

Light Quarks (u, d, s)

OMITTED FROM SUMMARY TABLE

u -QUARK MASS

The u -, d -, and s -quark masses are estimates of so-called “current-quark masses,” in a mass-independent subtraction scheme such as $\overline{\text{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s -quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of “Our Evaluation” were determined in part via Figures 1 and 2.

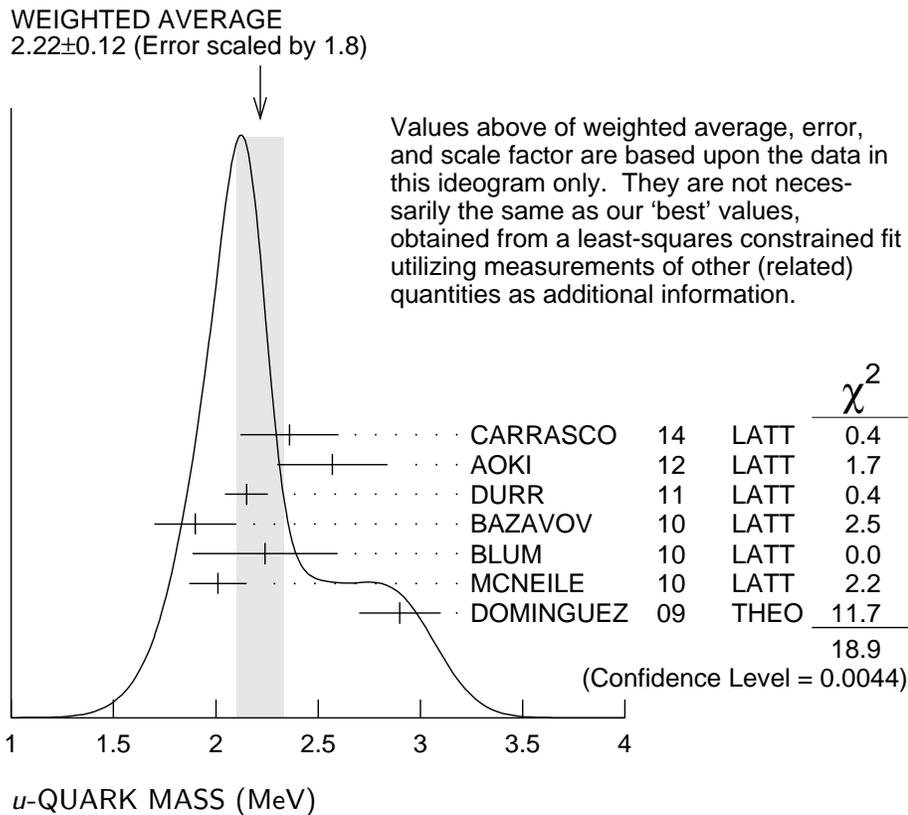
VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
2.2 $^{+0.6}_{-0.4}$ OUR EVALUATION	See the ideogram below.		
2.36 \pm 0.24	¹ CARRASCO	14	LATT $\overline{\text{MS}}$ scheme
2.57 \pm 0.26 \pm 0.07	² AOKI	12	LATT $\overline{\text{MS}}$ scheme
2.15 \pm 0.03 \pm 0.10	³ DURR	11	LATT $\overline{\text{MS}}$ scheme
1.9 \pm 0.2	⁴ BAZAVOV	10	LATT $\overline{\text{MS}}$ scheme
2.24 \pm 0.10 \pm 0.34	⁵ BLUM	10	LATT $\overline{\text{MS}}$ scheme
2.01 \pm 0.14	⁶ MCNEILE	10	LATT $\overline{\text{MS}}$ scheme
2.9 \pm 0.2	⁷ DOMINGUEZ	09	THEO $\overline{\text{MS}}$ scheme
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●			
2.01 \pm 0.14	⁶ DAVIES	10	LATT $\overline{\text{MS}}$ scheme
2.9 \pm 0.8	⁸ DEANDREA	08	THEO $\overline{\text{MS}}$ scheme
3.02 \pm 0.33	⁹ BLUM	07	LATT $\overline{\text{MS}}$ scheme
2.7 \pm 0.4	¹⁰ JAMIN	06	THEO $\overline{\text{MS}}$ scheme
1.9 \pm 0.2	¹¹ MASON	06	LATT $\overline{\text{MS}}$ scheme
2.8 \pm 0.2	¹² NARISON	06	THEO $\overline{\text{MS}}$ scheme
1.7 \pm 0.3	¹³ AUBIN	04A	LATT $\overline{\text{MS}}$ scheme

¹ CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

² AOKI 12 is a lattice computation using 1 + 1 + 1 dynamical quark flavors.

³ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed. The individual m_u , m_d values are obtained using the lattice determination of the average mass m_{ud} and of the ratio m_s/m_{ud} and the value of $Q = (m_s^2 - m_{ud}^2) / (m_d^2 - m_u^2)$ as determined from $\eta \rightarrow 3\pi$ decays.

