

## $B^0-\bar{B}^0$ MIXING

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There are two neutral  $B^0-\bar{B}^0$  meson systems,  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  (generically denoted  $B_q^0-\bar{B}_q^0$ ,  $q = s, d$ ), which exhibit particle-antiparticle mixing [1]. This mixing phenomenon is described in Ref. 2. In the following, we adopt the notation introduced in Ref. 2, and assume  $CPT$  conservation throughout. In each system, the light (L) and heavy (H) mass eigenstates,

$$|B_{L,H}\rangle = p|B_q^0\rangle \pm q|\bar{B}_q^0\rangle, \quad (1)$$

have a mass difference  $\Delta m_q = m_H - m_L > 0$ , and a total decay width difference  $\Delta\Gamma_q = \Gamma_L - \Gamma_H$ . In the absence of  $CP$  violation in the mixing,  $|q/p| = 1$ , these differences are given by  $\Delta m_q = 2|M_{12}|$  and  $|\Delta\Gamma_q| = 2|\Gamma_{12}|$ , where  $M_{12}$  and  $\Gamma_{12}$  are the off-diagonal elements of the mass and decay matrices [2]. The evolution of a pure  $|B_q^0\rangle$  or  $|\bar{B}_q^0\rangle$  state at  $t = 0$  is given by

$$|B_q^0(t)\rangle = g_+(t)|B_q^0\rangle + \frac{q}{p}g_-(t)|\bar{B}_q^0\rangle, \quad (2)$$

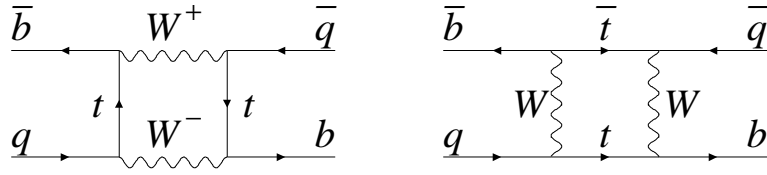
$$|\bar{B}_q^0(t)\rangle = g_+(t)|\bar{B}_q^0\rangle + \frac{p}{q}g_-(t)|B_q^0\rangle, \quad (3)$$

which means that the flavor states remain unchanged (+) or oscillate into each other (–) with time-dependent probabilities proportional to

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma_q t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma_q}{2}t\right) \pm \cos(\Delta m_q t) \right], \quad (4)$$

where  $\Gamma_q = (\Gamma_H + \Gamma_L)/2$ . In the absence of  $CP$  violation, the time-integrated mixing probability  $\int |g_-(t)|^2 dt / (\int |g_-(t)|^2 dt + \int |g_+(t)|^2 dt)$  is given by

$$\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2 + 1)}, \quad \text{where} \quad x_q = \frac{\Delta m_q}{\Gamma_q}, \quad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}. \quad (5)$$



**Figure 1:** Dominant box diagrams for the  $B_q^0 \rightarrow \bar{B}_q^0$  transitions ( $q = d$  or  $s$ ). Similar diagrams exist where one or both  $t$  quarks are replaced with  $c$  or  $u$  quarks.

### ***Standard Model predictions and phenomenology***

In the Standard Model, the transitions  $B_q^0 \rightarrow \bar{B}_q^0$  and  $\bar{B}_q^0 \rightarrow B_q^0$  are due to the weak interaction. They are described, at the lowest order, by box diagrams involving two  $W$  bosons and two up-type quarks (see Fig. 1), as is the case for  $K^0 - \bar{K}^0$  mixing. However, the long range interactions arising from intermediate virtual states are negligible for the neutral  $B$  meson systems, because the large  $B$  mass is off the region of hadronic resonances. The calculation of the dispersive and absorptive parts of the box diagrams yields the following predictions for the off-diagonal element of the mass and decay matrices [3],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} S_0(m_t^2/m_W^2) (V_{tq}^* V_{tb})^2, \quad (6)$$

$$\Gamma_{12} = \frac{G_F^2 m_b^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi} \times \left[ (V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb} \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) + (V_{cq}^* V_{cb})^2 \mathcal{O}\left(\frac{m_c^4}{m_b^4}\right) \right], \quad (7)$$

where  $G_F$  is the Fermi constant,  $m_W$  the  $W$  boson mass, and  $m_i$  the mass of quark  $i$ ;  $m_{B_q}$ ,  $f_{B_q}$  and  $B_{B_q}$  are the  $B_q^0$  mass, weak decay constant and bag parameter, respectively. The known function  $S_0(x_t)$  can be approximated very well by  $0.784 x_t^{0.76}$  [4], and  $V_{ij}$  are the elements of the CKM matrix [5]. The QCD corrections  $\eta_B$  and  $\eta'_B$  are of order unity. The only

non-negligible contributions to  $M_{12}$  are from box diagrams involving two top quarks. The phases of  $M_{12}$  and  $\Gamma_{12}$  satisfy

$$\phi_M - \phi_\Gamma = \pi + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right), \quad (8)$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the  $K^0-\bar{K}^0$  system, the heavy state is expected to have a smaller decay width than that of the light state:  $\Gamma_H < \Gamma_L$ . Hence,  $\Delta\Gamma = \Gamma_L - \Gamma_H$  is expected to be positive in the Standard Model.

