EXTRA DIMENSIONS

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I Introduction

Proposals for a spacetime with more than three spatial dimensions date back to the 1920’s, mainly through the work of Kaluza and Klein, in an attempt to unify the forces of nature [1]. Although their initial idea failed, the formalism that they and others developed is still useful nowadays. Around 1980, string theory proposed again to enlarge the number of space dimensions, this time as a requirement for describing a consistent theory of quantum gravity. The extra dimensions were supposed to be compactified at a scale close to the Planck scale, and thus not testable experimentally in the near future.

A different approach was given by Arkani-Hamed, Dimopoulos and Dvali (ADD) in their seminal paper in 1998 [2], where they showed that the weakness of gravity could be explained by postulating two or more extra dimensions in which only gravity could propagate. The size of these extra dimensions should range between roughly a millimeter and \(\sim 1/\text{TeV} \), leading to possible observable consequences in current and future experiments. A year later, Randall and Sundrum (RS) [3] found a new possibility using a warped geometry, postulating a five-dimensional Anti-de Sitter (AdS) spacetime with a compactification scale of order TeV. The origin of the smallness of the electroweak scale versus the Planck scale was explained by the gravitational redshift factor present in the warped AdS metric. As in the ADD model, originally only gravity was assumed to propagate in the extra dimensions, although it was soon clear that this was not necessary in warped extra-dimensions and also the SM gauge fields [4] and SM fermions [5,6] could propagate in the five-dimensional space.

The physics of warped extra-dimensional models has an alternative interpretation by means of the AdS/CFT correspondence [7]. Models with warped extra dimensions are related to four-dimensional strongly-interacting theories, allowing an understanding of the properties of five-dimensional fields as
those of four-dimensional composite states [8]. This approach has opened new directions for tackling outstanding questions in particle physics, such as the flavor problem, grand unification, and the origin of electroweak symmetry breaking or supersymmetry breaking.

**Kaluza-Klein Theories**: Field theories with compact extra dimensions can be written as theories in ordinary four dimensions (4D) by performing a Kaluza-Klein (KK) reduction. As an illustration, consider a simple example, namely a field theory of a complex scalar in flat five-dimensional (5D) spacetime. The action will be given by

\[ S_5 = -\int d^4x \, dy \, M_5 \left[ |\partial_\mu \phi|^2 + |\partial_y \phi|^2 + \lambda_5 |\phi|^4 \right], \]  

where \( y \) refers to the extra (fifth) dimension. A universal scale \( M_5 \) has been extracted in front of the action in order to keep the 5D field with the same mass-dimension as in 4D. This theory is perturbative for energies \( E \lesssim \ell_5 M_5 / \lambda_5 \) where \( \ell_5 = 24 \pi^3 \) [9].

Let us now consider that the fifth dimension is compact with the topology of a circle \( S^1 \) of radius \( R \), which corresponds to the identification of \( y \) with \( y + 2\pi R \). In such a case, the 5D complex scalar field can be expanded in a Fourier series:

\[ \phi(x, y) = \frac{1}{\sqrt{2\pi R M_5}} \sum_{n=-\infty}^{\infty} e^{iny/R} \phi^{(n)}(x), \]

that, inserted in Eq. (1) and integrating over \( y \), gives

\[ S_5 = S_4^{(0)} + S_4^{(n)}, \]

where

\[ S_4^{(0)} = -\int d^4x \left[ |\partial_\mu \phi^{(0)}|^2 + \lambda_4 |\phi^{(0)}|^4 \right], \]  

and

\[ S_4^{(n)} = -\int d^4x \sum_{n \neq 0} \left[ |\partial_\mu \phi^{(n)}|^2 + \left( \frac{n}{R} \right)^2 |\phi^{(n)}|^2 \right] + \text{quartic int}. \]

The \( n = 0 \) mode self-coupling is given by

\[ \lambda_4 = \frac{\lambda_5}{2\pi R M_5}. \]  

† Our convention for the metric is \( \eta_{MN} = \text{Diag}(-1, 1, 1, 1, 1). \)
The above action corresponds to a 4D theory with a massless scalar $\phi^{(0)}$, referred to as the zero-mode, and an infinite tower of massive modes $\phi^{(n)}$, known as KK modes. The KK reduction thus allows a treatment of 5D theories as 4D field theories with an infinite number of fields. At energies smaller than $1/R$, the KK modes can be neglected, leaving the zero-mode action of Eq. (2). The strength of the interaction of the zero-mode, given by Eq. (3), decreases as $R$ increases. Thus, for a large extra dimension $R \gg 1/M_5$, the massless scalar is weakly coupled.

**II Large Extra Dimensions for Gravity**

**II.1 The ADD Scenario**

The ADD scenario [2,10,11] assumes a $D = 4 + \delta$ dimensional spacetime, with $\delta$ compactified spatial dimensions. The weakness of gravity arises since it propagates in the higher-dimensional space. The SM is assumed to be localized in a 4D subspace, a 3-brane, as can be found in certain string constructions [12]. Gravity is described by the Einstein-Hilbert action in $D = 4 + \delta$ spacetime dimensions

$$S_D = -\frac{\tilde{M}_D^{2+\delta}}{2} \int d^4x d^\delta y \sqrt{-g} R + \int d^4x \sqrt{-g_{\text{ind}}} \mathcal{L}_{\text{SM}}, \quad (4)$$

