP}}|$ , the interference between decays with and without mixing is the only source of the asymmetry and  $\mathcal{A}_{f_{CP}}(t) = \text{Im}(\lambda_{f_{CP}}) \sin(x\Gamma t)$ .

Examples of these three types of *CP* violation will be given in Sections 12.4, 12.5, and 12.6.

## 12.2. Theoretical Interpretation: General Considerations

Consider the  $M \rightarrow f$  decay amplitude  $A_f$ , and the *CP* conjugate process,  $\overline{M} \rightarrow \overline{f}$ , with decay amplitude  $\overline{A}_{\overline{f}}$ . There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the *CP*-conjugate amplitude. Thus, their phases appear in  $A_f$  and  $\overline{A}_{\overline{f}}$  with opposite signs. In the Standard Model, these phases occur only in the couplings of the  $W^\pm$  bosons, and hence, are often

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called “weak phases.” The weak phase of any single term is convention-dependent. However, the difference between the weak phases in two different terms in  $A_f$  is convention-independent. A second type of phase can appear in scattering or decay amplitudes, even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by  $CP$ -invariant interactions, they are the same in  $A_f$  and  $\overline{A}_f$ . Usually the dominant rescattering is due to strong interactions; hence the designation “strong phases” for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The ‘weak’ and ‘strong’ phases discussed here appear in addition to the ‘spurious’  $CP$ -transformation phases of Eq. (12.16). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any  $CP$  violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution  $a_i$  to  $A_f$  in three parts: its magnitude  $|a_i|$ , its weak phase  $\phi_i$ , and its strong phase  $\delta_i$ . If, for example, there are two such contributions,  $A_f = a_1 + a_2$ , we have

$$\begin{aligned} A_f &= |a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)}, \\ \overline{A}_f &= |a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)}. \end{aligned} \quad (12.41)$$

Similarly, for neutral meson decays, it is useful to write

$$\mathbf{M}_{12} = |\mathbf{M}_{12}|e^{i\phi_M} \quad , \quad \mathbf{\Gamma}_{12} = |\mathbf{\Gamma}_{12}|e^{i\phi_\Gamma} . \quad (12.42)$$

Each of the phases appearing in Eqs. (12.41, 12.42) is convention-dependent, but combinations such as  $\delta_1 - \delta_2$ ,  $\phi_1 - \phi_2$ ,  $\phi_M - \phi_\Gamma$ , and  $\phi_M + \phi_1 - \overline{\phi}_1$  (where  $\overline{\phi}_1$  is a weak phase contributing to  $\overline{A}_f$ ) are physical.

It is now straightforward to evaluate the various asymmetries in terms of the theoretical parameters introduced here. We will do so with approximations that are often relevant to the most interesting measured asymmetries.

1. The  $CP$  asymmetry in charged meson decays [Eq. (12.35)] is given by

$$\mathcal{A}_{f^\pm} = -\frac{2|a_1a_2|\sin(\delta_2 - \delta_1)\sin(\phi_2 - \phi_1)}{|a_1|^2 + |a_2|^2 + 2|a_1a_2|\cos(\delta_2 - \delta_1)\cos(\phi_2 - \phi_1)}. \quad (12.43)$$

The quantity of most interest to theory is the weak phase difference  $\phi_2 - \phi_1$ . Its extraction from the asymmetry requires, however, that the amplitude ratio  $|a_2/a_1|$  and the strong phase difference  $\delta_2 - \delta_1$  are known. Both quantities depend on non-perturbative hadronic parameters that are difficult to calculate.

2. In the approximation that  $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}| \ll 1$  (valid for  $B$  and  $B_s$  mesons), the  $CP$  asymmetry in semileptonic neutral-meson decays [Eq. (12.37)] is given by

$$\mathcal{A}_{\text{SL}} = -\left|\frac{\mathbf{\Gamma}_{12}}{\mathbf{M}_{12}}\right|\sin(\phi_M - \phi_\Gamma). \quad (12.44)$$

The quantity of most interest to theory is the weak phase  $\phi_M - \phi_\Gamma$ . Its extraction from the asymmetry requires, however, that  $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}|$  is known. This quantity depends on long-distance physics that is difficult to calculate.

3. In the approximations that only a single weak phase contributes to decay,  $A_f = |a_f|e^{i(\delta_f + \phi_f)}$ , and that  $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}| = 0$ , we obtain  $|\lambda_f| = 1$ , and the *CP* asymmetries in decays to a final *CP* eigenstate  $f$  [Eq. (12.40)] with eigenvalue  $\eta_f = \pm 1$  are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f) \sin(\Delta mt) \quad \text{with} \quad \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f). \quad (12.45)$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from  $\mathcal{I}m(\lambda_f)$ .

The discussion above allows us to introduce another classification of *CP*-violating effects:

1. *Indirect CP violation* is consistent with taking  $\phi_M \neq 0$  and setting all other *CP* violating phases to zero. *CP* violation in mixing (type II) belongs to this class.
2. *Direct CP violation* cannot be accounted for by just  $\phi_M \neq 0$ . *CP* violation in decay (type I) belongs to this class.

As concerns type III *CP* violation, observing  $\eta_{f_1} \mathcal{I}m(\lambda_{f_1}) \neq \eta_{f_2} \mathcal{I}m(\lambda_{f_2})$  (for the same decaying meson and two different final *CP* eigenstates  $f_1$  and  $f_2$ ) would establish direct *CP* violation. The significance of this classification is related to theory. In superweak models [24], *CP* violation appears only in diagrams that contribute to  $\mathbf{M}_{12}$ , hence they predict that there is no direct *CP* violation. In most models and, in particular, in the Standard Model, *CP* violation is both direct and indirect. The experimental observation of  $\epsilon' \neq 0$  (see Section 12.4) excluded the superweak scenario.

### 12.3. Theoretical Interpretation: The KM Mechanism

Of all the Standard Model quark parameters, only the Kobayashi-Maskawa (KM) phase is *CP*-violating. Having a single source of *CP* violation, the Standard Model is very predictive for *CP* asymmetries: some vanish, and those that do not are correlated.

To be precise, *CP* could be violated also by strong interactions. The experimental upper bound on the electric-dipole moment of the neutron implies, however, that  $\theta_{\text{QCD}}$ , the non-perturbative parameter that determines the strength of this type of *CP* violation, is tiny, if not zero. (The smallness of  $\theta_{\text{QCD}}$  constitutes a theoretical puzzle, known as ‘the strong *CP* problem.’) In particular, it is irrelevant to our discussion of meson decays.

The charged current interactions (that is, the  $W^\pm$  interactions) for quarks are given by

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \overline{u_{Li}} \gamma^\mu (V_{\text{CKM}})_{ij} d_{Lj} W_\mu^\pm + \text{h.c.} \quad (12.46)$$

Here  $i, j = 1, 2, 3$  are generation numbers. The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix for quarks is a  $3 \times 3$  unitary matrix [25]. Ordering the quarks by their

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masses, *i.e.*,  $(u_1, u_2, u_3) \rightarrow (u, c, t)$  and  $(d_1, d_2, d_3) \rightarrow (d, s, b)$ , the elements of  $V_{\text{CKM}}$  are written as follows:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (12.47)$$

While a general  $3 \times 3$  unitary matrix depends on three real angles and six phases, the freedom to redefine the phases of the quark mass eigenstates can be used to remove five of the phases, leaving a single physical phase, the Kobayashi-Maskawa phase, that is responsible for all  $CP$  violation in meson decays in the Standard Model.