Furthermore, the quantity

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2} \frac{m_b^2}{m_W^2} \frac{1}{S_0(m_t^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right) \quad (9)$$

is small, and a power expansion of  $|q/p|^2$  yields

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\phi_M - \phi_\Gamma) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right). \quad (10)$$

Therefore, considering both Eqs. (8) and (9), the  $CP$ -violating parameter

$$1 - \left|\frac{q}{p}\right|^2 \simeq \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \quad (11)$$

is expected to be very small:  $\sim \mathcal{O}(10^{-3})$  for the  $B_d^0-\bar{B}_d^0$  system and  $\lesssim \mathcal{O}(10^{-4})$  for the  $B_s^0-\bar{B}_s^0$  system [6].

In the approximation of negligible  $CP$  violation in mixing, the ratio  $\Delta\Gamma_q/\Delta m_q$  is equal to the small quantity  $|\Gamma_{12}/M_{12}|$  of Eq. (9); it is hence independent of CKM matrix elements, *i.e.*, the same for the  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  systems. Calculations [7] yield  $\sim 5 \times 10^{-3}$  with a  $\sim 20\%$  uncertainty. Given the published experimental knowledge [8] on the mixing parameter  $x_q$

$$\begin{cases} x_d = 0.770 \pm 0.008 & (B_d^0-\bar{B}_d^0 \text{ system}) \\ x_s = 26.49 \pm 0.29 & (B_s^0-\bar{B}_s^0 \text{ system}) \end{cases}, \quad (12)$$

the Standard Model thus predicts that  $\Delta\Gamma_d/\Gamma_d$  is very small (below 1%), but  $\Delta\Gamma_s/\Gamma_s$  considerably larger ( $\sim 10\%$ ). These width differences are caused by the existence of final states to which both the  $B_q^0$  and  $\bar{B}_q^0$  mesons can decay. Such decays involve  $b \rightarrow c\bar{c}q$  quark-level transitions, which are Cabibbo-suppressed if  $q = d$  and Cabibbo-allowed if  $q = s$ .

A complete set of Standard Model predictions for all mixing parameters in both the  $B_d^0\text{--}\overline{B}_d^0$  and  $B_s^0\text{--}\overline{B}_s^0$  systems can be found in Ref. 7.

***Experimental issues and methods for oscillation analyses***

Time-integrated measurements of  $B^0\text{--}\overline{B}^0$  mixing were published for the first time in 1987 by UA1 [9] and ARGUS [10], and since then by many other experiments. These measurements are typically based on counting same-sign and opposite-sign lepton pairs from the semileptonic decay of the produced  $b\overline{b}$  pairs. Such analyses cannot easily separate the contributions from the different  $b$ -hadron species, therefore, the clean environment of  $\Upsilon(4S)$  machines (where only  $B_d^0$  and charged  $B_u$  mesons are produced) is in principle best suited to measure  $\chi_d$ .

However, better sensitivity is obtained from time-dependent analyses aiming at the direct measurement of the oscillation frequencies  $\Delta m_d$  and  $\Delta m_s$ , from the proper time distributions of  $B_d^0$  or  $B_s^0$  candidates identified through their decay in (mostly) flavor-specific modes, and suitably tagged as mixed or unmixed. This is particularly true for the  $B_s^0\text{--}\overline{B}_s^0$  system, where the large value of  $x_s$  implies maximal mixing, *i.e.*,  $\chi_s \simeq 1/2$ . In such analyses, the  $B_d^0$  or  $B_s^0$  mesons are either fully reconstructed, partially reconstructed from a charm meson, selected from a lepton with the characteristics of a  $b \rightarrow \ell^-$  decay, or selected from a reconstructed displaced vertex. At high-energy colliders (LEP, SLC, Tevatron, LHC), the proper time  $t = \frac{m_B}{p}L$  is measured from the distance  $L$  between the production vertex and the  $B$  decay vertex, and from an estimate of the  $B$  momentum  $p$ . At asymmetric  $B$  factories (KEKB, PEP-II), producing  $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d^0\overline{B}_d^0$  events with a boost  $\beta\gamma$  ( $= 0.425, 0.55$ ), the proper time difference between the two  $B$  candidates is estimated as  $\Delta t \simeq \frac{\Delta z}{\beta\gamma c}$ , where  $\Delta z$  is the spatial separation between the two  $B$  decay vertices along the boost direction. In all cases, the good resolution needed on the vertex positions is obtained with silicon detectors.