where $x$ labels the ordinary four coordinates, $y$ the $\delta$ extra coordinates, $g$ refers to the determinant of the $D$-dimensional metric whose Ricci scalar is defined by $R$, and $\tilde{M}_D$ is the reduced Planck scale of the $D$-dimensional theory. In the second term of Eq. (4), which gives the gravitational interactions of SM fields, the $D$-dimensional metric reduces to the induced metric on the 3-brane where the SM fields propagate. The extra dimensions are assumed to be flat and compactified in a volume $V_\delta$. As an example, consider a toroidal compactification of equal radii $R$ and volume $V_\delta = (2\pi R)^\delta$. After a KK reduction, one finds that the fields that couple to the SM are the spin-2 gravitational field $G_{\mu\nu}(x, y)$ and a tower of spin-1 KK graviscalars [13]. The graviscalars, however, only couple to SM fields through the trace of the energy-momentum tensor, resulting in weaker couplings.
to the SM fields. The Fourier expansion of the spin-2 field is given by

$$G_{\mu\nu}(x, y) = G_{\mu\nu}^{(0)}(x) + \frac{1}{\sqrt{V_\delta}} \sum_{\vec{n} \neq 0} e^{i\vec{n} \cdot \vec{y}/R} G_{\mu\nu}^{(\vec{n})}(x),$$  \hspace{1cm} (5)

where $\vec{y} = (y_1, y_2, ..., y_\delta)$ are the extra-dimensional coordinates and $\vec{n} = (n_1, n_2, ..., n_\delta)$. Eq. (5) contains a massless state, the 4D graviton, and its KK tower with masses $m_n^2 = |\vec{n}|^2/R^2$. At energies below $1/R$ the action is that of the zero-mode

$$S_4^{(0)} = -\frac{M_P^{2+\delta}}{2} \int d^4x \sqrt{-g^{(0)}} \sqrt{\mathcal{R}^{(0)}} + \int d^4x \sqrt{-g_{\text{MND}}^{(0)}} \mathcal{L}_{\text{SM}},$$

where we can identify the 4D reduced Planck mass, $M_P \equiv G_N/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV, as a function of the $D$-dimensional parameters:

$$M_P^2 = V_\delta M_D^{2+\delta} \equiv R^\delta M_D^{2+\delta}. \hspace{1cm} (6)$$

Fixing $M_D$ at around the electroweak scale $M_D \sim \text{TeV}$ to avoid introducing a new mass-scale in the model, Eq. (6) gives a prediction for $R$:

$$\delta = 1, 2, ..., 6 \rightarrow R \sim 10^9 \text{ km}, 0.5 \text{ mm}, ..., 0.1 \text{ MeV}^{-1}. \hspace{1cm} (7)$$

The option $\delta = 1$ is clearly ruled out. However this is not the case for $\delta \geq 2$, and possible observable consequences can be sought in present and future experiments.

Consistency of the model requires a stabilization mechanism for the radii of the extra dimensions, to the values shown in Eq. (7). The fact that we need $R \gg 1/M_D$ leads to a new hierarchy problem, the solution of which might require imposing supersymmetry in the extra-dimensional bulk [14].

II.2 Tests of the Gravitational Force Law at Sub-mm Distances

The KK modes of the graviton give rise to deviations from Newton’s law of gravitation for distances $\gtrsim 1/R$. Such deviations are usually parametrized by a modified Newtonian potential of the form

$$V(r) = -G_N \frac{m_1 m_2}{r} \left[1 + \alpha e^{-r/\lambda}\right]. \hspace{1cm} (8)$$
For a 2-torus compactification, $\alpha = 16/3$ and $\lambda = R$. Searches for deviations from Newton’s law of gravitation have been performed in several experiments. Ref. [15] gives the present constraints: $R < 37\mu$m at 95% CL for $\delta = 2$, corresponding to $M_D > 3.6$ TeV.

**II.3 Astrophysical and Cosmological Constraints**

The light KK gravitons could be copiously produced in stars, carrying away energy. Ensuring that the graviton luminosity is low enough to preserve the agreement of stellar models with observations provides powerful bounds on the scale $M_D$. The most stringent arises from supernova SN1987A, giving $M_D > 27 (2.4)$ TeV for $\delta = 2 (3)$ [16]. After a supernova explosion, most of the KK gravitons stay gravitationally trapped in the remnant neutron star. The requirement that neutron stars are not excessively heated by KK decays into photons leads to $M_D > 1700 (76)$ TeV for $\delta = 2 (3)$ [17].

Cosmological constraints are also quite stringent [18]. To avoid overclosure of the universe by relic gravitons one needs $M_D > 7$ TeV for $\delta = 2$. Relic KK gravitons decaying into photons contribute to the cosmic diffuse gamma radiation, from which one can derive the bound $M_D > 100$ TeV for $\delta = 2$.

We must mention however that bounds coming from the decays of KK gravitons into photons can be reduced if we assume that KK gravitons decay mainly into other non-SM states. This could happen, for example, if there were other 3-branes with hidden sectors residing on them [10].

**II.4 Collider Signals**

Collider limits on extra-dimensional models are dominated by Run I LHC results, which are based on total integrated luminosities of $\sim 5$ fb$^{-1}$ ($\sim 20$ fb$^{-1}$) collected in 2011 (2012) at a center-of-mass energy of 7 (8) TeV. This review focuses on the most recent limits, most of which are 8 TeV preliminary results which can be found on the WWW pages of public results of the ATLAS [19] and CMS [20] experiments; a more complete record of published results can be found in the PDG Listings.


**II.4a Graviton and Other Particle Production**

Although each KK graviton has a purely gravitational coupling, suppressed by $1/M_P$, inclusive processes in which one sums over the almost continuous spectrum of available gravitons have cross sections suppressed only by powers of $M_D$. Processes involving gravitons are therefore detectable in collider experiments if $M_D \sim \text{TeV}$. A number of experimental searches for evidence of large extra dimensions have been performed at colliders, and interpreted in the context of the ADD model.

