

## 2. ASTROPHYSICAL CONSTANTS AND PARAMETERS

**Table 2.1.** Revised November 2013 by D.E. Groom (LBNL). The figures in parentheses after some values give the  $1\text{-}\sigma$  uncertainties in the last digit(s). Physical constants are from Ref. 1. While every effort has been made to obtain the most accurate current values of the listed quantities, the table does not represent a critical review or adjustment of the constants, and is not intended as a primary reference.

The values and uncertainties for the cosmological parameters depend on the exact data sets, priors, and basis parameters used in the fit. Many of the derived parameters reported in this table have non-Gaussian likelihoods. Parameters may be highly correlated, so care must be taken in propagating errors. (But in multiplications by  $h^{-2}$  etc. in the table below, independent errors were assumed.) Unless otherwise specified, cosmological parameters are from six-parameter fits to a flat  $\Lambda$ CDM cosmology using CMB data alone: *Planck* temperature + WMAP polarization data + high-resolution data from ACT and SPT [2]. For more information see Ref. 3 and the original papers.

Quantity	Symbol, equation	Value	Reference, footnote
speed of light	$c$	299 792 458 m s <sup>-1</sup>	exact[4]
Newtonian gravitational constant	$G_N$	6.673 8(8) $\times 10^{-11}$ m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup>	[1,5]
Planck mass	$\sqrt{\hbar c/G_N}$	1.220 93(7) $\times 10^{19}$ GeV/ $c^2$ = 2.176 51(13) $\times 10^{-8}$ kg	[1]
Planck length	$\sqrt{\hbar G_N/c^3}$	1.616 20(10) $\times 10^{-35}$ m	[1]
standard gravitational acceleration	$g_N$	9.806 65 m s <sup>-2</sup>	exact[1]
jansky (flux density)	Jy	10 <sup>-26</sup> W m <sup>-2</sup> Hz <sup>-1</sup>	definition
tropical year (equinox to equinox) (2011)	yr	31 556 925.2 s $\approx \pi \times 10^7$ s	[6]
sidereal year (fixed star to fixed star) (2011)		31 558 149.8 s $\approx \pi \times 10^7$ s	[6]
mean sidereal day (2011) (time between vernal equinox transits)		23 <sup>h</sup> 56 <sup>m</sup> 04 <sup>s</sup> .090 53	[6]
astronomical unit	au	149 597 870 700 m	exact [7]
parsec (1 au/1 arc sec)	pc	3.085 677 581 49 $\times 10^{16}$ m = 3.262... ly	exact [8]
light year (deprecated unit)	ly	0.306 6... pc = 0.946 053... $\times 10^{16}$ m	
Schwarzschild radius of the Sun	$2G_N M_\odot/c^2$	2.953 250 077(2) km	[9]
Solar mass	$M_\odot$	1.988 5(2) $\times 10^{30}$ kg	[10]
Solar equatorial radius	$R_\odot$	6.9551(4) $\times 10^8$ m	[11]
Solar luminosity	$L_\odot$	3.828 $\times 10^{26}$ W	[12]
Schwarzschild radius of the Earth	$2G_N M_\oplus/c^2$	8.870 055 94(2) mm	[13]
Earth mass	$M_\oplus$	5.972 6(7) $\times 10^{24}$ kg	[14]
Earth mean equatorial radius	$R_\oplus$	6.378 137 $\times 10^6$ m	[6]
luminosity conversion (deprecated)	$L$	3.02 $\times 10^{28} \times 10^{-0.4 M_{\text{bol}}}$ W ( $M_{\text{bol}}$ = absolute bolometric magnitude = bolometric magnitude at 10 pc)	[15]
flux conversion (deprecated)	$\mathcal{F}$	2.52 $\times 10^{-8} \times 10^{-0.4 m_{\text{bol}}}$ W m <sup>-2</sup> ( $m_{\text{bol}}$ = apparent bolometric magnitude)	from above
ABsolute monochromatic magnitude	AB	-2.5 log <sub>10</sub> $f_\nu$ - 56.10 (for $f_\nu$ in W m <sup>-2</sup> Hz <sup>-1</sup> ) = -2.5 log <sub>10</sub> $f_\nu$ + 8.90 (for $f_\nu$ in Jy)	[16]
Solar angular velocity around the Galactic center	$\Theta_0/R_0$	30.3 $\pm$ 0.9 km s <sup>-1</sup> kpc <sup>-1</sup>	[17]
Solar distance from Galactic center	$R_0$	8.4(6) kpc	[17,18]
circular velocity at $R_0$	$v_0$ or $\Theta_0$	254(16) km s <sup>-1</sup>	[17]
local disk density	$\rho_{\text{disk}}$	3-12 $\times 10^{-24}$ g cm <sup>-3</sup> $\approx$ 2-7 GeV/ $c^2$ cm <sup>-3</sup>	[19]
local dark matter density	$\rho_\chi$	canonical value 0.3 GeV/ $c^2$ cm <sup>-3</sup> within factor 2-3	[20]
escape velocity from Galaxy	$v_{\text{esc}}$	498 km/s < $v_{\text{esc}}$ < 608 km/s	[21]
present day CMB temperature	$T_0$	2.7255(6) K	[22,23]
present day CMB dipole amplitude		3.355(8) mK	[22,24]
Solar velocity with respect to CMB		369(1) km/s towards $(\ell, b) = (263.99(14)^\circ, 48.26(3)^\circ)$	[22,24]
Local Group velocity with respect to CMB	$v_{\text{LG}}$	627(22) km/s towards $(\ell, b) = (276(3)^\circ, 30(3)^\circ)$	[22,24]
entropy density/Boltzmann constant	$s/k$	2 891.2 $(T/2.7255)^3$ cm <sup>-3</sup>	[25]
number density of CMB photons	$n_\gamma$	410.7 $(T/2.7255)^3$ cm <sup>-3</sup>	[25]
baryon-to-photon ratio	$\eta = n_b/n_\gamma$	6.05(7) $\times 10^{-10}$ (CMB)	[26]
present day Hubble expansion rate	$H_0$	5.7 $\times 10^{-10} \leq \eta \leq 6.7 \times 10^{-10}$ (95% CL)	[26]
scale factor for Hubble expansion rate	$h$	100 h km s <sup>-1</sup> Mpc <sup>-1</sup> = $h \times (9.777 752 \text{ Gyr})^{-1}$	[29]
Hubble length	$c/H_0$	0.673(12)	[2,3]
scale factor for cosmological constant	$c^2/3H_0^2$	0.925 0629 $\times 10^{26} h^{-1}$ m = 1.37(2) $\times 10^{26}$ m	
critical density of the Universe	$\rho_{\text{crit}} = 3H_0^2/8\pi G_N$	2.85247 $\times 10^{51} h^{-2}$ m <sup>2</sup> = 6.3(2) $\times 10^{51}$ m <sup>2</sup> = 2.775 366 27 $\times 10^{11} h^2 M_\odot \text{Mpc}^{-3}$ = 1.878 47(23) $\times 10^{-29} h^2$ g cm <sup>-3</sup> = 1.053 75(13) $\times 10^{-5} h^2$ (GeV/ $c^2$ ) cm <sup>-3</sup>	
number density of baryons	$n_b$	2.482(32) $\times 10^{-7}$ cm <sup>-3</sup> (2.1 $\times 10^{-7} < n_b < 2.7 \times 10^{-7}$ ) cm <sup>-3</sup> (95% CL)	[2,3,27,28] $\eta \times n_\gamma$
baryon density of the Universe	$\Omega_b = \rho_b/\rho_{\text{crit}}$	$\dagger 0.02207(27) h^{-2} = \dagger 0.0499(22)$	[2,3]
cold dark matter density of the universe	$\Omega_{\text{cdm}} = \rho_{\text{cdm}}/\rho_{\text{crit}}$	$\dagger 0.1198(26) h^{-2} = \dagger 0.265(11)$	[2,3]
100 $\times$ approx to $r_*/D_A$	100 $\times \theta_{\text{MC}}$	$\dagger 1.0413(6)$	[2,3]
reionization optical depth	$\tau$	$\dagger 0.091_{-0.014}^{+0.013}$	[2,3]
scalar spectral index	$n_s$	$\dagger 0.958(7)$	[2,3]
ln pwr primordial curvature pert. ( $k_0=0.05 \text{ Mpc}^{-1}$ )	$\ln(10^{10} \Delta_{\mathcal{R}}^2)$	$\dagger 3.090(25)$	[2,3]