The fact that one can parametrize  $V_{\text{CKM}}$  by three real and only one imaginary physical parameters can be made manifest by choosing an explicit parametrization. The Wolfenstein parametrization [26,27] is particularly useful:

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \quad (12.48)$$

Here  $\lambda \approx 0.23$  (not to be confused with  $\lambda_f$ ) plays the role of an expansion parameter, and  $\eta$  represents the  $CP$ -violating phase. Terms of  $\mathcal{O}(\lambda^6)$  were neglected.

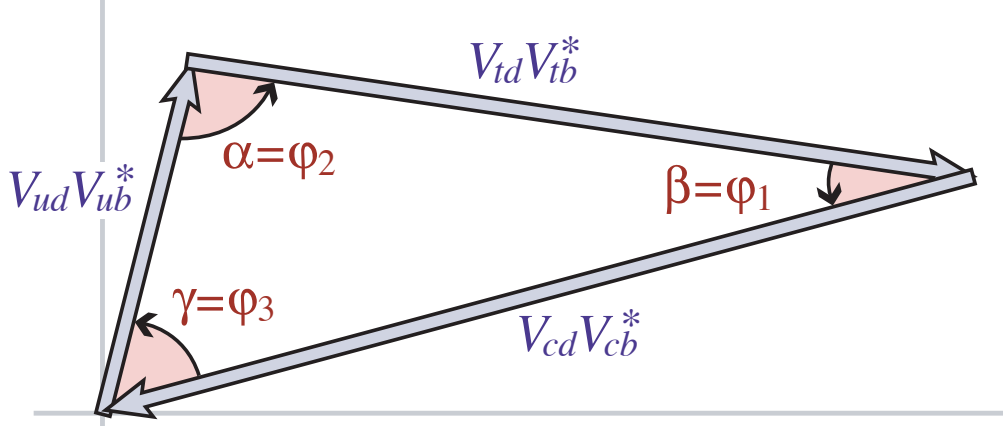
The unitarity of the CKM matrix,  $(VV^\dagger)_{ij} = (V^\dagger V)_{ij} = \delta_{ij}$ , leads to twelve distinct complex relations among the matrix elements. The six relations with  $i \neq j$  can be represented geometrically as triangles in the complex plane. Two of these,

$$\begin{aligned} V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \\ V_{td}V_{ud}^* + V_{ts}V_{us}^* + V_{tb}V_{ub}^* &= 0, \end{aligned}$$

have terms of equal order,  $\mathcal{O}(A\lambda^3)$ , and so have corresponding triangles whose interior angles are all  $\mathcal{O}(1)$  physical quantities that can, in principle, be independently measured. The angles of the first triangle (see Fig. 12.1) are given by

$$\begin{aligned} \alpha \equiv \varphi_2 &\equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \simeq \arg\left(-\frac{1 - \rho - i\eta}{\rho + i\eta}\right), \\ \beta \equiv \varphi_1 &\equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \simeq \arg\left(\frac{1}{1 - \rho - i\eta}\right), \\ \gamma \equiv \varphi_3 &\equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \simeq \arg(\rho + i\eta). \end{aligned} \quad (12.49)$$

The angles of the second triangle are equal to  $(\alpha, \beta, \gamma)$  up to corrections of  $\mathcal{O}(\lambda^2)$ . The notations  $(\alpha, \beta, \gamma)$  and  $(\varphi_1, \varphi_2, \varphi_3)$  are both in common usage but, for convenience, we only use the first convention in the following.



**Figure 12.1:** Graphical representation of the unitarity constraint  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$  as a triangle in the complex plane.

All unitarity triangles have the same area, commonly denoted by  $J/2$  [28]. If  $CP$  is violated,  $J$  is different from zero and can be taken as the single  $CP$ -violating parameter. In the Wolfenstein parametrization of Eq. (12.48),  $J \simeq \lambda^6 A^2 \eta$ .

## 12.4. *K* Decays

$CP$  violation was discovered in  $K \rightarrow \pi\pi$  decays in 1964 [1]. The same mode provided the first evidence for direct  $CP$  violation [4–6].

The decay amplitudes actually measured in neutral  $K$  decays refer to the mass eigenstates  $K_L$  and  $K_S$ , rather than to the  $K$  and  $\bar{K}$  states referred to in Eq. (12.13). The final  $\pi^+\pi^-$  and  $\pi^0\pi^0$  states are  $CP$ -even. In the  $CP$  limit,  $K_S(K_L)$  would be  $CP$ -even (odd), and therefore would (would not) decay to two pions. We define  $CP$ -violating amplitude ratios for two-pion final states,

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | \mathcal{H} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H} | K_S \rangle}, \quad \eta_{+-} \equiv \frac{\langle \pi^+\pi^- | \mathcal{H} | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H} | K_S \rangle}. \quad (12.50)$$

Another important observable is the asymmetry of time-integrated semileptonic decay rates:

$$\delta_L \equiv \frac{\Gamma(K_L \rightarrow \ell^+ \nu_\ell \pi^-) - \Gamma(K_L \rightarrow \ell^- \bar{\nu}_\ell \pi^+)}{\Gamma(K_L \rightarrow \ell^+ \nu_\ell \pi^-) + \Gamma(K_L \rightarrow \ell^- \bar{\nu}_\ell \pi^+)}. \quad (12.51)$$

$CP$  violation has been observed as an appearance of  $K_L$  decays to two-pion final states [19],

$$|\eta_{00}| = (2.221 \pm 0.011) \times 10^{-3} \quad |\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3} \quad (12.52)$$

$$|\eta_{00}/\eta_{+-}| = 0.9951 \pm 0.0008, \quad (12.53)$$

where the phase  $\phi_{ij}$  of the amplitude ratio  $\eta_{ij}$  has been determined both assuming  $CPT$  invariance:

$$\phi_{00} = (43.52 \pm 0.06)^\circ, \quad \phi_{+-} = (43.51 \pm 0.05)^\circ, \quad (12.54)$$

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and without assuming  $CPT$  invariance:

$$\phi_{00} = (43.7 \pm 0.8)^\circ, \quad \phi_{+-} = (43.4 \pm 0.7)^\circ. \quad (12.55)$$

$CP$  violation has also been observed in semileptonic  $K_L$  decays [19]

$$\delta_L = (3.32 \pm 0.06) \times 10^{-3}, \quad (12.56)$$

where  $\delta_L$  is a weighted average of muon and electron measurements, as well as in  $K_L$  decays to  $\pi^+\pi^-\gamma$  and  $\pi^+\pi^-e^+e^-$  [19].  $CP$  violation in  $K \rightarrow 3\pi$  decays has not yet been observed [19,29].