- 4 BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.
- 5 BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use 2+1 dynamical quark flavors.
- 6 DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_u is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratios, m_s/\overline{m} and m_u/m_d .
- 7 DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .
- 8 DEANDREA 08 determine $m_u - m_d$ from $\eta \rightarrow 3\pi^0$, and combine with the PDG 06 lattice average value of $m_u + m_d = 7.6 \pm 1.6$ to determine m_u and m_d .
- 9 BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- 10 JAMIN 06 determine $m_u(2 \text{ GeV})$ by combining the value of m_s obtained from the spectral function for the scalar $K\pi$ form factor with other determinations of the quark mass ratios.
- 11 MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order. The quark masses m_u and m_d were determined from their $(m_u + m_d)/2$ measurement and AUBIN 04A m_u/m_d value.
- 12 NARISON 06 uses sum rules for $e^+e^- \rightarrow \text{hadrons}$ to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.
- 13 AUBIN 04A employ a partially quenched lattice calculation of the pseudoscalar meson masses.



d -QUARK MASS

See the comment for the u quark above.

We have normalized the \overline{MS} masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
4.7^{+0.5}_{-0.4} OUR EVALUATION	See the ideogram below.		
5.03±0.26	¹ CARRASCO	14	LATT \overline{MS} scheme
3.68±0.29±0.10	² AOKI	12	LATT \overline{MS} scheme
4.79±0.07±0.12	³ DURR	11	LATT \overline{MS} scheme
4.6 ±0.3	⁴ BAZAVOV	10	LATT \overline{MS} scheme
4.65±0.15±0.32	⁵ BLUM	10	LATT \overline{MS} scheme
4.77±0.15	⁶ MCNEILE	10	LATT \overline{MS} scheme
5.3 ±0.4	⁷ DOMINGUEZ	09	THEO \overline{MS} scheme
• • • We do not use the following data for averages, fits, limits, etc. • • •			
4.79±0.16	⁶ DAVIES	10	LATT \overline{MS} scheme
4.7 ±0.8	⁸ DEANDREA	08	THEO \overline{MS} scheme
5.49±0.39	⁹ BLUM	07	LATT \overline{MS} scheme
4.8 ±0.5	¹⁰ JAMIN	06	THEO \overline{MS} scheme
4.4 ±0.3	¹¹ MASON	06	LATT \overline{MS} scheme
5.1 ±0.4	¹² NARISON	06	THEO \overline{MS} scheme
3.9 ±0.5	¹³ AUBIN	04A	LATT \overline{MS} scheme

¹ CARRASCO 14 is a lattice QCD computation of light quark masses using $2 + 1 + 1$ dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

² AOKI 12 is a lattice computation using $1 + 1 + 1$ dynamical quark flavors.

³ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed. The individual m_u , m_d values are obtained using the lattice determination of the average mass m_{ud} and of the ratio m_s/m_{ud} and the value of $Q = (m_s^2 - m_{ud}^2) / (m_d^2 - m_u^2)$ as determined from $\eta \rightarrow 3\pi$ decays.

⁴ BAZAVOV 10 is a lattice computation using $2+1$ dynamical quark flavors.

⁵ BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use $2+1$ dynamical quark flavors.

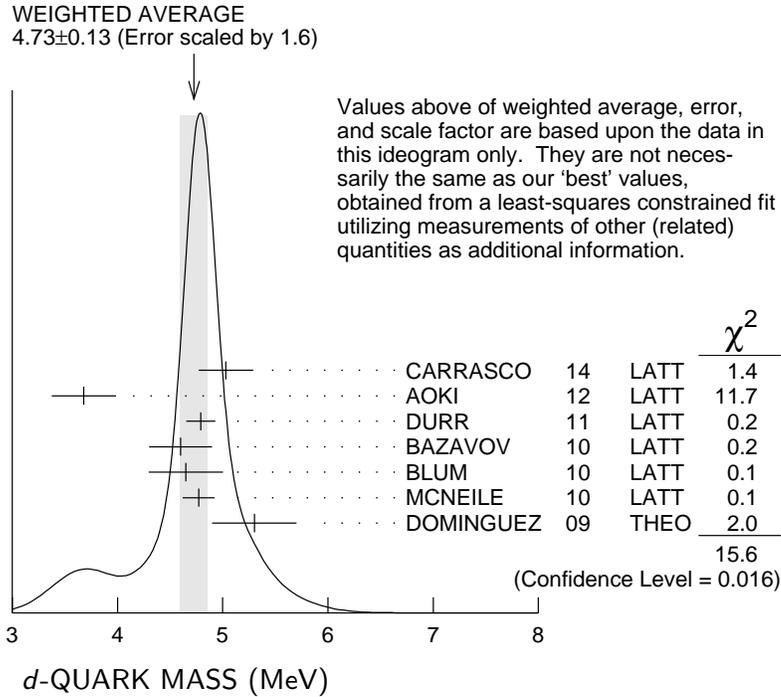
⁶ DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_d is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratios, m_s/\overline{m} and m_u/m_d .

⁷ DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .

⁸ DEANDREA 08 determine $m_u - m_d$ from $\eta \rightarrow 3\pi^0$, and combine with the PDG 06 lattice average value of $m_u + m_d = 7.6 \pm 1.6$ to determine m_u and m_d .

⁹ BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.

- 10 JAMIN 06 determine $m_d(2 \text{ GeV})$ by combining the value of m_s obtained from the spectral function for the scalar $K\pi$ form factor with other determinations of the quark mass ratios.
- 11 MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order. The quark masses m_u and m_d were determined from their $(m_u+m_d)/2$ measurement and AUBIN 04A m_u/m_d value.
- 12 NARISON 06 uses sum rules for $e^+e^- \rightarrow \text{hadrons}$ to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.
- 13 AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses, and one-loop perturbative renormalization constant.



$$\bar{m} = (m_u + m_d)/2$$

See the comments for the u quark above.

