DALITZ PLOT PARAMETERS FOR $K \to 3\pi$ DECAYS

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The Dalitz plot distribution for $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$, $K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$, and $K_L^{0} \to \pi^{+}\pi^{-}\pi^{0}$ can be parameterized by a series expansion such as that introduced by Weinberg [1]. We use the form

$$
\left| M \right|^2 \propto 1 + g \frac{(s_3 - s_0)}{m_{\pi^+}^2} + h \left[ \frac{s_3 - s_0}{m_{\pi^+}^2} \right]^2
$$

$$
+ j \frac{(s_2 - s_1)}{m_{\pi^+}^2} + k \left[ \frac{s_2 - s_1}{m_{\pi^+}^2} \right]^2
$$

$$
+ f \frac{(s_2 - s_1)(s_3 - s_0)}{m_{\pi^+}^2 m_{\pi^+}^2} + \cdots ,
$$

where $m_{\pi^+}^2$ has been introduced to make the coefficients $g$, $h$, $j$, and $k$ dimensionless, and

$$
s_i = (P_K - P_i)^2 = (m_K - m_i)^2 - 2m_KT_i , \quad i = 1, 2, 3 ,
$$

$$
s_0 = \frac{1}{3} \sum_i s_i = \frac{1}{3}(m_K^2 + m_1^2 + m_2^2 + m_3^2) .
$$

Here the $P_i$ are four-vectors, $m_i$ and $T_i$ are the mass and kinetic energy of the $i^{th}$ pion, and the index 3 is used for the odd pion.

The coefficient $g$ is a measure of the slope in the variable $s_3$ (or $T_3$) of the Dalitz plot, while $h$ and $k$ measure the quadratic dependence on $s_3$ and $(s_2 - s_1)$, respectively. The coefficient $j$ is related to the asymmetry of the plot and must be zero if $CP$ invariance holds. Note also that if $CP$ is good, $g$, $h$, and $k$ must be the same for $K^+ \to \pi^+\pi^+\pi^-$ as for $K^- \to \pi^-\pi^-\pi^+$.

Since different experiments use different forms for $\left| M \right|^2$, in order to compare the experiments we have converted to $g$, $h$, $j$, and $k$ whatever coefficients have been measured. Where such conversions have been done, the measured coefficient $a_y$, $a_t$, $a_u$, or $a_v$ is given in the comment at the right. For definitions of these coefficients, details of this conversion, and discussion of the data, see the April 1982 version of this note [2].

References