Historically,  $CP$  violation in neutral  $K$  decays has been described in terms of parameters  $\epsilon$  and  $\epsilon'$ . The observables  $\eta_{00}$ ,  $\eta_{+-}$ , and  $\delta_L$  are related to these parameters, and to those of Section 12.1, by

$$\begin{aligned} \eta_{00} &= \frac{1 - \lambda_{\pi^0\pi^0}}{1 + \lambda_{\pi^0\pi^0}} = \epsilon - 2\epsilon', \\ \eta_{+-} &= \frac{1 - \lambda_{\pi^+\pi^-}}{1 + \lambda_{\pi^+\pi^-}} = \epsilon + \epsilon', \\ \delta_L &= \frac{1 - |q/p|^2}{1 + |q/p|^2} = \frac{2\mathcal{R}e(\epsilon)}{1 + |\epsilon|^2}, \end{aligned} \quad (12.57)$$

where, in the last line, we have assumed that  $|A_{\ell^+\nu_\ell\pi^-}| = |\bar{A}_{\ell^-\bar{\nu}_\ell\pi^+}|$  and  $|A_{\ell^-\bar{\nu}_\ell\pi^+}| = |\bar{A}_{\ell^+\nu_\ell\pi^-}| = 0$ . (The convention-dependent parameter  $\tilde{\epsilon} \equiv (1 - q/p)/(1 + q/p)$ , sometimes used in the literature, is, in general, different from  $\epsilon$  but yields a similar expression,  $\delta_L = 2\mathcal{R}e(\tilde{\epsilon})/(1 + |\tilde{\epsilon}|^2)$ .) A fit to the  $K \rightarrow \pi\pi$  data yields [19]

$$\begin{aligned} |\epsilon| &= (2.228 \pm 0.011) \times 10^{-3}, \\ \mathcal{R}e(\epsilon'/\epsilon) &= (1.65 \pm 0.26) \times 10^{-3}. \end{aligned} \quad (12.58)$$

In discussing two-pion final states, it is useful to express the amplitudes  $A_{\pi^0\pi^0}$  and  $A_{\pi^+\pi^-}$  in terms of their isospin components via

$$\begin{aligned} A_{\pi^0\pi^0} &= \sqrt{\frac{1}{3}} |A_0| e^{i(\delta_0+\phi_0)} - \sqrt{\frac{2}{3}} |A_2| e^{i(\delta_2+\phi_2)}, \\ A_{\pi^+\pi^-} &= \sqrt{\frac{2}{3}} |A_0| e^{i(\delta_0+\phi_0)} + \sqrt{\frac{1}{3}} |A_2| e^{i(\delta_2+\phi_2)}, \end{aligned} \quad (12.59)$$

where we parameterize the amplitude  $A_I(\bar{A}_I)$  for  $K^0(\bar{K}^0)$  decay into two pions with total isospin  $I = 0$  or  $2$  as

$$\begin{aligned} A_I &\equiv \langle (\pi\pi)_I | \mathcal{H} | K^0 \rangle = |A_I| e^{i(\delta_I+\phi_I)}, \\ \bar{A}_I &\equiv \langle (\pi\pi)_I | \mathcal{H} | \bar{K}^0 \rangle = |A_I| e^{i(\delta_I-\phi_I)}. \end{aligned} \quad (12.60)$$

The smallness of  $|\eta_{00}|$  and  $|\eta_{+-}|$  allows us to approximate

$$\epsilon \simeq \frac{1}{2}(1 - \lambda_{(\pi\pi)_{I=0}}), \quad \epsilon' \simeq \frac{1}{6}(\lambda_{\pi^0\pi^0} - \lambda_{\pi^+\pi^-}). \quad (12.61)$$

The parameter  $\epsilon$  represents indirect *CP* violation, while  $\epsilon'$  parameterizes direct *CP* violation:  $\mathcal{R}e(\epsilon')$  measures *CP* violation in decay (type I),  $\mathcal{R}e(\epsilon)$  measures *CP* violation in mixing (type II), and  $\mathcal{I}m(\epsilon)$  and  $\mathcal{I}m(\epsilon')$  measure the interference between decays with and without mixing (type III).

The following expressions for  $\epsilon$  and  $\epsilon'$  are useful for theoretical evaluations:

$$\epsilon \simeq \frac{e^{i\pi/4} \mathcal{I}m(\mathbf{M}_{12})}{\sqrt{2} \Delta m}, \quad \epsilon' = \frac{i}{\sqrt{2}} \left| \frac{A_2}{A_0} \right| e^{i(\delta_2 - \delta_0)} \sin(\phi_2 - \phi_0). \quad (12.62)$$

The expression for  $\epsilon$  is only valid in a phase convention where  $\phi_2 = 0$ , corresponding to a real  $V_{ud}V_{us}^*$ , and in the approximation that also  $\phi_0 = 0$ . The phase of  $\epsilon$ ,  $\arg(\epsilon) \approx \arctan(-2\Delta m/\Delta\Gamma)$ , is independent of the electroweak model and is experimentally determined to be about  $\pi/4$ . The calculation of  $\epsilon$  benefits from the fact that  $\mathcal{I}m(\mathbf{M}_{12})$  is dominated by short distance physics. Consequently, the main source of uncertainty in theoretical interpretations of  $\epsilon$  are the values of matrix elements, such as  $\langle K^0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A} | \bar{K}^0 \rangle$ . The expression for  $\epsilon'$  is valid to first order in  $|A_2/A_0| \sim 1/20$ . The phase of  $\epsilon'$  is experimentally determined,  $\pi/2 + \delta_2 - \delta_0 \approx \pi/4$ , and is independent of the electroweak model. Note that, accidentally,  $\epsilon'/\epsilon$  is real to a good approximation.

A future measurement of much interest is that of *CP* violation in the rare  $K \rightarrow \pi\nu\bar{\nu}$  decays. The signal for *CP* violation is simply observing the  $K_L \rightarrow \pi^0\nu\bar{\nu}$  decay. The effect here is that of interference between decays with and without mixing (type III) [30]:

$$\frac{\Gamma(K_L \rightarrow \pi^0\nu\bar{\nu})}{\Gamma(K^+ \rightarrow \pi^+\nu\bar{\nu})} = \frac{1}{2} \left[ 1 + |\lambda_{\pi\nu\bar{\nu}}|^2 - 2 \mathcal{R}e(\lambda_{\pi\nu\bar{\nu}}) \right] \simeq 1 - \mathcal{R}e(\lambda_{\pi\nu\bar{\nu}}), \quad (12.63)$$

where in the last equation we neglect *CP* violation in decay and in mixing (expected, model-independently, to be of order  $10^{-5}$  and  $10^{-3}$ , respectively). Such a measurement would be experimentally very challenging and theoretically very rewarding [31]. Similar to the *CP* asymmetry in  $B \rightarrow J/\psi K_S$ , the *CP* violation in  $K \rightarrow \pi\nu\bar{\nu}$  decay is predicted to be large (that is, the ratio in Eq. (12.63) is neither CKM- nor loop-suppressed) and can be very cleanly interpreted.

Within the Standard Model, the  $K_L \rightarrow \pi^0\nu\bar{\nu}$  decay is dominated by an intermediate top quark contribution and, consequently, can be interpreted in terms of CKM parameters [32]. (For the charged mode,  $K^+ \rightarrow \pi^+\nu\bar{\nu}$ , the contribution from an intermediate charm quark is not negligible, and constitutes a source of hadronic uncertainty.) In particular,  $B(K_L \rightarrow \pi^0\nu\bar{\nu})$  provides a theoretically clean way to determine the Wolfenstein parameter  $\eta$  [33]:

$$B(K_L \rightarrow \pi^0\nu\bar{\nu}) = \kappa_L [X(m_t^2/m_W^2)]^2 A^4 \eta^2, \quad (12.64)$$

where  $\kappa_L \sim 2 \times 10^{-10}$  incorporates the value of the four-fermion matrix element which is deduced, using isospin relations, from  $B(K^+ \rightarrow \pi^0 e^+ \nu)$ , and  $X(m_t^2/m_W^2)$  is a known function of the top mass.

