16. GRAND UNIFIED THEORIES

Revised October 2011 by S. Raby (Ohio State University).

16.1. Grand Unification

16.1.1. *Standard Model : An Introduction* :

In spite of all the successes of the Standard Model [SM] it is unlikely to be the final theory. It leaves many unanswered questions. Why the local gauge interactions $SU(3)_C \times SU(2)_L \times U(1)_Y$ and why 3 families of quarks and leptons? Moreover why does one family consist of the states $[Q, u^c, d^c; L, e^c]$ transforming as $[(3, 2, 1/3)$, $(\bar{3}, 1, -4/3)$, $(\bar{3}, 1, 2/3)$; $(1, 2, -1)$, $(1, 1, 2)]$, where $Q = (u, d)$ and $L = (\nu, e)$ are $SU(2)_L$ doublets and u^c, d^c, e^c are charge conjugate $SU(2)_L$ singlet fields with the $U(1)_Y$ quantum numbers given? [We use the convention that electric charge $Q_{EM} = T_{3L} + Y/2$ and all fields are left handed Weyl spinors.] Note the SM gauge interactions of quarks and leptons are completely fixed by their gauge charges. Thus if we understood the origin of this charge quantization, we would also understand why there are no fractionally charged hadrons. Finally, what is the origin of quark and lepton masses or the apparent hierarchy of family masses and quark and leptonic mixing angles? Perhaps if we understood this, we would also know the origin of CP violation, the solution to the strong CP problem, the origin of the cosmological matter - antimatter asymmetry. In addition, it lacks an explanation for the observed dark matter and dark energy of the universe.

The SM has 19 arbitrary parameters; their values are chosen to fit the data. Three arbitrary gauge couplings: g_3, g, g' (where g, g' are the $SU(2)_L$, $U(1)_Y$ couplings, respectively) or equivalently $\alpha_s = (g_3^2/4\pi)$, $\alpha_{EM} = (e^2/4\pi)$ $(e = g \ \sin\theta_W)$ and $\sin^2\theta_W = (g')^2/(g^2 + (g')^2)$. In addition there are 13 parameters associated with the 9 charged fermion masses and the four mixing angles in the CKM matrix. The remaining 3 parameters are v, λ [the Higgs VEV and quartic coupling] (or equivalently M_Z, m_h^0) and the QCD θ parameter. In addition, data from neutrino oscillation experiments provide convincing evidence for neutrino masses. With 3 light Majorana neutrinos there are at least 9 additional parameters in the neutrino sector; 3 masses and 6 mixing angles and phases. In summary, the SM has too many arbitrary parameters and leaves open too many unresolved questions to be considered complete. These are the problems which grand unified theories hope to address.

16.1.2. *Charge Quantization* :

In the Standard Model, quarks and leptons are on an equal footing; both fundamental particles without substructure. It is now clear that they may be two faces of the same coin; unified, for example, by extending QCD (or $SU(3)_C$) to include leptons as the fourth color, $SU(4)_C$ [1]. The complete Pati-Salam gauge group is $SU(4)_C \times SU(2)_L \times SU(2)_R$ with the states of one family $[(Q, L), (Q^c, L^c)]$ transforming as $[(4, 2, 1)$, $(\bar{4}, 1, \bar{2})]$ where $Q^c = (d^c, u^c)$, $L^c = (e^c, \nu^c)$ are doublets under $SU(2)_R$. Electric charge is now given by the relation $Q_{EM} = T_{3L} + T_{3R} + 1/2(B - L)$ and $SU(4)_C$ contains the subgroup $SU(3)_C \times (B - L)$ where B (L) is baryon (lepton) number. Note ν^c has no SM quantum numbers and is thus completely "sterile". It is introduced to complete the $SU(2)_R$ lepton doublet. This additional state is desirable when considering neutrino masses.

Although quarks and leptons are unified with the states of one family forming two irreducible representations of the gauge group; there are still 3 independent gauge couplings (two if one also imposes parity, i.e. $L \leftrightarrow R$ symmetry). As a result the three low energy gauge couplings are still independent arbitrary parameters. This difficulty is resolved by embedding the SM gauge group into the simple unified gauge group, Georgi-Glashow $SU(5)$, with one universal gauge coupling α_G defined at the grand unification scale M_G [2]. Quarks and leptons still sit in two irreducible representations, as before, with a $\mathbf{10} = [Q, u^c, e^c]$ and $\bar{\mathbf{5}} = [d^c, L]$. Nevertheless, the three low energy gauge couplings are now determined in terms of two independent parameters : α_G and M_G. Hence there is one prediction.

In order to break the electroweak symmetry at the weak scale and give mass to quarks and leptons, Higgs doublets are needed which can sit in either a $\mathbf{5_H}$ or $\bar{\mathbf{5}}_\mathbf{H}$. The additional 3 states are color triplet Higgs scalars. The couplings of these color triplets violate baryon and lepton number and nucleons decay via the exchange of a single color triplet Higgs scalar. Hence in order not to violently disagree with the non-observation of nucleon decay, their mass must be greater than $\sim 10^{11}$ GeV [3]. Moreover, in supersymmetric GUTs, in order to cancel anomalies as well as give mass to both up and down quarks, both Higgs multiplets $\mathbf{5_H}$, $\bar{\mathbf{5}}_\mathbf{H}$ are required. As we shall discuss later, nucleon decay now constrains the color triplet Higgs states in a SUSY GUT to have mass significantly greater than M_G.

Complete unification is possible with the symmetry group $SO(10)$ with one universal gauge coupling α_G and one family of quarks and leptons sitting in the 16 dimensional spinor representation $\mathbf{16} = [\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}]$ [4]. The $SU(5)$ singlet $\mathbf{1}$ is identified with ν^c. In Table 1 we present the states of one family of quarks and leptons, as they appear in the **16**. It is an amazing and perhaps even profound fact that all the states of a single family of quarks and leptons can be represented digitally as a set of 5 zeros and/or ones or equivalently as the tensor product of 5 "spin" 1/2 states with $\pm = | \pm \frac{1}{2} >$ and with the condition that we have an even number of $|+>$ spins. The first three "spins" correspond to $SU(3)_C$ color quantum numbers, while the last two are $SU(2)_L$ weak quantum numbers. In fact an $SU(3)_C$ rotation just raises one color index and lowers another, thereby changing colors $\{r,\ b,\ y\}$. Similarly an $SU(2)_L$ rotation raises one weak index and lowers another, thereby flipping the weak isospin from up to down or vice versa. In this representation weak hypercharge Y is given by the simple relation $Y = -2/3(\sum \text{ color spins}) + (\sum \text{ weak spins})$. $SU(5)$ rotations [in particular, the ones NOT in $SU(3)_C \times SU(2)_L \times U(1)_Y$] then raise (or lower) a color index, while at the same time lowering (or raising) a weak index. It is easy to see that such rotations can mix the states $\{Q,\ u^c,\ e^c\}$ and $\{d^c, L\}$ among themselves and ν^c is a singlet. The new $SO(10)$ rotations [not in $SU(5)$] are then given by either raising or lowering any two spins. For example, by raising the two weak indices ν^c rotates into e^c, etc.

$SO(10)$ has two inequivalent maximal breaking patterns. $SO(10) \to SU(5) \times U(1)_X$ and $SO(10) \to SU(4)_C \times SU(2)_L \times SU(2)_R$. In the first case we obtain Georgi-Glashow $SU(5)$ if Q_{EM} is given in terms of $SU(5)$ generators alone or so-called flipped $SU(5)$ [5] if Q_{EM} is partly in $U(1)_X$. In the latter case we have the Pati-Salam symmetry. If $SO(10)$ breaks directly to the SM at M_G, then we retain the prediction for gauge

Table 16.1: The quantum numbers of the **16** dimensional representation of SO(10).

State	Y	Color	Weak
$\boldsymbol{\nu}^c$	0	$-\,-\,-$	$-\,-$
$\boldsymbol{e}^c$	2	$-\,-\,-$	$+\,+$
$\boldsymbol{u}_r$	1/3	$+\,-\,-$	$-\,+$
$\boldsymbol{d}_r$	1/3	$+\,-\,-$	$+\,-$
$\boldsymbol{u}_b$	1/3	$-\,+\,-$	$-\,+$
$\boldsymbol{d}_b$	1/3	$-\,+\,-$	$+\,-$
$\boldsymbol{u}_y$	1/3	$-\,-\,+$	$-\,+$
$\boldsymbol{d}_y$	1/3	$-\,-\,+$	$+\,-$
$\boldsymbol{u}^c_r$	−4/3	$-\,+\,+$	$-\,-$
$\boldsymbol{u}^c_b$	−4/3	$+\,-\,+$	$-\,-$
$\boldsymbol{u}^c_y$	−4/3	$+\,+\,-$	$-\,-$
$\boldsymbol{d}^c_r$	2/3	$-\,+\,+$	$+\,+$
$\boldsymbol{d}^c_b$	2/3	$+\,-\,+$	$+\,+$
$\boldsymbol{d}^c_y$	2/3	$+\,+\,-$	$+\,+$
$\boldsymbol{\nu}$	−1	$+\,+\,+$	$-\,+$
$\boldsymbol{e}$	−1	$+\,+\,+$	$+\,-$

coupling unification. However more possibilities for breaking (hence more breaking scales and more parameters) are available in $SO(10)$. Nevertheless with one breaking pattern $SO(10) \rightarrow SU(5) \rightarrow$ SM, where the last breaking scale is M_G, the predictions from gauge coupling unification are preserved. The Higgs multiplets in minimal $SO(10)$ are contained in the fundamental $\mathbf{10_H} = [\mathbf{5_H},\ \bar{\mathbf{5}}_\mathbf{H}]$ representation. Note, only in $SO(10)$ does the gauge symmetry distinguish quark and lepton multiplets from Higgs multiplets.

