

# REVIEW OF MULTIBODY CHARM ANALYSES

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***Kinematics & Models*** The differential decay rate to a point  $\mathbf{s} = (s_1, \dots, s_n)$  in  $n$  dimensional phase space can be expressed as

$$d\Gamma = |\mathcal{M}(\mathbf{s})|^2 \left| \frac{\partial^n \phi}{\partial(s_1 \dots s_n)} \right| d^n s \quad (1)$$

where  $\left| \frac{\partial^n \phi}{\partial(s_1 \dots s_n)} \right|$  represents the density of states at  $\mathbf{s}$ , and  $\mathcal{M}$  the matrix element for the decay at that point in phase space. For two-body decays,  $\left| \frac{\partial^n \phi}{\partial(s_1 \dots s_n)} \right|$  is a  $\delta$  function, while for  $D^0$  decays to 3, 4, 5, ... pseudoscalars, phase space is 2, 5, 8, ... dimensional, leading to a rich phenomenology. Additional parameters are required to fully describe decays with vector particles in the initial or final state.

For the important case of a pseudoscalar decaying to 3 pseudoscalars, the decay kinematics can be described in a two dimensional Dalitz plot [1]. The Dalitz plot of  $D \rightarrow abc$  is usually parametrized in terms of invariant-mass-squared variables  $s_1 = (p_a + p_b)^2$  and  $s_2 = (p_b + p_c)^2$ , where  $p_a, p_b, p_c$  are the four-momenta of particles  $a, b, c$ . In terms of these variables, phase-space density is constant across the kinematically allowed region, so that any structure seen in the Dalitz plot is a direct consequence of the dynamics encoded in  $|\mathcal{M}|^2$ .

An important difference between decays to two or three pseudoscalars compared to decays to four or more particles is the behavior under parity. In the former case, the operation of parity can also be expressed as a rotation, so no parity violating observables can be defined (unless they also violate rotational invariance). This is not the case for decays to four or more particles. This leads to the interesting possibility of using parity-odd observables in four body decays for  $CP$  violation searches, as discussed below. Another consequence of these considerations is that four-body-decay kinematics cannot be described unambiguously in terms of invariant-mass-squared variables, as these are all parity even.

The matrix element  $\mathcal{M}$  is usually modeled as a sum of interfering decay amplitudes, each proceeding through resonant two-body decays [2]. See Refs [2–4] for a review of resonance phenomenology. In most analyses, each resonance is described by a Breit–Wigner or Flatté lineshape, and the model includes a non-resonant term with a constant phase and magnitude across the Dalitz plot. This approach has well-known theoretical limitations, such as the violation of unitarity and analyticity, which tend to be particularly problematic for broad, overlapping resonances. This motivates the use of more sophisticated descriptions, especially for the broad, overlapping resonances that occur typically in the S-wave components. In charm analyses, these have included the K-matrix approach [5,6,7] which respects unitarity; the use of LASS scattering data [8]; dispersive methods [9,10]; methods based on chiral symmetry [11,12]; and quasi model-independent parametrizations [13,14]. An important example first analyzed by CLEO [15,16,17] is  $D^0 \rightarrow K_S \pi^+ \pi^-$ , which is a key channel in  $CP$  violation and charm mixing analyses. Belle models this final state as a superposition of 18 resonances (including 4 significant doubly Cabibbo suppressed amplitudes) described by Breit–Wigner or Flatté lineshapes, plus a non-resonant component [18]. CDF’s analysis follows a similar approach [19]. BaBar’s model for the same decay replaces the broad  $\pi\pi$  and  $K\pi$  S-wave resonances and the non-resonant component with a K-matrix description [20]. Belle’s and BaBar’s data have been re-analyzed by [21] in a QCD factorization framework, using line-shape parametrization for the S [11,12] and P wave [10] contributions (with input from  $\tau^- \rightarrow K_S \pi^- \nu_\tau$  data [22] for the latter) that preserve 2-body unitarity and analyticity. The measurements give compatible results for the components they share. All three approaches remain within the confines of the “isobar” framework which treats the decay as a series of independent two-body processes, ignoring long-range hadronic effects. Dispersive techniques that account for these hadronic effects and respect full 3 body unitarity and analyticity have been applied to regions of the  $D^- \rightarrow K^- \pi^+ \pi^+$  Dalitz plot below the  $\eta'K$  threshold [23].