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### 12.5. *D* Decays

First evidence for  $D^0\text{--}\bar{D}^0$  mixing has been recently obtained [34–36]. Long-distance contributions make it difficult to calculate the Standard Model prediction for the  $D^0\text{--}\bar{D}^0$  mixing parameters. Therefore, the goal of the search for  $D^0\text{--}\bar{D}^0$  mixing is not to constrain the CKM parameters, but rather to probe new physics. Here *CP* violation plays an important role. Within the Standard Model, the *CP*-violating effects are predicted to be negligibly small, since the mixing and the relevant decays are described, to an excellent approximation, by physics of the first two generations. Observation of *CP* violation in  $D^0\text{--}\bar{D}^0$  mixing (at a level much higher than  $\mathcal{O}(10^{-3})$ ) will constitute an unambiguous signal of new physics. At present, the most sensitive searches involve the  $D \rightarrow K^+K^-$  and  $D \rightarrow K^\pm\pi^\mp$  modes.

The neutral *D* mesons decay via a singly-Cabibbo-suppressed transition to the *CP* eigenstate  $K^+K^-$ . Since the decay proceeds via a Standard-Model tree diagram, it is very likely unaffected by new physics and, furthermore, dominated by a single weak phase. It is safe then to assume that direct *CP* violation plays no role here. In addition, given the experimental constraints [21,37],  $x \equiv \Delta m/\Gamma = 0.0100 \pm 0.0025$  and  $y \equiv \Delta\Gamma/(2\Gamma) = 0.0077 \pm 0.0018$ , we can expand the decay rates to first order in these parameters. Using Eq. (12.30) with these assumptions and approximations yields, for  $xt, yt \lesssim \Gamma^{-1}$ ,

$$\begin{aligned} \Gamma[D_{\text{phys}}^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |A_{KK}|^2 [1 - |q/p| (y \cos \phi_D - x \sin \phi_D) \Gamma t] , \\ \Gamma[\bar{D}_{\text{phys}}^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |A_{KK}|^2 [1 - |p/q| (y \cos \phi_D + x \sin \phi_D) \Gamma t] , \end{aligned} \tag{12.65}$$

where  $\phi_D$  is defined via  $\lambda_{K^+K^-} = -|q/p| e^{i\phi_D}$ . (In the limit of *CP* conservation, choosing  $\phi_D = 0$  is equivalent to defining the mass eigenstates by their *CP* eigenvalue:  $|D_\mp\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$ , with  $D_- (D_+)$  being the *CP*-odd (*CP*-even) state; that is, the state that does not (does) decay into  $K^+K^-$ .) Given the small values of  $x$  and  $y$ , the time dependencies of the rates in Eq. (12.65) can be recast into purely exponential forms, but with modified decay-rate parameters [38]:

$$\begin{aligned} \Gamma_{D^0 \rightarrow K^+K^-} &= \Gamma \times [1 + |q/p| (y \cos \phi_D - x \sin \phi_D)] , \\ \Gamma_{\bar{D}^0 \rightarrow K^+K^-} &= \Gamma \times [1 + |p/q| (y \cos \phi_D + x \sin \phi_D)] . \end{aligned} \tag{12.66}$$



One can define *CP*-conserving and *CP*-violating combinations of these two observables (normalized to the true width  $\Gamma$ ):

$$\begin{aligned}
 y_{CP} &\equiv \frac{\Gamma_{\bar{D}^0 \rightarrow K^+ K^-} + \Gamma_{D^0 \rightarrow K^+ K^-}}{2\Gamma} - 1 \\
 &= \frac{|q/p| + |p/q|}{2} y \cos \phi_D - \frac{|q/p| - |p/q|}{2} x \sin \phi_D , \\
 A_\Gamma &\equiv \frac{\Gamma_{D^0 \rightarrow K^+ K^-} - \Gamma_{\bar{D}^0 \rightarrow K^+ K^-}}{2\Gamma} \\
 &= \frac{|q/p| - |p/q|}{2} y \cos \phi_D - \frac{|q/p| + |p/q|}{2} x \sin \phi_D .
 \end{aligned} \tag{12.67}$$

In the limit of *CP* conservation (and, in particular, within the Standard Model),  $y_{CP} = (\Gamma_+ - \Gamma_-)/2\Gamma$  (where  $\Gamma_+(\Gamma_-)$  is the decay width of the *CP*-even (-odd) mass eigenstate) and  $A_\Gamma = 0$ . Indeed, present measurements imply that *CP* violation is small [21],

$$\begin{aligned}
 y_{CP} &= +1.07 \pm 0.26, \\
 A_\Gamma &= +0.12 \pm 0.25.
 \end{aligned}$$

The  $K^\pm \pi^\mp$  states are not *CP* eigenstates, but they are still common final states for  $D^0$  and  $\bar{D}^0$  decays. Since  $D^0(\bar{D}^0) \rightarrow K^- \pi^+$  is a Cabibbo-favored (doubly-Cabibbo-suppressed) process, these processes are particularly sensitive to  $x$  and/or  $y = \mathcal{O}(\lambda^2)$ . Taking into account that  $|\lambda_{K^- \pi^+}|, |\lambda_{K^+ \pi^-}^{-1}| \ll 1$  and  $x, y \ll 1$ , assuming that there is no direct *CP* violation (again, these are Standard Model tree-level decays dominated by a single weak phase), and expanding the time-dependent rates for  $xt, yt \lesssim \Gamma^{-1}$ , one obtains

$$\begin{aligned}
 \Gamma[D_{\text{phys}}^0(t) \rightarrow K^+ \pi^-] &= e^{-\Gamma t} |\bar{A}_{K^- \pi^+}|^2 \\
 &\times \left[ r_d^2 + r_d \left| \frac{q}{p} \right| (y' \cos \phi_D - x' \sin \phi_D) \Gamma t + \left| \frac{q}{p} \right|^2 \frac{y^2 + x^2}{4} (\Gamma t)^2 \right] , \\
 \Gamma[\bar{D}_{\text{phys}}^0(t) \rightarrow K^- \pi^+] &= e^{-\Gamma t} |\bar{A}_{K^- \pi^+}|^2 \\
 &\times \left[ r_d^2 + r_d \left| \frac{p}{q} \right| (y' \cos \phi_D + x' \sin \phi_D) \Gamma t + \left| \frac{p}{q} \right|^2 \frac{y^2 + x^2}{4} (\Gamma t)^2 \right] ,
 \end{aligned} \tag{12.68}$$

where

$$\begin{aligned}
 y' &\equiv y \cos \delta - x \sin \delta , \\
 x' &\equiv x \cos \delta + y \sin \delta .
 \end{aligned} \tag{12.69}$$

The weak phase  $\phi_D$  is the same as that of Eq. (12.65) (a consequence of the absence of direct *CP* violation),  $\delta$  is a strong-phase difference for these processes, and

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$r_d = \mathcal{O}(\tan^2 \theta_c)$  is the amplitude ratio,  $r_d = |\bar{A}_{K-\pi^+}/A_{K-\pi^+}| = |A_{K+\pi^-}/\bar{A}_{K+\pi^-}|$ , that is,  $\lambda_{K-\pi^+} = r_d(q/p)e^{-i(\delta-\phi_D)}$  and  $\lambda_{K+\pi^-}^{-1} = r_d(p/q)e^{-i(\delta+\phi_D)}$ . By fitting to the six coefficients of the various time-dependences, one can extract  $r_d$ ,  $|q/p|$ ,  $(x^2 + y^2)$ ,  $y' \cos \phi_D$ , and  $x' \sin \phi_D$ . In particular, finding  $CP$  violation ( $|q/p| \neq 1$  and/or  $\sin \phi_D \neq 0$ ) at a level higher than  $10^{-3}$  would constitute evidence for new physics.

A fit to all data [21], assuming no direct  $CP$  violation, yields no evidence for indirect  $CP$  violation:

$$1 - |q/p| = +0.06 \pm 0.14,$$

$$\phi_D = -0.05 \pm 0.09.$$

More details on theoretical and experimental aspects of  $D^0 - \bar{D}^0$  mixing can be found in [39].