We have normalized the \overline{MS} masses at a renormalization scale of $\mu = 2 \text{ GeV}$. Results quoted in the literature at $\mu = 1 \text{ GeV}$ have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
3.5 $\pm_{-0.3}^{+0.7}$	OUR EVALUATION		See the ideogram below.
3.70 ± 0.17	1 CARRASCO	14 LATT	\overline{MS} scheme
3.45 ± 0.12	2 ARTHUR	13 LATT	\overline{MS} scheme
3.59 ± 0.21	3 AOKI	11A LATT	\overline{MS} scheme
3.469 ± 0.047 ± 0.048	4 DURR	11 LATT	\overline{MS} scheme
3.6 ± 0.2	5 BLOSSIER	10 LATT	\overline{MS} scheme
3.39 ± 0.06	6 MCNEILE	10 LATT	\overline{MS} scheme

4.1 ± 0.2	7	DOMINGUEZ	09	THEO	\overline{MS} scheme
3.72 ± 0.41	8	ALLTON	08	LATT	\overline{MS} scheme
3.55 ^{+0.65} _{-0.28}	9	ISHIKAWA	08	LATT	\overline{MS} scheme
4.25 ± 0.35	10	BLUM	07	LATT	\overline{MS} scheme
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●					
3.40 ± 0.07	6	DAVIES	10	LATT	\overline{MS} scheme
3.85 ± 0.12 ± 0.4	11	BLOSSIER	08	LATT	\overline{MS} scheme
≥ 4.85 ± 0.20	12	DOMINGUEZ...08B	THEO	\overline{MS} scheme	
4.026 ± 0.048	13	NAKAMURA	08	LATT	\overline{MS} scheme
4.08 ± 0.25 ± 0.42	14	GOCKELER	06	LATT	\overline{MS} scheme
4.7 ± 0.2 ± 0.3	15	GOCKELER	06A	LATT	\overline{MS} scheme
3.2 ± 0.3	16	MASON	06	LATT	\overline{MS} scheme
3.95 ± 0.3	17	NARISON	06	THEO	\overline{MS} scheme
2.8 ± 0.3	18	AUBIN	04	LATT	\overline{MS} scheme
4.29 ± 0.14 ± 0.65	19	AOKI	03	LATT	\overline{MS} scheme
3.223 ± 0.3	20	AOKI	03B	LATT	\overline{MS} scheme
4.4 ± 0.1 ± 0.4	21	BECIREVIC	03	LATT	\overline{MS} scheme
4.1 ± 0.3 ± 1.0	22	CHIU	03	LATT	\overline{MS} scheme

¹ CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

² ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions. Masses at $\mu = 3$ GeV have been converted to $\mu = 2$ GeV using conversion factors given in their paper.

³ AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.

⁴ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.

⁵ BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.

⁶ DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass \overline{m} is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratio, m_s/\overline{m} .

⁷ DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .

⁸ ALLTON 08 use a lattice computation of the π , K , and Ω masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.

⁹ ISHIKAWA 08 use a lattice computation of the light meson spectrum with 2+1 dynamical flavors of $\mathcal{O}(a)$ improved Wilson quarks, and one-loop perturbative renormalization.

¹⁰ BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.

¹¹ BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.

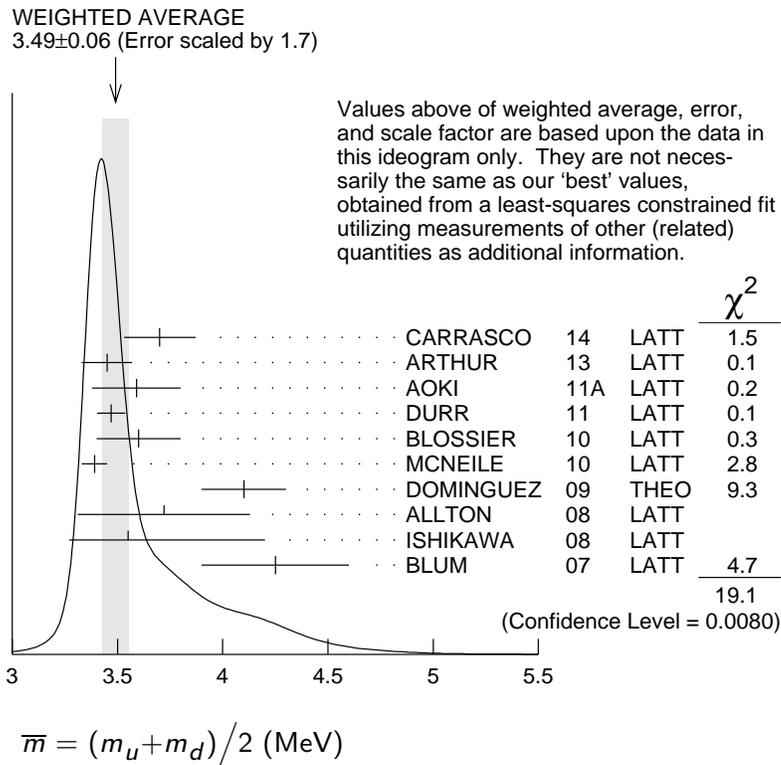
¹² DOMINGUEZ-CLARIMON 08B obtain an inequality from sum rules for the scalar two-point correlator.