One signature arises from direct graviton emission. By making a derivative expansion of Einstein gravity, one can construct an effective theory, valid for energies much lower than $M_D$, and use it to make predictions for graviton-emission processes at colliders [13,21,22]. Gravitons produced in the final state would escape detection, giving rise to missing transverse energy ($\not\!E_T$). The results quoted below are 95% CL lower limits on $M_D$ for a range of values of $\delta$ between 2 and 6, with more stringent limits corresponding to lower $\delta$ values.

At hadron colliders, experimentally sensitive channels include the $j + \not\!E_T$ and $\gamma + \not\!E_T$ final states. At the LHC, using the full 20 fb$^{-1}$ dataset at 8 TeV and assuming k-factors of 1.5 (1.4) for $\delta = 2, 3$ ($\delta = 4 - 6$) to account for next-to-leading order (NLO) contributions to the signal cross sections, CMS sets limits of $M_D > 3.12 - 5.67$ TeV [23] from analyzing the $j + \not\!E_T$ final state. ATLAS $j + \not\!E_T$ results with 10 fb$^{-1}$ of 8 TeV data provide limits of $M_D > 2.58 - 3.88$ TeV [24], using leading order (LO) cross sections. For these $j + \not\!E_T$ analyses, the LHC experiments handle somewhat differently the issue that the effective theory is only valid for energies much less than $M_D$: CMS suppresses the graviton cross section by a factor $M_D^4/\hat{s}^2$ for $\sqrt{\hat{s}} > M_D$, where $\sqrt{\hat{s}}$ is the parton-level center-of-mass energy of the hard collision. ATLAS considers the impact of simply truncating the differential cross section to remove the contribution from events where $\sqrt{\hat{s}} > M_D$, and shows that the effect of the truncation grows from a negligible impact for $\delta = 2$ up to a 50% reduction in the total cross section for $\delta = 6$. The ATLAS limits are quoted using the full phase space. Less stringent limits are obtained from analyses of the $\gamma + \not\!E_T$ final state.
state, where both ATLAS [25] and CMS [26] have published results using their full 7 TeV datasets of ∼5 fb⁻¹.

In models in which the ADD scenario is embedded in a string theory at the TeV scale [12], we expect the string scale $M_s$ to be smaller than $M_D$, and therefore expect production of string resonances at the LHC [27]. Analysis of the dijet invariant mass distribution has been interpreted by CMS for their 8 TeV data to exclude at 95% CL string excitations of quarks and gluons that decay predominantly to $q + g$ with masses in the range from 1.20 to 5.08 TeV [28]. An ATLAS dijet analysis [29] using a 13 fb⁻¹ dataset of 8 TeV collisions provides its results in the context of model-independent limits on the cross section times acceptance for generic resonances of a variety of possible widths.

### II.4b Virtual graviton effects

One can also search for virtual graviton effects, the calculation of which however depends on the ultraviolet cut-off of the theory and is therefore very model dependent. In the literature, several different formulations exist [13,22,30] for the dimension-eight operator for gravity exchange at tree level:

$$L_8 = \pm \frac{4}{M_{TT}^4} \left( T_{\mu\nu} T^{\mu\nu} - \frac{1}{\delta + 2} T^\mu_\mu T^\nu_\nu \right),$$ (9)

where $T_{\mu\nu}$ is the energy-momentum tensor and $M_{TT}$ is related to $M_D$ by some model-dependent coefficient [31]. The relations with the parametrizations of Refs. [30] and [13] are, respectively, $M_{TT} = M_S$ and $M_{TT} = (2/\pi)^{1/4} \Lambda_T$. The experimental results below are given as 95% CL lower limits on $M_{TT}$, including in some cases the possibility of both constructive or destructive interference, depending on the sign chosen in Eq. (9).

The most stringent limits arise from the CMS analysis of the dielectron [32] and dimuon [33] final states, using their full sample of 8 TeV collisions; the combined result corresponds to an approximate limit of $M_{TT} > 3.7$ TeV, assuming constructive interference. Using its full dataset at 7 TeV to analyse the $\gamma \gamma$ final state, ATLAS provides limits [34] of $M_{TT} > 2.94$ TeV.
(2.52 TeV) for constructive (destructive) interference; the ATLAS limit improves to $M_{TT} > 3.14$ TeV for constructive interference when they combine this diphoton result with their 7 TeV dilepton analysis [35].

At the one-loop level, gravitons can also generate dimension-six operators with coefficients that are also model dependent. Experimental bounds on these operators can also give stringent constraints on $M_D$ [31].

**II.4c Black Hole Production**

The physics at energies $\sqrt{s} \sim M_D$ is sensitive to the details of the unknown quantum theory of gravity. Nevertheless, in the transplanckian regime, $\sqrt{s} \gg M_D$, one can rely on a semiclassical description of gravity to obtain predictions. An interesting feature of transplanckian physics is the creation of black holes [36]. A black hole is expected to be formed in a collision in which the impact parameter is smaller than the Schwarzschild radius [37]:

$$R_S = \frac{1}{M_D} \left[ \frac{2\delta \pi^{(\delta-3)/2}}{\delta + 2} \frac{\Gamma \left( \frac{\delta + 3}{2} \right)}{\frac{\delta + 3}{2}} \frac{M_{BH}}{M_D} \right]^{1/(\delta+1)},$$

(10)

where $M_{BH}$ is the mass of the black hole, which would roughly correspond to the total energy in the collision. The cross section for black hole production can be estimated to be of the same order as the geometric area $\sigma \sim \pi R_S^2$. For $M_D \sim$ TeV, this gives a production of $\sim 10^7$ black holes at the $\sqrt{s} = 14$ TeV LHC with an integrated luminosity of $30$ fb$^{-1}$ [36]. A black hole would provide a striking experimental signature since it is expected to thermally radiate with a Hawking temperature $T_H = (\delta + 1)/(4\pi R_S)$, and therefore would evaporate democratically into all SM states. Nevertheless, given the present constraints on $M_D$, the LHC will not be able to reach energies much above $M_D$. This implies that predictions based on the semiclassical approximation could receive sizable modifications from model-dependent quantum-gravity effects.