### 12.6. $B$ and $B_s$ Decays

The upper bound on the  $CP$  asymmetry in semileptonic  $B$  decays [20] implies that  $CP$  violation in  $B^0 - \bar{B}^0$  mixing is a small effect (we use  $\mathcal{A}_{\text{SL}}/2 \approx 1 - |q/p|$ , see Eq. (12.37)):

$$\mathcal{A}_{\text{SL}} = (-0.4 \pm 5.6) \times 10^{-3} \implies |q/p| = 1.0002 \pm 0.0028. \quad (12.70)$$

The Standard Model prediction is

$$\mathcal{A}_{\text{SL}} = \mathcal{O} \left[ (m_c^2/m_t^2) \sin \beta \right] \lesssim 0.001. \quad (12.71)$$

In models where  $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$  is approximately real, such as the Standard Model, an upper bound on  $\Delta\Gamma/\Delta m \approx \mathcal{R}e(\mathbf{\Gamma}_{12}/\mathbf{M}_{12})$  provides yet another upper bound on the deviation of  $|q/p|$  from one. This constraint does not hold if  $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$  is approximately imaginary. (An alternative parameterization uses  $q/p = (1 - \tilde{\epsilon}_B)/(1 + \tilde{\epsilon}_B)$ , leading to  $\mathcal{A}_{\text{SL}} \simeq 4\mathcal{R}e(\tilde{\epsilon}_B)$ .)

The small deviation (less than one percent) of  $|q/p|$  from 1 implies that, at the present level of experimental precision,  $CP$  violation in  $B$  mixing is a negligible effect. Thus, for the purpose of analyzing  $CP$  asymmetries in hadronic  $B$  decays, we can use

$$\lambda_f = e^{-i\phi_{M(B)}} (\bar{A}_f/A_f), \quad (12.72)$$

where  $\phi_{M(B)}$  refers to the phase of  $\mathbf{M}_{12}$  appearing in Eq. (12.42) that is appropriate for  $B^0 - \bar{B}^0$  oscillations. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_{M(B)}} = (V_{tb}^* V_{td}) / (V_{tb} V_{td}^*). \quad (12.73)$$

Some of the most interesting decays involve final states that are common to  $B^0$  and  $\bar{B}^0$  [40,41]. It is convenient to rewrite Eq. (12.40) for  $B$  decays as [42–44]

$$A_f(t) = S_f \sin(\Delta mt) - C_f \cos(\Delta mt),$$

$$S_f \equiv \frac{2 \mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad (12.74)$$

where we assume that  $\Delta\Gamma = 0$  and  $|q/p| = 1$ . An alternative notation in use is  $A_f \equiv -C_f$ , but this  $A_f$  should not be confused with the  $A_f$  of Eq. (12.13).

A large class of interesting processes proceed via quark transitions of the form  $\bar{b} \rightarrow \bar{q}q\bar{q}'$  with  $q' = s$  or  $d$ . For  $q = c$  or  $u$ , there are contributions from both tree ( $t$ ) and penguin ( $p^{qu}$ , where  $q_u = u, c, t$  is the quark in the loop) diagrams (see Fig. 12.2) which carry different weak phases:

$$A_f = \left( V_{qb}^* V_{qq'} \right) t_f + \sum_{q_u=u,c,t} \left( V_{qub}^* V_{quq'} \right) p_f^{q_u}. \quad (12.75)$$

(The distinction between tree and penguin contributions is a heuristic one; the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, Ref. 45.) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations. For example, for  $f = \pi\pi$ , which proceeds via  $\bar{b} \rightarrow \bar{u}u\bar{d}$  transition, we can write

$$A_{\pi\pi} = (V_{ub}^* V_{ud}) T_{\pi\pi} + (V_{tb}^* V_{td}) P_{\pi\pi}^t, \quad (12.76)$$

where  $T_{\pi\pi} = t_{\pi\pi} + p_{\pi\pi}^u - p_{\pi\pi}^c$  and  $P_{\pi\pi}^t = p_{\pi\pi}^t - p_{\pi\pi}^c$ .  $CP$ -violating phases in Eq. (12.76) appear only in the CKM elements, so that

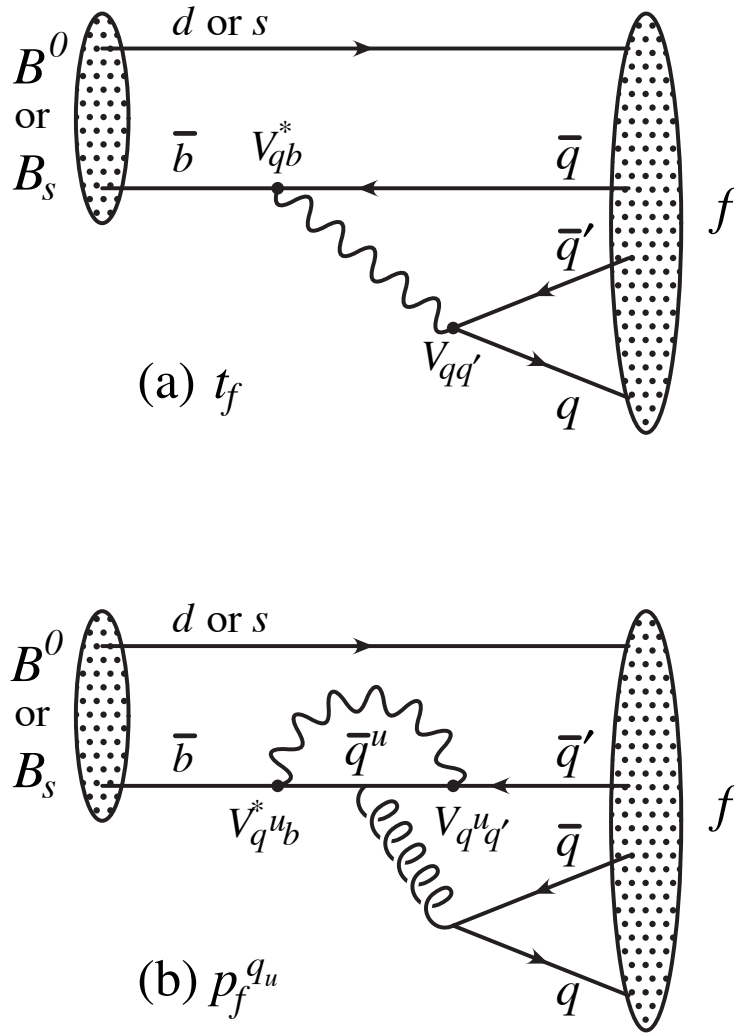
$$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{(V_{ub} V_{ud}^*) T_{\pi\pi} + (V_{tb} V_{td}^*) P_{\pi\pi}^t}{(V_{ub}^* V_{ud}) T_{\pi\pi} + (V_{tb}^* V_{td}) P_{\pi\pi}^t}. \quad (12.77)$$

For  $f = J/\psi K$ , which proceeds via  $\bar{b} \rightarrow \bar{c}c\bar{s}$  transition, we can write

$$A_{\psi K} = (V_{cb}^* V_{cs}) T_{\psi K} + (V_{ub}^* V_{us}) P_{\psi K}^u, \quad (12.78)$$

where  $T_{\psi K} = t_{\psi K} + p_{\psi K}^c - p_{\psi K}^t$  and  $P_{\psi K}^u = p_{\psi K}^u - p_{\psi K}^t$ . A subtlety arises in this decay that is related to the fact that  $B^0$  decays into a final  $J/\psi K^0$  state while  $\bar{B}^0$  decays into a final  $J/\psi \bar{K}^0$  state. A common final state, *e.g.*,  $J/\psi K_S$ , is reached only via  $K^0 - \bar{K}^0$  mixing. Consequently, the phase factor (defined in Eq. (12.42)) corresponding to neutral  $K$  mixing,  $e^{-i\phi_M(K)} = (V_{cd}^* V_{cs}) / (V_{cd} V_{cs}^*)$ , plays a role:

$$\frac{\bar{A}_{\psi K_S}}{A_{\psi K_S}} = - \frac{(V_{cb} V_{cs}^*) T_{\psi K} + (V_{ub} V_{us}^*) P_{\psi K}^u}{(V_{cb}^* V_{cs}) T_{\psi K} + (V_{ub}^* V_{us}) P_{\psi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}. \quad (12.79)$$



**Figure 12.2:** Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to  $B^0 \rightarrow f$  or  $B_s \rightarrow f$  via a  $\bar{b} \rightarrow \bar{q}q\bar{q}'$  quark-level process.

