RARE KAON DECAYS

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A. Introduction: There are several useful reviews on rare kaon decays and related topics [1–15]. Activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters
3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+ \nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focused on the decays $K_L \to \pi^0 \ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+ \ell^+ \ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Category 4 also includes $K_L \to \pi^0 \gamma\gamma$ and $K_L \to \ell^+ \ell^- \gamma$. The former is important in understanding a CP-conserving contribution to $K_L \to \pi^0 \ell^+ \ell^-$, whereas the latter could shed light on long distance contributions to $K_L \to \mu^+ \mu^-$.

The interplay between Categories 2-4 can be illustrated in Fig. 1. The modes $K \to \pi\nu\overline{\nu}$ are the cleanest ones theoretically. They can provide accurate determinations of certain CKM parameters (shown in the figure). In combination with alternate determinations of these parameters, they also constrain new interactions. The modes $K_L \to \pi^0 e^+ e^-$, $K_L \to \pi^0 \mu^+ \mu^-$ and $K_L \to \mu^+ \mu^-$ are also sensitive to CKM parameters. However, they suffer from a series of hadronic uncertainties that can be addressed, at least in part, through a systematic study of the additional modes indicated in the figure.

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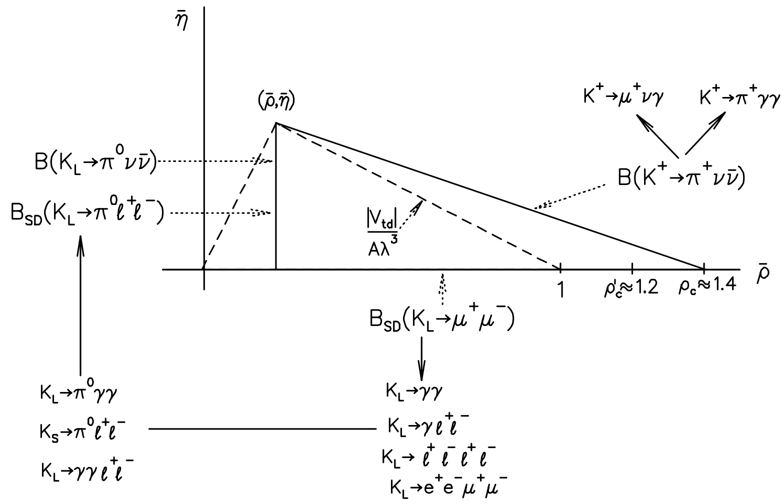

Figure 1: Role of rare kaon decays in determining the unitarity triangle. The solid arrows point to auxiliary modes needed to interpret the main results, or potential backgrounds to them.

B. Explicit violations of the Standard Model: Much activity has focussed on searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 4.7 \times 10^{-12}(148\ \mathrm{TeV}/M_X)^4$ [4]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of over 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV. The decays $K_L \to \mu^{\pm}e^{\mp}$ and $K^+ \to \pi^+ e^{\mp}\mu^{\pm}$ (or $K_L \to \pi^0 e^{\mp}\mu^{\pm}$) provide complementary information on potential family number violating interactions, since the former is sensitive to parity-odd couplings and the latter is sensitive to parity-even couplings. Limits on certain lepton-number violating kaon decays also exist, some recent ones being those of Refs. [16–18]. Related searches in μ and τ processes are discussed in our section "Tests of Conservation Laws."