Finally, larger symmetry groups have been considered. For example, $E(6)$ has a fundamental representation **27** which under $SO(10)$ transforms as a $[\mathbf{16} + \mathbf{10} + \mathbf{1}]$. The breaking pattern $E(6) \rightarrow SU(3)_C \times SU(3)_L \times SU(3)_R$ is also possible. With the additional permutation symmetry $Z(3)$ interchanging the three $SU(3)$s we obtain so-called "trinification" [6] with a universal gauge coupling. The latter breaking pattern has been used in phenomenological analyses of the heterotic string [7]. However, in larger symmetry groups, such as $E(6)$, $SU(6)$, etc., there are now many more states which have not been observed and must be removed from the effective low energy theory. In particular, three families of **27**s in $E(6)$ contain three Higgs type multiplets transforming as **10**s of $SO(10)$. This makes these larger symmetry groups unattractive starting points for model building.

16.1.3. *String Theory and Orbifold GUTs* :

Orbifold compactification of the heterotic string [8–10], and recent field theoretic constructions known as orbifold GUTs [11], contain grand unified symmetries realized in 5 and 6 dimensions. However, upon compactifying all but four of these extra dimensions, only the MSSM is recovered as a symmetry of the effective four dimensional field theory.[1] These theories can retain many of the nice features of four dimensional SUSY GUTs, such as charge quantization, gauge coupling unification and sometimes even Yukawa unification; while at the same time resolving some of the difficulties of 4d GUTs, in particular problems with unwieldy Higgs sectors necessary for spontaneously breaking the GUT symmetry, and problems with doublet-triplet Higgs splitting or rapid proton decay. We will comment further on the corrections to the four dimensional GUT picture due to orbifold GUTs in the following sections. Finally, recent progress has been made in finding MSSM-like theories in the string landscape. This success is made possible by incorporating SUSY GUTs at an intermediate step in the construction. For a brief discussion, see Sec. 16.1.

16.1.4. *Gauge coupling unification* :

The biggest paradox of grand unification is to understand how it is possible to have a universal gauge coupling g_G in a grand unified theory [GUT] and yet have three unequal gauge couplings at the weak scale with $g_3 > g > g'$. The solution is given in terms of the concept of an effective field theory [EFT] [18]. The GUT symmetry is spontaneously broken at the scale M_G and all particles not in the SM obtain mass of order M_G. When calculating Green's functions with external energies $E \gg M_G$, we can neglect the mass of all particles in the loop and hence all particles contribute to the renormalization group running of the universal gauge coupling. However, for $E \ll M_G$ one can consider an effective field theory including only the states with mass $< E \ll M_G$. The gauge symmetry of the EFT is $SU(3)_C \times SU(2)_L \times U(1)_Y$ and

---

1 Also, in recent years there has been a great deal of progress in constructing three and four family models in Type IIA string theory with intersecting D6 branes [12]. Although these models can incorporate SU(5) or a Pati-Salam symmetry group in four dimensions, they typically have problems with gauge coupling unification. In the former case this is due to charged exotics which affect the RG running, while in the latter case the $SU(4)\times SU(2)_L\times SU(2)_R$ symmetry never unifies. Local models, however, with D-branes at singularities have had some more success in obtaining gauge coupling unification [13]. Note, heterotic string theory models also exist whose low energy effective 4d field theory is a SUSY GUT [14]. These models have all the virtues and problems of 4d GUTs. Finally, many heterotic string models have been constructed with the standard model gauge symmetry in 4d and no intermediate GUT symmetry in less than 10d. Some minimal 3 family supersymmetric models have been constructed [15,16]. These theories may retain some of the symmetry relations of GUTs, however the unification scale would typically be the string scale, of order 5×10^{17} GeV, which is inconsistent with low energy data. A way out of this problem was discovered in the context of the strongly coupled heterotic string, defined in an effective 11 dimensions [17]. In this case the 4d Planck scale (which controls the value of the string scale) now unifies with the GUT scale.

the three gauge couplings renormalize independently. The states of the EFT include only those of the SM; 12 gauge bosons, 3 families of quarks and leptons and one or more Higgs doublets. At M_G the two effective theories [the GUT itself is most likely the EFT of a more fundamental theory defined at a higher scale] must give identical results; hence we have the boundary conditions $g_3 = g_2 = g_1 \equiv g_G$ where at any scale $\mu < M_G$ we have $g_2 \equiv g$ and $g_1 = \sqrt{5/3}\, g'$. Then using two low energy couplings, such as $\alpha_s(M_Z)$, $\alpha_{EM}(M_Z)$, the two independent parameters α_G, M_G can be fixed. The third gauge coupling, $\sin^2\theta_W$ in this case, is then predicted. This was the procedure up until about 1991 [19,20]. Subsequently, the uncertainties in $\sin^2\theta_W$ were reduced ten fold. Since then, $\alpha_{EM}(M_Z)$, $\sin^2\theta_W$ have been used as input to predict α_G, M_G and $\alpha_s(M_Z)$ [21].

We emphasize that the above boundary condition is only valid when using one loop renormalization group [RG] running. With precision electroweak data, however, it is necessary to use two loop RG running. Hence one must include one loop threshold corrections to gauge coupling boundary conditions at both the weak and GUT scales. In this case it is always possible to define the GUT scale as the point where $\alpha_1(M_G) = \alpha_2(M_G) \equiv \tilde{\alpha}_G$ and $\alpha_3(M_G) = \tilde{\alpha}_G\ (1 + \epsilon_3)$. The threshold correction ϵ_3 is a logarithmic function of all states with mass of order M_G and $\tilde{\alpha}_G = \alpha_G + \Delta$ where α_G is the GUT coupling constant above M_G and Δ is a one loop threshold correction. Note, the popular code "SOFTSUSY" [22] has defined the GUT scale in just this way. The value of ϵ_3 can be read off from the output data. To the extent that gauge coupling unification is perturbative, the GUT threshold corrections are small and calculable. This presumes that the GUT scale is sufficiently below the Planck scale or any other strong coupling extension of the GUT, such as a strongly coupled string theory.

Supersymmetric grand unified theories [SUSY GUTs] are an extension of non-SUSY GUTs [23]. The key difference between SUSY GUTs and non-SUSY GUTs is the low energy effective theory. The low energy effective field theory in a SUSY GUT is assumed to satisfy N=1 supersymmetry down to scales of order the weak scale in addition to the SM gauge symmetry. Hence the spectrum includes all the SM states plus their supersymmetric partners. It also includes one pair (or more) of Higgs doublets; one to give mass to up-type quarks and the other to down-type quarks and charged leptons. Two doublets with opposite hypercharge Y are also needed to cancel fermionic triangle anomalies. Finally, it is important to recognize that a low energy SUSY breaking scale (the scale at which the SUSY partners of SM particles obtain mass) is necessary to solve the gauge hierarchy problem.

Simple non-SUSY $SU(5)$ is ruled out; initially by the increased accuracy in the measurement of $\sin^2\theta_W$ and by early bounds on the proton lifetime (see below) [20]. However, by now LEP data [21] has conclusively shown that SUSY GUTs is the *new standard model*; by which we mean the theory used to guide the search for new physics beyond the present SM (see Fig. Fig. 16.1). SUSY extensions of the SM have the property that their effects decouple as the effective SUSY breaking scale is increased. Any theory beyond the SM must have this property simply because the SM works so well. However, the SUSY breaking scale cannot be increased with impunity, since this would reintroduce a gauge hierarchy problem. Unfortunately there is no clear-cut answer to the question,

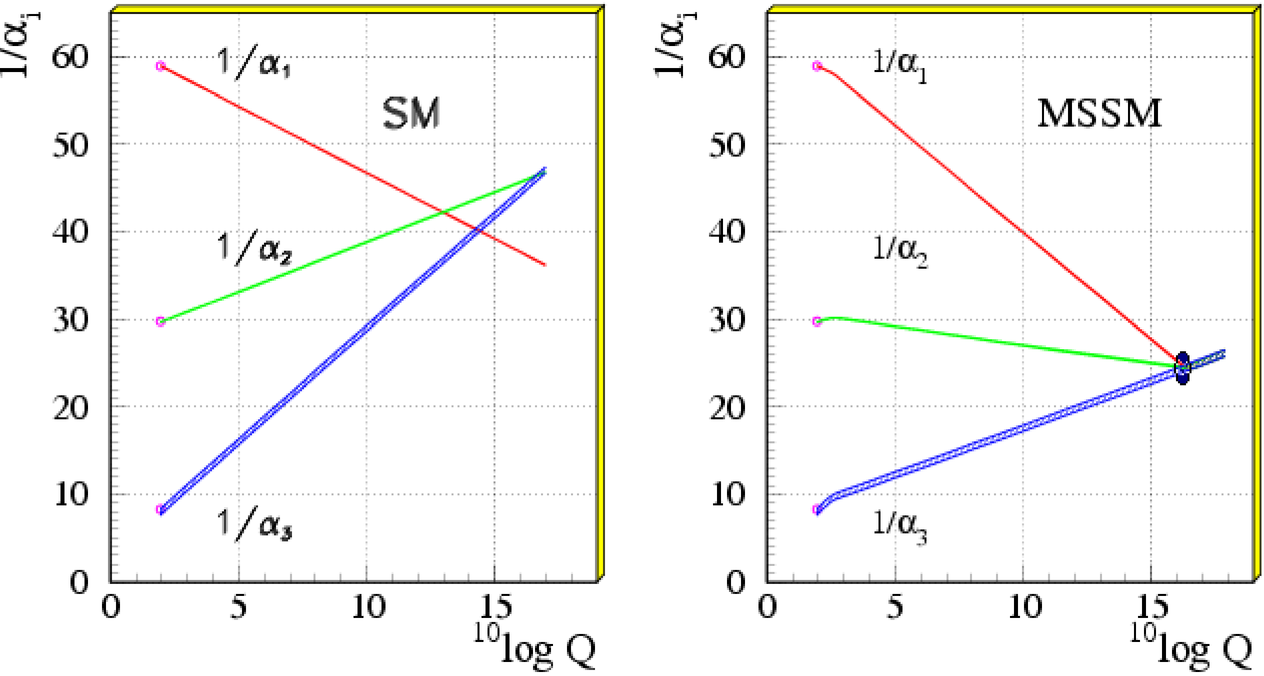

Figure 16.1: Gauge coupling unification in non-SUSY GUTs on the left vs. SUSY GUTs on the right using the LEP data as of 1991. Note, the difference in the running for SUSY is the inclusion of supersymmetric partners of standard model particles at scales of order a TeV (Fig. taken from Ref. 24). Given the present accurate measurements of the three low energy couplings, in particular $\alpha_s(M_Z)$, GUT scale threshold corrections are now needed to precisely fit the low energy data. The dark blob in the plot on the right represents these model dependent corrections.

when is the SUSY breaking scale too high. A conservative bound would suggest that the third generation quarks and leptons must be lighter than about 1 TeV, in order that the one loop corrections to the Higgs mass from Yukawa interactions remains of order the Higgs mass bound itself.