Limitations in the theoretical description of interfering resonances are the leading source of systematic uncertainty in many analyses. This is set to become increasingly problematic given the statistical precision achievable with the vast charm samples available at the B factories, LHCb, and their upgrades. Already now, clean data samples with millions of charm events are available even in suppressed decay modes, e.g. 2.4M  $D^0 \rightarrow \pi^- \pi^+ \pi^0$  events at LHCb [24]. In some cases, the model uncertainty can be removed through model-independent amplitude methods, often relying on input from the charm threshold, as discussed below. At the same time, increasingly sophisticated models are being developed, and applied to data.

***Applications of multibody charm analyses*** The interference between the decay paths via which multibody decays proceed provides sensitivity to both relative magnitudes and phases of the contributing decay amplitudes. It is especially this sensitivity to phases that makes amplitude analyses such a uniquely powerful tool for studying a wide range of phenomena. Here we concentrate on their use for  $CP$  violation measurements and mixing in charm, and charm inputs to  $CP$  violation analyses in  $B$  meson decays. The properties of light-meson resonances determined in  $D$ -meson amplitude analyses are reported in the light-unflavored-meson section of this *Review*.

***Time-integrated searches for  $CP$  violation in charm*** Comparing the results of amplitude fits for  $CP$ -conjugate decay modes provides a measure of  $CP$  violation. The advantage of this approach over the model-independent searches discussed in the next paragraph is the physical interpretation of any  $CP$  violation observation that such a fit result would allow. The disadvantage lies in the theoretical uncertainty intrinsic to such analyses due to the amplitude-model dependence. Recent  $CP$  violation searches using this method include CLEO-c's amplitude analysis of  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$  [25] and CDF's analysis of  $\sim 350,000 D^0 \rightarrow K_S \pi^+ \pi^-$  events [19].

The most common model-independent approach for searching for local  $CP$  violation across a Dalitz plot is based on performing a  $\chi^2$  comparison of the number of events in the bins of  $CP$ -conjugate Dalitz plots. This method was pioneered by

BaBar [26] and developed further in [27,28], with recent results in  $D^\pm \rightarrow K^+K^-\pi^\pm$  [29,30,31],  $D^0 \rightarrow K_S\pi^+\pi^-$  [19], and  $D^+ \rightarrow \pi^-\pi^+\pi^+$  [32]. These techniques have been generalized to four-body decays, and applied to  $D^0 \rightarrow K^+K^-\pi^+\pi^-$  and  $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$  [33]. Un-binned methods can increase the sensitivity [34]; two different unbinned methods have been applied by LHCb to  $D^+ \rightarrow \pi^-\pi^+\pi^+$  [32] and  $D^0 \rightarrow \pi^+\pi^-\pi^0$  [24]. None of these analyses have shown evidence of  $CP$  violation.

Another model-independent approach, providing complementary information, is based on constructing observables in four body decays that are odd under motion reversal (“naïve T”) [35–43], which is equivalent to  $P$  for scalar particles [43]. One such observable is  $C_T = \vec{p}_{K^+} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-})$  in  $D^0 \rightarrow K^+K^-\pi^+\pi^-$ . The rate asymmetry of positive and negative  $C_T$ ,  $A_T \equiv \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)}$ , is a  $P$  violating parameter. Comparing  $A_T$  with the  $C$ -conjugate asymmetry in  $\bar{D}^0$  decays,  $\bar{A}_T$ , provides sensitivity to  $CP$  violation. Searches for  $CP$  violation in this manner have been carried out by FOCUS in  $D^0 \rightarrow K^+K^-\pi^+\pi^-$  [44], BaBar in  $D^0 \rightarrow K^+K^-\pi^+\pi^-$ ,  $D^+ \rightarrow K^+K_S\pi^+\pi^-$ , and  $D_s^+ \rightarrow K^+K_S\pi^+\pi^-$  [45,46], and LHCb in  $D^0 \rightarrow K^+K^-\pi^+\pi^-$  [47]. In addition to a phase-space integrated result, LHCb’s analysis is also carried out locally in sub-regions of phase space to enhance the sensitivity of the method. All results so far have been consistent with  $CP$  conservation.