¹³ NAKAMURA 08 do a lattice computation using quenched domain wall fermions and non-perturbative renormalization.

¹⁴ GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization, to obtain

$\bar{m}(2 \text{ GeV}) = 4.08 \pm 0.25 \pm 0.19 \pm 0.23 \text{ MeV}$, where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.

- 15 GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization.
- 16 MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order.
- 17 NARISON 06 uses sum rules for $e^+e^- \rightarrow \text{hadrons}$ to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.
- 18 AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.
- 19 AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory.
- 20 The errors given in AOKI 03B were $^{+0.046}_{-0.069}$. We changed them to ± 0.3 for calculating the overall best values. AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the $\mathcal{O}(a)$ improved Wilson action.
- 21 BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses $\mathcal{O}(a)$ improved Wilson action and nonperturbative renormalization.
- 22 CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.

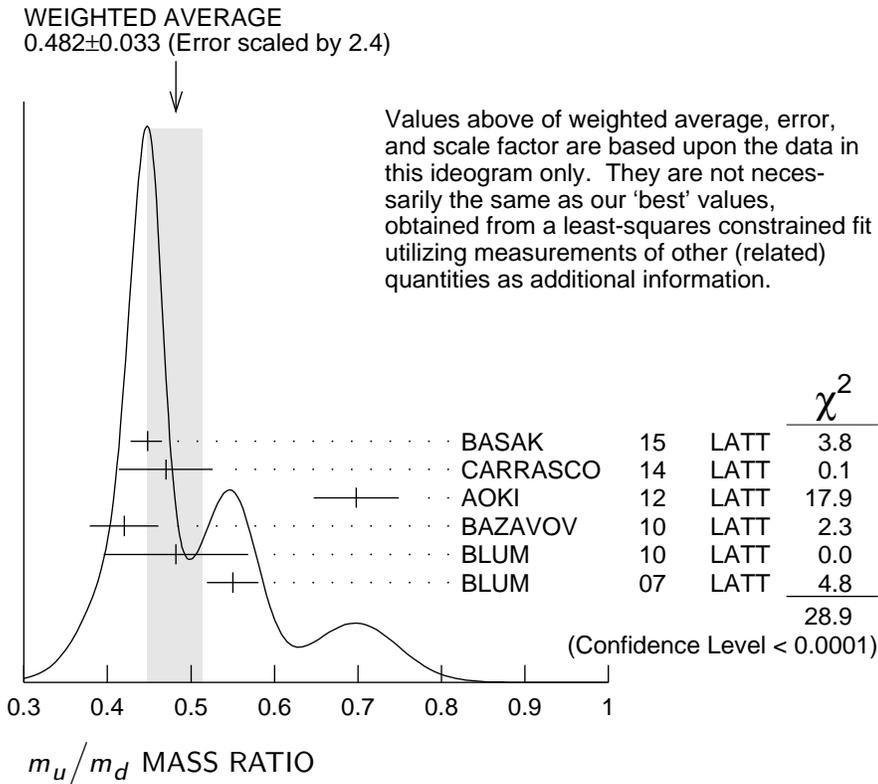


m_u/m_d MASS RATIO

VALUE	DOCUMENT ID	TECN	COMMENT
0.38–0.58 OUR EVALUATION	See the ideogram below.		
$0.4482^{+0.0173}_{-0.0206}$	¹ BASAK	15	LATT

0.470 ±0.056	2	CARRASCO	14	LATT
0.698 ±0.051	3	AOKI	12	LATT
0.42 ±0.01 ±0.04	4	BAZAVOV	10	LATT
0.4818±0.0096±0.0860	5	BLUM	10	LATT
0.550 ±0.031	6	BLUM	07	LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●				
0.43 ±0.08	7	AUBIN	04A	LATT
0.410 ±0.036	8	NELSON	03	LATT
0.553 ±0.043	9	LEUTWYLER	96	THEO Compilation

- ¹ BASAK 15 is a lattice computation using 2+1 dynamical quark flavors.
- ² CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.
- ³ AOKI 12 is a lattice computation using 1 + 1 + 1 dynamical quark flavors.
- ⁴ BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.
- ⁵ BLUM 10 is a lattice computation using 2+1 dynamical quark flavors.
- ⁶ BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- ⁷ AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses.
- ⁸ NELSON 03 computes coefficients in the order p^4 chiral Lagrangian using a lattice calculation with three dynamical flavors. The ratio m_u/m_d is obtained by combining this with the chiral perturbation theory computation of the meson masses to order p^4 .
- ⁹ LEUTWYLER 96 uses a combined fit to $\eta \rightarrow 3\pi$ and $\psi' \rightarrow J/\psi (\pi, \eta)$ decay rates, and the electromagnetic mass differences of the π and K .