At present gauge coupling unification within SUSY GUTs works extremely well. Exact unification at M_G, with two loop renormalization group running from M_G to M_Z, and one loop threshold corrections at the weak scale, fits to within 3 σ of the present precise low energy data. A small threshold correction at M_G ($\epsilon_3 \sim$ - 3% to - 4%) is sufficient to fit the low energy data precisely [25,26,27]. [2] This may be compared to non-SUSY GUTs where the fit misses by $\sim$ 12 σ and a precise fit requires new weak scale states in

[2] This result implicitly assumes universal GUT boundary conditions for soft SUSY breaking parameters at M_G. In the simplest case we have a universal gaugino mass $M_{1/2}$, a universal mass for squarks and sleptons m_{16} and a universal Higgs mass m_{10}, as motivated by $SO(10)$. In some cases, threshold corrections to gauge coupling unification can be exchanged for threshold corrections to soft SUSY parameters. See for example, Ref. 28 and references therein.

incomplete GUT multiplets or multiple GUT breaking scales.[3]

Following the analysis of Ref. 27 let us try to understand the need for the GUT threshold correction and its order of magnitude. The renormalization group equations relate the low energy gauge coupling constants $\alpha_i(M_Z)$, $i = 1, 2, 3$ to the value of the unification scale Λ_U and the GUT coupling α_U by the expression

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_U} + \frac{b_i}{2\pi} \log\left(\frac{\Lambda_U}{M_Z}\right) + \delta_i \tag{16.1}$$

where Λ_U is the GUT scale evaluated at one loop and the threshold corrections, δ_i, are given by $\delta_i = \delta_i^{(2)} + \delta_i^{(l)} + \delta_i^{(g)}$ with $\delta_i^{(2)}$ representing two loop running effects, $\delta_i^{(l)}$ the light threshold corrections at the SUSY breaking scale and $\delta_i^{(g)} = \delta_i^{(h)} + \delta_i^{(b)}$ representing GUT scale threshold corrections. Note, in this analysis, the two loop RG running is treated on the same footing as weak and GUT scale threshold corrections. One then obtains the prediction

$$(\alpha_3(M_Z) - \alpha_3^{LO}(M_Z))/\alpha_3^{LO}(M_Z) = -\alpha_3^{LO}(M_Z)\ \delta_s \tag{16.2}$$

where $\alpha_3^{LO}(M_Z)$ is the leading order one loop RG result and $\delta_s = \frac{1}{7}(5\delta_1 - 12\delta_2 + 7\delta_3)$ is the net threshold correction. [A similar formula applies at the GUT scale with the GUT threshold correction, ϵ_3, given by $\epsilon_3 = -\tilde{\alpha}_G\ \delta_s^{(g)}$.] Given the experimental inputs [31,32]:

$$\begin{aligned} \alpha_{em}^{-1}(M_Z) &= 127.916 \pm 0.015 \\ sin^2\theta_W(M_Z) &= 0.23116 \pm 0.00013 \\ \alpha_3(M_Z) &= 0.1184 \pm 0.0007 \end{aligned} \tag{16.3}$$

and taking into account the light threshold corrections, assuming an ensemble of 10 SUSY spectra [27](corresponding to the Snowmass benchmark points), we have

$$\alpha_3^{LO}(M_Z) \approx 0.118 \tag{16.4}$$

and

$$\begin{aligned} \delta_s^{(2)} &\approx -0.82 \\ \delta_s^{(l)} &\approx -0.50 + \frac{19}{28\pi} \log\frac{M_{SUSY}}{M_Z}. \end{aligned}$$

[3] Non-SUSY GUTs with a more complicated breaking pattern can still fit the data. For example, non-SUSY $SO(10) \to SU(4)_C \times SU(2)_L \times SU(2)_R \to SM$ with the second breaking scale of order an intermediate scale, determined by light neutrino masses using the see-saw mechanism, can fit the low energy data for gauge couplings [29] and at the same time survive nucleon decay bounds [30], discussed in the following section.

For $M_{SUSY} = 1$ TeV, we have $\delta_s^{(2)} + \delta_s^{(l)} \approx -0.80$. Since the one loop result $\alpha_3^{LO}(M_Z)$ is very close to the experimental value, we need $\delta_s \approx 0$ or equivalently, $\delta_s^{(g)} \approx 0.80$. This corresponds, at the GUT scale, to $\epsilon_3 \approx -3\%$. Note, this result depends implicitly on the assumption of universal soft SUSY breaking masses at the GUT scale, which directly affect the spectrum of SUSY particles at the weak scale. For example, if gaugino masses were not unified at M_G and, in particular, gluinos were lighter than winos at the weak scale, then it is possible that, due to weak scale threshold corrections, a much smaller or even slightly positive threshold correction at the GUT scale would be consistent with gauge coupling unification [34].

In four dimensional SUSY GUTs, the threshold correction ϵ_3 receives a positive contribution from Higgs doublets and triplets.[4] Thus a larger, negative contribution must come from the GUT breaking sector of the theory. This is certainly possible in specific SO(10) [35] or SU(5) [36] models, but it is clearly a significant constraint on the 4d GUT sector of the theory. In five or six dimensional orbifold GUTs, on the other hand, the "GUT scale" threshold correction comes from the Kaluza-Klein modes between the compactification scale, M_c, and the effective cutoff scale M_*.[5] Thus, in orbifold GUTs, gauge coupling unification at two loops is only consistent with the low energy data with a fixed value for M_c and M_*.[6] Typically, one finds $M_c < M_G = 3 \times 10^{16}$ GeV, where M_G is the 4d GUT scale. Since the grand unified gauge bosons, responsible for nucleon decay, get mass at the compactification scale, the result $M_c < M_G$ for orbifold GUTs has significant consequences for nucleon decay.

A few final comments are in order. We do not consider the scenario of split supersymmetry [39] in this review. In this scenario squarks and sleptons have mass at a scale $\tilde{m} \gg M_Z$, while gauginos and Higgsinos have mass of order the weak scale. Gauge coupling unification occurs at a scale of order 10^{16} GeV, *provided that the scale $\tilde{m}$ lies in the range $10^3 - 10^{11}$ GeV* [40]. A serious complaint concerning the split SUSY scenario is that it does not provide a solution to the gauge hierarchy problem. Moreover, it is only consistent with grand unification if it also postulates an "intermediate" scale, $\tilde{m}$, for scalar masses. In addition, it is in conflict with $b-\tau$ Yukawa unification, unless $\tan\beta$ is fine-tuned to be close to 1 [40]. [7]

---

[4] Note, the Higgs contribution is given by $\epsilon_3 = \frac{3\tilde{\alpha}_G}{5\pi} \log|\frac{\tilde{M}_t \ \gamma}{M_G}|$ where $\tilde{M}_t$ is the effective color triplet Higgs mass (setting the scale for dimension 5 baryon and lepton number violating operators) and $\gamma = \lambda_b/\lambda_t$ at M_G. Since $\tilde{M}_t$ is necessarily greater than M_G, the Higgs contribution to ϵ_3 is positive.

[5] In string theory, the cutoff scale is the string scale.

[6] It is interesting to note that a ratio $M_*/M_c \sim 100$, needed for gauge coupling unification to work in orbifold GUTs is typically the maximum value for this ratio consistent with perturbativity [37]. In addition, in orbifold GUTs brane-localized gauge kinetic terms may destroy the successes of gauge coupling unification. However, for values of $M_*/M_c = M_* \pi R \gg 1$ the unified bulk gauge kinetic terms can dominate over the brane-localized terms [38].

[7] $b-\tau$ Yukawa unification only works for $\tilde{m} < 10^4$ for $\tan\beta \geq 1.5$. This is because the

We have also neglected to discuss non-supersymmetric GUTs in four dimensions which still survive once one allows for several scales of GUT symmetry breaking [29]. Finally, it has been shown that non-supersymmetric GUTs in warped 5 dimensional orbifolds can be consistent with gauge coupling unification, assuming that the right-handed top quark and the Higgs doublets are composite-like objects with a compositeness scale of order a TeV [42]. However perturbative unification seems to fail.

16.1.5. *Nucleon Decay* :

Baryon number is necessarily violated in any GUT [43]. In $SU(5)$, nucleons decay via the exchange of gauge bosons with GUT scale masses, resulting in dimension 6 baryon number violating operators suppressed by $(1/M_G^2)$. The nucleon lifetime is calculable and given by $\tau_N \propto M_G^4/(\alpha_G^2\, m_p^5)$. The dominant decay mode of the proton (and the baryon violating decay mode of the neutron), via gauge exchange, is $p \rightarrow e^+\,\pi^0$ $(n \rightarrow e^+\,\pi^-)$. In any simple gauge symmetry, with one universal GUT coupling and scale $(\alpha_G,\ M_G)$, the nucleon lifetime from gauge exchange is calculable. Hence, the GUT scale may be directly observed via the extremely rare decay of the nucleon. Experimental searches for nucleon decay began with the Kolar Gold Mine, Homestake, Soudan, NUSEX, Frejus, HPW, and IMB detectors [19]. The present experimental bounds come from Super-Kamiokande and Soudan II. We discuss these results shortly. Non-SUSY GUTs are also ruled out by the non-observation of nucleon decay [20]. In SUSY GUTs, the GUT scale is of order 3×10^{16} GeV, as compared to the GUT scale in non-SUSY GUTs which is of order 10^{15} GeV. Hence the dimension 6 baryon violating operators are significantly suppressed in SUSY GUTs [23] with $\tau_p \sim 10^{34-38}$ yrs.