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.
$K^+ \to \pi^+ e^- \mu^+$	1.2×10^{-11}	BNL-865	2005/Ref. 19
$K^+ \to \pi^+ e^+ \mu^-$	5.2×10^{-10}	BNL-865	2000/Ref. 16
$K_L \to \mu e$	4.7×10^{-12}	BNL-871	1998/Ref. 20
$K_L \to \pi^0 e\mu$	7.6×10^{-11}	KTeV	2008/Ref. 21
$K_L \to \pi^0\pi^0 e\mu$	1.7×10^{-10}	KTeV	2008/Ref. 21

Physics beyond the SM is also pursued through the search for $K^+ \to \pi^+ X^0$, where X^0 is a new light particle. The searches cover both long-lived particles (*e.g.*, hyperphoton, axion, familon, *etc.*), and short lived ones that decay to muon, electron or photon pairs. The 90% CL upper limit on $K^+ \to \pi^+ X^0$ is 7.3×10^{-11} [22]. Recent new bounds for a short lived pseudoscalar X^0 decaying to muons or photons are $B(K_L \to \pi^0\pi^0\mu^+\mu^-) < 1\times10^{-10}$ [23] and $B(K_L \to \pi^0\pi^0\gamma\gamma) < 2.4\times10^{-7}$ [24].

C. Measurements of Standard Model parameters:

In the SM, the decay $K^+ \to \pi^+\nu\overline{\nu}$ is dominated by one-loop diagrams with top-quark intermediate states and long-distance contributions are known to be quite small [2,25]. This permits a precise calculation of this rate in terms of SM parameters. Studies of this process are thus motivated by the possibility of detecting non-SM physics when comparing with the results of global fits [28,29].

BNL-787 observed two candidate events [30,31] in the clean high π^+ momentum and one event [32] in the low-momentum region. The successor experiment BNL-949 observed one more in the high-momentum region [22] and three more in the low-momentum region [33] yielding a branching ratio of $(1.73^{+1.15}_{-1.05})\times 10^{-10}$ [34]. A new experiment, NA62, with a sensitivity goal of $\sim 10^{-12}$/event was proposed [35] at CERN in 2005. It has been approved and is scheduled to run with a partial detector in autumn 2012. In the future, this mode may provide grounds for precision tests of flavor dynamics [36]. The branching ratio can be written in a compact form that exhibits the different ingredients that go into the calculation [37],

$$\mathrm{B}(K^+ \to \pi^+\nu\overline{\nu}(\gamma)) = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\frac{\mathrm{Im}(V_{ts}^\star V_{td})}{\lambda^5}X_t\right)^2 + \left(\frac{\mathrm{Re}(V_{cs}^\star V_{cd})}{\lambda}(P_c+\delta P_{c,u}) + \frac{\mathrm{Re}(V_{ts}^\star V_{td})}{\lambda^5}X_t\right)^2\right]. \quad (1)$$

The parameters in Eq. (1) incorporate the *a priori* unknown hadronic matrix element in terms of the very well-measured K_{e3} rate [2] in κ_+; long distance QED corrections in Δ_{EM} [27]; the Inami-Lim function for the short distance top-quark contribution [38] including NLO QCD corrections [39] and the two-loop electroweak correction [37], all in X_t; and the charm-quark contributions due to short distance effects including NNLO QCD corrections [40] and NLO electroweak corrections via P_c [41], as well as certain long distance effects via $\delta P_{c,u}$ [26]. An interesting approximate way to cast this result in terms of the CKM parameters λ, V_{cb}, $\overline{\rho}$ and $\overline{\eta}$ (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [11] is:

$$\mathrm{B}(K^+ \to \pi^+\nu\overline{\nu}) \approx 1.6\times 10^{-5}|V_{cb}|^4[\sigma\overline{\eta}^2 + (\rho_c - \overline{\rho})^2], \quad (2)$$

where $\rho_c \approx 1.45$ and $\sigma \equiv 1/(1-\frac{1}{2}\lambda^2)^2$. Thus, $\mathrm{B}(K^+ \to \pi^+\nu\overline{\nu})$ determines an ellipse in the $\overline{\rho}$, $\overline{\eta}$ plane with center $(\rho_c, 0)$ and semiaxes $\approx \frac{1}{|V_{cb}|^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.6\times10^{-5}}}$ and $\frac{1}{\sigma|V_{cb}|^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.6\times10^{-5}}}$. The latest numerical study leads to a predicted branching ratio $(7.81^{+0.80}_{-0.71} \pm 0.29)\times 10^{-11}$ [37], near the lower end of the measurement of BNL-787 and 949.