s-QUARK MASS

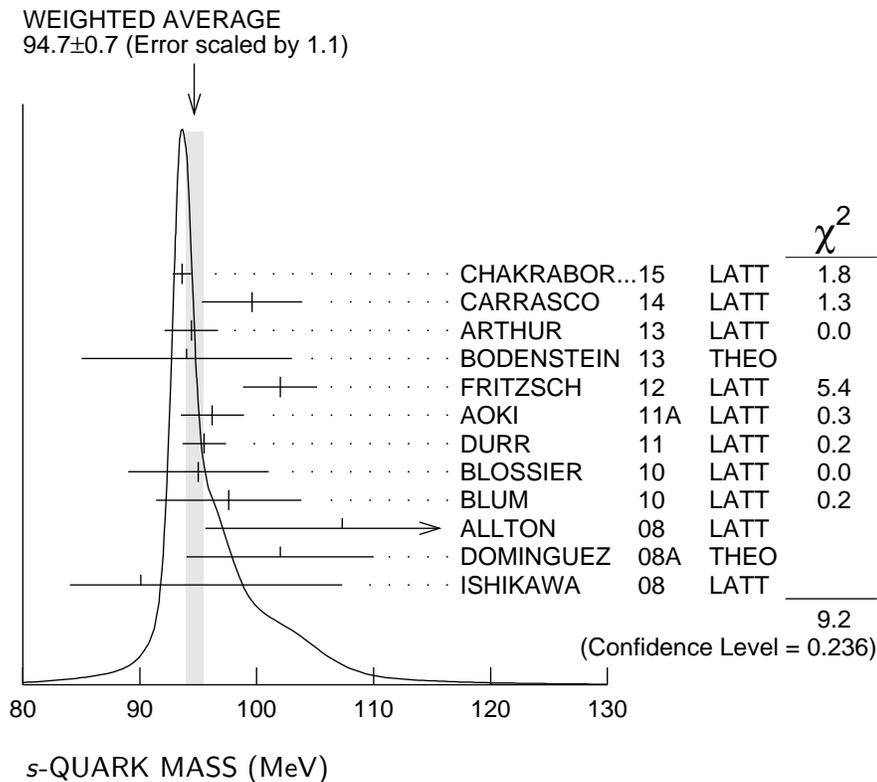
See the comment for the u quark above.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
96 \pm 8 - 4	OUR EVALUATION		See the ideogram below.
93.6 \pm 0.8	1 CHAKRABOR..15	LATT	$\overline{\text{MS}}$ scheme
99.6 \pm 4.3	2 CARRASCO 14	LATT	$\overline{\text{MS}}$ scheme
94.4 \pm 2.3	3 ARTHUR 13	LATT	$\overline{\text{MS}}$ scheme
94 \pm 9	4 BODENSTEIN 13	THEO	$\overline{\text{MS}}$ scheme
102 \pm 3 \pm 1	5 FRITZSCH 12	LATT	$\overline{\text{MS}}$ scheme
96.2 \pm 2.7	6 AOKI 11A	LATT	$\overline{\text{MS}}$ scheme
95.5 \pm 1.1 \pm 1.5	7 DURR 11	LATT	$\overline{\text{MS}}$ scheme
95 \pm 6	8 BLOSSIER 10	LATT	$\overline{\text{MS}}$ scheme
97.6 \pm 2.9 \pm 5.5	9 BLUM 10	LATT	$\overline{\text{MS}}$ scheme
107.3 \pm 11.7	10 ALLTON 08	LATT	$\overline{\text{MS}}$ scheme
102 \pm 8	11 DOMINGUEZ 08A	THEO	$\overline{\text{MS}}$ scheme
90.1 \pm 17.2 - 6.1	12 ISHIKAWA 08	LATT	$\overline{\text{MS}}$ scheme
• • • We do not use the following data for averages, fits, limits, etc. • • •			
92.4 \pm 1.5	13 DAVIES 10	LATT	$\overline{\text{MS}}$ scheme
92.2 \pm 1.3	13 MCNEILE 10	LATT	$\overline{\text{MS}}$ scheme
105 \pm 3 \pm 9	14 BLOSSIER 08	LATT	$\overline{\text{MS}}$ scheme
105.6 \pm 1.2	15 NAKAMURA 08	LATT	$\overline{\text{MS}}$ scheme
119.5 \pm 9.3	16 BLUM 07	LATT	$\overline{\text{MS}}$ scheme
105 \pm 6 \pm 7	17 CHETYRKIN 06	THEO	$\overline{\text{MS}}$ scheme
111 \pm 6 \pm 10	18 GOCKELER 06	LATT	$\overline{\text{MS}}$ scheme
119 \pm 5 \pm 8	19 GOCKELER 06A	LATT	$\overline{\text{MS}}$ scheme
92 \pm 9	20 JAMIN 06	THEO	$\overline{\text{MS}}$ scheme
87 \pm 6	21 MASON 06	LATT	$\overline{\text{MS}}$ scheme
104 \pm 15	22 NARISON 06	THEO	$\overline{\text{MS}}$ scheme
$\geq 71 \pm 4, \leq 151 \pm 14$	23 NARISON 06	THEO	$\overline{\text{MS}}$ scheme
96 \pm 5 \pm 16 - 3 - 18	24 BAIKOV 05	THEO	$\overline{\text{MS}}$ scheme
81 \pm 22	25 GAMIZ 05	THEO	$\overline{\text{MS}}$ scheme
125 \pm 28	26 GORBUNOV 05	THEO	$\overline{\text{MS}}$ scheme
93 \pm 32	27 NARISON 05	THEO	$\overline{\text{MS}}$ scheme
76 \pm 8	28 AUBIN 04	LATT	$\overline{\text{MS}}$ scheme
116 \pm 6 \pm 0.65	29 AOKI 03	LATT	$\overline{\text{MS}}$ scheme
84.5 \pm 12 - 1.7	30 AOKI 03B	LATT	$\overline{\text{MS}}$ scheme
106 \pm 2 \pm 8	31 BECIREVIC 03	LATT	$\overline{\text{MS}}$ scheme
92 \pm 9 \pm 16	32 CHIU 03	LATT	$\overline{\text{MS}}$ scheme
117 \pm 17	33 GAMIZ 03	THEO	$\overline{\text{MS}}$ scheme
103 \pm 17	34 GAMIZ 03	THEO	$\overline{\text{MS}}$ scheme

