## $V_{cb}$ and $V_{ub}$ CKM Matrix Elements

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### $V_{cb}$ MEASUREMENTS

For the discussion of $V_{cb}$ measurements, which is not repeated here, see the review on “Determination of $|V_{cb}|$ and $|V_{ub}|$.”

The CKM matrix element $|V_{cb}|$ can be determined by studying the rate of the semileptonic decay $B \rightarrow D^{(*)} \ell \nu$ as a function of the recoil kinematics of $D^{(*)}$ mesons. Taking advantage of theoretical constraints on the normalization and a linear $\omega$ dependence of the form factors ($F(\omega), G(\omega)$) provided by Heavy Quark Effective Theory (HQET), the $|V_{cb}| \times F(\omega)$ and $\rho^2 (\omega^2)$ can be simultaneously extracted from data, where $\omega$ is the scalar product of the two-meson four velocities, $F(1)$ is the form factor at zero recoil ($\omega=1$) and $\rho^2$ is the slope, sometimes denoted as $\alpha^2$. Using the theoretical input of $F(1)$, a value of $|V_{cb}|$ can be obtained.

“OUR EVALUATION” is an average using rescaled values of the data listed below. The average and rescaling were performed by the Heavy Flavor Averaging Group (HFAG) and are described at http://www.slac.stanford.edu/xorg/hfag/. The averaging/rescaling procedure takes into account correlations between the measurements.

$$|V_{cb}| \times F(1) \text{ (from } B^0 \rightarrow D^{*-} \ell^+ \nu)$$

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<tr>
<td>0.03581±0.00045</td>
<td>OUR EVALUATION</td>
<td>with $\rho^2=1.207 \pm 0.026$ and a correlation 0.324.</td>
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The fitted $\chi^2$ is 30.0 for 23 degrees of freedom.

**0.0360 ± 0.0009 OUR AVERAGE** Error includes scale factor of 1.5. See the ideogram below.

0.0346 ± 0.0002 ± 0.0010 1 DUNGEL 10 BELL $e^+ e^- \rightarrow \Upsilon(4S)$
0.0350 ± 0.0002 ± 0.0012 2 AUBERT 09A BABR $e^+ e^- \rightarrow \Upsilon(4S)$
0.0350 ± 0.0006 ± 0.0014 3 AUBERT 08AT BABR $e^+ e^- \rightarrow \Upsilon(4S)$
0.0392 ± 0.0018 ± 0.0023 4 ABDALLAH 04D DLPH $e^+ e^- \rightarrow \Upsilon(4S)$
0.0431 ± 0.0013 ± 0.0018 5 ADAM 03 CLE2 $e^+ e^- \rightarrow \Upsilon(4S)$
0.0385 ± 0.0014 ± 0.0023 6 ABREU 01H DLPH $e^+ e^- \rightarrow \Upsilon(4S)$
0.0371 ± 0.0010 ± 0.0020 7 ABBIENDI 00Q OPAL $e^+ e^- \rightarrow Z^0$
0.0319 ± 0.0018 ± 0.0019 8 BUSKULIC 97 ALEP $e^+ e^- \rightarrow Z$

• • • We do not use the following data for averages, fits, limits, etc. • • •

0.0344 ± 0.0003 ± 0.0011 9 AUBERT 08R BABR Repl. by AUBERT 09A
0.0355 ± 0.0003 ± 0.0016 10 AUBERT 05E BABR Repl. by AUBERT 08R
0.0377 ± 0.0011 ± 0.0019 11 ABDALLAH 04D DLPH $e^+ e^- \rightarrow \Upsilon(4S)$
0.0354 ± 0.0019 ± 0.0018 12 ABE 02F BELL Repl. by DUNGEL 10
0.0431 ± 0.0013 ± 0.0018 13 BRIERE 02 CLE2 $e^+ e^- \rightarrow \Upsilon(4S)$
0.0328 ± 0.0019 ± 0.0022 14 ACKERSTAFF 97G OPAL Repl. by ABBIENDI 00Q
0.0350 ± 0.0019 ± 0.0023 15 ABREU 96P DLPH Repl. by ABREU 01H
0.0351 ± 0.0019 ± 0.0020 15 BARISH 95 CLE2 Repl. by ADAM 03
0.0314 ± 0.0023 ± 0.0025 15 BUSKULIC 95N ALEP Repl. by BUSKULIC 97