***D mixing and CP violation*** Time-dependent amplitude analyses in decays to final states that are accessible to both  $D^0$  and  $\bar{D}^0$  have unique sensitivity to mixing parameters. A Dalitz plot analysis of a self-conjugate final state, such as  $K_S\pi^+\pi^-$  and  $K_S K^+K^-$ , allows the measurement of the phase difference between the relevant  $D^0$  and  $\bar{D}^0$  decay amplitudes, and thus a direct measurement of the mixing parameters  $x, y$  (rather than the decay-specific parameters  $x'^2, y'$  measured for example in  $D^0 \rightarrow K\pi$ ) [17]. These analyses are also sensitive to  $CP$  violation in mixing and in the interference between mixing and decay. These results are summarized in Ref. [48]. The important role from charm threshold data as input to such measurements is discussed below.

**Charm amplitude analyses for measuring  $\gamma/\phi_3$**  Neutral  $D$  mesons originating from  $B^- \rightarrow DK^-$  (which we denote with  $D_{B^-}$ ) are a superposition of  $D^0$  and  $\overline{D}^0$  with a relative phase that depends on  $\gamma/\phi_3$ :

$$D_{B^-} \propto D^0 + r_B e^{i(\delta_B - \gamma)} \overline{D}^0,$$

where  $\delta_B$  is a  $CP$  conserving strong phase, and  $r_B \sim 0.1$ . In the corresponding  $CP$ -conjugate expression,  $\gamma/\phi_3$  changes sign. An amplitude analysis of the subsequent decay of the  $D_{B^\pm}$  allows an extraction of  $\gamma/\phi_3$  [49–54]. The method generalizes to similar  $B$  hadron decays, such as  $B^0 \rightarrow DK^{*0}$ . Measurements based on this technique have been reported by BaBar, Belle and LHCb using both model-dependent approaches and model-independent ones based on CLEO-c input [18,55–61,65–67]. The most precise individual results come from the study of the  $D_{B^-} \rightarrow K_S \pi^+ \pi^-$  and  $D_{B^-} \rightarrow K_S K^+ K^-$  with an uncertainty of approximately  $15^\circ$  [18,55,59,67].

**Model independent methods for  $\gamma/\phi_3$  and charm mixing**

The theoretical uncertainty on amplitude models of multibody  $D^0$  decays potentially limits the precision of measurements of  $\gamma/\phi_3$  in  $B^\pm \rightarrow DK^\pm$  and related decay modes. Model-independent methods to measure  $\gamma/\phi_3$  require input related to the relative phases of the  $D^0$  and  $\overline{D}^0$  decay amplitudes across the phase-space distribution. The same considerations apply to measurements of  $D^0$  mixing and  $CP$  violation parameters in time-dependent Dalitz plot analyses. The required phase information is accessible at the charm threshold, where CLEO-c and BES III operate [48,52,68–74]. There,  $D$  mesons originate from the decay  $\psi(3770) \rightarrow D\overline{D}$ . The two  $D$  mesons are quantum-correlated which can be used to identify decays of well-defined  $D^0 - \overline{D}^0$  superpositions to the final state of interest. The resulting interference of  $D^0$  and  $\overline{D}^0$  amplitudes provides the desired model-independent phase information. For decays to non-self-conjugate decays such as  $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ , analysing  $D^0 - \overline{D}^0$  superpositions provides the only way of measuring the relative phase between the  $D^0$  and  $\overline{D}^0$  amplitudes.