The most stringent limits on microscopic black holes arise from LHC searches which observed no excesses above the SM background in high-multiplicity final states. The results are usually quoted as model-independent limits on the cross section
for new physics in the final state and kinematic region analyzed. These results can then be used to provide constraints of models of low-scale gravity and weakly-coupled string theory. In addition, limits are sometimes quoted on particular implementations of models, which are used as benchmarks to illustrate the sensitivity. For example, the ATLAS analysis [38] of the track multiplicity in same-sign dimuon events, using their full 20 fb$^{-1}$ sample at 8 TeV, excludes semiclassical black holes below masses in the range of 5.1 - 5.7 TeV, fixing $M_D = 1.5$ TeV and depending on details of the model and also the number of extra dimensions. A CMS analysis [39] of multi-object final states using 12 fb$^{-1}$ of 8 TeV data provides similar limits, but extending out to values of $M_D \sim 5$ TeV.

For black hole masses near $M_D$, the semi-classical approximation is not valid, and one instead expects quantum black holes that decay primarily into two-body final states [40]. LHC results provide lower limits on quantum black hole masses of order 5 TeV, depending on the details of the model, including from the CMS multi-object analysis [39] and from an ATLAS search in the photon+jet final state [41] using their full 8 TeV dataset.

In weakly-coupled string models the semiclassical description of gravity fails in the energy range between $M_s$ and $M_s/g_s^2$ where stringy effects are important. In this regime one expects, instead of black holes, the formation of string balls, made of highly excited long strings, that could be copiously produced at the LHC for $M_s \sim$ TeV [42], and would evaporate thermally at the Hagedorn temperature giving rise to high-multiplicity events. The same analyses used to search for black holes can be interpreted in the context of string balls. For example, the ATLAS same-sign dimuon analysis [38] excludes string balls with minimal masses below 5.3 TeV, for the case of $\delta = 6$ and with model parameters fixed to values of $g_s = 0.4$, $M_D = 1.5$ TeV, and $M_s = M_D/1.26 = 1.2$ TeV. The CMS multi-object [39] analysis excludes the production of string balls with a minimum mass below $\sim 5.5$ TeV for $g_s = 0.4$, $M_D$ in the range of 1.4 – 2.1 TeV, and $M_s = M_D/1.25$. 


III TeV-Scale Extra Dimensions

III.1 Warped Extra Dimensions

The RS model [3] is the most attractive setup of warped extra dimensions at the TeV scale, since it provides an alternative solution to the hierarchy problem. The RS model is based on a 5D theory with the extra dimension compactified in an orbifold, $S^1/Z_2$, a circle $S^1$ with the extra identification of $y$ with $-y$. This corresponds to the segment $y \in [0, \pi R]$, a manifold with boundaries at $y = 0$ and $y = \pi R$. Let us now assume that this 5D theory has a cosmological constant in the bulk $\Lambda$, and on the two boundaries $\Lambda_0$ and $\Lambda_{\pi R}$:

$$S_5 = -\int d^4x dy \left\{ \sqrt{-g} \left[ \frac{1}{2} M_5^3 R + \Lambda \right] + \sqrt{-g_0} \delta(y) \Lambda_0 + \sqrt{-g_{\pi R}} \delta(y - \pi R) \Lambda_{\pi R} \right\} ,$$

(11)

where $g_0$ and $g_{\pi R}$ are the values of the determinant of the induced metric on the two respective boundaries. Einstein’s equations can be solved, giving in this case the metric

$$ds^2 = a(y)^2 dx^\mu dx^\nu \eta_{\mu\nu} + dy^2 , \quad a(y) = e^{-ky} ,$$

(12)

where $k = \sqrt{-\Lambda/6M_5^3}$. Consistency of the solution requires $\Lambda_0 = -\Lambda_{\pi R} = -\Lambda/k$. The metric in Eq. (12) corresponds to a 5D AdS space. The factor $a(y)$ is called the “warp” factor and determines how 4D scales change as a function of the position in the extra dimension. In particular, this implies that energy scales for 4D fields localized at the boundary at $y = \pi R$ are red-shifted by a factor $e^{-k\pi R}$ with respect to those localized at $y = 0$. For this reason, the boundaries at $y = 0$ and $y = \pi R$ are usually referred to as the ultraviolet (UV) and infrared (IR) boundaries, respectively.

As in the ADD case, we can perform a KK reduction and obtain the low-energy effective theory of the 4D massless graviton. In this case we obtain

$$M_P^2 = \int_0^{\pi R} dy e^{-2ky} M_5^3 = \frac{M_5^3}{2k} \left( 1 - e^{-2k\pi R} \right) .$$

(13)

Taking $M_5 \sim k \sim M_P$, we can generate an IR-boundary scale of order $ke^{-k\pi R} \sim \text{TeV}$ for an extra dimension of radius $R \simeq 11/k$. 

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Mechanisms to stabilize $R$ to this value have been proposed [43] that, contrary to the ADD case, do not require introducing any new small or large parameter. Therefore a natural solution to the hierarchy problem can be achieved in this framework if the Higgs field, whose vacuum expectation value (VEV) is responsible for electroweak symmetry breaking, is localized at the IR-boundary where the effective mass scales are of order TeV.

