

## TESTS OF CONSERVATION LAWS

Updated May 2010 by L. Wolfenstein (Carnegie-Mellon University), T.G. Trippe (LBNL), and C.-J. Lin (LBNL).

In keeping with the current interest in tests of conservation laws, we collect together a Table of experimental limits on all weak and electromagnetic decays, mass differences, and moments, and on a few reactions, whose observation would violate conservation laws. The Table is given only in the full *Review of Particle Physics*, not in the Particle Physics Booklet. For the benefit of Booklet readers, we include the best limits from the Table in the following text. Limits in this text are for CL=90% unless otherwise specified. The Table is in two parts: “Discrete Space-Time Symmetries,” *i.e.*,  $C$ ,  $P$ ,  $T$ ,  $CP$ , and  $CPT$ ; and “Number Conservation Laws,” *i.e.*, lepton, baryon, hadronic flavor, and charge conservation. The references for these data can be found in the the Particle Listings in the *Review*. A discussion of these tests follows.

### $CPT$ INVARIANCE

General principles of relativistic field theory require invariance under the combined transformation  $CPT$ . The simplest tests of  $CPT$  invariance are the equality of the masses and lifetimes of a particle and its antiparticle. The best test comes from the limit on the mass difference between  $K^0$  and  $\bar{K}^0$ . Any such difference contributes to the  $CP$ -violating parameter  $\epsilon$ . Assuming  $CPT$  invariance,  $\phi_\epsilon$ , the phase of  $\epsilon$  should be very close to  $44^\circ$ . (See the review “ $CP$  Violation in  $K_L$  decay” in this edition.) In contrast, if the entire source of  $CP$  violation in  $K^0$  decays were a  $K^0 - \bar{K}^0$  mass difference,  $\phi_\epsilon$  would be  $44^\circ + 90^\circ$ .

Assuming that there is no other source of  $CPT$  violation than this mass difference, it is possible to deduce that[1]

$$m_{\bar{K}^0} - m_{K^0} \approx \frac{2(m_{K_L^0} - m_{K_S^0})|\eta|(\frac{2}{3}\phi_{+-} + \frac{1}{3}\phi_{00} - \phi_{SW})}{\sin \phi_{SW}},$$

where  $\phi_{SW} = (43.51 \pm 0.05)^\circ$ , the superweak angle. Using our best values of the  $CP$ -violation parameters, we get  $|(m_{\bar{K}^0} - m_{K^0})/m_{K^0}| \leq 0.8 \times 10^{-18}$  at CL=90%. Limits can also be

placed on specific  $CPT$ -violating decay amplitudes. Given the small value of  $(1 - |\eta_{00}/\eta_{+-}|)$ , the value of  $\phi_{00} - \phi_{+-}$  provides a measure of  $CPT$  violation in  $K_L^0 \rightarrow 2\pi$  decay. Results from CERN [1] and Fermilab [2] indicate no  $CPT$ -violating effect.

### $CP$ AND $T$ INVARIANCE

Given  $CPT$  invariance,  $CP$  violation and  $T$  violation are equivalent. The original evidence for  $CP$  violation came from the measurement of  $|\eta_{+-}| = |A(K_L^0 \rightarrow \pi^+\pi^-)/A(K_S^0 \rightarrow \pi^+\pi^-)| = (2.232 \pm 0.011) \times 10^{-3}$ . This could be explained in terms of  $K^0$ - $\bar{K}^0$  mixing, which also leads to the asymmetry  $[\Gamma(K_L^0 \rightarrow \pi^-e^+\nu) - \Gamma(K_L^0 \rightarrow \pi^+e^-\bar{\nu})]/[\text{sum}] = (0.334 \pm 0.007)\%$ . Evidence for  $CP$  violation in the kaon decay amplitude comes from the measurement of  $(1 - |\eta_{00}/\eta_{+-}|)/3 = \text{Re}(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3}$ . In the Standard Model much larger  $CP$ -violating effects are expected. The first of these, which is associated with  $B$ - $\bar{B}$  mixing, is the parameter  $\sin(2\beta)$  now measured quite accurately to be  $0.671 \pm 0.023$ . A number of other  $CP$ -violating observables are being measured in  $B$  decays; direct evidence for  $CP$  violation in the  $B$  decay amplitude comes from the asymmetry  $[\Gamma(\bar{B}^0 \rightarrow K^-\pi^+) - \Gamma(B^0 \rightarrow K^+\pi^-)]/[\text{sum}] = -0.098 \pm 0.013$ . Direct tests of  $T$  violation are much more difficult; a measurement by CPLEAR of the difference between the oscillation probabilities of  $K^0$  to  $\bar{K}^0$  and  $\bar{K}^0$  to  $K^0$  is related to  $T$  violation [3]. Other searches for  $CP$  or  $T$  violation involve effects that are expected to be unobservable in the Standard Model. The most sensitive are probably the searches for an electric dipole moment of the neutron, measured to be  $< 2.9 \times 10^{-26}$  e cm, and the electron  $(0.07 \pm 0.07) \times 10^{-26}$  e cm. A nonzero value requires both  $P$  and  $T$  violation.

