THE CHARGED KAON MASS
Revised 1994 by T.G. Trippe (LBNL).

The average of the six charged kaon mass measurements which we use in the Particle Listings is

\[ m_{K^\pm} = 493.677 \pm 0.013 \text{ MeV} \ (S = 2.4), \]  

(1)

where the error has been increased by the scale factor S. The large scale factor indicates a serious disagreement between different input data. The average before scaling the error is

\[ m_{K^\pm} = 493.677 \pm 0.005 \text{ MeV}, \]

\[ \chi^2 = 22.9 \text{ for 5 D.F., Prob.} = 0.04\%, \]  

(2)

where the high \( \chi^2 \) and correspondingly low \( \chi^2 \) probability further quantify the disagreement.

The main disagreement is between the two most recent and precise results,

\[ m_{K^\pm} = 493.696 \pm 0.007 \text{ MeV} \quad \text{DENISOV 91} \]
\[ m_{K^\pm} = 493.636 \pm 0.011 \text{ MeV} \ (S = 1.5) \quad \text{GALL 88} \]

Average = 493.679 \pm 0.006 MeV

\[ \chi^2 = 21.2 \text{ for 1 D.F., Prob.} = 0.0004\% , \]  

(3)

both of which are measurements of x-ray energies from kaonic atoms. Comparing the average in Eq. (3) with the overall average in Eq. (2), it is clear that DENISOV 91 and GALL 88 dominate the overall average, and that their disagreement is responsible for most of the high \( \chi^2 \).

The GALL 88 measurement was made using four different kaonic atom transitions, \( K^- \text{Pb} \ (9 \rightarrow 8) \), \( K^- \text{Pb} \ (11 \rightarrow 10) \), \( K^- \text{W} \ (9 \rightarrow 8) \), and \( K^- \text{W} \ (11 \rightarrow 10) \). The \( m_{K^\pm} \) values they obtain from each of these transitions is shown in the Particle Listings and in Fig. 1. Their \( K^- \text{Pb} \ (9 \rightarrow 8) \) \( m_{K^\pm} \) is below and somewhat inconsistent with their other three transitions. The average of their four measurements is

\[ m_{K^\pm} = 493.636 \pm 0.007, \]

\[ \chi^2 = 7.0 \text{ for 3 D.F., Prob.} = 7.2\% . \]  

(4)
This is a low but acceptable $\chi^2$ probability so, to be conservative, GALL 88 scaled up the error on their average by $S=1.5$ to obtain their published error $\pm 0.011$ shown in Eq. (3) above and used in the Particle Listings average.

\[ \chi^2 \]

Figure 1: Ideogram of $m_{K^\pm}$ mass measurements. GALL 88 and CHENG 75 measurements are shown separately for each transition they measured.

The ideogram in Fig. 1 shows that the DENISOV 91 measurement and the GALL 88 $K^-$ Pb ($9 \rightarrow 8$) measurement yield two well-separated peaks. One might suspect the GALL 88 $K^-$ Pb ($9 \rightarrow 8$) measurement since it is responsible both for the internal inconsistency in the GALL 88 measurements and the disagreement with DENISOV 91.

To see if the disagreement could result from a systematic problem with the $K^-$ Pb ($9 \rightarrow 8$) transition, we have separated the CHENG 75 data, which also used $K^-$ Pb, into its separate transitions. Figure 1 shows that the CHENG 75 and GALL 88
$K^-$ Pb (9 → 8) values are consistent, suggesting the possibility of a common effect such as contaminant nuclear $\gamma$ rays near the $K^-$ Pb (9 → 8) transition energy, although the CHENG 75 errors are too large to make a strong conclusion. The average of all 13 measurements has a $\chi^2$ of 52.6 as shown in Fig. 1 and the first line of Table 1, yielding an unacceptable $\chi^2$ probability of 0.00005%. The second line of Table 1 excludes both the GALL 88 and CHENG 75 measurements of the $K^-$ Pb (9 → 8) transition and yields a $\chi^2$ probability of 43%. The third [fourth] line of Table 1 excludes only the GALL 88 $K^-$ Pb (9 → 8) [DENISOV 91] measurement and yields a $\chi^2$ probability of 20% [8.6%]. Table 1 shows that removing both measurements of the $K^-$ Pb (9 → 8) transition produces the most consistent set of data, but that excluding only the GALL 88 $K^-$ Pb (9 → 8) transition or DENISOV 91 also produces acceptable probabilities.