In the RS model [3], all the SM fields were assumed to be localized on the IR-boundary. Nevertheless, for the hierarchy problem, only the Higgs field has to be localized there. SM gauge bosons and fermions can propagate in the 5D bulk [4,5,6,44]. By performing a KK reduction from the 5D action of a gauge boson, we find [4]

$$\frac{1}{g_4^2} = \int_0^{\pi R} dy \frac{1}{g_5^2} = \frac{\pi R}{g_5^2},$$

where $g_D$ ($D = 4, 5$) is the gauge coupling in $D$-dimensions. Therefore the 4D gauge couplings can be of order one, as is the case of the SM, if one demands $g_5^2 \sim \pi R$. Using $kR \sim 10$ and $g_4 \sim 0.5$, one obtains the 5D gauge coupling

$$g_5 \sim \frac{4}{\sqrt{k}}. \quad (14)$$

Boundary kinetic terms for the gauge bosons can modify this relation, allowing for larger values of $g_5\sqrt{k}$.

Fermions propagating in a warped extra-dimension have 4D massless zero-modes with wavefunctions which vary as $f_0 \sim \text{Exp}[(1/2 - c_f)ky]$, where $c_f k$ is their 5D mass [45,6]. Depending on the free parameter $c_f k$, fermions can be localized either towards the UV-boundary ($c_f > 1/2$) or IR-boundary ($c_f < 1/2$). Since the Higgs is localized on the IR-boundary, one can generate exponentially suppressed Yukawa couplings by having the fermion zero-modes localized towards the UV-boundary, generating naturally the light SM fermion spectrum [6]. A large overlap with the wavefunction of the Higgs is needed for the top quark, in order to generate its large mass, thus requiring it to be localized towards the IR-boundary. In conclusion, the large mass hierarchies present in the SM fermion
spectrum can be easily obtained in warped models via suitable choices of the order-one parameters $c_f$ [46]. In these scenarios, deviations in flavor physics from the SM predictions are expected to arise from flavor-changing KK gluon couplings [47], putting certain constraints on the parameters of the models and predicting new physics effects to be observed in $B$-physics processes [48].

The masses of the KK states can also be calculated. One finds [6]

$$m_n \simeq \left(n + \frac{\alpha}{2} - \frac{1}{4}\right) \pi k e^{-\pi k R},$$

(15)

where $n = 1, 2, \ldots$ and $\alpha = \{|c_f - 1/2|, 0, 1\}$ for KK fermions, KK gauge bosons and KK gravitons, respectively. Their masses are of order $ke^{-\pi k R} \sim \text{TeV}$; the first KK state of the gauge bosons would be the lightest, while gravitons are expected to be the heaviest.

**III.1a Models of Electroweak Symmetry Breaking**

Theories in warped extra dimensions can be used to implement symmetry breaking at low energies by boundary conditions [49]. For example, for a $U(1)$ gauge symmetry in the 5D bulk, this can be easily achieved by imposing a Dirichlet boundary condition on the IR-boundary for the gauge-boson field, $A_\mu|_{y=\pi R} = 0$. This makes the zero-mode gauge boson get a mass, given by $m_A = g_4 \sqrt{2k/g_5^2} e^{-\pi k R}$. A very different situation occurs if the Dirichlet boundary condition is imposed on the UV-boundary, $A_\mu|_{y=0} = 0$. In this case the zero-mode gauge boson disappears from the spectrum. Finally, if a Dirichlet boundary condition is imposed on the two boundaries, one obtains a massless 4D scalar corresponding to the fifth component of the 5D gauge boson, $A_5$. Thus, different scenarios can be implemented by appropriately choosing the 5D bulk gauge symmetry, $G_5$, and the symmetries to which it reduces on the UV and IR-boundary, $H_{UV}$ and $H_{IR}$ respectively. In all cases the KK spectrum comes in representations of the group $G_5$.

The recent discovery of a light Higgs with $m_H \sim 125 \text{ GeV}$ [50] rules out Higgsless 5D models for electroweak symmetry breaking [51]. This discovery however is consistent with
5D composite Higgs model where a light Higgs is present in the spectrum.

**Composite Higgs models:** Warped extra dimensions can give rise to scenarios, often called gauge-Higgs unified models, where the Higgs appears as the fifth component of a 5D gauge boson, $A_5$. The Higgs mass is protected by the 5D gauge invariance and can only get a nonzero value from non-local one-loop effects [52]. To guarantee the relation $M_W^2 \simeq M_Z^2 \cos^2 \theta_W$, a custodial $SU(2)_V$ symmetry is needed in the bulk and IR-boundary [53]. The simplest realization [54] has

$$
G_5 = SU(3)_c \times SO(5) \times U(1)_X, \\
\mathcal{H}_{IR} = SU(3)_c \times SO(4) \times U(1)_X, \\
\mathcal{H}_{UV} = G_{SM}.
$$

The Higgs gets a potential at the one-loop level that triggers a VEV, breaking the electroweak symmetry. In these models there is a light Higgs whose mass can be around 125 GeV, as required by the recently discovered Higgs boson [50]. This state, as will be explained in Sec. III.2, behaves as a composite pseudo-Goldstone boson with couplings that deviate from the SM Higgs [55]. The lightest KK modes of the model are color fermions with charges $Q = -1/3, 2/3$ and $5/3$ [56].