### CONSERVATION OF LEPTON NUMBERS

Present experimental evidence and the standard electroweak theory are consistent with the absolute conservation of three separate lepton numbers: electron number  $L_e$ , muon number  $L_\mu$ , and tau number  $L_\tau$ , except for the effect of neutrino mixing associated with neutrino masses. Searches for violations are of the following types:

**a)  $\Delta L = 2$  for one type of charged lepton.** The best limit comes from the search for neutrinoless double beta decay  $(Z, A) \rightarrow (Z + 2, A) + e^- + e^-$ . The best laboratory limit is  $t_{1/2} > 1.9 \times 10^{25}$  yr (CL=90%) for  $^{76}\text{Ge}$ .

**b) Conversion of one charged-lepton type to another.**

For purely leptonic processes, the best limits are on  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$ , measured as  $\Gamma(\mu \rightarrow e\gamma)/\Gamma(\mu \rightarrow \text{all}) < 1.2 \times 10^{-11}$  and  $\Gamma(\mu \rightarrow 3e)/\Gamma(\mu \rightarrow \text{all}) < 1.0 \times 10^{-12}$ . For semileptonic processes, the best limit comes from the coherent conversion process in a muonic atom,  $\mu^- + (Z, A) \rightarrow e^- + (Z, A)$ , measured as  $\Gamma(\mu^- \text{Ti} \rightarrow e^- \text{Ti})/\Gamma(\mu^- \text{Ti} \rightarrow \text{all}) < 4.3 \times 10^{-12}$ . Of special interest is the case in which the hadronic flavor also changes, as in  $K_L \rightarrow e\mu$  and  $K^+ \rightarrow \pi^+ e^- \mu^+$ , measured as  $\Gamma(K_L \rightarrow e\mu)/\Gamma(K_L \rightarrow \text{all}) < 4.7 \times 10^{-12}$  and  $\Gamma(K^+ \rightarrow \pi^+ e^- \mu^+)/\Gamma(K^+ \rightarrow \text{all}) < 1.3 \times 10^{-11}$ . Limits on the conversion of  $\tau$  into  $e$  or  $\mu$  are found in  $\tau$  decay and are much less stringent than those for  $\mu \rightarrow e$  conversion, *e.g.*,  $\Gamma(\tau \rightarrow \mu\gamma)/\Gamma(\tau \rightarrow \text{all}) < 4.4 \times 10^{-8}$  and  $\Gamma(\tau \rightarrow e\gamma)/\Gamma(\tau \rightarrow \text{all}) < 3.3 \times 10^{-8}$ .

**c) Conversion of one type of charged lepton into another type of charged antilepton.** The case most studied is  $\mu^- + (Z, A) \rightarrow e^+ + (Z - 2, A)$ , the strongest limit being  $\Gamma(\mu^- \text{Ti} \rightarrow e^+ \text{Ca})/\Gamma(\mu^- \text{Ti} \rightarrow \text{all}) < 3.6 \times 10^{-11}$ .

**d) Neutrino oscillations.** It is expected even in the standard electroweak theory that the lepton numbers are not separately conserved, as a consequence of lepton mixing analogous to Cabibbo-Kobayashi-Maskawa quark mixing. However, if the only source of lepton-number violation is the mixing of low-mass neutrinos then processes such as  $\mu \rightarrow e\gamma$  are expected to have extremely small unobservable probabilities. For small neutrino masses, the lepton-number violation would be observed first in neutrino oscillations, which have been the subject of extensive experimental searches. Strong evidence for neutrino mixing has come from atmospheric and solar neutrinos. The SNO experiment has detected the total flux of neutrinos from the sun measured via neutral current interactions and found it greater than the flux of  $\nu_e$ . This confirms previous indications of a deficit of  $\nu_e$ . Furthermore, evidence for such oscillations

for reactor  $\bar{\nu}$  has been found by the KAMLAND detector. A global analysis combining all solar neutrino data (SNO, Borexino, Super-Kamiokande, Chlorine, Gallium) and the KamLAND data yields  $\Delta(m^2) = (7.59 \pm 0.20) \times 10^{-5} \text{ eV}^2$ [4].

Underground detectors observing neutrinos produced by cosmic rays in the atmosphere have found a factor of 2 deficiency of upward going  $\nu_\mu$  compared to downward. This provides compelling evidence for  $\nu_\mu$  disappearance, for which the most probable explanation is  $\nu_\mu \rightarrow \nu_\tau$  oscillations with nearly maximal mixing. This mixing space can also be explored by accelerator-based long-baseline experiments. The most recent result from MINOS gives  $\Delta(m^2) = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2$ [5].