For  $q = s$  or  $d$ , there are only penguin contributions to  $A_f$ , that is,  $t_f = 0$  in Eq. (12.75). (The tree  $\bar{b} \rightarrow \bar{u}u\bar{q}'$  transition followed by  $\bar{u}u \rightarrow \bar{q}q$  rescattering is included below in the  $P^u$  terms.) Again, CKM unitarity allows us to write  $A_f$  in terms of two CKM combinations. For example, for  $f = \phi K_S$ , which proceeds via  $\bar{b} \rightarrow \bar{s}s\bar{s}$  transition, we can write

$$\frac{\bar{A}_{\phi K_S}}{A_{\phi K_S}} = -\frac{(V_{cb}V_{cs}^*)P_{\phi K}^c + (V_{ub}V_{us}^*)P_{\phi K}^u}{(V_{cb}^*V_{cs})P_{\phi K}^c + (V_{ub}^*V_{us})P_{\phi K}^u} \times \frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}, \quad (12.80)$$

where  $P_{\phi K}^c = p_{\phi K}^c - p_{\phi K}^t$  and  $P_{\phi K}^u = p_{\phi K}^u - p_{\phi K}^t$ .

Since the amplitude  $A_f$  involves two different weak phases, the corresponding decays can exhibit both  $CP$  violation in the interference of decays with and without mixing,  $S_f \neq 0$ , and  $CP$  violation in decays,  $C_f \neq 0$ . (At the present level of experimental precision, the contribution to  $C_f$  from  $CP$  violation in mixing is negligible, see Eq. (12.70).) If the contribution from a second weak phase is suppressed, then the interpretation of  $S_f$  in terms of Lagrangian  $CP$ -violating parameters is clean, while  $C_f$  is small. If such a second contribution is not suppressed,  $S_f$  depends on hadronic parameters and, if the relevant strong phase is large,  $C_f$  is large.

A summary of  $\bar{b} \rightarrow \bar{q}q'q'$  modes with  $q' = s$  or  $d$  is given in Table 12.1. The  $\bar{b} \rightarrow \bar{d}d\bar{q}$  transitions lead to final states that are similar to the  $\bar{b} \rightarrow \bar{u}u\bar{q}$  transitions and have similar phase dependence. Final states that consist of two-vector mesons ( $\psi\phi$  and  $\phi\phi$ ) are not  $CP$  eigenstates, and angular analysis is needed to separate the  $CP$ -even from the  $CP$ -odd contributions.

**Table 12.1:** Summary of  $\bar{b} \rightarrow \bar{q}q'q'$  modes with  $q' = s$  or  $d$ . The second and third columns give examples of final hadronic states. The fourth column gives the CKM dependence of the amplitude  $A_f$ , using the notation of Eqs. (12.76, 12.78, 12.80), with the dominant term first and the subdominant second. The suppression factor of the second term compared to the first is given in the last column. “Loop” refers to a penguin versus tree-suppression factor (it is mode-dependent and roughly  $\mathcal{O}(0.2 - 0.3)$ ) and  $\lambda = 0.23$  is the expansion parameter of Eq. (12.48).

$\bar{b} \rightarrow \bar{q}q'q'$	$B^0 \rightarrow f$	$B_s \rightarrow f$	CKM dependence of $A_f$	Suppression
$\bar{b} \rightarrow \bar{c}c\bar{s}$	$\psi K_S$	$\psi\phi$	$(V_{cb}^*V_{cs})T + (V_{ub}^*V_{us})P^u$	loop $\times \lambda^2$
$\bar{b} \rightarrow \bar{s}s\bar{s}$	$\phi K_S$	$\phi\phi$	$(V_{cb}^*V_{cs})P^c + (V_{ub}^*V_{us})P^u$	$\lambda^2$
$\bar{b} \rightarrow \bar{u}u\bar{s}$	$\pi^0 K_S$	$K^+K^-$	$(V_{cb}^*V_{cs})P^c + (V_{ub}^*V_{us})T$	$\lambda^2/\text{loop}$
$\bar{b} \rightarrow \bar{c}c\bar{d}$	$D^+D^-$	$\psi K_S$	$(V_{cb}^*V_{cd})T + (V_{tb}^*V_{td})P^t$	loop
$\bar{b} \rightarrow \bar{s}s\bar{d}$	$\phi\pi$	$\phi K_S$	$(V_{tb}^*V_{td})P^t + (V_{cb}^*V_{cd})P^c$	$\lesssim 1$
$\bar{b} \rightarrow \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\pi^0 K_S$	$(V_{ub}^*V_{ud})T + (V_{tb}^*V_{td})P^t$	loop

The cleanliness of the theoretical interpretation of  $S_f$  can be assessed from the information in the last column of Table 12.1. In case of small uncertainties, the expression for  $S_f$  in terms of CKM phases can be deduced from the fourth column of Table 12.1 in combination with Eq. (12.73) (and, for  $b \rightarrow q\bar{q}s$  decays, the example in Eq. (12.79)). Here we consider several interesting examples.

For  $B \rightarrow J/\psi K_S$  and other  $\bar{b} \rightarrow \bar{c}c\bar{s}$  processes, we can neglect the  $P^u$  contribution to  $A_f$ , in the Standard Model, to an approximation that is better than one percent:

$$\lambda_{\psi K_S} = -e^{-2i\beta} \Rightarrow S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0. \quad (12.81)$$

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In the presence of new physics,  $A_f$  is still likely to be dominated by the  $T$  term, but the mixing amplitude might be modified. We learn that, model-independently,  $C_f \approx 0$  while  $S_f$  cleanly determines the mixing phase ( $\phi_M - 2 \arg(V_{cb}V_{cd}^*)$ ). The experimental measurement [21],  $S_{\psi K} = 0.673 \pm 0.023$ , gave the first precision test of the Kobayashi-Maskawa mechanism, and its consistency with the predictions for  $\sin 2\beta$  makes it very likely that this mechanism is indeed the dominant source of  $CP$  violation in meson decays.