The average statistical significance  $\mathcal{S}$  of a  $B_d^0$  or  $B_s^0$  oscillation signal can be approximated as [11]

$$\mathcal{S} \approx \sqrt{N/2} f_{\text{sig}} (1 - 2\eta) e^{-(\Delta m \sigma_t)^2/2}, \quad (13)$$

where  $N$  is the number of selected and tagged candidates,  $f_{\text{sig}}$  is the fraction of signal in that sample,  $\eta$  is the total mistag probability, and  $\sigma_t$  is the resolution on proper time (or proper time difference). The quantity  $\mathcal{S}$  decreases very quickly as  $\Delta m$  increases; this dependence is controlled by  $\sigma_t$ , which is therefore a critical parameter for  $\Delta m_s$  analyses. At high-energy colliders, the proper time resolution  $\sigma_t \sim \frac{m_B}{\langle p \rangle} \sigma_L \oplus t \frac{\sigma_p}{p}$  includes a constant contribution due to the decay length resolution  $\sigma_L$  (typically 0.04–0.3 ps), and a term due to the relative momentum resolution  $\sigma_p/p$  (typically 10–20% for partially reconstructed decays), which increases with proper time. At  $B$  factories, the boost of the  $B$  mesons is estimated from the known beam energies, and the term due to the spatial resolution dominates (typically 1–1.5 ps because of the much smaller  $B$  boost).

In order to tag a  $B$  candidate as mixed or unmixed, it is necessary to determine its flavor both in the initial state and in the final state. The initial and final state mistag probabilities,  $\eta_i$  and  $\eta_f$ , degrade  $\mathcal{S}$  by a total factor  $(1 - 2\eta) = (1 - 2\eta_i)(1 - 2\eta_f)$ . In lepton-based analyses, the final state is tagged by the charge of the lepton from  $b \rightarrow \ell^-$  decays; the largest contribution to  $\eta_f$  is then due to  $\bar{b} \rightarrow \bar{c} \rightarrow \ell^-$  decays. Alternatively, the charge of a reconstructed charm meson ( $D^{*-}$  from  $B_d^0$  or  $D_s^-$  from  $B_s^0$ ), or that of a kaon hypothesized to come from a  $b \rightarrow c \rightarrow s$  decay [12], can be used. For fully-inclusive analyses based on topological vertexing, final-state tagging techniques include jet-charge [13] and charge-dipole [14,15] methods. At high-energy colliders, the methods to tag the initial state (*i.e.*, the state at production), can be divided into two groups: the ones that tag the initial charge of the  $\bar{b}$  quark contained in the  $B$  candidate itself (same-side tag), and the ones that tag the initial charge of the other  $b$  quark produced in the event (opposite-side tag). On the same side, the sign of a charged pion or kaon from the primary vertex is correlated with the production state of the  $B_d^0$  or  $B_s^0$  if that particle is a decay product of a  $B^{**}$  state or

the first in the fragmentation chain [16,17]. Jet- and vertex-charge techniques work on both sides and on the opposite side, respectively. Finally, the charge of a lepton from  $b \rightarrow \ell^-$  or of a kaon from  $b \rightarrow c \rightarrow s$  can be used as opposite side tags, keeping in mind that their performance is degraded due to integrated mixing. At SLC, the beam polarization produced a sizeable forward-backward asymmetry in the  $Z \rightarrow b\bar{b}$  decays, and provided another very interesting and effective initial state tag based on the polar angle of the  $B$  candidate [14]. Initial state tags have also been combined to reach  $\eta_i \sim 26\%$  at LEP [17,18], or even 22% at SLD [14] with full efficiency. In the case  $\eta_f = 0$ , this corresponds to an effective tagging efficiency  $Q = \epsilon D^2 = \epsilon(1 - 2\eta)^2$ , where  $\epsilon$  is the tagging efficiency, in the range 23 – 31%. The equivalent figure achieved by CDF during Tevatron Run I was  $\sim 3.5\%$  [19], reflecting the fact that tagging is more difficult at hadron colliders. The current CDF and DØ analyses of Tevatron Run II data reach  $\epsilon D^2 = (1.8 \pm 0.1)\%$  [20] and  $(2.5 \pm 0.2)\%$  [21] for opposite-side tagging, while same-side kaon tagging (for  $B_s^0$  analyses) is contributing an additional 3.7 – 4.8% at CDF [20], and pushes the combined performance to  $(4.7 \pm 0.5)\%$  at DØ [22]. LHCb, operating in the forward region at the LHC where the environment is different in terms of track multiplicity and  $b$ -hadron production kinematics, has reported  $\epsilon D^2 = (2.10 \pm 0.25)\%$  [23] for opposite-side tagging and  $(1.3 \pm 0.4)\%$  [24] for same-side kaon tagging.

At  $B$  factories, the flavor of a  $B_d^0$  meson at production cannot be determined, since the two neutral  $B$  mesons produced in a  $\Upsilon(4S)$  decay evolve in a coherent  $P$ -wave state where they keep opposite flavors at any time. However, as soon as one of them decays, the other follows a time-evolution given by Eqs. (2) or (3), where  $t$  is replaced with  $\Delta t$  (which will take negative values half of the time). Hence, the “initial state” tag of a  $B$  can be taken as the final-state tag of the other  $B$ . Effective tagging efficiencies  $Q$  of 30% are achieved by BaBar and Belle [25], using different techniques including  $b \rightarrow \ell^-$  and  $b \rightarrow c \rightarrow s$  tags. It is worth noting that, in this case, mixing of the other  $B$  (*i.e.*, the coherent mixing occurring before the first  $B$  decay) does not contribute to the mistag probability.