- ¹ CHAKRABORTY 15 is a lattice QCD computation that determines m_c and m_c/m_s using pseudoscalar mesons masses tuned on gluon field configurations with 2+1+1 dynamical flavors of HISQ quarks with u/d masses down to the physical value.
- ² CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.
- ³ ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions. Masses at $\mu = 3$ GeV have been converted to $\mu = 2$ GeV using conversion factors given in their paper.
- ⁴ BODENSTEIN 13 determines m_s from QCD finite energy sum rules, and the perturbative computation of the pseudoscalar correlator to five-loop order.
- ⁵ FRITZSCH 12 determine m_s using a lattice computation with $N_f = 2$ dynamical flavors.
- ⁶ AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.
- ⁷ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.
- ⁸ BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.
- ⁹ BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use 2+1 dynamical quark flavors.
- ¹⁰ ALLTON 08 use a lattice computation of the π , K , and Ω masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- ¹¹ DOMINGUEZ 08A make determination from QCD finite energy sum rules for the pseudoscalar two-point function computed to order α_s^4 .
- ¹² ISHIKAWA 08 use a lattice computation of the light meson spectrum with 2+1 dynamical flavors of $\mathcal{O}(a)$ improved Wilson quarks, and one-loop perturbative renormalization.
- ¹³ DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_s is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10.
- ¹⁴ BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- ¹⁵ NAKAMURA 08 do a lattice computation using quenched domain wall fermions and non-perturbative renormalization.
- ¹⁶ BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- ¹⁷ CHETYRKIN 06 use QCD sum rules in the pseudoscalar channel to order α_s^4 .
- ¹⁸ GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization, to obtain $\overline{m}_s(2 \text{ GeV}) = 111 \pm 6 \pm 4 \pm 6 \text{ MeV}$, where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.
- ¹⁹ GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization.
- ²⁰ JAMIN 06 determine $\overline{m}_s(2 \text{ GeV})$ from the spectral function for the scalar $K\pi$ form factor.
- ²¹ MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order.
- ²² NARISON 06 uses sum rules for $e^+e^- \rightarrow \text{hadrons}$ to order α_s^3 .
- ²³ NARISON 06 obtains the quoted range from positivity of the spectral functions.

- 24 BAIKOV 05 determines $\overline{m}_s(M_\tau) = 100^{+5+17}_{-3-19}$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^3 , with an estimate of the α_s^4 terms. We have converted the result to $\mu = 2$ GeV.
- 25 GAMIZ 05 determines $\overline{m}_s(2 \text{ GeV})$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^2 , with an estimate of the α_s^3 terms.
- 26 GORBUNOV 05 use hadronic tau decays to N³LO, including power corrections.
- 27 NARISON 05 determines $\overline{m}_s(2 \text{ GeV})$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^3 .
- 28 AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.
- 29 AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory. Determines $m_s = 113.8 \pm 2.3^{+5.8}_{-2.9}$ using K mass as input and $m_s = 142.3 \pm 5.8^{+22}_0$ using ϕ mass as input. We have performed a weighted average of these values.
- 30 AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the $\mathcal{O}(a)$ improved Wilson action.
- 31 BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses $\mathcal{O}(a)$ improved Wilson action and nonperturbative renormalization. They also quote $\overline{m}/m_s = 24.3 \pm 0.2 \pm 0.6$.
- 32 CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.
- 33 GAMIZ 03 determines m_s from SU(3) breaking in the τ hadronic width. The value of V_{us} is chosen to satisfy CKM unitarity.
- 34 GAMIZ 03 determines m_s from SU(3) breaking in the τ hadronic width. The value of V_{us} is taken from the PDG.



OTHER LIGHT QUARK MASS RATIOS

 m_s/m_d MASS RATIO

<u>VALUE</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>COMMENT</u>
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17–22 OUR EVALUATION

• • • We do not use the following data for averages, fits, limits, etc. • • •

20.0	¹ GAO	97	THEO
18.9±0.8	² LEUTWYLER	96	THEO Compilation
21	³ DONOGHUE	92	THEO
18	⁴ GERARD	90	THEO
18 to 23	⁵ LEUTWYLER	90B	THEO

¹ GAO 97 uses electromagnetic mass splittings of light mesons.

² LEUTWYLER 96 uses a combined fit to $\eta \rightarrow 3\pi$ and $\psi' \rightarrow J/\psi (\pi, \eta)$ decay rates, and the electromagnetic mass differences of the π and K .