For  $B \rightarrow \phi K_S$  and other  $\bar{b} \rightarrow \bar{s}s\bar{s}$  processes (as well as some  $\bar{b} \rightarrow \bar{u}u\bar{s}$  processes), we can neglect the subdominant contributions, in the Standard Model, to an approximation that is good on the order of a few percent:

$$\lambda_{\phi K_S} = -e^{-2i\beta} \Rightarrow S_{\phi K_S} = \sin 2\beta, \quad C_{\phi K_S} = 0. \quad (12.82)$$

In the presence of new physics, both  $A_f$  and  $\mathbf{M}_{12}$  can get contributions that are comparable in size to those of the Standard Model and carry new weak phases. Such a situation gives several interesting consequences for penguin-dominated  $b \rightarrow q\bar{q}s$  decays ( $q = u, d, s$ ) to a final state  $f$ :

1. The value of  $-\eta_f S_f$  may be different from  $S_{\psi K_S}$  by more than a few percent, where  $\eta_f$  is the  $CP$  eigenvalue of the final state.
2. The values of  $\eta_f S_f$  for different final states  $f$  may be different from each other by more than a few percent (for example,  $S_{\phi K_S} \neq S_{\eta' K_S}$ ).
3. The value of  $C_f$  may be different from zero by more than a few percent.

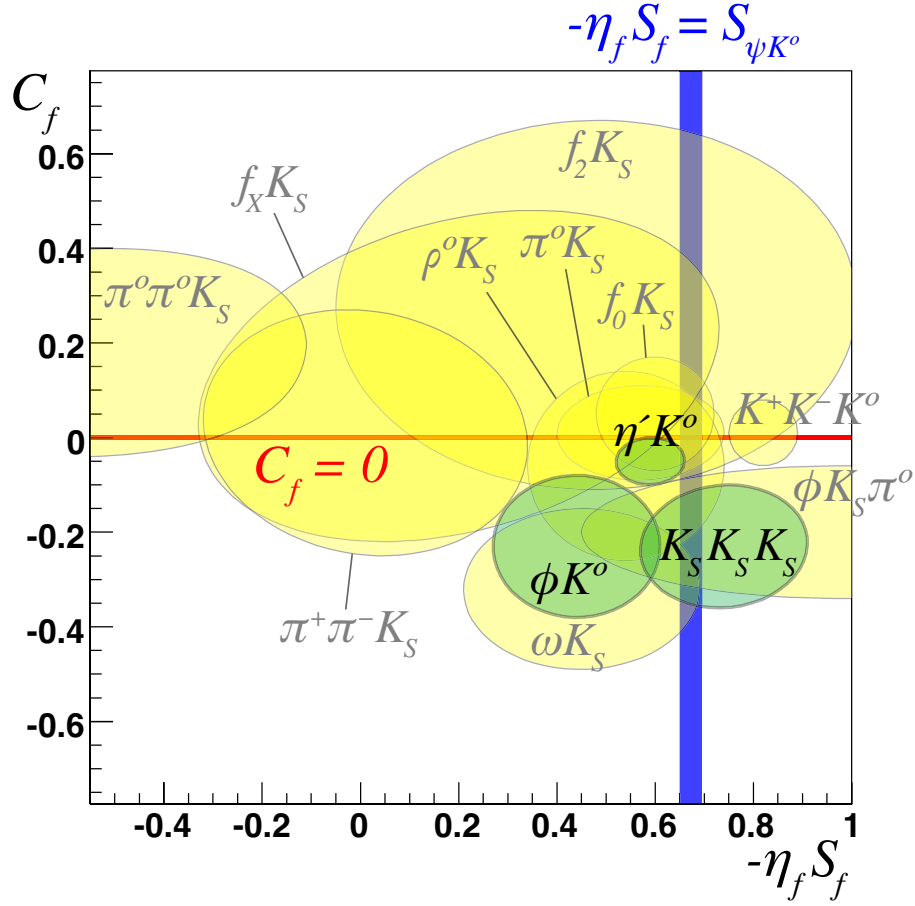
While a clear interpretation of such signals in terms of Lagrangian parameters will be difficult because, under these circumstances, hadronic parameters do play a role, any of the above three options will clearly signal new physics. Fig. 12.3 summarizes the present experimental results: none of the possible signatures listed above is unambiguously established, but there is definitely still room for new physics.

For  $B \rightarrow \pi\pi$  and other  $\bar{b} \rightarrow \bar{u}u\bar{d}$  processes, the penguin-to-tree ratio can be estimated using  $SU(3)$  relations and experimental data on related  $B \rightarrow K\pi$  decays. The result is that the suppression is on the order of  $0.2 - 0.3$  and so cannot be neglected. The expressions for  $S_{\pi\pi}$  and  $C_{\pi\pi}$  to leading order in  $R_{PT} \equiv (|V_{tb}V_{td}| P_{\pi\pi}^t) / (|V_{ub}V_{ud}| T_{\pi\pi})$  are:

$$\lambda_{\pi\pi} = e^{2i\alpha} \left[ (1 - R_{PT}e^{-i\alpha}) / (1 - R_{PT}e^{+i\alpha}) \right] \Rightarrow$$

$$S_{\pi\pi} \approx \sin 2\alpha + 2 \operatorname{Re}(R_{PT}) \cos 2\alpha \sin \alpha, \quad C_{\pi\pi} \approx 2 \operatorname{Im}(R_{PT}) \sin \alpha. \quad (12.83)$$

Note that  $R_{PT}$  is mode-dependent and, in particular, could be different for  $\pi^+\pi^-$  and  $\pi^0\pi^0$ . If strong phases can be neglected, then  $R_{PT}$  is real, resulting in  $C_{\pi\pi} = 0$ . The size of  $C_{\pi\pi}$  is an indicator of how large the strong phase is. The present experimental range is  $C_{\pi\pi} = -0.38 \pm 0.06$  [21]. As concerns  $S_{\pi\pi}$ , it is clear from Eq. (12.83) that the relative size or strong phase of the penguin contribution must be known to extract  $\alpha$ . This is the problem of penguin pollution.



**Figure 12.3:** Summary of the results [21] of time-dependent analyses of  $b \rightarrow q\bar{q}s$  decays, which are potentially sensitive to new physics. Subdominant corrections are expected to be smallest for the modes shown in green (darker). Results for final states including  $K^0$  mesons combine  $CP$ -conjugate  $K_S$  and  $K_L$  measurements. The final state  $K^+K^-K^0$  is not a  $CP$  eigenstate; the mixture of  $CP$ -even and  $CP$ -odd components is taken into account in obtaining an effective value for  $\eta_f S_f$ . Correlations between  $C_f$  and  $S_f$  are included when available.

The cleanest solution involves isospin relations among the  $B \rightarrow \pi\pi$  amplitudes [46]:

$$\frac{1}{\sqrt{2}}A_{\pi^+\pi^-} + A_{\pi^0\pi^0} = A_{\pi^+\pi^0}. \quad (12.84)$$

The method exploits the fact that the penguin contribution to  $P_{\pi\pi}^t$  is pure  $\Delta I = \frac{1}{2}$  (this is not true for the electroweak penguins which, however, are expected to be small), while the tree contribution to  $T_{\pi\pi}$  contains pieces which are both  $\Delta I = \frac{1}{2}$  and  $\Delta I = \frac{3}{2}$ . A simple geometric construction then allows one to find  $R_{PT}$  and extract  $\alpha$  cleanly from  $S_{\pi^+\pi^-}$ . The key experimental difficulty is that one must measure accurately the separate rates for  $B^0, \bar{B}^0 \rightarrow \pi^0\pi^0$ .