In the absence of experimental observation of a decay-width difference, oscillation analyses typically neglected  $\Delta\Gamma$  in Eq. (4), and described the data with the physics functions  $\Gamma e^{-\Gamma t}(1 \pm \cos(\Delta m t))/2$  (high-energy colliders) or  $\Gamma e^{-\Gamma|\Delta t|}(1 \pm \cos(\Delta m \Delta t))/4$  (asymmetric  $\Upsilon(4S)$  machines). As can be seen from Eq. (4), a non-zero value of  $\Delta\Gamma$  would effectively reduce the oscillation amplitude with a small time-dependent factor that would be very difficult to distinguish from time resolution effects. Measurements of  $\Delta m$  are usually extracted from the data using a maximum likelihood fit.

### **$\Delta m_d$ and $\Delta\Gamma_d$ measurements**

Many  $B_d^0\text{-}\bar{B}_d^0$  oscillations analyses have been published [26] by the ALEPH [27], DELPHI [15,28], L3 [29], OPAL [30,31] BaBar [32], Belle [33], CDF [16], DØ [21], and LHCb [34] collaborations. Although a variety of different techniques have been used, the individual  $\Delta m_d$  results obtained at high-energy colliders have remarkably similar precision. Their average is compatible with the recent and more precise measurements from asymmetric  $B$  factories. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or  $b$ -hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the  $b$ -hadron lifetimes and fractions published in this *Review*. Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fragmentation fractions, lifetimes, branching ratios of  $b$  hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, tagging, background description). Combining all published measurements [15,16,21,27–34] and accounting for all identified correlations yields  $\Delta m_d = 0.507 \pm 0.003(\text{stat}) \pm 0.003(\text{syst}) \text{ ps}^{-1}$  [8], a result dominated by the  $B$  factories.

On the other hand, ARGUS and CLEO have published time-integrated measurements [35–37], which average to  $\chi_d = 0.182 \pm 0.015$ . Following Ref. 37, the width difference  $\Delta\Gamma_d$  could in principle be extracted from the measured value of  $\Gamma_d$  and the above averages for  $\Delta m_d$  and  $\chi_d$  (see Eq. (5)), provided that

$\Delta\Gamma_d$  has a negligible impact on the  $\Delta m_d$  measurements. However, direct time-dependent studies published by DELPHI [15], BaBar [38] and Belle [39] provide stronger constraints, which can be combined to yield [8]

$$\text{sign}(\text{Re}\lambda_{\text{CP}})\Delta\Gamma_d/\Gamma_d = 0.015 \pm 0.018,$$

where  $\text{sign}(\text{Re}\lambda_{\text{CP}}) = +1$  is expected in the Standard Model.

Assuming  $\Delta\Gamma_d = 0$  and no  $CP$  violation in mixing, and using the measured  $B_d^0$  lifetime of  $1.519 \pm 0.007$  ps, the  $\Delta m_d$  and  $\chi_d$  results are combined to yield the world average

$$\Delta m_d = 0.507 \pm 0.004 \text{ ps}^{-1} \quad (14)$$

or, equivalently,

$$\chi_d = 0.1862 \pm 0.0023. \quad (15)$$

This  $\Delta m_d$  value provides an estimate of  $2|M_{12}|$ , and can be used with Eq. (6) to extract  $|V_{td}|$  within the Standard Model [40]. The main experimental uncertainties on the result come from  $m_t$  and  $\Delta m_d$ , but are completely negligible with respect to the uncertainty due to the hadronic matrix element  $f_{B_d}\sqrt{B_{B_d}} = 211 \pm 12$  MeV obtained from lattice QCD calculations [41].

### ***$\Delta m_s$ and $\Delta\Gamma_s$ measurements***

After many years of intense search at LEP and SLC,  $B_s^0-\bar{B}_s^0$  oscillations were first observed in 2006 by CDF using  $1 \text{ fb}^{-1}$  of Tevatron Run II data [20]. A year later DØ reported an independent preliminary evidence using  $2.4 \text{ fb}^{-1}$  of data [46]. Recently LHCb obtained the most precise results using  $0.036 \text{ fb}^{-1}$  [34] and  $0.34 \text{ fb}^{-1}$  [24] of data collected at the LHC in 2010 and 2011, respectively. While the average of the published measurements of  $\Delta m_s$  [20,34] is

$$\Delta m_s = 17.69 \pm 0.08 \text{ ps}^{-1}, \quad (16)$$

including also the preliminary LHCb measurement [24] and taking systematic correlations into account yields

$$\Delta m_s = 17.719 \pm 0.036(\text{stat}) \pm 0.023(\text{syst}) \text{ ps}^{-1}. \quad (17)$$