Modes with an extra pion, $K \to \pi\pi\nu\bar{\nu}$, could also be used in the extraction of CKM parameters as they are also dominated by short distance contributions [42]. However, they occur at much lower rates with branching rations of order 10^{-13}, and the current best bound from E391a is $B(K_L \to \pi^0\pi^0\nu\bar{\nu}) < 8.1\times 10^{-7}$ at 90% c.l. [43]. There is also an older bound of $B(K^+ \to \pi^+\pi^0\nu\bar{\nu}) < 4.3\times 10^{-5}$ at 90% c.l. [44] from BNL E787.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter $\overline{\rho}$, given by [11]:

$$\mathrm{B_{SD}}(K_L \to \mu^+\mu^-) \approx 2.7 \times 10^{-4} |V_{cb}|^4 (\rho'_c - \overline{\rho})^2 \tag{3}$$

where ρ'_c depends on the charm quark mass and is approximately 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is determined by the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B_{abs}}(K_L \to \mu^+\mu^-) = (6.64 \pm 0.07) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$ [45]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. The latter cannot be derived directly from experiment [46], but can be estimated with certain assumptions [47,48]. The decay $K_L \to e^+e^-$ is completely dominated by long distance physics and is easier to estimate. The result, $B(K_L \to e^+e^-) \sim 9 \times 10^{-12}$ [46,49], is in good agreement with the BNL-871 measurement, $(8.7^{+5.7}_{-4.1}) \times 10^{-12}$ [50].

D. Searches for direct CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [2,51,52]. In the Standard Model, this mode is dominated by an intermediate top-quark state and does not suffer from the small uncertainty associated with the charm-quark intermediate state that affects the mode $K^+ \to \pi^+\nu\overline{\nu}$. The branching ratio is given by Ref. 11:

$$\begin{aligned} \mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) &= \kappa_L \left(\frac{\mathrm{Im}(V^\star_{ts} V_{td})}{\lambda^5} X_t \right)^2 \\ &\approx 7.6 \times 10^{-5} |V_{cb}|^4 \overline{\eta}^2 \ . \end{aligned} \tag{4}$$

The hadronic matrix element can be related to that measured in $K_{\ell 3}$ decay and is parameterized in κ_L. The latest numerical evaluation leads to a predicted branching ratio $(2.43^{+0.40}_{-0.37} \pm 0.06) \times 10^{-11}$ [37]. The 90% CL bound on $K^+ \to \pi^+\nu\overline{\nu}$ provides a nearly model-independent bound $B(K_L \to \pi^0\nu\overline{\nu}) < 1.46 \times 10^{-9}$ [53]. KEK-391a, which took data in 2004 and

2005, has published a 90% CL upper bound of $\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \leq 2.6 \times 10^{-8}$ [54] The KOTO experiment, whose initial goal is to reach the 10^{-11}/event level, is in the final stages of construction at J-PARC [55].

There has been much theoretical work on possible contributions to rare K decays beyond the SM. A comprehensive discussion of these can be found in Refs. [14] and [56].

The decay $K_L \to \pi^0 e^+ e^-$ also has sensitivity to the CKM parameter η through its CP-violating component. There are both direct and indirect CP-violating amplitudes which can interfere. The direct CP-violating amplitude is short distance dominated and has been calculated in detail within the SM [8]. The indirect CP-violating amplitude can be inferred from a measurement of $K_S \to \pi^0 e^+ e^-$. The complete CP-violating contribution to the rate can be written as [57,58]:

$$\mathrm{B}_{\mathrm{CPV}} \approx 10^{-12}\left[15.7|a_S|^2 \pm 1.4\left(\frac{|V_{cb}|^2\overline{\eta}}{10^{-4}}\right)|a_S| + 0.12\left(\frac{|V_{cb}|^2\overline{\eta}}{10^{-4}}\right)^2\right] \qquad (5)$$