Citation: K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: http://pdg.lbl.gov)
1 Uses fully reconstructed $D^* \ell^+ \nu$ events ($\ell = e$ or $\mu$).
2 Obtained from a global fit to $B \to D^*(\ell) \ell \nu_\ell$ events, with reconstructed $D^0 \ell$ and $D^+ \ell$
final states and $\rho^2 = 1.22 \pm 0.02 \pm 0.07$.
3 Measured using the dependence of $B^- \to D^{*0} e^- \tau e$ decay differential rate and the
form factor description by CAPRINI 98 with $\rho^2 = 1.16 \pm 0.06 \pm 0.08$.
4 Measurement using fully reconstructed $D^*$ sample with $\rho^2 = 1.32 \pm 0.15 \pm 0.33$.
5 Average of the $B^0 \to D^{*0}(2010)\to e^+ \nu$ and $B^+ \to D^{*}(2007)\to e^+ \nu$ modes with $\rho^2 = 1.61 \pm 0.09 \pm 0.21$ and $f_{\pm^-} = 0.521 \pm 0.012$.
6 ABREU 01H measured using about 5000 partial reconstructed $D^*$ sample with a
$\rho^2 = 1.34 \pm 0.14 \pm 0.24$.
7 ABBIENDI 00Q measured using both inclusively and exclusively reconstructed $D^{*\pm}$
samples with a $\rho^2 = 1.21 \pm 0.12 \pm 0.20$. The statistical and systematic correlations
between $|V_{cb}| F(1)$ and $\rho^2$ are 0.90 and 0.54 respectively.
8 BUSKULIC 97: measured using exclusively reconstructed $D^{*\pm}$ with $\rho^2 = 0.31 \pm 0.17 \pm
0.08$. The statistical correlation is 0.92.
9 Measured using fully reconstructed $D^*$ sample and a simultaneous fit to the Caprini-
Lellouch-Neubert form factor parameters: $\rho^2 = 1.191 \pm 0.048 \pm 0.028$, $R_1(1) = 1.429 \pm 0.061 \pm 0.044$, and $R_2(1) = 0.827 \pm 0.038 \pm 0.022$.
10 Measurement using fully reconstructed $D^*$ sample with $\rho^2 = 1.29 \pm 0.03 \pm 0.27$.
11 Combines with previous partial reconstructed $D^*$ measurement with $\rho^2 = 1.39 \pm 0.10 \pm
0.33$.
12 Measured using exclusive $B^0 \to D^{*0}(892)\to e^+ \nu$ decays with $\rho^2 = 1.35 \pm 0.17 \pm 0.19$
and a correlation of 0.91.
13 BRIERE 02 result is based on the same analysis and data sample reported in ADAM 03.
14 ABREU 96P: measured using both inclusively and exclusively reconstructed $D^{*\pm}$ samples.
15 BARISH 95: measured using both exclusive reconstructed $B^0 \to D^{*0} e^+ \nu$ and $B^+ \to
D^{*0} e^+ \nu$ samples. They report their experiment’s uncertainties $\pm 0.0019 \pm 0.0018 \pm
0.0008$, with the first error is statistical, the second is systematic, and the third is the
uncertainty in the lifetimes. We combine the last two in quadrature.

\begin{tabular}{cccc}
\hline
Experiment & Sample & \(\rho^2\) & \(\chi^2\) \\
\hline
DUNGEL & 10 & BELL & 1.8 \\
AUBERT & 09A & BABR & 0.0 \\
AUBERT & 08AT & BABR & 0.0 \\
ABDALLAH & 04D & DLPH & 1.2 \\
ADAM & 03 & CLE2 & 10.3 \\
ABREU & 01H & DLPH & 0.0 \\
ABBIENDI & 00Q & OPAL & 0.3 \\
BUSKULIC & 97 & ALEP & 2.4 \\
\hline
\end{tabular}

\(\frac{|V_{cb}| \times F(1)}{\text{(from } B^0 \to D^{*-} e^+ \nu)}\)

\(\chi^2\) with \(\text{(Confidence Level } = 0.025)\)

\(0.0360 \pm 0.0009\) (Error scaled by 1.5)
$|V_{cb}| \times G(1) \text{ (from } B \to D^- \ell^+ \nu)$

**VALUE** | **DOCUMENT ID** | **TECN** | **COMMENT**
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0.04265 ±0.00153 | OUR EVALUATION | with $\rho^2 = 1.190 \pm 0.054$ and a correlation 0.83.

The fitted $\chi^2$ is 0.5 for 8 degrees of freedom.

0.0421 ±0.0016 | OUR AVERAGE

- 0.0423 ± 0.0019 ± 0.0014
- 0.0431 ± 0.0008 ± 0.0023
- 0.0411 ± 0.0044 ± 0.0052
- 0.0416 ± 0.0047 ± 0.0037
- 0.0278 ± 0.0068 ± 0.0065

16 Obtained from a fit to the combined $B \to D \ell^+ \nu$ sample in which a hadronic decay of the second $B$ meson is fully reconstructed and $\rho^2 = 1.20 \pm 0.09 \pm 0.04$.

17 Obtained from a global fit to $B \to D^\pm \ell \nu$ events, with reconstructed $D^0 \ell$ and $D^+ \ell$ final states and $\rho^2 = 1.20 \pm 0.04 \pm 0.07$.

18 Using the missing energy and momentum to extract kinematic information about the undetected neutrino in the $B^0 \to D^- \ell^+ \nu$ decay.

19 BARTEL T 99: measured using both exclusive reconstructed $B^0 \to D^- \ell^+ \nu$ and $B^+ \to D^+ \ell^+ \nu$ samples.

20 BUSKULIC 97: measured using exclusively reconstructed $D^\pm$ with a $a^2 = -0.05 \pm 0.53 \pm 0.38$. The statistical correlation is 0.99.

21 ATHANAS 97: measured using both exclusive reconstructed $B^0 \to D^- \ell^+ \nu$ and $B^+ \to D^0 \ell^+ \nu$ samples with a $\rho^2 = 0.59 \pm 0.22 \pm 0.12 \pm 0.59$. They report their experiment’s uncertainties $\pm 0.0044 \pm 0.0048 \pm 0.0053 \pm 0.0012$, where the first error is statistical, the second is systematic, and the third is the uncertainty due to the form factor model variations. We combine the last two in quadrature.

### $V_{ub}$ MEASUREMENTS

For the discussion of $V_{ub}$ measurements, which is not repeated here, see the review on “Determination of $|V_{cb}|$ and $|V_{ub}|$.”

The CKM matrix element $|V_{ub}|$ can be determined by studying the rate of the charmless semileptonic decay $b \to u \ell \nu$. The relevant branching ratio measurements based on exclusive and inclusive decays can be found in the $B$ Listings, and are not repeated here.

### $V_{cb}$ and $V_{ub}$ CKM Matrix Elements REFERENCES

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