**CPT INVARIANCE TESTS IN NEUTRAL KAON DECAY**

Updated October 2013 by M. Antonelli (LNF-INFN, Frascati) and G. D’Ambrosio (INFN Sezione di Napoli).

CPT theorem is based on three assumptions: quantum field theory, locality, and Lorentz invariance, and thus it is a fundamental probe of our basic understanding of particle physics. Strangeness oscillation in $K^0 - \bar{K}^0$ system, described by the equation

$$i \frac{d}{dt} \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = [M - i\Gamma/2] \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix},$$

where $M$ and $\Gamma$ are hermitian matrices (see PDG review [1], references [2,3], and KLOE paper [5] for notations and previous literature), allows a very accurate test of CPT symmetry; indeed since CPT requires $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, the mass and width eigenstates, $K_{S,L}$, have a CPT-violating piece, $\delta$, in addition to the usual CPT-conserving parameter $\epsilon$:

$$K_{S,L} = \frac{1}{\sqrt{2 (1 + |\epsilon_{S,L}|^2)}} \begin{bmatrix} (1 + \epsilon_{S,L}) K^0 + (1 - \epsilon_{S,L}) \bar{K}^0 \end{bmatrix}$$

$$\epsilon_{S,L} = \frac{-i \Im (M_{12}) - \frac{1}{2} \Im (\Gamma_{12}) \mp \frac{1}{2} \left[ M_{11} - M_{22} - \frac{i}{2} (\Gamma_{11} - \Gamma_{22}) \right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2}$$

$$\equiv \epsilon \pm \delta. \quad (1)$$

Using the phase convention $\Im (\Gamma_{12}) = 0$, we determine the phase of $\epsilon$ to be $\phi_{SW} \equiv \arctan \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$. Imposing unitarity to an arbitrary combination of $K^0$ and $\bar{K}^0$ wave functions, we obtain the Bell-Steinberger relation [4] connecting CP and CPT violation in the mass matrix to CP and CPT violation in the decay; in fact, neglecting $O(\epsilon)$ corrections to the coefficient of the CPT-violating parameter, $\delta$, we can write [5]

$$\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{SW} \left[ \frac{\Re(\epsilon)}{1 + |\epsilon|^2} - i \Im(\delta) \right] =$$

$$\frac{1}{\Gamma_S - \Gamma_L} \sum_f A_L(f) A_S^*(f), \quad (2)$$
where $A_{L,S}(f) \equiv A(K_{L,S} \to f)$. We stress that this relation is phase-convention-independent. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in Eq. (2); in fact, defining for the hadronic modes

$$\alpha_i \equiv \frac{1}{\Gamma_S} \langle A_L(i)A_S^*(i) \rangle = \eta_i \, \mathcal{B}(K_S \to i),$$

\[ i = \pi^0\pi^0, \pi^+\pi^-(\gamma), 3\pi^0, \pi^0\pi^+\pi^-(\gamma), \] (3)

the recent data from CPLEAR, KLOE, KTeV, and NA48 have led to the following determinations (the analysis described in Ref. 5 has been updated by using the recent measurements of $K_L$ branching ratios from KTeV [6,7], NA48 [8,9], and the results described in the CP violation in $K_L$ decays minireview, and the recent KLOE result [10])

$$\alpha_{\pi^+\pi^-} = ((1.112 \pm 0.010) + i(1.061 \pm 0.010)) \times 10^{-3},$$
$$\alpha_{\pi^0\pi^0} = ((0.493 \pm 0.005) + i(0.471 \pm 0.005)) \times 10^{-3},$$
$$\alpha_{\pi^+\pi^-\pi^0} = ((0 \pm 2) + i(0 \pm 2)) \times 10^{-6},$$
$$|\alpha_{\pi^0\pi^0\pi^0}| < 1.5 \times 10^{-6} \text{ at 95\% CL.}$$ (4)

The semileptonic contribution to the right-handed side of Eq. (2) requires the determination of several observables: we define [2,3]

$$\mathcal{A}(K^0 \to \pi^-l^+\nu) = A_0(1 - y),$$
$$\mathcal{A}(K^0 \to \pi^+l^-\nu) = A_0^*(1 + y^*)(x_+ - x_-)^*,$$
$$\mathcal{A}(\overline{K}^0 \to \pi^+l^-\nu) = A_0^*(1 + y^*),$$
$$\mathcal{A}(\overline{K}^0 \to \pi^-l^+\nu) = A_0(1 - y)(x_+ + x_-),$$ (5)

where $x_+$ ($x_-$) describes the violation of the $\Delta S = \Delta Q$ rule in CPT-conserving (violating) decay amplitudes, and $y$ parametrizes CPT violation for $\Delta S = \Delta Q$ transitions. Taking advantage of their tagged $K^0(\overline{K}^0)$ beams, CPLEAR has measured $\Im(x_+)$, $\Re(x_-)$, $\Im(\delta)$, and $\Re(\delta)$ [11]. These determinations have been improved in Ref. 5 by including the
information \( A_S - A_L = 4[\Re(\delta) + \Re(x_-)] \), where \( A_{L,S} \) are the \( K_L \) and \( K_S \) semileptonic charge asymmetries, respectively, from the PDG [12] and KLOE [13]. Here we are also including the \( T \)-violating asymmetry measurement from CPLEAR [14].