**Table 1:** $m_{K^\pm}$ averages for some combinations of Fig. 1 data.

<table>
<thead>
<tr>
<th>$m_{K^\pm}$ (MeV)</th>
<th>$\chi^2$</th>
<th>D.F.</th>
<th>Prob. (%)</th>
<th>Measurements used</th>
</tr>
</thead>
<tbody>
<tr>
<td>493.664 ± 0.004</td>
<td>52.6</td>
<td>12</td>
<td>0.00005</td>
<td>all 13 measurements</td>
</tr>
<tr>
<td>493.690 ± 0.006</td>
<td>10.1</td>
<td>10</td>
<td>43</td>
<td>no $K^-$ Pb (9→8)</td>
</tr>
<tr>
<td>493.687 ± 0.006</td>
<td>14.6</td>
<td>11</td>
<td>20</td>
<td>no GALL 88 $K^-$ Pb (9→8)</td>
</tr>
<tr>
<td>493.642 ± 0.006</td>
<td>17.8</td>
<td>11</td>
<td>8.6</td>
<td>no DENISOV 91</td>
</tr>
</tbody>
</table>

Yu.M. Ivanov, representing DENISOV 91, has estimated corrections needed for the older experiments because of improved $^{192}\text{Ir}$ and $^{198}\text{Au}$ calibration $\gamma$-ray energies. He estimates that CHENG 75 and BACKENSTOSS 73 $m_{K^\pm}$ values could be raised by about 15 keV and 22 keV, respectively. With these estimated corrections, Table 1 becomes Table 2. The last line of Table 2 shows that if such corrections are assumed, then GALL 88 $K^-$ Pb (9 → 8) is inconsistent with the rest of the data even when DENISOV 91 is excluded. Yu.M. Ivanov warns that these are rough estimates. Accordingly, we do not use Table 2 to reject the GALL 88 $K^-$ Pb (9 → 8) transition, but we note that a future reanalysis of the CHENG 75 data could
be useful because it might provide supporting evidence for such a rejection.

**Table 2:** \(m_{K^\pm}\) averages for some combinations of Fig. 1 data after raising CHENG 75 and BACKENSTOSS 73 values by 0.015 and 0.022 MeV respectively.

<table>
<thead>
<tr>
<th>(m_{K^\pm}) (MeV)</th>
<th>(\chi^2)</th>
<th>D.F.</th>
<th>Prob. (%)</th>
<th>Measurements used</th>
</tr>
</thead>
<tbody>
<tr>
<td>493.666 ± 0.004</td>
<td>53.9</td>
<td>12</td>
<td>0.00003</td>
<td>all 13 measurements</td>
</tr>
<tr>
<td>493.693 ± 0.006</td>
<td>9.0</td>
<td>10</td>
<td>53</td>
<td>no (K^-) Pb(9→8)</td>
</tr>
<tr>
<td>493.690 ± 0.006</td>
<td>11.5</td>
<td>11</td>
<td>40</td>
<td>no GALL 88 (K^-) Pb(9→8)</td>
</tr>
<tr>
<td>493.645 ± 0.006</td>
<td>23.0</td>
<td>11</td>
<td>1.8</td>
<td>no DENISOV 91</td>
</tr>
</tbody>
</table>

The GALL 88 measurement uses a Ge semiconductor spectrometer which has a resolution of about 1 keV, so they run the risk of some contaminant nuclear \(\gamma\) rays. Studies of \(\gamma\) rays following stopped \(\pi^-\) and \(\Sigma^-\) absorption in nuclei (unpublished) do not show any evidence for contaminants according to GALL 88 spokesperson, B.L. Roberts. The DENISOV 91 measurement uses a crystal diffraction spectrometer with a resolution of 6.3 eV for radiation at 22.1 keV to measure the 4f-3d transition in \(K^-\) \(^{12}\)C. The high resolution and the light nucleus reduce the probability for overlap by contaminant \(\gamma\) rays, compared with the measurement of GALL 88. The DENISOV 91 measurement is supported by their high-precision measurement of the 4d-2p transition energy in \(\pi^-\) \(^{12}\)C, which is good agreement with the calculated energy.

While we suspect that the GALL 88 \(K^-\) Pb (9 → 8) measurements could be the problem, we are unable to find clear grounds for rejecting it. Therefore, we retain their measurement in the average and accept the large scale factor until further information can be obtained from new measurements and/or from reanalysis of GALL 88 and CHENG 75 data.

We thank B.L. Roberts (Boston Univ.) and Yu.M. Ivanov (Petersburg Nuclear Physics Inst.) for their extensive help in understanding this problem.