

# INTRODUCTION

## 1. Overview

The *Review of Particle Physics* is a review of the field of Particle Physics and of related areas in Cosmology. It consists of “Summary Tables”, “Particle Listings”, and “Reviews, Tables, and Plots”. The latter covers a wide variety of theoretical and experimental topics and provides a quick reference for the practicing particle physicist.

The Summary Tables give our best values and limits for particle properties such as masses, widths or lifetimes, and branching fractions, as well as an extensive summary of searches for hypothetical particles and a summary of experimental tests of conservation laws.

The Particle Listings are a compilation/evaluation of data on particle properties. They contain all the data used to get the values given in the Summary Tables. The Particle Listings also give information on unconfirmed particles and on particle searches, as well as reviews on subjects of particular interest or controversy. In this edition, the Particle Listings include 3,062 new measurements from 721 papers, in addition to the 35,436 measurements from 9,843 papers that first appeared in previous editions [1]. Because of the large quantity of data, the Particle Listings are not an archive of all published data on particle properties. We refer interested readers to earlier editions for data now considered to be obsolete.

The book version of the *Review* is published in even-numbered years. This edition is an updating through January 2016 (and, in some areas, well into 2016). The content of this *Review* is available on the web and is updated between printed editions.

We organize the particles into six categories:

Gauge and Higgs bosons

Leptons

Quarks

Mesons

Baryons

Searches for monopoles, supersymmetry,  
compositeness, extra dimensions, *etc.*

The last category only includes searches for particles that do not belong to the previous groups; searches for heavy charged leptons and massive neutrinos, by contrast, are with the leptons.

In Sec. 2 of this Introduction, we list the main areas of responsibility of the authors of the Particle Listings. Our many consultants, without whom we would not have been able to produce this *Review*, are acknowledged in Sec. 3. In Sec. 4, we mention briefly the naming scheme for hadrons. In Sec. 5, we discuss our procedures for choosing among measurements of particle properties and for obtaining best values of the properties from the measurements.

The accuracy and usefulness of this *Review* depend in large part on interaction between its users and the authors. We appreciate comments, criticisms, and suggestions for improvements of any kind. Please send them to the appropriate author, according to the list of responsibilities in Sec. 2 below, or to [pdg@lbl.gov](mailto:pdg@lbl.gov).

The complete *Review* is published online in a journal and on the PDG website

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(<http://pdg.lbl.gov>). In addition to the online publication, the Review is available in different formats:

- The printed *PDG Book* contains the Summary Tables and all review articles. In contrast to previous editions, the detailed tables from the Particle Listings are no longer printed.
- The *Particle Physics Booklet* includes the Summary Tables and abbreviated versions of some of the review articles in a pocket format.
- *pdgLive* (<http://pdgLive.lbl.gov>) is a web application for online access to the PDG database.
- Files that can be downloaded from the PDG website include a table of masses, widths, and PDG Monte Carlo particle ID numbers; PDF files of the entire PDG Book and Booklet; individual review articles; all figures; and an archive file containing the complete PDG website (except for *pdgLive*).

Copies of the *PDG Book* or the *Particle Physics Booklet* can be ordered from our website or directly at <http://pdg.lbl.gov/order>. For special requests only, please email [pdg@lbl.gov](mailto:pdg@lbl.gov) in North and South America, Australia, and the Far East, and [pdg-products@cern.ch](mailto:pdg-products@cern.ch) in all other areas.

## 2. Particle Listings responsibilities

\* Asterisk indicates the people to contact with questions or comments about Particle Listings sections.

### Gauge and Higgs bosons

|              |                                       |
|--------------|---------------------------------------|
| $\gamma$     | C. Grab, D.E. Groom*                  |
| Gluons       | R.M. Barnett,* A.V. Manohar           |
| Graviton     | D.E. Groom*                           |
| $W, Z$       | A. Gurtu,* M. Grünewald*              |
| Higgs bosons | K. Hikasa, G. Weiglein*               |
| Heavy bosons | S. Pagan Griso,* M. Tanabashi         |
| Axions       | K.A. Olive, F. Takahashi, G. Raffelt* |

### Leptons

|           |                                                                         |
|-----------|-------------------------------------------------------------------------|
| Neutrinos | M. Goodman, C.-J. Lin,* K. Nakamura,<br>K.A. Olive, A. Piepke, P. Vogel |
| $e, \mu$  | C. Grab, C.-J. Lin*                                                     |
| $\tau$    | K.G. Hayes, K. Mönig*                                                   |