However, in SUSY GUTs there are additional sources for baryon number violation – dimension 4 and 5 operators [44]. Although our notation does not change, when discussing SUSY GUTs all fields are implicitly chiral superfields and the operators considered are the so-called F terms which contain two fermionic components and the rest scalars or products of scalars. Within the context of $SU(5)$ the dimension 4 and 5 operators have the form $(\mathbf{10}\ \bar{\mathbf{5}}\ \bar{\mathbf{5}}) \supset (u^c\ d^c\ d^c) + (Q\ L\ d^c) + (e^c\ L\ L)$ and $(\mathbf{10}\ \mathbf{10}\ \mathbf{10}\ \bar{\mathbf{5}}) \supset (Q\ Q\ Q\ L) + (u^c\ u^c\ d^c\ e^c) +$ B and L conserving terms, respectively. The dimension 4 operators are renormalizable with dimensionless couplings; similar to Yukawa couplings. On the other hand, the dimension 5 operators have a dimensionful coupling of order $(1/M_G)$.

The dimension 4 operators violate baryon number or lepton number, respectively, but not both. The nucleon lifetime is extremely short if both types of dimension 4 operators are present in the low energy theory. However both types can be eliminated by requiring R parity. In $SU(5)$ the Higgs doublets reside in a $\mathbf{5_H}$, $\bar{\mathbf{5}}_{\mathbf{H}}$ and R parity distinguishes the $\bar{\mathbf{5}}$ (quarks and leptons) from $\bar{\mathbf{5}}_{\mathbf{H}}$ (Higgs). R parity [45](or its cousin, family reflection symmetry (or *matter parity*) (see Dimopoulos and Georgi [23] and DRW

---

effective theory between the gaugino mass scale and $\tilde{m}$ includes only one Higgs doublet, as in the standard model. In this case, the large top quark Yukawa coupling tends to increase the ratio λ_b/λ_τ as one runs down in energy below $\tilde{m}$. This is opposite to what happens in MSSM where the large top quark Yukawa coupling decreases the ratio λ_b/λ_τ [41].

[46]) takes $F \to -F$, $H \to H$ with $F = \{\mathbf{10}, \bar{\mathbf{5}}\}$, $H = \{\bar{\mathbf{5}}_{\mathbf{H}}, \mathbf{5}_{\mathbf{H}}\}$. This forbids the dimension 4 operator $(\mathbf{10}\ \bar{\mathbf{5}}\ \bar{\mathbf{5}})$, but allows the Yukawa couplings of the form $(\mathbf{10}\ \bar{\mathbf{5}}\ \bar{\mathbf{5}}_{\mathbf{H}})$ and $(\mathbf{10}\ \mathbf{10}\ \mathbf{5}_{\mathbf{H}})$. It also forbids the dimension 3, lepton number violating, operator $(\bar{\mathbf{5}}\ \mathbf{5}_{\mathbf{H}}) \supset (L\ H_u)$ with a coefficient with dimensions of mass which, like the μ parameter, could be of order the weak scale and the dimension 5, baryon number violating, operator $(\mathbf{10}\ \mathbf{10}\ \mathbf{10}\ \bar{\mathbf{5}}_{\mathbf{H}}) \supset (Q\ Q\ Q\ H_d) + \cdots$.

Note, in the MSSM it is possible to retain R parity violating operators at low energy as long as they violate either baryon number or lepton number only but not both. Such schemes are natural if one assumes a low energy symmetry, such as lepton number, baryon number, baryon triality [47] or proton hexality [48]. However these symmetries cannot be embedded in a GUT. Thus, in a SUSY GUT, only R parity can prevent all the dimension three and four baryon and lepton number violating operators. This does not mean to say that R parity is guaranteed to be satisfied in any GUT. For example the authors of Refs. [51,52] use constrained matter content to selectively generate safe effective R parity violating operators in a GUT. For a review on R parity violating interactions, see [53]. In Ref. [52], the authors show how to obtain the effective R parity violating operator $O^{ijk} = (\bar{5}^j \cdot \bar{5}^k)_{\overline{15}} \cdot (10^i \cdot \Sigma)_{15}$ where Σ is an $SU(5)$ adjoint field and the subscripts $\overline{15}, 15$ indicate that the product of fields in parentheses have been projected into these $SU(5)$ directions. As a consequence the operator O^{ijk} is symmetric under interchange of the two $\bar{5}$ states, $O^{ijk} = O^{ikj}$, and out of $\mathbf{10}\ \bar{\mathbf{5}}\ \bar{\mathbf{5}}$ only the lepton number/R parity violating operator $QL\bar{D}$ survives.

Note also, R parity distinguishes Higgs multiplets from ordinary families. In $SU(5)$, Higgs and quark/lepton multiplets have identical quantum numbers; while in $E(6)$, Higgs and families are unified within the fundamental **27** representation. Only in SO(10) are Higgs and ordinary families distinguished by their gauge quantum numbers. Moreover the Z(4) center of SO(10) distinguishes **10**s from **16**s and can be associated with R parity [49].

Dimension 5 baryon number violating operators may be forbidden at tree level by symmetries in $SU(5)$, etc. These symmetries are typically broken however by the VEVs responsible for the color triplet Higgs masses. Consequently these dimension 5 operators are generically generated via color triplet Higgsino exchange. Hence, the color triplet partners of Higgs doublets must necessarily obtain mass of order the GUT scale. [It is also important to note that Planck or string scale physics may independently generate dimension 5 operators, even without a GUT. These contributions must be suppressed by some underlying symmetry; for example, the same flavor symmetry which may be responsible for hierarchical fermion Yukawa matrices.]

The dominant decay modes from dimension 5 operators are $p \to K^+\ \bar{\nu}$ $(n \to K^0\ \bar{\nu})$. This is due to a simple symmetry argument; the operators $(Q_i\ Q_j\ Q_k\ L_l)$, $(u^c_i\ u^c_j\ d^c_k\ e^c_l)$ (where $i,\ j,\ k,\ l = 1, 2, 3$ are family indices and color and weak indices are implicit) must be invariant under $SU(3)_C$ and $SU(2)_L$. As a result their color and weak doublet indices must be anti-symmetrized. However since these operators are given by bosonic superfields, they must be totally symmetric under interchange of all indices. Thus the first operator vanishes for $i = j = k$ and the second vanishes for $i = j$. Hence a second or third generation member must exist in the final state [46].

Recent Super-Kamiokande bounds on the proton lifetime severely constrain these dimension 6 and 5 operators with (172.8 kt-yr) of data they find $\tau_{(p\rightarrow e^+\pi^0)} > 1.0 \times 10^{34}$ yrs, $\tau_{(p\rightarrow K^+\bar{\nu})} > 3.3 \times 10^{33}$ yrs and $\tau_{(n\rightarrow e^+\pi^-)} > 2 \times 10^{33}$ yrs at (90% CL) [54]. These constraints are now sufficient to rule out minimal SUSY $SU(5)$ [55]. [8] Non-minimal Higgs sectors in $SU(5)$ or $SO(10)$ theories still survive [26,36]. The upper bound on the proton lifetime from these theories are approximately a factor of 10 above the experimental bounds. They are also being pushed to their theoretical limits. Hence if SUSY GUTs are correct, then nucleon decay must be seen soon.

Is there a way out of this conclusion? Orbifold GUTs and string theories, see Sec. 16.1, contain grand unified symmetries realized in higher dimensions. In the process of compactification and GUT symmetry breaking, color triplet Higgs states are removed (projected out of the massless sector of the theory). In addition, the same projections typically rearrange the quark and lepton states so that the massless states which survive emanate from different GUT multiplets. In these models, proton decay due to dimension 5 operators can be severely suppressed or eliminated completely. However, proton decay due to dimension 6 operators may be enhanced, since the gauge bosons mediating proton decay obtain mass at the compactification scale, M_c, which is typically less than the 4d GUT scale (see the discussion at the end of Sec. 16.1), or suppressed, if the states of one family come from different irreducible representations. Which effect dominates is a model dependent issue. In some complete 5d orbifold GUT models [59,27] the lifetime for the decay $\tau(p \rightarrow e^+\pi^0)$ can be near the excluded bound of 1×10^{34} years with, however, large model dependent and/or theoretical uncertainties. In other cases, the modes $p \rightarrow K^+\bar{\nu}$ and $p \rightarrow K^0\mu^+$ may be dominant [27]. To summarize, in either 4d or orbifold string/field theories, nucleon decay remains a premier signature for SUSY GUTs. Moreover, the observation of nucleon decay may distinguish extra-dimensional orbifold GUTs from four dimensional ones.

As a final note, in orbifold GUTs or string theory new discrete symmetries consistent with SUSY GUTs can forbid all dimension 3 and 4 baryon [B] and lepton [L] number violating operators and even forbid the mu term and dimension 5 B and L violating operators to all orders in perturbation theory [50]. The mu term and dimension 5 B and L violating operators may then be generated, albeit sufficiently suppressed, via non-perturbative effects. The simplest example of this is a Z_4^R symmetry which is the unique discrete R symmetry consistent with SO(10) [50]. In this case, proton decay is

---

[8] This conclusion relies on the mild assumption that the three-by-three matrices diagonalizing squark and slepton mass matrices are not so different from their fermionic partners. It has been shown that if this caveat is violated, then dimension five proton decay in minimal SUSY $SU(5)$ may not be ruled out [56]. This is however a very fine-tuned resolution of the problem. Another possible way out is to allow for a more complicated $SU(5)$ breaking Higgs sector in the otherwise minimal model [57]. I have also implicitly assumed a hierarchical structure for Yukawa matrices in this analysis. It is however possible to fine-tune a hierarchical structure for quarks and leptons which baffles the family structure. In this case it is possible to avoid the present constraints on minimal SUSY $SU(5)$, for example see [58].

completely dominated by dimension 6 operators.