The information on  $|V_{ts}|$  obtained in the framework of the Standard Model is hampered by the hadronic uncertainty, as



in the  $B_d^0$  case. However, several uncertainties cancel in the frequency ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2, \quad (18)$$

where  $\xi = (f_{B_s} \sqrt{B_{B_s}})/(f_{B_d} \sqrt{B_{B_d}}) = 1.237 \pm 0.032$  is an SU(3) flavor-symmetry breaking factor obtained from lattice QCD calculations [41]. Using the measurements of Eqs. (14) and (16), one can extract

$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.2111 \pm 0.0010(\text{exp}) \pm 0.0055(\text{lattice}), \quad (19)$$

in good agreement with (but much more precise than) the value obtained from the ratio of the  $b \rightarrow d\gamma$  and  $b \rightarrow s\gamma$  transition rates observed at the  $B$  factories [40].

The CKM matrix can be constrained using experimental results on observables such as  $\Delta m_d$ ,  $\Delta m_s$ ,  $|V_{ub}/V_{cb}|$ ,  $\epsilon_K$ , and  $\sin(2\beta)$  together with theoretical inputs and unitarity conditions [40,47,48]. The constraint from our knowledge on the ratio  $\Delta m_s/\Delta m_d$  is more effective in limiting the position of the apex of the CKM unitarity triangle than the one obtained from the  $\Delta m_d$  measurements alone, due to the reduced hadronic uncertainty in Eq. (18). We also note that the measured value of  $\Delta m_s$  is consistent with the Standard Model prediction obtained from CKM fits where no experimental information on  $\Delta m_s$  is used, *e.g.*,  $19.0 \pm 1.5 \text{ ps}^{-1}$  [47] or  $18.1^{+2.2}_{-2.1} \text{ ps}^{-1}$  [48].

Information on  $\Delta\Gamma_s$  can be obtained from the study of the proper time distribution of untagged  $B_s^0$  samples [49]. In the case of an inclusive  $B_s^0$  selection [50], or a semileptonic (or flavor-specific)  $B_s^0$  decay selection [18,51], both the short- and long-lived components are present, and the proper time distribution is a superposition of two exponentials with decay constants  $\Gamma_{L,H} = \Gamma_s \pm \Delta\Gamma_s/2$ . In principle, this provides sensitivity to both  $\Gamma_s$  and  $(\Delta\Gamma_s/\Gamma_s)^2$ . Ignoring  $\Delta\Gamma_s$  and fitting for a single exponential leads to an estimate of  $\Gamma_s$  with a relative bias proportional to  $(\Delta\Gamma_s/\Gamma_s)^2$ . An alternative approach, which is directly sensitive to first order in  $\Delta\Gamma_s/\Gamma_s$ , is to determine the effective lifetime of untagged  $B_s^0$  candidates decaying to (fairly pure)  $CP$  eigenstates; measurements

exist for  $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$  [52],  $B_s^0 \rightarrow K^+ K^-$  [53], and  $B_s^0 \rightarrow J/\psi f_0(980)$  [54]. The extraction of  $1/\Gamma_s$  and  $\Delta\Gamma_s$  from such measurements, discussed in detail in Ref. [55], requires additional information in the form of theoretical assumptions or external inputs on weak phases and hadronic parameters. In what follows, only the effective lifetimes from the decays to the pure CP eigenstates  $K^+ K^-$  and  $J/\psi f_0(980)$  will be used, under the assumption that these decays are dominated by a single weak phase.

The best sensitivity to  $1/\Gamma_s$  and  $\Delta\Gamma_s$  is achieved by the recent time-dependent measurements of the  $B_s^0 \rightarrow J/\psi\phi$  decay rates performed at CDF [56,57], DØ [58] and LHCb [59,60], where the CP-even and CP-odd amplitudes are separated statistically through a full angular analysis. In particular LHCb obtained the first observation of a non-zero value of  $\Delta\Gamma_s$  [60]. These studies use both untagged and tagged  $B_s^0$  candidates and are optimized for the measurement of the CP-violating phase  $\phi_s$ , defined as the weak phase difference between the  $B_s^0$ - $\bar{B}_s^0$  mixing amplitude and the  $b \rightarrow c\bar{c}s$  decay amplitude. The Standard Model prediction for  $\phi_s$ , if Penguin pollution is neglected, is equal to  $-2\beta_s = -2 \arg(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)) = -0.0363^{+0.0016}_{-0.0015}$  [48]. With a Gaussian constraint on  $\phi_s$  to this Standard Model expectation, a combination [8] of the published  $B_s^0 \rightarrow J/\psi\phi$  analyses [56,58,59] and of the lifetime measurements with flavor-specific [18,51] and pure CP [53,54] final states yields

$$\Delta\Gamma_s = +0.100 \pm 0.013 \text{ ps}^{-1} \quad \text{and} \quad 1/\Gamma_s = 1.497 \pm 0.015 \text{ ps}, \quad (20)$$

or, equivalently,

$$1/\Gamma_L = 1.393 \pm 0.019 \text{ ps} \quad \text{and} \quad 1/\Gamma_H = 1.618 \pm 0.024 \text{ ps}, \quad (21)$$

in good agreement with the Standard Model prediction  $\Delta\Gamma_s = 0.087 \pm 0.021 \text{ ps}^{-1}$  [7].