### Quarks

|            |                             |
|------------|-----------------------------|
| Quarks     | R.M. Barnett,* A.V. Manohar |
| Top quark  | R.M. Barnett,* Y. Sumino    |
| $b', t'$   | R.M. Barnett,* Y. Sumino    |
| Free quark | S. Pagan Griso*             |

Mesons $\pi, \eta$ 

Unstable mesons

D.A. Dwyer,\* C. Grab

C. Amsler, M. Doser,\* S. Eidelman,\*

T. Gutsche, C. Hanhart, B. Heltsley,

J.J. Hernández-Rey, A. Masoni,

R.E. Mitchell, S. Navas, C. Patrignani,

S. Spanier, N.A. Törnqvist,

G. Venanzoni

 $K$  (stable)

G. D'Ambrosio, C.-J. Lin\*

 $D$  (stable, no mix.)

J. Rademacker, C.G. Wohl\*

 $D^0$  mixing

D.M. Asner, W.-M. Yao\*

Baryons $B$  (stable)

A. Cerri,\* P. Eerola, M. Kreps,

Y. Kwon, W.-M. Yao\*

Stable baryons

C. Grab, C.G. Wohl\*

Unstable baryons

V. Burkert, E. Klempt, M. Pennington,

L. Tiator, R.L. Workman\*

Charmed baryons

J. Rademacker, C.G. Wohl\*

Bottom baryons

A. Cerri,\* P. Eerola, M. Kreps,

Y. Kwon, W.-M. Yao\*

Miscellaneous searches

Monopole

D. Milstead\*

Supersymmetry

H.K. Dreiner,\* A. de Gouvêa,

M. D'Onofrio, F. Moortgat,

K.A. Olive

Technicolor

K. Agashe,\* M. Tanabashi

Compositeness

M. Tanabashi, J. Terning\*

Extra Dimensions

D.A. Dwyer,\* T. Gherghetta

WIMPs and Other

K. Hikasa\*

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### 3. Consultants

The Particle Data Group benefits greatly from the assistance of some 700 physicists who are asked to verify every piece of data entered into this *Review*. Of special value is the advice of the PDG Advisory Committee which meets biennially and thoroughly reviews all aspects of our operation. The members of the 2016 committee are:

A. Seiden (UCSC)  
T. Carli (CERN)  
L. Hall (UC Berkeley/LBNL)  
J. Olson (Princeton)  
A. Slosar (BNL)  
J. Tanaka (Tokyo)

We have especially relied on the expertise of the following people for advice on particular topics:

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- A. Ali (DESY)
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**4. Naming scheme for hadrons**

We introduced in the 1986 edition [2] a new naming scheme for the hadrons. Changes from older terminology affected mainly the heavier mesons made of  $u$ ,  $d$ , and  $s$  quarks. Otherwise, the only important change to known hadrons was that the  $F^\pm$  became the  $D_s^\pm$ . None of the lightest pseudoscalar or vector mesons changed names, nor did the  $c\bar{c}$  or  $b\bar{b}$  mesons (we do, however, now use  $\chi_c$  for the  $c\bar{c}$   $\chi$  states), nor did any of the established baryons. The Summary Tables give both the new and old names whenever a change has occurred.

The scheme is described in “Naming Scheme for Hadrons” (p. 130) of this *Review*. We give here our conventions on type-setting style. Particle symbols are italic (or slanted) characters:  $e^-$ ,  $p$ ,  $\Lambda$ ,  $\pi^0$ ,  $K_L$ ,  $D_s^+$ ,  $b$ . Charge is indicated by a superscript:  $B^-$ ,  $\Delta^{++}$ . Charge is not normally indicated for  $p$ ,  $n$ , or the quarks, and is optional for neutral isosinglets:  $\eta$  or  $\eta^0$ . Antiparticles and particles are distinguished by charge for charged leptons and mesons:  $\tau^+$ ,  $K^-$ . Otherwise, distinct antiparticles are indicated by a bar (overline):  $\bar{\nu}_\mu$ ,  $\bar{t}$ ,  $\bar{p}$ ,  $\bar{K}^0$ , and  $\bar{\Sigma}^+$  (the antiparticle of the  $\Sigma^-$ ).

## 5. Procedures

**5.1. *Selection and treatment of data*** : The Particle Listings contain all relevant data known to us that are published in journals. With very few exceptions, we do not include results from preprints or conference reports. Nor do we include data that are of historical importance only (the Listings are not an archival record). We search every volume of 20 journals through our cutoff date for relevant data. We also include later published papers that are sent to us by the authors (or others).