These analyses can be performed in sub-regions/bins of phase space, or integrated across phase space. The relevant

result can be expressed in terms of one complex parameter  $\mathcal{Z} = Re^{-i\delta}$  per pair of  $CP$ -conjugate phase space bins, with magnitude  $R \leq 1$ . The larger  $R$ , the higher the sensitivity to interference effects, and thus to  $\gamma/\phi_3$ . The sensitivity of the binned analyses can be optimized by using amplitude model-dependent information to maximize  $R$  in each bin, without introducing a model-dependent bias in the result. CLEO-c data have been analyzed in this way to provide binned results for the self-conjugate decays  $D^0 \rightarrow K_S\pi\pi$  and  $D^0 \rightarrow K_S K K$  [75,76]. The phase-space integrated analyses for  $D^0, \bar{D}^0 \rightarrow K_S K^+ \pi^-$ ,  $K^- \pi^+ \pi^0$ , and  $K^- \pi^+ \pi^- \pi^+$  have yielded  $\mathcal{Z}^{K_S K \pi} = (0.73 \pm 0.8)e^{-i(8.3^\circ \pm 15.2^\circ)}$ ,  $\mathcal{Z}^{K \pi \pi^0} = (0.82 \pm 0.07)e^{-i(164^\circ_{-14^\circ} + 20^\circ)}$ ,  $\mathcal{Z}^{K^3 \pi} = (0.32^{+0.20}_{-0.28})e^{-i(225^\circ_{-78^\circ} + 21^\circ)}$ , respectively [77,78,79]. These results follow the usual convention for  $\gamma/\phi_3$ -related studies where  $CP|D^0\rangle = +|\bar{D}^0\rangle$ , while in charm mixing measurements, one usually takes  $CP|D^0\rangle = -|\bar{D}^0\rangle$ , leading to a phase-shift in  $\delta$  of  $\pi$ . Restricting the analysis to a bin around the  $K^*K$  resonance in the  $K_S K \pi$  Dalitz plot, [77] find  $R = 1.00 \pm 0.16$ , illustrating the benefit in dividing phase space into bins.

The corresponding phase space-integrated input for self-conjugate decays such as  $D^0 \rightarrow \pi^+ \pi^- \pi^0$  takes the form of a single real parameter, the  $CP$ -even fraction  $F_+$ , defined such that a  $CP$  even eigenstate has  $F_+ = 1$ , while a  $CP$ -odd eigenstate has  $F_+ = 0$  [72]. A recent analysis of CLEO-c data revealed that  $D^0 \rightarrow \pi^+ \pi^- \pi^0$  is compatible with being completely  $CP$ -even with  $F_+ = 1.014 \pm 0.045 \pm 0.022$ , while  $D^0 \rightarrow K^+ K^- \pi^0$  has  $F_+ = 0.734 \pm 0.106 \pm 0.054$  and  $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  has  $F_+ = 0.737 \pm 0.028$  [73].

The charm system itself provides, through mixing, a well-defined, time-dependent superposition of  $D^0$  and  $\bar{D}^0$ . Using mixing parameters measured independently as input, this can be used to obtain the relevant information for  $\gamma/\phi_3$  measurements. This method is expected to be particularly powerful in doubly Cabibbo-suppressed decays such as  $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ , and when used in conjunction with information from charm threshold [80,81].

**Summary** Multibody charm decays offer a rich phenomenology, including unique sensitivity to  $CP$  violation and charm

mixing. This is a highly dynamic field with many new results (some of which we presented here) and rapidly increasing, high quality datasets. These datasets constitute a huge opportunity, but also a challenge to improve the theoretical descriptions of soft hadronic effects in multibody decays. For some measurements, model-independent methods, many relying on input from the charm threshold, provide a way of removing model-induced uncertainties. At the same time, work is ongoing to improve the theoretical description of multibody decays.

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