Before concluding the topic of baryon number violation, consider the status of $\Delta B = 2$ neutron- anti-neutron oscillations. Generically the leading operator for this process is the dimension 9 six quark operator $G_{(\Delta B=2)}$ $(u^c\ d^c\ d^c\ u^c\ d^c\ d^c)$ with dimensionful coefficient $G_{(\Delta B=2)} \sim 1/M^5$. The present experimental bound $\tau_{n-\bar{n}} \geq 0.86 \times 10^8$ sec. at 90% CL [60] probes only up to the scale $M \leq 10^6$ GeV. For $M \sim M_G$, $n - \bar{n}$ oscillations appear to be unobservable for any GUT (for a recent discussion see [61]) .

16.1.6. *Yukawa coupling unification* :

16.1.6.1. *3rd generation, $b - \tau$ or $t - b - \tau$ unification:*

If quarks and leptons are two sides of the same coin, related by a new grand unified gauge symmetry, then that same symmetry relates the Yukawa couplings (and hence the masses) of quarks and leptons. In $SU(5)$, there are two independent renormalizable Yukawa interactions given by λ_t $(\mathbf{10\ 10\ 5_H})$ + λ $(\mathbf{10\ \bar{5}\ \bar{5}_H})$. These contain the SM interactions λ_t $(\mathbf{Q\ u^c\ H_u})$ + λ $(\mathbf{Q\ d^c\ H_d}\ +\ \mathbf{e^c\ L\ H_d})$. Hence, at the GUT scale we have the tree level relation, $\lambda_b = \lambda_\tau \equiv \lambda$ [41]. In $SO(10)$ there is only one independent renormalizable Yukawa interaction given by λ $(\mathbf{16\ 16\ 10_H})$ which gives the tree level relation, $\lambda_t = \lambda_b = \lambda_\tau \equiv \lambda$ [62,63]. Note, in the discussion above we assume the minimal Higgs content with Higgs in $\mathbf{5}$, $\bar{\mathbf{5}}$ for $SU(5)$ and $\mathbf{10}$ for $SO(10)$. With Higgs in higher dimensional representations there are more possible Yukawa couplings [75,76,77].

In order to make contact with the data, one now renormalizes the top, bottom and τ Yukawa couplings, using two loop RG equations, from M_G to M_Z. One then obtains the running quark masses $m_t(M_Z) = \lambda_t(M_Z)\ v_u$, $m_b(M_Z) = \lambda_b(M_Z)\ v_d$ and $m_\tau(M_Z) = \lambda_\tau(M_Z)\ v_d$ where $< H_u^0 > \equiv v_u = \sin\beta\ v/\sqrt{2}$, $< H_d^0 > \equiv v_d = \cos\beta\ v/\sqrt{2}$, $v_u/v_d \equiv \tan\beta$ and $v \sim 246$ GeV is fixed by the Fermi constant, G_μ.

Including one loop threshold corrections at M_Z and additional RG running, one finds the top, bottom and τ pole masses. In SUSY, $b - \tau$ unification has two possible solutions with $\tan\beta \sim 1$ or $40 - 50$. The small $\tan\beta$ solution is now disfavored by the LEP limit, $\tan\beta > 2.4$ [64]. [9] The large $\tan\beta$ limit overlaps the $SO(10)$ symmetry relation.

When $\tan\beta$ is large there are significant weak scale threshold corrections to down quark and charged lepton masses from either gluino and/or chargino loops [66]. Yukawa unification (consistent with low energy data) is only possible in a restricted region of SUSY parameter space with important consequences for SUSY searches [67]. More recent analyses of Yukawa unification can be found in Refs. [68,69,70,71]. There seems to be at least four possible choices of soft SUSY breaking parameters which fit the data, possibly more. Each case then leads to a distinct sparticle spectrum and phenomenology for LHC and dark matter experiments. They correspond to:

- universal squark and slepton masses (m_{16}), universal A parameter (A_0) and gaugino masses ($M_{1/2}$), and non-universal Higgs masses (m_{H_u}, m_{H_d}) with "just-so" splitting [67,68].

---

[9] However, this bound disappears if one takes $M_{SUSY} = 2$ TeV and $m_t = 180$ GeV [65]. This apparent loop hole is now inconsistent with the observed top quark mass.

- a universal squark and slepton mass term for the first two families ($m_{16_{1,2}}$) which is larger than the universal scalar mass for the third family (m_{16_3}), universal A parameter (A_0) and gaugino masses ($M_{1/2}$) and universal Higgs mass term (m_{10}). However all scalar masses then receive a D-term contribution to their masses given by the $U(1)$ from $SO(10)$ which commutes with $SU(5)$. This is of the form

$$\begin{aligned} m_Q^2 = m_E^2 = m_U^2 &= m_{16}^2 + M_D^2, \\ m_D^2 = m_L^2 &= m_{16}^2 - 3M_D^2, \\ m_{\bar{\nu}}^2 &= m_{16}^2 + 5M_D^2, \\ m_{H_{u,d}}^2 &= m_{10}^2 \mp 2M_D^2. \end{aligned}$$

  This is the so-called "DR3 splitting" [69]. The R is associated with taking into account the renormalization group [RG] running of the right-handed neutrino from the GUT scale to the nominal value of its mass of order 10^{10-14} GeV, as indicated by light neutrino masses via the See-Saw mechanism. This RG running contributes to an additional splitting of the H_u and H_d masses [67].
- universal squark and slepton masses (m_0), split Higgs masses and non-universal gaugino masses satisfying ($M_1 = \frac{3}{5}M_2 + \frac{2}{5}M_3$), and $\mu, M_2 < 0$ [70], and
- universal squark and slepton mass term (m_{16}), A parameter (A_0), Higgs mass term (m_{10}). All scalar masses then receive a D-term contribution to their masses given by the $U(1)$ from $SO(10)$ which commutes with $SU(5)$, as above. Finally, non-universal gaugino masses satisfying ($M_3 : M_2 : M_1 = 2 : -3 : -1$) with $M_3 > 0$ and $\mu < 0$ [71].

16.1.6.2. *Three families:*

Simple Yukawa unification is not possible for the first two generations of quarks and leptons. Consider the $SU(5)$ GUT scale relation $\lambda_b = \lambda_\tau$. If extended to the first two generations one would have $\lambda_s = \lambda_\mu$, $\lambda_d = \lambda_e$ which gives $\lambda_s/\lambda_d = \lambda_\mu/\lambda_e$. The last relation is a renormalization group invariant and is thus satisfied at any scale. In particular, at the weak scale one obtains $m_s/m_d = m_\mu/m_e$ which is in serious disagreement with the data with $m_s/m_d \sim 20$ and $m_\mu/m_e \sim 200$. An elegant solution to this problem was given by Georgi and Jarlskog [72]. For a recent anaylsis in the context of supersymmetric GUTs, see Ref. [73]. Of course, a three family model must also give the observed CKM mixing in the quark sector. Note, although there are typically many more parameters in the GUT theory above M_G, it is possible to obtain effective low energy theories with many fewer parameters making strong predictions for quark and lepton masses.

Three family models which make significant predictions for low energy experiments have been constructed in the context of supersymmetric GUTs. It is important to note that grand unification alone is not sufficient to obtain predictive theories of fermion masses and mixing angles. Other ingredients are needed. In one approach additional global family symmetries are introduced (non-abelian family symmetries can significantly reduce the number of arbitrary parameters in the Yukawa matrices). These family symmetries constrain the set of effective higher dimensional fermion mass operators. In addition, sequential breaking of the family symmetry is correlated with the hierarchy of fermion masses. Three-family models exist which fit all the data, including neutrino masses and mixing [74]. In a completely separate approach for $SO(10)$ models, the Standard Model Higgs bosons are contained in the higher dimensional Higgs representations including the **10**, $\overline{\mathbf{126}}$ and/or **120**. Such theories have been shown to make predictions for neutrino masses and mixing angles [75–77]. A recent paper on this subject argues the necessity of split supersymmetry [78].

16.1.7. *Neutrino Masses* :

Atmospheric and solar neutrino oscillations, along with long baseline accelerator and reactor experiments, require neutrino masses. Adding three "sterile" neutrinos ν^c with the Yukawa coupling λ_ν ($\nu^{\mathbf{c}}$ $\mathbf{L}$ $\mathbf{H_u}$), one easily obtains three massive Dirac neutrinos with mass $m_\nu = \lambda_\nu\ v_u$.[10] However in order to obtain a tau neutrino with mass of order 0.1 eV, one needs $\lambda_{\nu_\tau}/\lambda_\tau \leq 10^{-10}$. The see-saw mechanism, on the other hand, can naturally explain such small neutrino masses [79,80]. Since ν^c has no SM quantum numbers, there is no symmetry (other than global lepton number) which prevents the mass term $\frac{1}{2}\ \nu^c\ M\ \nu^c$. Moreover one might expect $M \sim M_G$. Heavy "sterile" neutrinos can be integrated out of the theory, defining an effective low energy theory with only light active Majorana neutrinos with the effective dimension 5 operator $\frac{1}{2}\ (\mathbf{L}\ \mathbf{H_u})\ \lambda_\nu^T\ M^{-1}\ \lambda_\nu\ (\mathbf{L}\ \mathbf{H_u})$. This then leads to a 3×3 Majorana neutrino mass matrix $\mathbf{m} = m_\nu^T\ M^{-1}\ m_\nu$.

Atmospheric neutrino oscillations require neutrino masses with $\Delta m_\nu^2 \sim 3 \times 10^{-3}$ eV^2 with maximal mixing, in the simplest two neutrino scenario. With hierarchical neutrino masses $m_{\nu_\tau} = \sqrt{\Delta m_\nu^2} \sim 0.055$ eV. Moreover via the "see-saw" mechanism $m_{\nu_\tau} = m_t(m_t)^2/(3M)$. Hence one finds $M \sim 2 \times 10^{14}$ GeV; remarkably close to the GUT scale. Note we have related the neutrino Yukawa coupling to the top quark Yukawa coupling $\lambda_{\nu_\tau} = \lambda_t$ at M_G as given in $SO(10)$ or $SU(4) \times SU(2)_L \times SU(2)_R$. However at low energies they are no longer equal and we have estimated this RG effect by $\lambda_{\nu_\tau}(M_Z) \approx \lambda_t(M_Z)/\sqrt{3}$.