**III.1b Constraints from Electroweak Precision Tests**

Models in which the SM gauge bosons propagate in 1/TeV-sized extra dimensions give generically large corrections to electroweak observables. When the SM fermions are confined on a boundary these corrections are universal and can be parametrized by four quantities: $\hat{S}$, $\hat{T}$, $W$ and $Y$, as defined in Ref. [57]. For warped models, where the 5D gauge coupling of Eq. (14) is large, the most relevant parameter is $\hat{T}$, which gives the bound $m_{KK} \gtrsim 10 \text{ TeV}$ [44]. When a custodial symmetry is imposed [53], the main constraint comes from the $\hat{S}$ parameter, requiring $m_{KK} \gtrsim 3 \text{ TeV}$, independent of the value of $g_5$. Corrections to the $Zb_L\bar{b}_L$ coupling can also be important [44], especially in warped models for electroweak symmetry breaking as the ones described above.
**III.1c Kaluza-Klein Searches**

The main prediction of 1/TeV-sized extra dimensions is the presence of a discretized KK spectrum, with masses around the TeV scale, associated with the SM fields that propagate in the extra dimension.

In the RS model [3], only gravity propagates in the 5D bulk. Experimental searches have been performed for the lightest KK graviton through its decay to a variety of SM particle-antiparticle pairs. The results are usually interpreted in the plane of the dimensionless coupling $k/M_P$ versus $m_1$, where $M_P$ is the reduced Planck mass defined previously and $m_1$ is the mass of the lightest KK excitation of the graviton. Since the AdS curvature $\sim k$ cannot exceed the cut-off scale of the model, which is estimated to be $\ell_5^{1/3} M_5$ [31], one must demand $k \ll \sqrt{2\ell_5 M_P}$. The results quoted below are 95% CL lower limits on the KK graviton mass for a coupling $k/M_P = 0.1$.

The most stringent limits currently arise from searches for dilepton resonances, combining results from the $ee$ and $\mu\mu$ final states. The ATLAS dilepton analysis [58] uses their full sample of 8 TeV collisions and excludes gravitons with masses below 2.47 TeV. The CMS dilepton analysis [59] combines the full 7 TeV dataset with the first 4 $fb^{-1}$ of 8 TeV data to exclude graviton masses below 2.39 TeV. The $\gamma\gamma$ final state is quite powerful, with a branching ratio twice that of any individual lepton flavor, plus lower backgrounds. The ATLAS $\gamma\gamma$ analysis [34] using the full dataset at 7 TeV provides, in combination with $ee/\mu\mu$ results of the same 7 TeV dataset, a lower limit on the graviton mass of 2.23 TeV; this result is dominated by the $\gamma\gamma$ channel, which on its own provides a limit of 2.06 TeV. Less stringent limits on the KK graviton mass come from the $WW$ [60,61] and $ZZ$ [62] final states.

In warped extra-dimensional models in which the SM fields propagate in the 5D bulk, the couplings of the KK graviton to $ee/\mu\mu/\gamma\gamma$ are suppressed [63], and the above bounds do not apply. Furthermore, the KK graviton is the heaviest KK state (see Eq. (15)), and therefore experimental searches for KK gauge bosons and fermions are more appropriate discovery
channels in these scenarios. For the scenarios discussed above in which only the Higgs and the top quark are localized close to the IR-boundary, the KK gauge bosons mainly decay into top quarks, longitudinal $W/Z$ bosons, and Higgs bosons. Couplings to light SM fermions are suppressed by a factor $g/\sqrt{g_5^2 k} \sim 0.2$ [6] for the value of Eq. (14) that is considered from now on. Searches have been made for evidence of the lightest KK excitation of the gluon, through its decay to $t\bar{t}$ pairs. The searches need to take into account the natural KK gluon width, which is typically $\sim 15\%$ of its mass. The decay of a heavy particle to $t\bar{t}$ would tend to produce highly boosted (anti-)top quarks in the final state. Products of the subsequent top decays would therefore tend to be close to each other in the detector. In the case of $t \rightarrow Wb \rightarrow jjb$ decays, the three jets could overlap one another and not be individually reconstructed with the standard jet algorithms, while $t \rightarrow Wb \rightarrow \ell\nu b$ decays could result in the lepton failing standard isolation cuts due to its proximity to the $b$-jet; in both cases, the efficiency for properly reconstructing the final state would fall as the mass of the original particle increases. To avoid the loss in sensitivity which would result, a number of techniques, known generally as “top tagging”, have been developed to reconstruct and identify highly boosted top quarks, for example by using a single “fat” jet to contain all the decay products of a hadronic top decay. The large backgrounds from QCD jets can then be reduced by requiring the “jet mass” be consistent with that of a top quark, and also by examining the substructure of the large jet for indication that it resulted from the hadronic decay of a top quark. These techniques are key to extending to very high masses the range of accessible resonances decaying to $t\bar{t}$ pairs. The most stringent current limits result from analyses of the lepton-plus-jets channel, with less stringent limits from analyses of the fully hadronic channel. CMS analyses of the lepton-plus-jets [64]( fully hadronic [65]) final state using the full 8 TeV dataset of 20 fb$^{-1}$ exclude KK gluons with masses below 2.54 TeV (1.8 TeV), assuming a k-factor of 1.3 (without using the techniques for boosted top reconstruction, the lepton-plus-jets limit would be 1.7 TeV). An ATLAS analysis [66] of
the lepton-plus-jets final state, using 14 fb$^{-1}$ of 8 TeV data, excludes KK gluon masses below 2.0 TeV, whereas the ATLAS fully hadronic analysis [67], using the full 7 TeV dataset, yields a lower limit of 1.62 TeV; both of the ATLAS analyses assume LO cross section values. The results are not directly comparable between the two experiments, since they employ in their respective analyses different implementations of the theoretical model.