where the three terms correspond to the indirect CP violation, the interference, and the direct CP violation respectively. The parameter a_S has been extracted by NA48 from a measurement of the decay $K_S \to \pi^0 e^+ e^-$ with the result $|a_S| = 1.06^{+0.26}_{-0.21} \pm 0.07$ [59], as well as from a measurement of the decay $K_S \to \pi^0 \mu^+ \mu^-$ with the result $|a_s| = 1.54^{+0.40}_{-0.32} \pm 0.06$ [60]. With current constraints on the CKM parameters, and assuming a positive sign for the interference term [58,61], this implies that $\mathrm{B}_{\mathrm{CPV}}(K_L \to \pi^0 e^+ e^-) \approx (3.1 \pm 0.9) \times 10^{-11}$, and that the indirect CP violation is larger than the direct CP violation. The complete CP violating amplitude for the related mode $K_L \to \pi^0 \mu^+ \mu^-$ is predicted to be $\mathrm{B}_{\mathrm{CPV}}(K_L \to \pi^0 \mu^+ \mu^-) \approx (1.4 \pm 0.5) \times 10^{-11}$ [62,15].

$K_L \to \pi^0 \gamma\gamma$ also has a CP-conserving component dominated by a two-photon intermediate state. This component can be decomposed into an absorptive and a dispersive part. The absorptive part can be extracted from the measurement of the low $m_{\gamma\gamma}$ region of the $K_L \to \pi^0 \gamma\gamma$ spectrum. The rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$ in $K_L \to \pi^0\gamma\gamma$ are well described in chiral perturbation theory in terms of three (*a priori*) unknown parameters [63,64].

Both KTeV and NA48 have studied the mode $K_L \to \pi^0\gamma\gamma$, reporting similar results. KTeV finds $B(K_L \to \pi^0\gamma\gamma) = (1.29 \pm 0.03_{\text{stat}} \pm 0.05_{\text{sys}}) \times 10^{-6}$ [65], while NA48 finds $B(K_L \to \pi^0\gamma\gamma) = (1.36 \pm 0.03_{\text{stat}} \pm 0.03_{\text{sys}} \pm 0.03_{\text{norm}}) \times 10^{-6}$ [66]. Both experiments are consistent with a negligible rate in the low $m_{\gamma\gamma}$ region, suggesting a very small CP-conserving component $B_{\text{CP}}(K_L \to \pi^0 e^+e^-) \sim \mathcal{O}(10^{-13})$ [58,64,66]. There remains some model dependence in the estimate of the dispersive part of the CP-conserving $K_L \to \pi^0 e^+e^-$ [58].

The related process, $K_L \to \pi^0\gamma e^+e^-$, is potentially an additional background in some region of phase space [67]. This process has been observed with a branching ratio of $(1.62 \pm 0.14_{\text{stat}} \pm 0.09_{\text{sys}}) \times 10^{-8}$ [68].

The decay $K_L \to \gamma\gamma e^+e^-$ constitutes the dominant background to $K_L \to \pi^0 e^+e^-$. It was first observed by BNL-845 [69], and subsequently confirmed with a much larger sample by FNAL-799 [70]. It has been estimated that this background will enter at about the 10^{-10} level [71,72], comparable to or larger than the signal level. Because of this, the observation of $K_L \to \pi^0 e^+e^-$ at the SM level will depend on background subtraction with good statistics. Possible alternative strategies are discussed in Ref. 58 and references cited therein.

The 90% CL upper bound for the process $K_L \to \pi^0 e^+e^-$ is 2.8×10^{-10} [72]. For the closely related muonic process, the published upper bound is $\text{B}(K_L \to \pi^0\mu^+\mu^-) \leq 3.8 \times 10^{-10}$ [73], compared with the SM prediction of $(1.5 \pm 0.3) \times 10^{-11}$ [62] (assuming positive interference between the direct- and indirect-CP violating components).