The positive sign of  $\Delta\Gamma_s$  is due to the constraint applied on  $\phi_s$ . In absence of such constraint, there would be two mirror solutions related by the transformation  $(\Delta\Gamma_s, \phi_s) \rightarrow (-\Delta\Gamma_s, \pi - \phi_s)$ . Recently the LHCb collaboration analyzed

the  $B_s^0 \rightarrow J/\psi K^+ K^-$  decay, considering that the  $K^+ K^-$  system can be in a P-wave or S-wave state, and measured the dependence of the strong phase difference between the P-wave and S-wave amplitudes as a function of the  $K^+ K^-$  invariant mass [61]. This allowed, for the first time, the unambiguous determination of the sign of  $\Delta\Gamma_s$ , which was found to be positive at the  $4.7\sigma$  level.

Independent estimates of  $\Delta\Gamma_s/\Gamma_s$  obtained from measurements of the  $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$  branching fractions [52,62] are no longer included in the average, since they are based on the questionable [7] assumption that these decays account for all  $CP$ -even final states.

***Average  $b$ -hadron mixing probability and  $b$ -hadron production fractions at high energy***

Mixing measurements can significantly improve our knowledge on the fractions  $f_u$ ,  $f_d$ ,  $f_s$ , and  $f_{\text{baryon}}$ , defined as the fractions of  $B_u$ ,  $B_d^0$ ,  $B_s^0$ , and  $b$ -baryons in an unbiased sample of weakly decaying  $b$  hadrons produced in high-energy collisions. Indeed, time-integrated mixing analyses using lepton pairs from  $b\bar{b}$  events at high energy measure the quantity

$$\bar{\chi} = f'_d \chi_d + f'_s \chi_s, \quad (22)$$

where  $f'_d$  and  $f'_s$  are the fractions of  $B_d^0$  and  $B_s^0$  hadrons in a sample of semileptonic  $b$ -hadron decays. Assuming that all  $b$  hadrons have the same semileptonic decay width implies  $f'_q = f_q/(\Gamma_q \tau_b)$  ( $q = s, d$ ), where  $\tau_b$  is the average  $b$ -hadron lifetime. Hence  $\bar{\chi}$  measurements performed at LEP [63] and Tevatron [64,65,66], together with the  $\chi_d$  average of Eq. (15) and the very good approximation  $\chi_s = 1/2$  (in fact  $\chi_s = 0.499292 \pm 0.000016$  from Eqs. (5), (16) and (20)), provide constraints on the fractions  $f_d$  and  $f_s$ . In what follows, we use the preliminary  $\bar{\chi}$  result from CDF [65] instead of the published one [64]. Averages based on published data only can be found in the full listings of this *Review*.

The LEP experiments have measured  $\mathcal{B}(\bar{b} \rightarrow B_s^0) \times \mathcal{B}(B_s^0 \rightarrow D_s^- \ell^+ \nu_\ell X)$  [67],  $\mathcal{B}(b \rightarrow \Lambda_b^0) \times \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell X)$  [68], and  $\mathcal{B}(b \rightarrow \Xi_b^-) \times \mathcal{B}(\Xi_b^- \rightarrow \Xi^- \ell^- \bar{\nu}_\ell X)$  [69] from partially reconstructed final states including a lepton,  $f_{\text{baryon}}$  from protons

identified in  $b$  events [70], and the production rate of charged  $b$  hadrons [71]. The  $b$ -hadron fraction ratios measured at CDF are based on double semileptonic  $K^*\mu\mu$  and  $\phi\mu\mu$  final states [72] and lepton-charm final states [73]; in addition CDF and DØ have both measured strange  $b$ -baryon production [74]. On the other hand, fraction ratios have been studied by LHCb using fully reconstructed hadronic  $B_s^0$  and  $B_d^0$  decays [75], as well as semileptonic decays [76]. Both CDF and LHCb observe that the ratio  $f_{\Lambda_b^0}/(f_u + f_d)$  decreases with the transverse momentum of the lepton+charm system, indicating that the  $b$ -hadron fractions are not the same in different environments. We therefore provide sets of fractions separately for LEP and Tevatron (and no complete set for LHCb, where strange  $b$ -baryon production has not been measured yet). A combination of all the available information under the constraints  $f_u = f_d$ ,  $f_u + f_d + f_s + f_{\text{baryon}} = 1$ , and Eq. (22), yields the averages shown in Table 1.