³ DONOGHUE 92 result is from a combined analysis of meson masses, $\eta \rightarrow 3\pi$ using second-order chiral perturbation theory including nonanalytic terms, and $(\psi(2S) \rightarrow J/\psi(1S)\pi)/(\psi(2S) \rightarrow J/\psi(1S)\eta)$.

⁴ GERARD 90 uses large N and η - η' mixing.

⁵ LEUTWYLER 90B determines quark mass ratios using second-order chiral perturbation theory for the meson and baryon masses, including nonanalytic corrections. Also uses Weinberg sum rules to determine L_7 .

 m_s/\bar{m} MASS RATIO

$$\bar{m} \equiv (m_u + m_d)/2$$

<u>VALUE</u>	<u>DOCUMENT ID</u>	<u>TECN</u>
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27.3 ±0.7 OUR EVALUATION See the ideogram below.

27.35±0.05 ^{+0.10} _{-0.07}	¹ BAZAVOV	14A	LATT
26.66±0.32	² CARRASCO	14	LATT
27.36±0.54	³ ARTHUR	13	LATT
26.8 ±1.4	⁴ AOKI	11A	LATT
27.53±0.20±0.08	⁵ DURR	11	LATT
27.3 ±0.9	⁶ BLOSSIER	10	LATT
28.8 ±1.65	⁷ ALLTON	08	LATT
27.3 ±0.3 ±1.2	⁸ BLOSSIER	08	LATT
23.5 ±1.5	⁹ OLLER	07A	THEO

• • • We do not use the following data for averages, fits, limits, etc. • • •

27.4 ±0.4	¹⁰ AUBIN	04	LATT
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¹ BAZAVOV 14A is a lattice computation using 4 dynamical flavors of HISQ fermions.

² CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

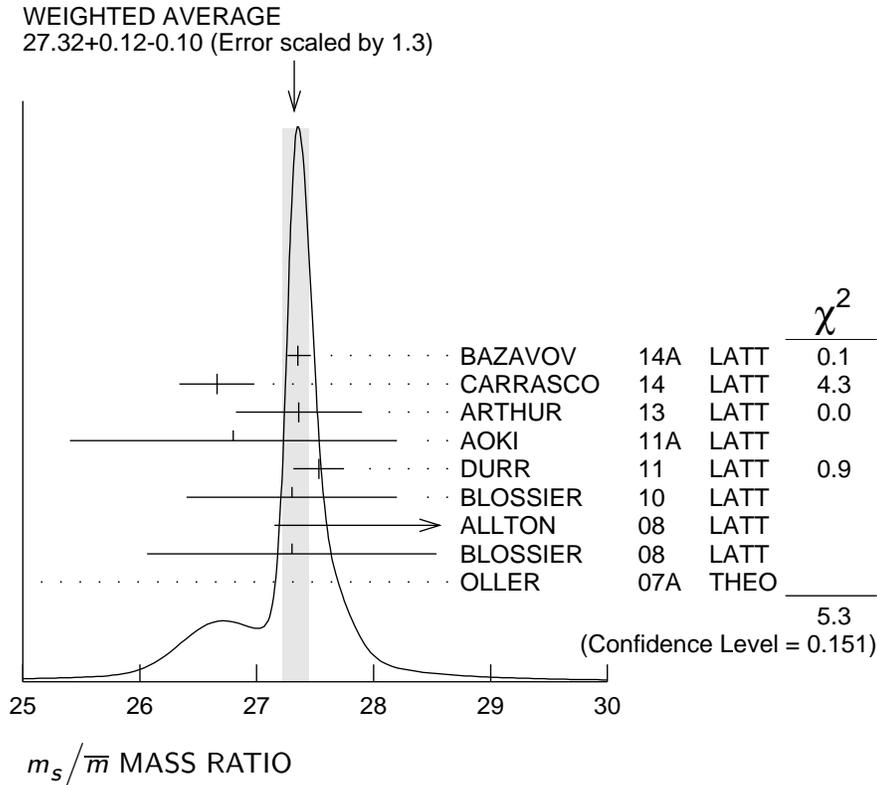
³ ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions.

⁴ AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.

⁵ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.

⁶ BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.

- ⁷ ALLTON 08 use a lattice computation of the π , K , and Ω masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- ⁸ BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- ⁹ OLLER 07A use unitarized chiral perturbation theory to order p^4 .
- ¹⁰ Three flavor dynamical lattice calculation of pseudoscalar meson masses.



Q MASS RATIO

$$Q \equiv \sqrt{(m_s^2 - \bar{m}^2)/(m_d^2 - m_u^2)}; \quad \bar{m} \equiv (m_u + m_d)/2$$

VALUE	DOCUMENT ID	TECN
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• • • We do not use the following data for averages, fits, limits, etc. • • •

22.8±0.4	¹ MARTEMYA... 05	THEO
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22.7±0.8	² ANISOVICH 96	THEO
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¹ MARTEMYANOV 05 determine Q from $\eta \rightarrow 3\pi$ decay.

² ANISOVICH 96 find Q from $\eta \rightarrow \pi^+ \pi^- \pi^0$ decay using dispersion relations and chiral perturbation theory.