A study of $K_L \to \pi^0\mu^+\mu^-$ has indicated that it might be possible to extract the direct CP-violating contribution by a joint study of the Dalitz plot variables and the components of the μ^+ polarization [74]. The latter tends to be quite substantial so that large statistics may not be necessary.

Combined information from the two $K_L \to \pi^0\ell^+\ell^-$ modes complements the $K \to \pi\nu\overline{\nu}$ measurements in constraining physics beyond the SM [75].

E. Other long distance dominated modes:

The decays $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e$ or μ) have received considerable attention. The rate and spectrum have been measured for both the electron and muon modes [76,77,18]. Ref. 57 has proposed a parametrization inspired by chiral perturbation theory, which provides a successful description of data but indicates the presence of large corrections beyond leading order. More work is needed to fully understand the origin of these large corrections.

Much information has been recorded by KTeV and NA48 on the rates and spectrum for the Dalitz pair conversion modes $K_L \to \ell^+\ell^-\gamma$ [78,79], and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ [17,80–82]. All these results are used to test hadronic models and could further our understanding of the long distance component in $K_L \to \mu^+\mu^-$.

References

1. D. Bryman, Int. J. Mod. Phys. **A4**, 79 (1989).
2. J. Hagelin and L. Littenberg, Prog. in Part. Nucl. Phys. **23**, 1 (1989).
3. L. Littenberg and G. Valencia, Ann. Rev. Nucl. and Part. Sci. **43**, 729 (1993).
4. J. Ritchie and S. Wojcicki, Rev. Mod. Phys. **65**, 1149 (1993).
5. B. Winstein and L. Wolfenstein, Rev. Mod. Phys. **65**, 1113 (1993).
6. G. D'Ambrosio *et al.*, *Radiative Non-Leptonic Kaon Decays*, in The DAΦNE Physics Handbook (second edition), eds. L. Maiani, G. Pancheri, and N. Paver (Frascati), Vol. I, 265 (1995).
7. A. Pich, Rept. on Prog. in Phys. **58**, 563 (1995).
8. G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
9. G. D'Ambrosio and G. Isidori, Int. J. Mod. Phys. **A13**, 1 (1996).
10. P. Buchholz and B. Renk, Prog. in Part. Nucl. Phys. **39**, 253 (1997).
11. A.J. Buras and R. Fleischer, TUM-HEP-275-97, `hep-ph/9704376`, *Heavy Flavours II*, World Scientific, eds. A.J. Buras and M. Lindner (1997), 65–238.

12. A.J. Buras, TUM-HEP-349-99, Lectures given at Lake Louise Winter Institute: Electroweak Physics, Lake Louise, Alberta, Canada, 14–20 Feb. 1999.

13. A.R. Barker and S.H. Kettell, Ann. Rev. Nucl. and Part. Sci. **50**, 249 (2000).

14. A.J. Buras, F. Schwab, and S. Uhlig, Rev. Mod. Phys. **80**, 965 (2008).

15. V. Cirigliano *et al.*, "Kaon Decays in the Standard Model," [`arXiv:1107.6001 [hep-ph]`].

16. R. Appel *et al.*, Phys. Rev. Lett. **85**, 2877 (2000).

17. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **90**, 141801 (2003).

18. J.R. Batley *et al.*, Phys. Lett. **B697**, 107 (2011).

19. A. Sher *et al.*, Phys. Rev. **D72**, 012005 (2005).

20. D. Ambrose *et al.*, Phys. Rev. Lett. **81**, 5734 (1998).

21. E. Abouzaid *et al.*, Phys. Rev. Lett. **100**, 131803 (2008).

22. V.V. Anisimovsky *et al.*, Phys. Rev. Lett. **93**, 031801 (2004).

23. E. Abouzaid *et al.*, [`arXiv:1105.4800 [hep-ex]`] see also, D.G. Phillips II, "Search for the Rare Decay $K_L \to \pi^0\pi^0\mu^+\mu^-$," University of Virginia thesis, May 2009.