## CONSERVATION OF HADRONIC FLAVORS

In strong and electromagnetic interactions, hadronic flavor is conserved, *i.e.* the conversion of a quark of one flavor ( $d, u, s, c, b, t$ ) into a quark of another flavor is forbidden. In the Standard Model, the weak interactions violate these conservation laws in a manner described by the Cabibbo-Kobayashi-Maskawa mixing (see the section “Cabibbo-Kobayashi-Maskawa Mixing Matrix”). The way in which these conservation laws are violated is tested as follows:

**(a)  $\Delta S = \Delta Q$  rule.** In the strangeness-changing semileptonic decay of strange particles, the strangeness change equals the change in charge of the hadrons. Tests come from limits on decay rates such as  $\Gamma(\Sigma^+ \rightarrow ne^+\nu)/\Gamma(\Sigma^+ \rightarrow \text{all}) < 5 \times 10^{-6}$ , and from a detailed analysis of  $K_L \rightarrow \pi e \nu$ , which yields the parameter  $x$ , measured to be  $(\text{Re } x, \text{Im } x) = (-0.002 \pm 0.006, 0.0012 \pm 0.0021)$ . Corresponding rules are  $\Delta C = \Delta Q$  and  $\Delta B = \Delta Q$ .

**(b) Change of flavor by two units.** In the Standard Model this occurs only in second-order weak interactions. The classic example is  $\Delta S = 2$  via  $K^0 - \bar{K}^0$  mixing, which is directly measured by  $m(K_L) - m(K_S) = (0.5292 \pm 0.0009) \times 10^{10} \text{ } \hbar s^{-1}$ . The  $\Delta B = 2$  transitions in the  $B^0$  and  $B_s^0$  systems via mixing are also well established. The measured mass differences between

the eigenstates are  $(m_{B_H^0} - m_{B_L^0}) = (0.507 \pm 0.005) \times 10^{12} \hbar s^{-1}$  and  $(m_{B_{sH}^0} - m_{B_{sL}^0}) = (17.77 \pm 0.12) \times 10^{12} \hbar s^{-1}$ . There is now strong evidence of  $\Delta C = 2$  transition in the charm sector with the mass difference  $m_{D_H^0} - m_{D_L^0} = (2.39_{-0.63}^{+0.59}) \times 10^{10} \hbar s^{-1}$ . All results are consistent with the second-order calculations in the Standard Model.

**(c) Flavor-changing neutral currents.** In the Standard Model the neutral-current interactions do not change flavor. The low rate  $\Gamma(K_L \rightarrow \mu^+ \mu^-)/\Gamma(K_L \rightarrow \text{all}) = (6.84 \pm 0.11) \times 10^{-9}$  puts limits on such interactions; the nonzero value for this rate is attributed to a combination of the weak and electromagnetic interactions. The best test should come from  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , which occurs in the Standard Model only as a second-order weak process with a branching fraction of  $(0.4 \text{ to } 1.2) \times 10^{-10}$ . Combining results from BNL-E787 and BNL-E949 experiments yield  $\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu})/\Gamma(K^+ \rightarrow \text{all}) = (1.7 \pm 1.1) \times 10^{-10}$ [6]. Limits for charm-changing or bottom-changing neutral currents are much less stringent:  $\Gamma(D^0 \rightarrow \mu^+ \mu^-)/\Gamma(D^0 \rightarrow \text{all}) < 1.3 \times 10^{-6}$  and  $\Gamma(B^0 \rightarrow \mu^+ \mu^-)/\Gamma(B^0 \rightarrow \text{all}) < 1.5 \times 10^{-8}$ . One cannot isolate flavor-changing neutral current (FCNC) effects in non leptonic decays. For example, the FCNC transition  $s \rightarrow d + (\bar{u} + u)$  is equivalent to the charged-current transition  $s \rightarrow u + (\bar{u} + d)$ . Tests for FCNC are therefore limited to hadron decays into lepton pairs. Such decays are expected only in second-order in the electroweak coupling in the Standard Model.

## References

1. R. Carosi *et al.*, Phys. Lett. **B237**, 303 (1990).
2. A. Alavi-Harati *et al.*, Phys. Rev. **D67**, 012005 (2003);  
B. Schwingenheuer *et al.*, Phys. Rev. Lett. **74**, 4376 (1995).
3. A. Angelopoulos *et al.*, Phys. Lett. **B444**, 43 (1998);  
L. Wolfenstein, Phys. Rev. Lett. **83**, 911 (1999).
4. B. Aharmim *et al.*, Phys. Rev. Lett. **101**, 111301 (2008).
5. P. Adamson *et al.*, Phys. Rev. Lett. **101**, 131802 (2008).
6. A.V. Artamonov *et al.*, Phys. Rev. Lett. **101**, 191802 (2008).