Quantity	Symbol, equation	Value	Reference, footnote
dark energy density of the $\Lambda$ CDM Universe	$\Omega_\Lambda$	$0.685^{+0.017}_{-0.016}$	[2,3]
pressureless matter density of the Universe	$\Omega_m = \Omega_{\text{cdm}} + \Omega_b$	$0.315^{+0.016}_{-0.017}$ (From $\Omega_\Lambda$ and flatness constraint)	[2,3]
dark energy equation of state parameter	$w$	$\# -1.10^{+0.08}_{-0.07}$ ( <i>Planck</i> +WMAP+BAO+SN)	[32]
CMB radiation density of the Universe	$\Omega_\gamma = \rho_\gamma/\rho_c$	$2.473 \times 10^{-5} (T/2.7255)^4 h^{-2} = 5.46(19) \times 10^{-5}$	[25]
effective number of neutrinos	$N_{\text{eff}}$	$\dagger 3.36 \pm 0.34$	[2]
sum of neutrino masses	$\sum m_\nu$	$< 0.23$ eV (95% CL; CMB+BAO) $\Rightarrow \Omega_\nu h^2 < 0.0025$	[2,30,31]
neutrino density of the Universe	$\Omega_\nu$	$< 0.0025 h^{-2} \Rightarrow < 0.0055$ (95% CL; CMB+BAO)	[2,30,31]
curvature	$\Omega_{\text{tot}} = \Omega_m + \dots + \Omega_\Lambda$	$\# 0.96^{+0.4}_{-0.5}$ (95%CL)	[2]
		$\# 1.000(7)$ (95% CL; CMB+BAO)	[2]
fluctuation amplitude at $8 h^{-1}$ Mpc scale	$\sigma_8$	$\dagger 0.828 \pm 0.012$	[2,3]
running spectral index slope, $k_0 = 0.002$ Mpc $^{-1}$	$dn_s/d \ln k$	$\# -0.015(9)$	[2]
tensor-to-scalar field perturbations ratio, $k_0=0.002$ Mpc $^{-1}$	$r = T/S$	$\# < 0.11$ at 95% CL; no running	[2,3]
redshift at decoupling	$z_{\text{dec}}$	$\dagger 1090.2 \pm 0.7$	[2]
age at decoupling	$t_*$	$\dagger 3.72 \times 10^5$ yr	
sound horizon at decoupling	$r_s(z_*)$	$\dagger 147.5 \pm 0.6$ Mpc ( <i>Planck</i> CMB)	[32]
redshift of matter-radiation equality	$z_{\text{eq}}$	$\dagger 3360 \pm 70$	[2]
redshift at half reionization	$z_{\text{reion}}$	$\dagger 11.1 \pm 1.1$	[2]
age at half reionization	$t_{\text{reion}}$	$\dagger 462$ Myr	
age of the Universe	$t_0$	$\dagger 13.81 \pm 0.05$ Gyr	[2]