24. Y.C. Tung *et al.*, Phys. Rev. Lett. **102**, 051802 (2009).

25. M. Lu and M.B. Wise, Phys. Lett. **B324**, 461 (1994); A.F. Falk, A. Lewandowski, and A.A. Petrov, Phys. Lett. **B505**, 107 (2001).

26. G. Isidori, F. Mescia, and C. Smith, Nucl. Phys. **B718**, 319 (2005);
A.F. Falk, A. Lewandowski, and A.A. Petrov, Phys. Lett. **B505**, 107 (2001).

27. F. Mescia and C. Smith, Phys. Rev. **D76**, 034017 (2007).

28. CKMfitter Group (J. Charles *et al.*), Phys. Rev. **D84**, 1 (2011), [`arXiv:1106.4041[hep-ph]`], updated results and plots available at: `http://ckmfitter.in2p3.fr`.

29. M. Bona *et al.*, [UTfit Collaboration] "Model-independent constraints on Delta F=2 operators and the scale of New Physics," `arXiv:0707.0636` [hep-ph].

30. S. Adler *et al.*, Phys. Rev. Lett. **88**, 041803 (2002).

31. S. Adler *et al.*, Phys. Rev. Lett. **84**, 3768 (2000).

32. S. Adler *et al.*, Phys. Lett. **B537**, 237 (2002).

33. A.V. Artamonov *et al.*, Phys. Rev. Lett. **101**, 191802 (2008).

34. A.V. Artamonov *et al.*, Phys. Rev. **D79**, 092004 (2009).

35. G. Anelli *et al.*, CERN-SPSC-2005-013, 11 June 2005.

36. G. D'Ambrosio and G. Isidori, Phys. Lett. **B530**, 108 (2002).

37. J. Brod, M. Gorbahn, and E. Stamou, Phys. Rev. **D83**, 034030 (2011).

38. T. Inami and C.S. Lim, Prog. Theor. Phys. **65**, 297 (1981); Erratum Prog. Theor. Phys. **65**, 172 (1981).

39. G. Buchalla and A.J. Buras, Nucl. Phys. **B548**, 309 (1999);
M. Misiak and J. Urban, Phys. Lett. **B451**, 161 (1999).

40. A.J. Buras *et al.*, Phys. Rev. Lett. **95**, 261805 (2005);
A.J. Buras *et al.*, JHEP **0611**, 002 (2006).

41. J. Brod and M. Gorbahn, Phys. Rev. **D78**, 034006 (2008).

42. L. Littenberg and G. Valencia, Phys. Lett. **B385**, 379 (1996);
C.-W. Chiang and F.J. Gilman, Phys. Rev. **D62**, 094026 (2000);
C.Q. Geng, I.J. Hsu, and Y.C. Lin, Phys. Rev. **D50**, 5744 (1994).

43. R. Ogata, *et al.*, `arXiv:1106.3404(2011).`

44. S. Adler, *et al.*, Phys. Rev. **D63**, 032004 (2001).

45. D. Ambrose *et al.*, Phys. Rev. Lett. **84**, 1389 (2000).

46. G. Valencia, Nucl. Phys. **B517**, 339 (1998).

47. G. D'Ambrosio, G. Isidori, and J. Portoles, Phys. Lett. **B423**, 385 (1998).

48. G. Isidori and R. Unterdorfer, JHEP **0401**, 009 (2004).

49. D. Gomez-Dumm and A. Pich, Phys. Rev. Lett. **80**, 4633 (1998).

50. D. Ambrose *et al.*, Phys. Rev. Lett. **81**, 4309 (1998).

51. L. Littenberg, Phys. Rev. **D39**, 3322 (1989).

52. G. Buchalla and G. Isidori, Phys. Lett. **B440**, 170 (1998).

53. Y. Grossman and Y. Nir, Phys. Lett. **B398**, 163 (1997).

54. J.K. Ahn *et al.*, Phys. Rev. **D81**, 072004 (2010).

55. J. Comfort *et al.*, "Proposal for $K_L^0 \rightarrow \pi^0 \nu\overline{\nu}$ Experiment at J-Parc," J-PARC Proposal 14 (2006).