**Table 1:**  $\bar{\chi}$  and  $b$ -hadron fractions (see text).

	in $Z$ decays [8]	at Tevatron [8]	at LHCb [76]
$\bar{\chi}$	$0.1259 \pm 0.0042$	$0.127 \pm 0.008$	
$f_u = f_d$	$0.403 \pm 0.009$	$0.330 \pm 0.030$	
$f_s$	$0.103 \pm 0.009$	$0.103 \pm 0.012$	
$f_{\text{baryon}}$	$0.090 \pm 0.015$	$0.236 \pm 0.067$	
$f_s/f_d$	$0.256 \pm 0.025$	$0.311 \pm 0.037$	$0.267^{+0.021}_{-0.020}$

### *CP-violation studies*

Evidence for  $CP$  violation in  $B_q^0\text{-}\bar{B}_q^0$  mixing has been searched for, both with flavor-specific and inclusive  $B_q^0$  decays, in samples where the initial flavor state is tagged, usually with a lepton from the other  $b$ -hadron in the event. In the case of semileptonic (or other flavor-specific) decays, where the final-state tag is also available, the following asymmetry [2]

$$\mathcal{A}_{\text{SL}}^q = \frac{N(\bar{B}_q^0(t) \rightarrow \ell^+ \nu_\ell X) - N(B_q^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)}{N(\bar{B}_q^0(t) \rightarrow \ell^+ \nu_\ell X) + N(B_q^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)} \simeq 1 - |q/p|_q^2 \quad (23)$$

has been measured either in time-integrated analyses at CLEO [37,77], CDF [78] and DØ [79], or in time-dependent analyses at LEP [31,80], BaBar [38,81], Belle [82] and DØ [83]. In the inclusive case, also investigated at LEP [80,84], no final-state tag is used, and the asymmetry [85]

$$\begin{aligned} & \frac{N(B_q^0(t) \rightarrow \text{all}) - N(\overline{B}_q^0(t) \rightarrow \text{all})}{N(B_q^0(t) \rightarrow \text{all}) + N(\overline{B}_q^0(t) \rightarrow \text{all})} \\ & \simeq \mathcal{A}_{\text{SL}}^q \left[ \frac{x_q}{2} \sin(\Delta m_q t) - \sin^2 \left( \frac{\Delta m_q t}{2} \right) \right] \end{aligned} \quad (24)$$

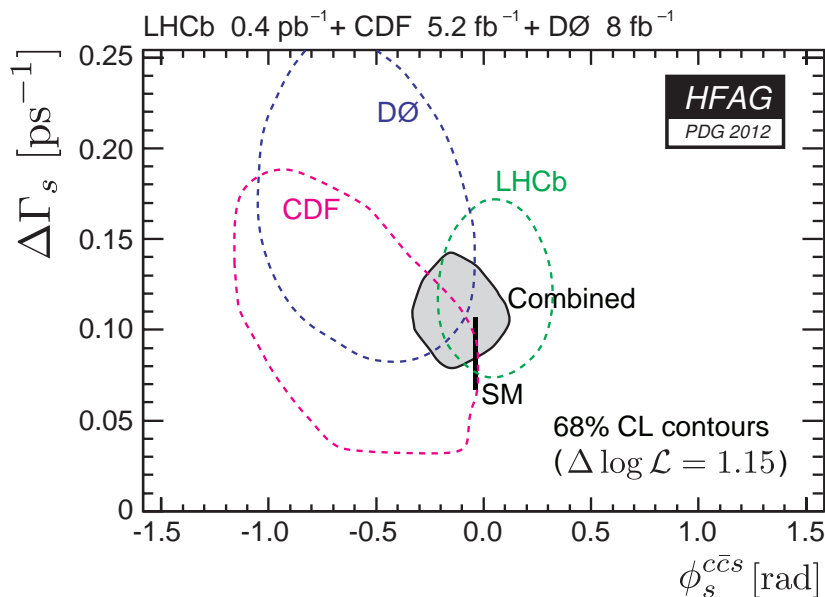
must be measured as a function of the proper time to extract information on  $CP$  violation.

The DØ collaboration measures a like-sign dimuon charge asymmetry in semileptonic  $b$  decays that deviates by  $3.9\sigma$  from the Standard Model prediction [79]. In all other cases, asymmetries compatible with zero (and the Standard Model) have been found, with a precision limited by the available statistics. Most of the analyses at high energy don't disentangle the  $B_d^0$  and  $B_s^0$  contributions, and either quote a mean asymmetry or a measurement of  $\mathcal{A}_{\text{SL}}^d$  assuming  $\mathcal{A}_{\text{SL}}^s = 0$ : we no longer include these in the average. An exception is the latest dimuon DØ analysis [79], which separates the two contributions by exploiting their dependence on the muon impact parameter cut. The resulting measurements of  $\mathcal{A}_{\text{SL}}^d$  and  $\mathcal{A}_{\text{SL}}^s$  are then both compatible with the Standard Model. They are also correlated. We therefore perform a two-dimensional average of all published measurements [37,38,77,79,81–83] and obtain [8]

$$\mathcal{A}_{\text{SL}}^d = -0.0033 \pm 0.0033, \quad \text{or } |q/p|_d = 1.0017 \pm 0.0017, \quad (25)$$

$$\mathcal{A}_{\text{SL}}^s = -0.0105 \pm 0.0064, \quad \text{or } |q/p|_s = 1.0052 \pm 0.0032, \quad (26)$$

with a correlation coefficient of  $-0.57$  between  $\mathcal{A}_{\text{SL}}^d$  and  $\mathcal{A}_{\text{SL}}^s$ . These results show no evidence of  $CP$  violation and don't constrain yet the Standard Model.