A gauge boson KK excitation could be also sought through its decay to longitudinal $W/Z$ bosons. While searches for $WZ$ resonances have been used to set limits on sequential SM $W^\prime$ bosons [68] or other models, as yet no $WZ$ experimental results have been interpreted in the context of warped extra dimensions. The decay to a pair of intermediate vector bosons has, however, been exploited to search for KK gravitons in models in which the SM fields propagate in the 5D bulk. ATLAS analyses of the full 5 fb$^{-1}$ dataset at 7 TeV searching for $G^* \rightarrow WW \rightarrow \ell\nu\ell\nu$ (\ell\nuqq') [60]( [61]) exclude gravitons with masses below 0.84 (0.71) TeV, both for $k/M_P = 1.0$ and using LO cross sections. Results from searches for $G^* \rightarrow ZZ \rightarrow \ell\ellqq$ include CMS [69]( ATLAS [70]) lower limits on the graviton mass of 0.71 TeV (0.85 TeV) for $k/M_P = 0.5$ (1.0), using 20 fb$^{-1}$ (7 fb$^{-1}$) of 8 TeV data. CMS has searched, using their full 8 TeV dataset, for $G^* \rightarrow W/WZ$ in the fully hadronic decay mode [71], reconstructing each hadronic $W/Z$ decay using the boosted techniques mentioned previously; the results, which are approximate since the finite graviton width is not taken into account, are exclusions of the graviton mass in the range between 1 TeV and 1.59 TeV (1.17 TeV) for decays to $WW$ ($ZZ$), for $k/M_P = 0.1$.

The lightest KK states are, in certain models, the partners of the top quark. For example, in 5D composite Higgs models these are colored states with charges $Q = -1/3, 2/3$ and $5/3$, and masses expected to be below the TeV [56]. They can be either singly- or pair-produced, and mainly decay into a combination of $W/Z$ with top/bottom [72]. Of particular note, the $Q = 5/3$ state decays mainly into $W^+t \rightarrow W^+W^+b$, giving a pair of same-sign leptons in the final state. CMS has
used a same-sign dilepton analysis [73] of their full 20 fb$^{-1}$ dataset at 8 TeV to search for pair-production of the $Q = 5/3$ state, excluding masses below 770 GeV. A search by ATLAS, using their full 7 TeV dataset and requiring in addition to a pair of same-sign leptons at least one b-tagged jet in the event [74], provides a lower mass limit on the $Q = 5/3$ state of 670 GeV from pair production, and up to 700 GeV from single production, the cross section for which is model-dependent [75]. Both LHC experiments have searched for pair-production of vector-like quarks $T$ and $B$ of charges $Q = 2/3$ and $-1/3$ respectively, assuming the allowable decays are $T \to Wb/Zt/Ht$ and $B \to Wt/Zb/Hb$. In each case, it is assumed the branching ratios of the three decay modes sum to unity, but the individual branching ratios, which are model-dependent, are allowed to vary within this constraint. CMS has performed inclusive searches, using their full 8 TeV data sample, for $T$ [76] and $B$ [77] pair-production, providing lower limits on the masses in the range of $687 - 782$ GeV ($582 - 732$ GeV) for $T$ ($B$) vector-like quarks, depending on the values of the individual branching ratios. ATLAS has presented results from searches for pair-production of vector-like quarks, with 14 fb$^{-1}$ of 8 TeV data, using same-sign dileptons with $b$-jets [78] as well as analyses targeting final states that include $Z + b/t$ [79], $H + t$ [80], and $W + b$ [81]; the first two analyses are relevant for both $T$ and $B$ searches, while the latter two apply only for $T$. The ATLAS results are similar to the CMS limits quoted above, though ATLAS does not provide a statistical combination of their results but has presented summary plots [19] which show the overlap of the separate results as a function of the individual branching ratio values.

III.2 Connection with Strongly-Coupled Models via the AdS/CFT Correspondence

The AdS/CFT correspondence [7] provides a connection between warped extra-dimensional models and strongly-coupled theories in ordinary 4D. Although the exact connection is only known for certain cases, the AdS/CFT techniques have been very useful to obtain, at the qualitative level, a 4D
holographic description of the various phenomena in warped extra-dimensional models [8].

The connection goes as follows. The physics of the bulk AdS$_5$ models can be interpreted as that of a 4D conformal field theory (CFT) which is strongly-coupled. The extra-dimensional coordinate $y$ plays the role of the renormalization scale $\mu$ of the CFT by means of the identification $\mu \equiv ke^{-ky}$. Therefore the UV-boundary corresponds in the CFT to a UV cut-off scale at $\Lambda_{UV} = k \sim M_P$, breaking explicitly conformal invariance, while the IR-boundary can be interpreted as a spontaneous breaking of the conformal symmetry at energies $ke^{-k\pi R} \sim$ TeV. Fields localized on the UV-boundary are elementary fields external to the CFT, while fields localized on the IR-boundary and KK states corresponds to composite resonances of the CFT. Furthermore, local gauge symmetries in the 5D models, $G_5$, correspond to global symmetries of the CFT, while the UV-boundary symmetry can be interpreted as a gauging of the subgroup $\mathcal{H}_{UV}$ of $G_5$ in the CFT. Breaking gauge symmetries by IR-boundary conditions corresponds to the spontaneous breaking $G_5 \rightarrow \mathcal{H}_{IR}$ in the CFT at energies $\sim ke^{-k\pi R}$. Using this correspondence one can easily derive the 4D massless spectrum of the compactified AdS$_5$ models. One also has the identification $k^3/M_5^3 \approx 16\pi^2/N^2$ and $g_5^2k \approx 16\pi^2/N^r$ ($r = 1$ or 2 for CFT fields in the fundamental or adjoint representation of the gauge group), where $N$ plays the role of the number of colors of the CFT. Therefore the weak-coupling limit in AdS$_5$ corresponds to a large-$N$ expansion in the CFT.