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$CP$  asymmetries in  $B \rightarrow \rho\pi$  and  $B \rightarrow \rho\rho$  can also be used to determine  $\alpha$ . In particular, the  $B \rightarrow \rho\rho$  measurements are presently very significant in constraining  $\alpha$ . The extraction proceeds via isospin analysis similar to that of  $B \rightarrow \pi\pi$ . There are, however, several important differences. First, due to the finite width of the  $\rho$  mesons, a final  $(\rho\rho)_{I=1}$  state is possible [47]. The effect is, however, small, on the order of  $(\Gamma_\rho/m_\rho)^2 \sim 0.04$ . Second, due to the presence of three helicity states for the two-vector mesons, angular analysis is needed to separate the  $CP$ -even and  $CP$ -odd components. The theoretical expectation is, however, that the  $CP$ -odd component is small. This expectation is supported by experiments which find that the  $\rho^+\rho^-$  and  $\rho^\pm\rho^0$  modes are dominantly longitudinally polarized. Third, an important advantage of the  $\rho\rho$  modes is that the penguin contribution is expected to be small due to different hadronic dynamics. This expectation is confirmed by the smallness of the upper bound on  $\mathcal{B}(B^0 \rightarrow \rho^0\rho^0)$ . Thus,  $S_{\rho^+\rho^-}$  is not far from  $\sin 2\alpha$ . Finally, both  $S_{\rho^0\rho^0}$  and  $C_{\rho^0\rho^0}$  are experimentally accessible, which may allow a precision determination of  $\alpha$ . The consistency between the range of  $\alpha$  determined by the  $B \rightarrow \pi\pi, \rho\pi, \rho\rho$  measurements and the range allowed by CKM fits (excluding these direct determinations) provides further support to the Kobayashi-Maskawa mechanism.

An interesting class of decay modes is that of the tree level decays  $B^\pm \rightarrow D^{(*)0}K^\pm$ . These decays provide golden methods for a clean determination of the angle  $\gamma$  [48–51]. The method uses the decays  $B^+ \rightarrow D^0K^+$ , which proceeds via the quark transition  $\bar{b} \rightarrow \bar{u}c\bar{s}$ , and  $B^+ \rightarrow \bar{D}^0K^+$ , which proceeds via the quark transition  $\bar{b} \rightarrow \bar{c}u\bar{s}$ , with the  $D^0$  and  $\bar{D}^0$  decaying into a common final state. The decays into common final states, such as  $(\pi^0K_S)_D K^+$ , involve interference effects between the two amplitudes, with sensitivity to the relative phase,  $\delta + \gamma$  ( $\delta$  is the relevant strong phase). The  $CP$ -conjugate processes are sensitive to  $\delta - \gamma$ . Measurements of branching ratios and  $CP$  asymmetries allow an extraction of  $\gamma$  and  $\delta$  from amplitude triangle relations. The extraction suffers from discrete ambiguities but involves no hadronic uncertainties. However, the smallness of the CKM-suppressed  $b \rightarrow u$  transitions makes it difficult at present to use the simplest methods [48–50] to determine  $\gamma$ . These difficulties are overcome by performing a Dalitz plot analysis for multi-body  $D$  decays [51]. The consistency between the range of  $\gamma$  determined by the  $B \rightarrow DK$  measurements and the range allowed by CKM fits (excluding these direct determinations) provides further support to the Kobayashi-Maskawa mechanism.

For  $B_s$  decays, one has to replace Eq. (12.73) with  $e^{-i\phi_{M(B_s)}} = (V_{tb}^*V_{ts})/(V_{cb}V_{cs}^*)$ . Note that one expects  $\Delta\Gamma/\Gamma = \mathcal{O}(0.1)$ , and therefore,  $y$  should not be put to zero in Eqs. (12.30, 12.31), but  $|q/p| = 1$  is expected to hold to an even better approximation than for  $B$  mesons. The  $CP$  asymmetry in  $B_s \rightarrow J/\psi\phi$  will determine (with angular analysis to disentangle the  $CP$ -even and  $CP$ -odd components of the final state)  $\sin 2\beta_s$ , where

$$\beta_s \equiv \arg \left( -\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right). \quad (12.85)$$

Other observables, such as the width difference between the neutral  $B_s$ -mesons and the semileptonic asymmetry in their decay, are also sensitive to  $\phi_{M(B_s)}$ . The CDF and D0



experiments are now providing first constraints on these observables.

## 12.7. Summary and Outlook

*CP* violation has been experimentally established in neutral *K* and *B* meson decays:

1. All three types of *CP* violation have been observed in  $K \rightarrow \pi\pi$  decays:

$$\mathcal{R}e(\epsilon') = \frac{1}{6} \left( \left| \frac{\overline{A}_{\pi^0\pi^0}}{A_{\pi^0\pi^0}} \right| - \left| \frac{\overline{A}_{\pi^+\pi^-}}{A_{\pi^+\pi^-}} \right| \right) = (2.5 \pm 0.4) \times 10^{-6} \quad (\text{I})$$

$$\mathcal{R}e(\epsilon) = \frac{1}{2} \left( 1 - \left| \frac{q}{p} \right| \right) = (1.66 \pm 0.02) \times 10^{-3} \quad (\text{II})$$

$$\mathcal{I}m(\epsilon) = -\frac{1}{2} \mathcal{I}m(\lambda_{(\pi\pi)_{I=0}}) = (1.57 \pm 0.02) \times 10^{-3} . \quad (\text{III})$$

(12.86)

2. Direct *CP* violation has been observed, first in  $B^0 \rightarrow K^+\pi^-$  decays (and more recently also in  $B \rightarrow \pi^+\pi^-$ ,  $B^0 \rightarrow \eta K^{*0}$ , and  $B^+ \rightarrow \rho^0 K^+$  decays), and *CP* violation in interference of decays with and without mixing has been observed, first in  $B \rightarrow J/\psi K_S$  decays and related modes (as well as other final *CP* eigenstates:  $\eta' K_S$ ,  $K^+ K^- K_S$ ,  $J/\psi \pi^0$  and  $\pi^+\pi^-$ ):

$$\mathcal{A}_{K^+\pi^-} = \frac{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 - 1}{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 + 1} = -0.098 \pm 0.013 \quad (\text{I})$$

$$S_{\psi K} = \mathcal{I}m(\lambda_{\psi K}) = 0.673 \pm 0.023 . \quad (\text{III})$$

(12.87)

Searches for additional *CP* asymmetries are ongoing in *B*, *D*, and *K* decays, and current limits are consistent with Standard Model expectations.

Based on Standard Model predictions, further observation of *CP* violation in *B* decays seems promising for the near future, both at LHCb [52] and a possible higher-luminosity asymmetric-energy B factory [53,54]. Observables that are subject to clean theoretical interpretation, such as  $S_{\psi K_S}$  and  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ , are of particular value for constraining the values of the CKM parameters and probing the flavor sector of extensions to the Standard Model. Other probes of *CP* violation now being pursued experimentally include the electric dipole moments of the neutron and electron, and the decays of tau leptons. Additional processes that are likely to play an important role in future *CP* studies include top-quark production and decay, and neutrino oscillations.

All measurements of *CP* violation to date are consistent with the predictions of the Kobayashi-Maskawa mechanism of the Standard Model. Actually, it is now established that the KM mechanism plays a major role in the *CP* violation measured in meson decays. However, a dynamically-generated matter-antimatter asymmetry of the universe requires additional sources of *CP* violation, and such sources are naturally generated by

extensions to the Standard Model. New sources might eventually reveal themselves as small deviations from the predictions of the KM mechanism in meson decay rates, or else might not be observable in meson decays at all, but observable with future probes such as neutrino oscillations or electric dipole moments. We cannot guarantee that new sources of  $CP$  violation will ever be found experimentally, but the fundamental nature of  $CP$  violation demands a vigorous effort.

A number of excellent reviews of  $CP$  violation are available [55–62], where the interested reader may find a detailed discussion of the various topics that are briefly reviewed here.

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