**Table 1:** Values, errors, and correlation coefficients for \( \Re(\delta) \), \( \Im(\delta) \), \( \Re(x_-) \), \( \Im(x_+) \), and \( A_S + A_L \) obtained from a combined fit, including KLOE [5] and CPLEAR [14].

<table>
<thead>
<tr>
<th>value</th>
<th>Correlations coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Re(\delta) )</td>
<td>(3.0 ± 2.3) \times 10^{-4}</td>
</tr>
<tr>
<td>( \Im(\delta) )</td>
<td>(-0.66 ± 0.65) \times 10^{-2}</td>
</tr>
<tr>
<td>( \Re(x_-) )</td>
<td>(-0.30 ± 0.21) \times 10^{-2}</td>
</tr>
<tr>
<td>( \Im(x_+) )</td>
<td>(0.02 ± 0.22) \times 10^{-2}</td>
</tr>
<tr>
<td>( A_S + A_L )</td>
<td>(-0.40 ± 0.83) \times 10^{-2}</td>
</tr>
</tbody>
</table>

The value \( A_S + A_L \) in Table 1 can be directly included in the semileptonic contributions to the Bell Steinberger relations in Eq. (2)

\[
\sum_{\pi \ell \nu} \langle A_L(\pi \ell \nu)A_S^*(\pi \ell \nu) \rangle = 2\Gamma(K_L \rightarrow \pi \ell \nu)(\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\delta))) = 2\Gamma(K_L \rightarrow \pi \ell \nu)((A_S + A_L)/4 - i(\Im(x_+) + \Im(\delta))) . \tag{6}
\]

Defining

\[
\alpha_{\pi \ell \nu} \equiv \frac{1}{\Gamma_S} \sum_{\pi \ell \nu} \langle A_L(\pi \ell \nu)A_S^*(\pi \ell \nu) \rangle + 2i\frac{\tau_{K_S}}{\tau_{K_L}} B(K_L \rightarrow \pi \ell \nu) \Im(\delta) , \tag{7}
\]

we find:

\[
\alpha_{\pi \ell \nu} = ((-0.2 \pm 0.5) + i(0.1 \pm 0.5)) \times 10^{-5} .
\]

Inserting the values of the \( \alpha \) parameters into Eq. (2), we find

\[
\Re(\epsilon) = (161.1 \pm 0.5) \times 10^{-5} ,
\]

\[
\Im(\delta) = (-0.7 \pm 1.4) \times 10^{-5} . \tag{8}
\]

The complete information on Eq. (8) is given in Table 2.
Table 2: Summary of results: values, errors, and correlation coefficients for $\Re(\epsilon)$, $\Im(\delta)$, $\Re(\delta)$, and $\Re(x_-)$.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Re(\epsilon)$</td>
<td>$(161.1 \pm 0.5) \times 10^{-5}$</td>
<td>1</td>
</tr>
<tr>
<td>$\Im(\delta)$</td>
<td>$(-0.7 \pm 1.4) \times 10^{-5}$</td>
<td>0.09 1</td>
</tr>
<tr>
<td>$\Re(\delta)$</td>
<td>$(2.4 \pm 2.3) \times 10^{-4}$</td>
<td>0.08 −0.12 1</td>
</tr>
<tr>
<td>$\Re(x_-)$</td>
<td>$(-4.1 \pm 1.7) \times 10^{-3}$</td>
<td>0.14 0.22 −0.43 1</td>
</tr>
</tbody>
</table>

Now the agreement with CPT conservation, $\Im(\delta) = \Re(\delta) = \Re(x_-) = 0$, is at 18% C.L.

The allowed region in the $\Re(\epsilon) - \Im(\delta)$ plane at 68% CL and 95% C.L. is shown in the top panel of Fig. 1.

The process giving the largest contribution to the size of the allowed region is $K_L \rightarrow \pi^+\pi^-$, through the uncertainty on $\phi_{\pi^-}$.

The limits on $\Im(\delta)$ and $\Re(\delta)$ can be used to constrain the $K^0 - \bar{K}^0$ mass and width difference

$$\delta = \frac{i(m_{K^0} - m_{\bar{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\bar{K}^0})}{\Gamma_S - \Gamma_L} \cos \phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)] .$$

The allowed region in the $\Delta M = (m_{K^0} - m_{\bar{K}^0}), \Delta \Gamma = (\Gamma_{K^0} - \Gamma_{\bar{K}^0})$ plane is shown in the bottom panel of Fig. 1. As a result, we improve on the previous limits (see for instance, P. Bloch in Ref. 12) and in the limit $\Gamma_{K^0} - \Gamma_{\bar{K}^0} = 0$ we obtain

$$-4.0 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\bar{K}^0} < 4.0 \times 10^{-19} \text{ GeV} \text{ at } 95\% \text{ C.L.}$$
Figure 1: Top: allowed region at 68% and 95% C.L. in the $\Re(\epsilon), \Im(\delta)$ plane. Bottom: allowed region at 68% and 95% C.L. in the $\Delta M, \Delta \Gamma$ plane.
References

1. See the “CP Violation in Meson Decays,” in this Review.


9. We thank G. Isidori and M. Palutan for their contribution to the original analysis [5] performed with KLOE data.


15. We thank M. Palutan for the collaboration in this analysis.