Neutrinos pose a special problem for SUSY GUTs. The question is why are the quark mixing angles in the CKM matrix small, while there are two large lepton mixing angles in the PMNS matrix. For a recent discussion of neutrino masses and mixing angles, see Refs. [81] and [82]. For SUSY GUT models which fit quark and lepton masses, see Ref. [74]. Finally, for a compilation of the range of SUSY GUT predictions for neutrino mixing, see [83].

[10] Note, these "sterile" neutrinos are quite naturally identified with the right-handed neutrinos necessarily contained in complete families of $SO(10)$ or Pati-Salam.

16.1.8. *Selected Topics* :

16.1.8.1. *Magnetic Monopoles:*

In the broken phase of a GUT there are typically localized classical solutions carrying magnetic charge under an unbroken $U(1)$ symmetry [84]. These magnetic monopoles with mass of order M_G/α_G are produced during the GUT phase transition in the early universe. The flux of magnetic monopoles is experimentally found to be less than $\sim 10^{-16}$ cm^{-2} s^{-1} sr^{-1} [85]. Many more are however predicted, hence the GUT monopole problem. In fact, one of the original motivations for an inflationary universe is to solve the monopole problem by invoking an epoch of rapid inflation after the GUT phase transition [86]. This would have the effect of diluting the monopole density as long as the reheat temperature is sufficiently below M_G. Other possible solutions to the monopole problem include: sweeping them away by domain walls [87], U(1) electromagnetic symmetry breaking at high temperature [88] or GUT symmetry non-restoration [89]. Parenthetically, it was also shown that GUT monopoles can catalyze nucleon decay [90]. A significantly lower bound on the monopole flux can then be obtained by considering X-ray emission from radio pulsars due to monopole capture and the subsequent nucleon decay catalysis [91].

16.1.8.2. *Baryogenesis via Leptogenesis:*

Baryon number violating operators in $SU(5)$ or $SO(10)$ preserve the global symmetry $B - L$. Hence the value of the cosmological $B - L$ density is an initial condition of the theory and is typically assumed to be zero. On the other hand, anomalies of the electroweak symmetry violate $B + L$ while also preserving $B - L$. Hence thermal fluctations in the early universe, via so-called sphaleron processes, can drive $B + L$ to zero, washing out any net baryon number generated in the early universe at GUT temperatures.

One way out of this dilemma is to generate a net $B - L$ dynamically in the early universe. We have just seen that neutrino oscillations suggest a new scale of physics of order 10^{14} GeV. This scale is associated with heavy Majorana neutrinos with mass M. If in the early universe, the decay of the heavy neutrinos is out of equilibrium and violates both lepton number and CP, then a net lepton number may be generated. This lepton number will then be partially converted into baryon number via electroweak processes [92].

16.1.8.3. *GUT symmetry breaking:*

The grand unification symmetry is necessarily broken spontaneously. Scalar potentials (or superpotentials) exist whose vacua spontaneously break $SU(5)$ and $SO(10)$. These potentials are ad hoc (just like the Higgs potential in the SM) and therefore it is hoped that they may be replaced with better motivated sectors. Gauge coupling unification now tests GUT breaking sectors, since it is one of the two dominant corrections to the GUT threshold correction ϵ_3. The other dominant correction comes from the Higgs sector and doublet-triplet splitting. This latter contribution is always positive $\epsilon_3 \propto \ln(M_T/M_G)$ (where M_T is an effective color triplet Higgs mass), while the low energy data typically requires $\epsilon_3 < 0$. Hence the GUT breaking sector must provide a significant (of order

-8%) contribution to ϵ_3 to be consistent with the Super-K bound on the proton lifetime [35,26,36,74].

In string theory (and GUTs in extra-dimensions), GUT breaking may occur due to boundary conditions in the compactified dimensions [8,11]. This is still ad hoc. The major benefit is that it does not require complicated GUT breaking sectors.

16.1.8.4. *Doublet-triplet splitting:*

The minimal supersymmetric standard model has a μ problem; why is the coefficient of the bilinear Higgs term in the superpotential $\mu\,(\mathbf{H_u}\,\mathbf{H_d})$ of order the weak scale when, since it violates no low energy symmetry, it could be as large as M_G. In a SUSY GUT, the μ problem is replaced by the problem of *doublet-triplet* splitting — giving mass of order M_G to the color triplet Higgs and mass μ to the Higgs doublets. Several mechanisms for natural doublet-triplet splitting have been suggested, such as the sliding singlet [93], missing partner or missing VEV [94], and pseudo-Nambu-Goldstone boson mechanisms. Particular examples of the missing partner mechanism for $SU(5)$ [36], the missing VEV mechanism for $SO(10)$ [74,26] and the pseudo-Nambu-Goldstone boson mechanism for $SU(6)$ [95] have been shown to be consistent with gauge coupling unification and proton decay. There are also several mechanisms for explaining why μ is of order the SUSY breaking scale [96]. Finally, for a recent review of the μ problem and some suggested solutions in SUSY GUTs and string theory, see Ref. [97,10,98,50] and references therein.

Once again, in string theory (and orbifold GUTs), the act of breaking the GUT symmetry via orbifolding projects certain states out of the theory. It has been shown that it is possible to remove the color triplet Higgs while retaining the Higgs doublets in this process. Hence the doublet-triplet splitting problem is finessed. As discussed earlier (see Sec. 16.1), this can have the effect of eliminating the contribution of dimension 5 operators to nucleon decay.

16.1.9. ***String theory*** :

String theory has made significant progress in locating the minimal supersymmetric standard model [MSSM] in the string landscape. Random searches for MSSM-like models have found some success, see for example Ref. 99. However, recently a solid leap forward has been made by imposing a supersymmetric GUT locally in the extra dimensions of the string. Many MSSM-like models have been found in $E(8) \times E(8)$ heterotic orbifold constructions [100–103] or more recently on smooth Calabi-Yau three-folds [104]. See also in F theory constructions [105–107]. There appear, however, to be some problems associated with large threshold corrections to gauge coupling unification in the F theory constructions which make use of a non-vanishing hypercharge field strength to break $SU(5)$ to $SU(3)_C \times SU(2)_L \times U(1)_Y$ [108]. Nevertheless, a SUSY GUT guarantees the correct particle content of the Standard Model and also allows for reasonable looking hierarchical Yukawa matrices. For a more detailed discussion, see [109].

16.2. Conclusion

Grand unification of the strong and electroweak interactions requires that the three low energy gauge couplings unify (up to small threshold corrections) at a unique scale, M_G. Supersymmetric grand unified theories provide, by far, the most predictive and economical framework allowing for perturbative unification.

The three pillars of SUSY GUTs are:

- gauge coupling unification at $M_G \sim 3 \times 10^{16}$ GeV;
- low-energy supersymmetry [with a large SUSY desert], and
- nucleon decay.

The first prediction has already been verified (see Fig. Fig. 16.1). Perhaps the next two will soon appear. Whether or not Yukawa couplings unify is more model dependent. Nevertheless, the "digital" 16 dimensional representation of quarks and leptons in $SO(10)$ is very compelling and may yet lead to an understanding of fermion masses and mixing angles.

In any event, the experimental verification of the first three pillars of SUSY GUTs would forever change our view of Nature. Moreover, the concomitant evidence for a vast SUSY desert would expose a huge lever arm for discovery. For then it would become clear that experiments probing the TeV scale could reveal physics at the GUT scale and perhaps beyond. Of course, some questions will still remain: Why do we have three families of quarks and leptons? How is the grand unified symmetry and possible family symmetries chosen by Nature? At what scale might stringy physics become relevant? Etc.

References:

1. J. Pati and A. Salam, Phys. Rev. **D8**, 1240 (1973);
For more discussion on the standard charge assignments in this formalism, see A. Davidson, Phys. Rev. **D20**, 776 (1979); and R.N. Mohapatra and R.E. Marshak, Phys. Lett. **B91**, 222 (1980).
2. H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
3. E. Golowich, Phys. Rev. **D24**, 2899 (1981).
4. H. Georgi, Particles and Fields, *Proceedings of the APS Div. of Particles and Fields*, ed. C. Carlson, p. 575 (1975);
H. Fritzsch and P. Minkowski, Ann. Phys. **93**, 193 (1975).
5. S.M. Barr, Phys. Lett. **B112**, 219 (1982).
6. A. de Rujula *et al.*, *5th Workshop on Grand Unification*, ed. K. Kang *et al.*, World Scientific, Singapore (1984), p. 88;
See also earlier paper by Y. Achiman and B. Stech, p. 303, "New Phenomena in Lepton-Hadron Physics," ed. D.E.C. Fries and J. Wess, Plenum, NY (1979).
7. B.R. Greene *et al.*, Nucl. Phys. **B278**, 667 (1986);
ibid., Nucl. Phys. **B292**, 606 (1987);
B.R. Greene *et al.*, Nucl. Phys. **B325**, 101 (1989);
J.E. Kim, Phys. Lett. **B591**, 119 (2004).