LIGHT QUARKS (u, d, s) REFERENCES

BASAK	15	JPCS 640 012052	S. Basak <i>et al.</i>	(MILC Collab.)
CHAKRABOR...	15	PR D91 054508	B. Chakraborty <i>et al.</i>	(HPQCD Collab.)
BAZAVOV	14A	PR D90 074509	A. Bazavov <i>et al.</i>	(Fermi-LAT and MILC Collabs.)
CARRASCO	14	NP B887 19	N. Carrasco <i>et al.</i>	(European Twisted Mass Collab.)
ARTHUR	13	PR D87 094514	R. Arthur <i>et al.</i>	(RBC and UKQCD Collabs.)
BODENSTEIN	13	JHEP 1307 138	S. Bodenstein, C.A. Dominguez, K. Schilcher	(MANZ+)
AOKI	12	PR D86 034507	S. Aoki <i>et al.</i>	(PACS-CS Collab.)
FRITZSCH	12	NP B865 397	P. Fritzscht <i>et al.</i>	(ALPHA Collab.)
AOKI	11A	PR D83 074508	Y. Aoki <i>et al.</i>	(RBC-UKQCD Collab.)
DURR	11	PL B701 265	S. Durr <i>et al.</i>	(BMW Collab.)

BAZAVOV	10	RMP 82 1349	A. Bazavov <i>et al.</i>	(MILC Collab.)
BLOSSIER	10	PR D82 114513	B. Blossier <i>et al.</i>	(ETM Collab.)
BLUM	10	PR D82 094508	T. Blum <i>et al.</i>	
DAVIES	10	PRL 104 132003	C.T.H. Davies <i>et al.</i>	(HPQCD Collab.)
MCNEILE	10	PR D82 034512	C. McNeile <i>et al.</i>	(HPQCD Collab.)
DOMINGUEZ	09	PR D79 014009	C.A. Dominguez <i>et al.</i>	
ALLISON	08	PR D78 054513	I. Allison <i>et al.</i>	(HPQCD Collab.)
ALLTON	08	PR D78 114509	C. Allton <i>et al.</i>	(RBC and UKQCD Collabs.)
BLOSSIER	08	JHEP 0804 020	B. Blossier <i>et al.</i>	(ETM Collab.)
DEANDREA	08	PR D78 034032	A. Deandrea, A. Nehme, P. Talavera	
DOMINGUEZ	08A	JHEP 0805 020	C.A. Dominguez <i>et al.</i>	
DOMINGUEZ...	08B	PL B660 49	A. Dominguez-Clarimon, E. de Rafael, J. Taron	
ISHIKAWA	08	PR D78 011502	T. Ishikawa <i>et al.</i>	(CP-PACS and JLQCD Collabs.)
NAKAMURA	08	PR D78 034502	Y. Nakamura <i>et al.</i>	(CP-PACS Collab.)
BLUM	07	PR D76 114508	T. Blum <i>et al.</i>	(RBC Collab.)
OLLER	07A	EPJ A34 371	J.A. Oller, L. Roca	
CHETYRKIN	06	EPJ C46 721	K.G. Chetyrkin, A. Khodjamirian	
GOCKELER	06	PR D73 054508	M. Gockeler <i>et al.</i>	(QCDSF, UKQCD Collabs)
GOCKELER	06A	PL B639 307	M. Gockeler <i>et al.</i>	(QCDSF, UKQCD Collabs)
JAMIN	06	PR D74 074009	M. Jamin, J.A. Oller, A. Pich	
MASON	06	PR D73 114501	Q. Mason <i>et al.</i>	(HPQCD Collab.)
NARISON	06	PR D74 034013	S. Narison	
PDG	06	JP G33 1	W.-M. Yao <i>et al.</i>	(PDG Collab.)
BAIKOV	05	PRL 95 012003	P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn	
GAMIZ	05	PRL 94 011803	E. Gamiz <i>et al.</i>	
GORBUNOV	05	PR D71 013002	D.S. Gorbunov, A.A. Pivovarov	
MARTEMYA...	05	PR D71 017501	B.V. Martemyanov, V.S. Sopov	
NARISON	05	PL B626 101	S. Narison	
AUBIN	04	PR D70 031504	C. Aubin <i>et al.</i>	(HPQCD, MILC, UKQCD Collabs.)
AUBIN	04A	PR D70 114501	C. Aubin <i>et al.</i>	(MILC Collab.)
AOKI	03	PR D67 034503	S. Aoki <i>et al.</i>	(CP-PACS Collab.)
AOKI	03B	PR D68 054502	S. Aoki <i>et al.</i>	(CP-PACS Collab.)
BECIREVIC	03	PL B558 69	D. Becirevic, V. Lubicz, C. Tarantino	
CHIU	03	NP B673 217	T.-W. Chiu, T.-H. Hsieh	
GAMIZ	03	JHEP 0301 060	E. Gamiz <i>et al.</i>	
NELSON	03	PRL 90 021601	D. Nelson, G.T. Fleming, G.W. Kilcup	
GAO	97	PR D56 4115	D.-N. Gao, B.A. Li, M.-L. Yan	
ANISOVICH	96	PL B375 335	A.V. Anisovich, H. Leutwyler	
LEUTWYLER	96	PL B378 313	H. Leutwyler	
DONOGHUE	92	PRL 69 3444	J.F. Donoghue, B.R. Holstein, D. Wyler	(MASA+)
GERARD	90	MPL A5 391	J.M. Gerard	(MPIM)
LEUTWYLER	90B	NP B337 108	H. Leutwyler	(BERN)