$\ddagger$  Parameter in six-parameter  $\Lambda$ CDM fit [2].

$\dagger$  Derived parameter in six-parameter  $\Lambda$ CDM fit [2].

$\#$  Extended model parameter [2].

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- Astronomical\_Constants\_2014.pdf*, downloaded from [asa.usno.navy.mil/SecK/Constants.html](http://asa.usno.navy.mil/SecK/Constants.html); also see [www.iau.org/static/resolutions/IAU2012\\_English.pdf](http://www.iau.org/static/resolutions/IAU2012_English.pdf). The Gaussian gravitational constant  $k$  is now deleted from the system of astronomical constants.
- The distance at which 1 au subtends 1 arc sec: 1 au divided by  $\pi/648000$ .
- Product of  $2/c^2$  and the observationally determined Solar mass parameter  $G_N M_\odot$  [7] (TDB time scale).
- $G_N M_\odot$  [7]  $\div G_N$  [1].
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- $4\pi (1 \text{ au})^2 \times (1361 \text{ W m}^{-2})$ , assuming isotropic irradiance; G. Kopp and J.L. Lean, *Geophys. Res. Lett.* **38**, L01706 (2011) give  $1360.8 \pm 0.6 \text{ W m}^{-2}$ , but given the scatter in the data we use the rounded value without quoting an error.
- Product of  $2/c^2$  and the geocentric gravitational constant  $G_N M_\oplus$  [7] (TDB time scale).
- $G_N M_\oplus$  [7]  $\div G_N$  [1].
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- $n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{kT}{hc}\right)^3$ ;  $\rho_\gamma = \frac{\pi^2 kT}{15 c^2} \left(\frac{kT}{hc}\right)^3$ ;  $s/k = \frac{2 \cdot 43 \cdot \pi^2}{11 \cdot 45} \left(\frac{kT}{hc}\right)^3$ ;  $kT_0/hc = 11.902(4)/\text{cm}$ .
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- $n_b$  depends only upon the measured  $\Omega_b h^2$ , the average baryon mass at the present epoch [28], and  $G_N$ :  $n_b = (\Omega_b h^2) h^{-2} \rho_{\text{crit}} / (0.93711 \text{ GeV}/c^2 \text{ per baryon})$ .
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- Conversion using length of sidereal year.
- $\Omega_\nu h^2 = \sum m_{\nu_j} / 93.04 \text{ eV}$ , where the sum is over all neutrino mass eigenstates. The lower limit follows from neutrino mixing results reported in this *Review* combined with the assumptions that there are three light neutrinos ( $m_\nu < 45 \text{ GeV}/c^2$ ) and that the lightest neutrino is substantially less massive than the others:  $\Delta m_{32}^2 = (2.32^{+0.12}_{-0.08}) \times 10^{-3} \text{ eV}^2$ , so  $\sum m_{\nu_j} \geq m_{\nu_3} \approx \sqrt{\Delta m_{32}^2} = 0.05 \text{ eV}$ . (This becomes 0.10 eV if the mass hierarchy is inverted, with  $m_{\nu_1} \approx m_{\nu_2} \gg m_{\nu_3}$ .) Alternatively, if the limit obtained from tritium decay experiments ( $m_\nu < 2 \text{ eV}$ ) is used for the upper limit, then  $\Omega_\nu < 0.04$ .
- Astrophysical determinations of  $\sum m_{\nu_j}$ , reported in the Full Listings of this *Review* under “Sum of the neutrino masses,” range from  $< 0.17 \text{ eV}$  to  $< 2.3 \text{ eV}$  in papers published since 2003.
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