8. P. Candelas *et al.*, Nucl. Phys. **B258**, 46 (1985);
L.J. Dixon *et al.*, Nucl. Phys. **B261**, 678 (1985);
ibid., Nucl. Phys. **B274**, 285 (1986);
L. E. Ibanez *et al.*, Phys. Lett. **B187**, 25 (1987);
ibid., Phys. Lett. **B191**, 282 (1987);
J.E. Kim *et al.*, Nucl. Phys. **B712**, 139 (2005).
9. T.Kobayashi *et al.*, Phys. Lett. **B593**, 262 (2004);
S. Forste *et al.*, Phys. Rev. **D70**, 106008 (2004);
T. Kobayashi *et al.*, Nucl. Phys. **B704**, 3 (2005);
W. Buchmuller *et al.*, Nucl. Phys. **B712**, 139 (2005);
W. Buchmuller *et al.*, Phys. Rev. Lett. **96**, 121602 (2006);
ibid., Nucl. Phys. **B785**, 149 (2007);
O. Lebedev, *et al.*, Phys. Lett. **B645**, 88 (2007);
J. E. Kim, J. H. Kim and B. Kyae, JHEP **0706**, 034 (2007);
O. Lebedev, *et al.*, Phys. Rev. **D77**, 046013 (2008).
10. E. Witten, [hep-ph/0201018];
M. Dine *et al.*, Phys. Rev. **D66**, 115001 (2002), [hep-ph/0206268].
11. Y. Kawamura, Prog. Theor. Phys. **103**, 613 (2000);
ibid., **105**, 999 (2001);
G. Altarelli *et al.*, Phys. Lett. **B5111**, 257 (2001);
L.J. Hall *et al.*, Phys. Rev. **D64**, 055003 (2001);
A. Hebecker and J. March-Russell, Nucl. Phys. **B613**, 3 (2001);
T. Asaka *et al.*, Phys. Lett. **B523**, 199 (2001);
L.J. Hall *et al.*, Phys. Rev. **D65**, 035008 (2002);
R. Dermisek and A. Mafi, Phys. Rev. **D65**, 055002 (2002);
H.D. Kim and S. Raby, JHEP **0301**, 056 (2003).
12. For a recent review see, R. Blumenhagen *et al.*, “Toward realistic intersecting D-brane models,” hep-th/0502005.
13. M.J. Dolan, S. Krippendorf, and F. Quevedo, JHEP **1110**, 024 (2011).
14. G. Aldazabal *et al.*, Nucl. Phys. **B452**, 3 (1995);
Z. Kakushadze and S.H.H. Tye, Phys. Rev. **D54**, 7520 (1996);
Z. Kakushadze *et al.*, Int. J. Mod. Phys. **A13**, 2551 (1998).
15. G. B. Cleaver *et al.*, Int. J. Mod. Phys. **A16**, 425 (2001), hep-ph/9904301;
ibid., Nucl. Phys. **B593**, 471 (2001) hep-ph/9910230.
16. V. Braun *et al.*, Phys. Lett. **B618**, 252 (2005), hep-th/0501070;
ibid., JHEP **506**, 039 (2005), [hep-th/0502155].
17. E. Witten, Nucl. Phys. **B471**, 135 (1996), [hep-th/9602070].
18. H. Georgi *et al.*, Phys. Rev. Lett. **33**, 451 (1974);
See also the definition of effective field theories by S. Weinberg, Phys. Lett. **91B**, 51 (1980).
19. See talks on proposed and running nucleon decay experiments, and theoretical talks by P. Langacker, p. 131, and W.J. Marciano and A. Sirlin, p. 151, in *The Second Workshop on Grand Unification*, eds. J.P. Leveille *et al.*, Birkhäuser, Boston (1981).

20. W.J. Marciano, p. 190, *Eighth Workshop on Grand Unification*, ed. K. Wali, World Scientific Publishing Co., Singapore (1987).
21. U. Amaldi *et al.*, Phys. Lett. **B260**, 447 (1991);
J. Ellis *et al.*, Phys. Lett. **B260**, 131 (1991);
P. Langacker and M. Luo, Phys. Rev. **D44**, 817 (1991);
P. Langacker and N. Polonsky, Phys. Rev. **D47**, 4028 (1993);
M. Carena *et al.*, Nucl. Phys. **B406**, 59 (1993);
see also the review by S. Dimopoulos *et al.*, Physics Today, p. 25 October (1991).
22. B.C. Allanach, Comput. Phys. Commun. **143**, 305 (2002) [hep-ph/0104145].
23. S. Dimopoulos *et al.*, Phys. Rev. **D24**, 1681 (1981);
S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981);
L. Ibanez and G.G. Ross, Phys. Lett. **105B**, 439 (1981);
N. Sakai, Z. Phys. **C11**, 153 (1981);
M.B. Einhorn and D.R.T. Jones, Nucl. Phys. **B196**, 475 (1982);
W.J. Marciano and G. Senjanovic, Phys. Rev. **D25**, 3092 (1982).
24. D.I. Kazakov, Lectures given at the European School on High Energy Physics, Aug.-Sept. 2000, Caramulo, Portugal [hep-ph/0012288v2].
25. V. Lucas and S. Raby, Phys. Rev. **D54**, 2261 (1996) [hep-ph/9601303];
T. Blazek *et al.*, Phys. Rev. **D56**, 6919 (1997) [hep-ph/9611217];
G. Altarelli *et al.*, JHEP **0011**, 040 (2000) [hep-ph/0007254].
26. R. Dermíšek *et al.*, Phys. Rev. **D63**, 035001 (2001);
K.S. Babu *et al.*, Nucl. Phys. **B566**, 33 (2000).
27. M. L. Alciati *et al.*, JHEP **0503**, 054 (2005) [hep-ph/0501086].
28. G. Anderson *et al.*, eConf **C960625**, SUP107 (1996) [hep-ph/9609457].
29. R.N. Mohapatra and M.K. Parida, Phys. Rev. **D47**, 264 (1993).
30. D.G. Lee *et al.*, Phys. Rev. **D51**, 229 (1995).
31. Eur. Phys. J. **C64**, 689 (2009).
32. K. Nakamura *et al.*, (Particle Data Group), J. Phys. **G37**, 075021 (2010).
33. K. Nakamura *et al.*, (Particle Data Group), J. Phys. **G37**, 075021 (2010).
34. S. Raby *et al.*, Phys. Lett. **B687**, 342 (2010) [arXiv:0911.4249 [hep-ph]].
35. K.S. Babu and S.M. Barr, Phys. Rev. **D48**, 5354 (1993);
V. Lucas and S. Raby, Phys. Rev. **D54**, 2261 (1996);
S.M. Barr and S. Raby, Phys. Rev. Lett. **79**, 4748 (1997) and references therein.
36. G. Altarelli *et al.*, JHEP **0011**, 040 (2000) See also earlier papers by A. Masiero *et al.*, Phys. Lett. **B115**, 380 (1982);
B. Grinstein, Nucl. Phys. **B206**, 387 (1982).
37. K.R. Dienes *et al.*, Phys. Rev. Lett. **91**, 061601 (2003).
38. L. J. Hall *et al.*, Phys. Rev. **D64**, 055003 (2001).
39. N. Arkani-Hamed and S. Dimopoulos, JHEP **0506**, 073 (2005) [hep-th/0405159].
40. G.F. Giudice and A. Romanino, Nucl. Phys. **B699**, 65 (2004) [Erratum: *ibid.*, Nucl. Phys. **B706**, 65 (2005)] [hep-ph/0406088].
41. M. Chanowitz *et al.*, Nucl. Phys. **B135**, 66 (1978);
For the corresponding SUSY analysis, see M. Einhorn and D.R.T. Jones, Nucl. Phys. **B196**, 475 (1982);

K. Inoue *et al.*, Prog. Theor. Phys. **67**, 1889 (1982);
L. E. Ibanez and C. Lopez, Phys. Lett. **B126**, 54 (1983);
ibid., Nucl. Phys. **B233**, 511 (1984).

42. K. Agashe *et al.*, [hep-ph/0502222].
43. M. Gell-Mann *et al.*, in *Supergravity*, eds. P. van Nieuwenhuizen and D.Z. Freedman, North-Holland, Amsterdam, 1979, p. 315.
44. S. Weinberg, Phys. Rev. **D26**, 287 (1982);
N. Sakai and T Yanagida, Nucl. Phys. **B197**, 533 (1982).
45. G. Farrar and P. Fayet, Phys. Lett. **B76**, 575 (1978).
46. S. Dimopoulos *et al.*, Phys. Lett. **112B**, 133 (1982);
J. Ellis *et al.*, Nucl. Phys. **B202**, 43 (1982).
47. L.E. Ibanez and G.G. Ross, Nucl. Phys. **B368**, 3 (1992).
48. H. K. Dreiner, C. Luhn, and M. Thormeier, Phys. Rev. **D73**, 075007 (2006).
49. For a recent discussion, see C.S. Aulakh *et al.*, Nucl. Phys. **B597**, 89 (2001).
50. H. M. Lee *et al.*, Phys. Lett. **B694**, 491 (2011);
R. Kappl *et al.*, Nucl. Phys. **B847**, 325 (2011);
H. M. Lee *et al.*, Nucl. Phys. **B850**, 1 (2011).
51. R. Barbieri *et al.*, Phys. Lett. **B407**, 250 (1997).
52. G. F. Giudice and R. Rattazzi, Phys. Lett. **B406**, 321 (1997).
53. R. Barbier *et al.*, Phys. Rev. **420**, 1 (2005).
54. Makoto Miura [Super-Kamiokande Collab.], ICHEP 2010.
55. T. Goto and T. Nihei, Phys. Rev. **D59**, 115009 (1999) [hep-ph/9808255];
H. Murayama and A. Pierce, Phys. Rev. **D65**, 055009 (2002) hep-ph/0108104.
56. B. Bajc *et al.*, Phys. Rev. **D66**, 075005 (2002) [hep-ph/0204311].
57. J.L. Chkareuli and I.G. Gogoladze, Phys. Rev. **D58**, 055011 (1998) [hep-ph/9803335].
58. K.S. Choi, Phys. Lett. **B668**, 392 (2008).
59. L.J. Hall and Y. Nomura, Phys. Rev. **D66**, 075004 (2002);
H.D. Kim *et al.*, JHEP **0505**, 036 (2005).
60. M. Baldoceolin et al., Z. Phys. **C63**, 409 (1994).
61. K. S. Babu and R. N. Mohapatra, Phys. Lett. **B518**, 269 (2001) [hep-ph/0108089].
62. H. Georgi and D.V. Nanopoulos, Nucl. Phys. **B159**, 16 (1979);
J. Harvey *et al.*, Phys. Lett. **B92**, 309 (1980);
ibid., Nucl. Phys. **B199**, 223 (1982).
63. T. Banks, Nucl. Phys. **B303**, 172 (1988);
M. Olechowski and S. Pokorski, Phys. Lett. **B214**, 393 (1988);
S. Pokorski, Nucl. Phys. (Proc. Supp.) **B13**, 606 (1990);
B. Ananthanarayan *et al.*, Phys. Rev. **D44**, 1613 (1991);
Q. Shafi and B. Ananthanarayan, ICTP Summer School lectures (1991);
S. Dimopoulos *et al.*, Phys. Rev. Lett. **68**, 1984 (1992);
ibid., Phys. Rev. **D45**, 4192 (1992);
G. Anderson *et al.*, Phys. Rev. **D47**, 3702 (1993);
B. Ananthanarayan *et al.*, Phys. Lett. **B300**, 245 (1993);

G. Anderson *et al.*, Phys. Rev. **D49**, 3660 (1994);
B. Ananthanarayan *et al.*, Phys. Rev. **D50**, 5980 (1994).

64. LEP Higgs Working Group and ALEPH Collab. and DELPHI Collab. and L3 Collab. and OPAL Collab., Preliminary results, [hep-ex/0107030] (2001).