56. D. Bryman *et al.*, Int. J. Mod. Phys. **A21**, 487 (2006).

57. G. D'Ambrosio *et al.*, JHEP **9808**, 004 (1998);
C.O. Dib, I. Dunietz, and F.J. Gilman, Phys. Rev. **D39**, 2639 (1989).

58. G. Buchalla, G. D'Ambrosio, and G. Isidori, Nucl. Phys. **B672**, 387 (2003).

59. J.R. Batley *et al.*, Phys. Lett. **B576**, 43 (2003).

60. J.R. Batley *et al.*, Phys. Lett. **B599**, 197 (2004).
61. S. Friot, D. Greynat, and E. de Rafael, Phys. Lett. **B595**, 301 (2004).
62. G. Isidori, C. Smith, and R. Unterdorfer, Eur. Phys. J. **C36**, 57 (2004).
63. G. Ecker, A. Pich, and E. de Rafael, Phys. Lett. **237B**, 481 (1990);
L. Cappiello, G. D'Ambrosio, and M. Miragliuolo, Phys. Lett. **B298**, 423 (1993);
A. Cohen, G. Ecker, and A. Pich, Phys. Lett. **B304**, 347 (1993).
64. F. Gabbiani and G. Valencia, Phys. Rev. **D66**, 074006 (2002).
65. E. Abouzaid *et al.*, Phys. Rev. **D77**, 112004 (2008).
66. A. Lai *et al.*, Phys. Lett. **B536**, 229 (2002).
67. J. Donoghue and F. Gabbiani, Phys. Rev. **D56**, 1605 (1997).
68. E. Abouzaid *et al.*, Phys. Rev. **D76**, 052001 (2007).
69. W.M. Morse *et al.*, Phys. Rev. **D45**, 36 (1992).
70. A. Alavi-Harati *et al.*, Phys. Rev. **D64**, 012003 (2001).
71. H.B. Greenlee, Phys. Rev. **D42**, 3724 (1990).
72. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **93**, 021805 (2004).
73. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **84**, 5279 (2000).
74. M.V. Diwan, H. Ma, and T.L. Trueman, Phys. Rev. **D65**, 054020 (2002).
75. F. Mescia, C. Smith, and S. Trine, JHEP **0608**, 088 (2006).
76. R. Appel *et al.*, Phys. Rev. Lett. **83**, 4482 (1999);
J.R. Batley *et al.*, Phys. Lett. **B677**, 246 (2009).
77. S.C. Adler *et al.*, Phys. Rev. Lett. **79**, 4756 (1997);
R. Appel *et al.*, Phys. Rev. Lett. **84**, 2580 (2000);
H.K. Park *et al.*, Phys. Rev. Lett. **88**, 111801 (2002).
78. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **87**, 071801 (2001).
79. A. Abouzaid *et al.*, Phys. Rev. Lett. **99**, 051804 (2007).
80. J.R. LaDue "Understanding Dalitz Decays of the K_L in particular the decays of $K_L \rightarrow e^+e^-\gamma$ and $K_L \rightarrow e^+e^-e^+e^-$" University of Colorado Thesis, May 2003. The preliminary result for $K_L \rightarrow e^+e^-\gamma$ in this thesis has been superseded by the final result in [79].
81. A. Alavi-Harati *et al.*, Phys. Rev. Lett. **86**, 5425 (2001).
82. V. Fanti *et al.*, Phys. Lett. **B458**, 458 (1999).