Following the above AdS/CFT dictionary one can understand the RS solution to the hierarchy problem from a 4D viewpoint. The equivalent 4D model is a CFT with a TeV mass-gap and a Higgs emerging as a composite state. In the particular case where the Higgs is the fifth-component of the gauge-boson, $A_5$, this corresponds to models, similar to those proposed in Ref. [82], where the Higgs is a composite pseudo-Goldstone boson arising from the spontaneous breaking $G_5 \rightarrow \mathcal{H}_{IR}$ in the CFT. The AdS/CFT dictionary tells us that KK states must behave as composite resonances. For example, if the SM gauge bosons propagate in the 5D bulk, the lowest KK $SU(2)_L$-gauge
boson must have properties similar to those of the Techni-rho $\rho_T$ \cite{rhoT} with a coupling to longitudinal $W/Z$ bosons given by $g_5\sqrt{k} \approx g_{\rho_T}$, while the coupling to elementary fermions is $g^2/\sqrt{g_5^2k} \approx g^2 F_{\rho_T}/M_{\rho_T}$.

Fermions in compactified AdS$_5$ also have a simple 4D holographic interpretation. The 4D massless mode described in Sec. III.1 corresponds to an external fermion $\psi_i$ linearly coupled to a fermionic CFT operator $O_i$: $L_{\text{int}} = \lambda_i \bar{\psi}_i O_i + h.c.$.

The dimension of the operator $O_i$ is related to the 5D fermion mass according to $\text{Dim}[O_i] = |c_f + 1/2| - 1$. Therefore, by varying $c_f$ one varies $\text{Dim}[O_i]$, making the coupling $\lambda_i$ irrelevant ($c_f > 1/2$), marginal ($c_f = 1/2$) or relevant ($c_f < 1/2$). When irrelevant, the coupling is exponentially suppressed at low energies, and then the coupling of $\psi_i$ to the CFT (and eventually to the composite Higgs) is very small. When relevant, the coupling grows in the IR and become as large as $g_5$ (in units of $k$), meaning that the fermion is as strongly coupled as the CFT states \cite{CDF}. In this latter case $\psi_i$ behaves as a composite fermion.

### III.3 Flat Extra Dimensions

Models with quantum-gravity at the TeV scale, as in the ADD scenario, can have extra (flat) dimensions of $1/$TeV size, as happens in string scenarios \cite{ADD}. All SM fields may propagate in these extra dimensions, leading to the possibility of observing their corresponding KK states.

A simple example is to assume that the SM gauge bosons propagate in a flat five-dimensional orbifold $S^1/Z_2$ of radius $R$, with the fermions localized on a 4D boundary. The KK gauge bosons behave as sequential SM gauge bosons with a coupling to fermions enhanced by a factor $\sqrt{2}$ \cite{ADD}. The experimental limits on such sequential gauge bosons could therefore be recast as limits on KK gauge bosons. Such an interpretation of the ATLAS 7 TeV dilepton analysis \cite{ATLAS} yielded the bound $1/R > 4.16$ TeV. Indirect bounds from LEP2 require however $1/R \gtrsim 6$ TeV \cite{LEP2}.

An alternative scenario, known as Universal Extra Dimensions (UED) \cite{UED}, assumes that all SM fields propagate universally in a flat orbifold $S^1/Z_2$ with an extra $Z_2$ parity,
called KK-parity, that interchanges the two boundaries. In this case, the lowest KK state is stable and is a Dark Matter candidate. At colliders, KK particles would have to be created in pairs, and would then cascade decay to the lightest KK particle (LKP), which would be stable and escape detection. Experimental signatures, such as jets or leptons and $E_T$, would be similar to those of typical $R$-parity conserving SUSY searches. Theoretical studies of the trilepton final state [87] suggest a potential bound from the LHC at 8 TeV with 20 fb$^{-1}$ of $1/R \gtrsim 1.3$ TeV for $\Lambda R = 10$, where $\Lambda$ is the cut-off scale of the model. The experimental searches have not yet been interpreted in the general UED scenario; for example, the ATLAS trilepton analysis [88] of their full 8 TeV dataset provides upper limits on the visible cross section for new physics that could be utilized to determine UED limits.

Experimental limits have been provided on two specific UED models which include KK parity violation. In one case, KK parity is violated by gravitational interactions [89], and the LKP can decay via $\gamma^* \rightarrow \gamma + G$. Beginning with strong production of a pair of KK quarks and/or gluons [90,91], the final state would be $\gamma\gamma + E_T + X$. Using their full 7 TeV datasets, ATLAS [92] and CMS [93] each determine a limit of $1/R \gtrsim 1.4$ TeV for $\Lambda R = 20$. In a second model, that involves two UEDs, the breaking of the KK parity allows the decay of the KK photon to $t\bar{t}$ [94]. The ATLAS same-sign lepton plus b-jet analysis [74], applied for searches of pair-produced KK photons, excludes KK masses below 0.9 TeV in this model.

Finally, realistic models of electroweak symmetry breaking can also be constructed with flat extra spatial dimensions, similarly to those in the warped case, requiring, however, the presence of sizeable boundary kinetic terms [95]. There is also the possibility of breaking supersymmetry by boundary conditions [96]. Models of this type could explain naturally the presence of a Higgs boson lighter than $M_D \sim$ TeV [97].

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