65. M. Carena and H.E. Haber, Prog. in Part. Nucl. Phys. **50**, 63 (2003) [hep-ph/0208209].

66. L.J. Hall *et al.*, Phys. Rev. **D50**, 7048 (1994);
M. Carena *et al.*, Nucl. Phys. **B419**, 213 (1994);
R. Rattazzi and U. Sarid, Nucl. Phys. **B501**, 297 (1997).

67. Blazek *et al.*, Phys. Rev. Lett. **88**, 111804 (2002) [hep-ph/0107097];
ibid., Phys. Rev. **D65**, 115004 (2002) [hep-ph/0201081];
K. Tobe and J. D. Wells, Nucl. Phys. **B663**, 123 (2003) [hep-ph/0301015];
D. Auto *et al.*, JHEP **0306**, 023 (2003) [hep-ph/0302155];
R. Dermisek *et al.*, JHEP **0304**, 037 (2003);
ibid., JHEP **0509**, 029 (2005).

68. W. Altmannshofer *et al.*, Phys. Lett. **B668**, 385 (2008);
D. Guadagnoli *et al.*, JHEP **0910**, 059 (2009).

69. H. Baer *et al.*, JHEP **0810**, 079 (2008);
H. Baer *et al.*, JHEP **0909**, 005 (2009).

70. I. Gogoladze *et al.*, JHEP **1012**, 055 (2010);
S. Dar *et al.*, [arXiv:1105.5122 [hep-ph]];
I. Gogoladze *et al.*, [arXiv:1107.1228 [hep-ph]].

71. M. Badziak, M. Olechowski, and S. Pokorski, JHEP **1108**, 147 (2011).

72. H. Georgi and C. Jarlskog, Phys. Lett. **B86**, 297 (1979).

73. S. Antusch and M. Spinrath, Phys. Rev. **D79**, 095004 (2009).

74. K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **74**, 2418 (1995);
V. Lucas and S. Raby, Phys. Rev. **D54**, 2261 (1996);
T. Blažek *et al.*, Phys. Rev. **D56**, 6919 (1997);
R. Barbieri *et al.*, Nucl. Phys. **B493**, 3 (1997);
T. Blazek *et al.*, Phys. Rev. **D60**, 113001 (1999);
ibid., Phys. Rev. **D62**, 055001 (2000);
Q. Shafi and Z. Tavartkiladze, Phys. Lett. **B487**, 145 (2000);
C.H. Albright and S.M. Barr, Phys. Rev. Lett. **85**, 244 (2000);
K.S. Babu *et al.*, Nucl. Phys. **B566**, 33 (2000);
M. -C. Chen and K. T. Mahanthappa, Phys. Rev. **D62**, 113007 (2000);
G. Altarelli *et al.*, Ref. 36;
Z. Berezhiani and A. Rossi, Nucl. Phys. **B594**, 113 (2001);
C. H. Albright and S. M. Barr, Phys. Rev. **D64**, 073010 (2001);
M. -C. Chen, K. T. Mahanthappa, Int. J. Mod. Phys. **A18**, 5819 (2003);
R. Dermisek and S. Raby, Phys. Lett. **B622**, 327 (2005);
R. Dermisek *et al.*, Phys. Rev. **D74**, 035011 (2006);
M. Albrecht *et al.*, JHEP **0710**, 055 (2007);
K. S. Babu *et al.*, JHEP **1006**, 084 (2010).

75. G. Lazarides *et al.*, Nucl. Phys. **B181**, 287 (1981);
T. E. Clark *et al.*, Phys. Lett. **B115**, 26 (1982);
K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993).
76. B. Bajc *et al.*, Phys. Rev. Lett. **90**, 051802 (2003).
77. H. S. Goh *et al.*, Phys. Lett. **B570**, 215 (2003) [hep-ph/0303055];
ibid., Phys. Rev. **D68**, 115008 (2003) [hep-ph/0308197];
B. Dutta *et al.*, Phys. Rev. **D69**, 115014 (2004) [hep-ph/0402113];
S. Bertolini and M. Malinsky, [hep-ph/0504241];
K. S. Babu and C. Macesanu, [hep-ph/0505200].
78. B. Bajc *et al.*, JHEP **0811**, 007 (2008).
79. P. Minkowski, Phys. Lett. **B67**, 421 (1977).
80. T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number of the Universe*, eds. O. Sawada and A. Sugamoto, KEK report No. 79-18, Tsukuba, Japan, 1979;
S. Glashow, Quarks and leptons, published in *Proceedings of the Carg'ese Lectures,*M. Levy (ed.), Plenum Press, New York, (1980);
M. Gell-Mann *et al.*, in *Supergravity,*ed. P. van Nieuwenhuizen *et al.*, North-Holland, Amsterdam, (1979), p. 315;
R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
81. G.L. Fogli *et al.*, Phys. Rev. **D84**, 053007 (2011).
82. T. Schwetz, M. Tortola, and J.W.F. Valle, New J. Phys. **13**, 109401 (2011), [arXiv:1108.1376 [hep-ph]].
83. C.H. Albright and M.-C. Chen, Phys. Rev. **D74**, 113006 (2006).
84. G. 't Hooft, Nucl. Phys. **B79**, 276 (1974);
A.M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 430 (1974) [JETP Lett. **20**, 194 (1974)];
For a pedagogical introduction, see S. Coleman, in *Aspects of Symmetry*, Selected Erice Lectures, Cambridge University Press, Cambridge, (1985), and P. Goddard and D. Olive, Rep. Prog. Phys. **41**, 1357 (1978).
85. I. De Mitri, (MACRO Collab.), Nucl. Phys. (Proc. Suppl.) **B95**, 82 (2001).
86. For a review, see A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic, Switzerland (1990).
87. G. R. Dvali *et al.*, Phys. Rev. Lett. **80**, 2281 (1998) [hep-ph/9710301].
88. P. Langacker and S. Y. Pi, Phys. Rev. Lett. **45**, 1 (1980).
89. G. R. Dvali *et al.*, Phys. Rev. Lett. **75**, 4559 (1995) [hep-ph/9507230].
90. V. Rubakov, Nucl. Phys. **B203**, 311 (1982), Institute of Nuclear Research Report No. P-0211, Moscow (1981), unpublished;
C. Callan, Phys. Rev. **D26**, 2058 (1982);
F. Wilczek, Phys. Rev. Lett. **48**, 1146 (1982);
See also, S. Dawson and A.N. Schellekens, Phys. Rev. **D27**, 2119 (1983).
91. K. Freese *et al.*, Phys. Rev. Lett. **51**, 1625 (1983).
92. M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986);
See also the recent review by W. Buchmuller *et al.*, hep-ph/0502169 and references therein.

93. E. Witten, Phys. Lett. **105B**, 267 (1981).
94. S. Dimopoulos and F. Wilczek, *Proceedings Erice Summer School*, ed. A. Zichichi (1981);
K.S. Babu and S.M. Barr, Phys. Rev. **D50**, 3529 (1994).
95. R. Barbieri *et al.*, Nucl. Phys. **B391**, 487 (1993);
Z. Berezhiani *et al.*, Nucl. Phys. **B444**, 61 (1995);
Q. Shafi and Z. Tavartkiladze, Phys. Lett. **B522**, 102 (2001).
96. G.F. Giudice and A. Masiero, Phys. Lett. **B206**, 480 (1988);
J.E. Kim and H.P. Nilles, Mod. Phys. Lett. **A9**, 3575 (1994).
97. L. Randall and C. Csaki, `hep-ph/9508208`.
98. A. Hebecker *et al.*, [`arXiv:0801.4101 [hep-ph]`];
F. Brümmer *et al.*, JHEP **08**, 011 (2009);
F. Brümmer *et al.*, JHEP **04**, 006 (2010).
99. T. P. T. Dijkstra *et al.*, Phys. Lett. **B609**, 408 (2005).
100. T. Kobayashi *et al.*, Phys. Lett. **B593**, 262 (2004);
ibid., Nucl. Phys. **B704**, 3 (2005).
101. W. Buchmuller *et al.*, Phys. Rev. Lett. **96**, 121602 (2006);
ibid., Nucl. Phys. **B785**, 149 (2007).
102. O. Lebedev *et al.*, Phys. Lett. **B645**, 88 (2007);
ibid., [`arXiv:0708.2691[hep-th]`];
O. Lebedev *et al.*, Phys. Lett. **B668**, 331 (2008).
103. J. E. Kim *et al.*, JHEP **0706**, 034 (2007);
J. E. Kim and B. Kyae, Phys. Rev. **D77**, 106008 (2008).
104. L. B. Anderson *et al.*, [`arXiv:1106.4804 [hep-th]`].
105. C. Beasley *et al.*, JHEP **01**, 058 (2009);
R. Donagi and M. Wijnholt, [`arXiv:0802.2969 [hep-th]`];
C. Beasley *et al.*, JHEP **01**, 059 (2009);
R. Donagi and M. Wijnholt, [`arXiv:0808.2223 [hep-th]`];
R. Blumenhagen *et al.*, [`arXiv:0811.2936 [hep-th]`].
106. C. M. Chen and Y. C. Chung, Nucl. Phys. **B824**, 273 (2010).
107. J. Marsano *et al.*, JHEP **0908**, 030 (2009).
108. R. Blumenhagen, Phys. Rev. Lett. **102**, 071601 (2009).
109. S. Raby, Rept. on Prog. in Phys. **74**, 036901 (2011).