**Figure 2:** 68% CL contours in the  $(\phi_s, \Delta\Gamma_s)$  plane, showing the measurements from CDF [56], DØ [58] and LHCb [59,86], with their combination [8]. The unphysical mirror solution with  $\Delta\Gamma_s < 0$  [61] is not shown. The thin rectangle represents the Standard Model predictions of  $\phi_s$  [48] and  $\Delta\Gamma_s$  [7].

$CP$  violation induced by  $B_s^0\text{-}\bar{B}_s^0$  mixing in  $b \rightarrow c\bar{c}s$  decays has been a field of very active study in the past couple of years. In addition to the previously mentioned  $B_s^0 \rightarrow J/\psi\phi$  studies, the recently observed  $CP$ -odd decay mode  $B_s^0 \rightarrow J/\psi f_0(980)$ ,  $f_0(980) \rightarrow \pi^+\pi^-$  has also been analyzed by LHCb to measure  $\phi_s$ , without the need for an angular analysis [86]. A two-dimensional fit [8] of all published analyses [56,58,59,86] in the  $(\phi_s, \Delta\Gamma_s)$  plane, shown on Fig. 2, yields  $\phi_s = -0.14^{+0.16}_{-0.11}$ . This is consistent with the Standard Model expectation, although with a large uncertainty. The fit is then repeated, but using the external constraint provided by the measured semileptonic asymmetry of Eq. (26), which depends on  $\Delta\Gamma_s$  and on the mixing phase difference  $\phi_M - \phi_\Gamma - \pi = \arg(-M_{12}/\Gamma_{12})$  of Eq. (8). Since New Physics is expected to affect  $\arg(-M_{12}/\Gamma_{12})$  and  $\phi_s$  in the same way, the constraint is implemented under the assumption that a new phase in  $B_s^0$  mixing would not change the difference  $\arg(-M_{12}/\Gamma_{12}) - \phi_s$  from its Standard Model value [7]. The result is [8]

$$\phi_s = -0.17^{+0.14}_{-0.11}, \quad (27)$$

still not showing any sign of  $CP$  violation or New Physics. The precision on this average will soon improve significantly. Indeed it does not include yet the preliminary updates of CDF [57] and LHCb [60], nor a new preliminary LHCb measurement using  $B_s^0 \rightarrow J/\psi\pi^+\pi^-$  decays [87]. The LHCb preliminary combined value is  $\phi_s = -0.002 \pm 0.083(\text{stat}) \pm 0.027(\text{syst})$  [60].

### *Summary*

$B^0$ – $\bar{B}^0$  mixing has been and still is a field of intense study. While relatively little experimental progress was achieved in the  $B_d^0$  sector during the past years, impressive new  $B_s^0$  results became available from CDF, DØ and LHCb. The mass difference in the  $B_s^0$ – $\bar{B}_s^0$  system is now known to a relative precision which is significantly better than that in the  $B_d^0$ – $\bar{B}_d^0$  system. The non-zero decay width difference in the  $B_s^0$ – $\bar{B}_s^0$  system is now firmly established, with a relative difference of  $(15 \pm 2)\%$ . Its sign has been determined: the heavy state of the  $B_s^0$ – $\bar{B}_s^0$  system lives longer than the light state. In contrast, the relative decay width difference in the  $B_d^0$ – $\bar{B}_d^0$  system,  $(1.5 \pm 1.8)\%$ , is still consistent with zero.  $CP$  violation in mixing has not been observed yet, with precisions on the semileptonic asymmetries below 1%. A quantum step has been achieved in the measurement of the mixing-induced phase  $\phi_s$  in  $B_s^0$  decays proceeding through the  $b \rightarrow c\bar{c}s$  transition, with a Gaussian uncertainty reaching the 0.1 radian level. Despite these significant improvements, all observations remain consistent with the Standard Model expectations.

However, the measurements where New Physics might show up are still statistically limited. More results are expected in the future, especially from LHCb in the  $B_s^0$  sector, with promising prospects for the investigation of the  $CP$ -violating phase  $\arg(-M_{12}/\Gamma_{12})$  and an expected uncertainty on  $\phi_s$  of  $\sim 0.05$  radian by the time of the next edition of this *Review*.

Mixing studies have clearly reached the stage of precision measurements, where much effort is needed, both on the experimental and theoretical sides, in particular to further reduce the hadronic uncertainties of lattice QCD calculations. In the long term, a stringent check of the consistency of the  $B_d^0$  and  $B_s^0$  mixing amplitudes (magnitudes and phases) with all other

measured flavor-physics observables will be possible within the Standard Model, leading to very tight limits on (or otherwise a long-awaited surprize about) New Physics.

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