In the Particle Listings, we clearly separate measurements that are used to calculate or estimate values given in the Summary Tables from measurements that are not used. We give explanatory comments in many such cases. Among the reasons a measurement might be excluded are the following:

- It is superseded by or included in later results.
- No error is given.
- It involves assumptions we question.
- It has a poor signal-to-noise ratio, low statistical significance, or is otherwise of poorer quality than other data available.
- It is clearly inconsistent with other results that appear to be more reliable. Usually we then state the criterion, which sometimes is quite subjective, for selecting “more reliable” data for averaging. See Sec. 5.4.
- It is not independent of other results.
- It is not the best limit (see below).
- It is quoted from a preprint or a conference report.

In some cases, *none* of the measurements is entirely reliable and no average is calculated. For example, the masses of many of the baryon resonances, obtained from partial-wave analyses, are quoted as estimated ranges thought to probably include the true values, rather than as averages with errors. This is discussed in the Baryon Particle Listings.

For upper limits, we normally quote in the Summary Tables the strongest limit. We do not average or combine upper limits except in a very few cases where they may be re-expressed as measured numbers with Gaussian errors.

As is customary, we assume that particle and antiparticle share the same spin, mass, and mean life. The Tests of Conservation Laws table, following the Summary Tables, lists tests of *CPT* as well as other conservation laws.

We use the following indicators in the Particle Listings to tell how we get values from the tabulated measurements:

- OUR AVERAGE—From a weighted average of selected data.
- OUR FIT—From a constrained or overdetermined multiparameter fit of selected data.
- OUR EVALUATION—Not from a direct measurement, but evaluated from measurements of related quantities.
- OUR ESTIMATE—Based on the observed range of the data. Not from a formal statistical procedure.
- OUR LIMIT—For special cases where the limit is evaluated by us from measured ratios or other data. Not from a direct measurement.

An experimentalist who sees indications of a particle will of course want to know what has been seen in that region in the past. Hence we include in the Particle Listings all reported states that, in our opinion, have sufficient statistical merit and that have not been disproved by more reliable data. However, we promote to the Summary Tables only those states that we feel are well established. This judgment is, of course, somewhat subjective and no precise criteria can be given. For more detailed discussions, see the minireviews in the Particle Listings.

**5.2. Averages and fits :** We divide discussion on obtaining averages and errors into three sections: (1) treatment of errors; (2) unconstrained averaging; (3) constrained fits.

**5.2.1. Treatment of errors:** In what follows, the “error”  $\delta x$  means that the range  $x \pm \delta x$  is intended to be a 68.3% confidence interval about the central value  $x$ . We treat this error as if it were Gaussian. Thus when the error *is* Gaussian,  $\delta x$  is the usual one standard deviation ( $1\sigma$ ). Many experimenters now give statistical and systematic errors separately, in which case we usually quote both errors, with the statistical error first. For averages and fits, we then add the the two errors in quadrature and use this combined error for  $\delta x$ .

When experimenters quote asymmetric errors  $(\delta x)^+$  and  $(\delta x)^-$  for a measurement  $x$ , the error that we use for that measurement in making an average or a fit with other measurements is a continuous function of these three quantities. When the resultant average or fit  $\bar{x}$  is less than  $x - (\delta x)^-$ , we use  $(\delta x)^-$ ; when it is greater than  $x + (\delta x)^+$ , we use  $(\delta x)^+$ . In between, the error we use is a linear function of  $x$ . Since the errors we use are functions of the result, we iterate to get the final result. Asymmetric output errors are determined from the input errors assuming a linear relation between the input and output quantities.

In fitting or averaging, we usually do not include correlations between different measurements, but we try to select data in such a way as to reduce correlations. Correlated errors are, however, treated explicitly when there are a number of results of the form  $A_i \pm \sigma_i \pm \Delta$  that have identical systematic errors  $\Delta$ . In this case, one can first average the  $A_i \pm \sigma_i$  and then combine the resulting statistical error with  $\Delta$ . One obtains, however, the same result by averaging  $A_i \pm (\sigma_i^2 + \Delta_i^2)^{1/2}$ , where  $\Delta_i = \sigma_i \Delta [\sum (1/\sigma_j^2)]^{1/2}$ . This procedure has the advantage that, with the modified systematic errors  $\Delta_i$ , each measurement may be treated as independent and averaged in the usual way with other data. Therefore, when appropriate, we adopt this procedure. We tabulate  $\Delta$  and invoke an automated procedure that computes  $\Delta_i$  before averaging and we include a note saying that there are common systematic errors.

Another common case of correlated errors occurs when experimenters measure two quantities and then quote the two and their difference, *e.g.*,  $m_1$ ,  $m_2$ , and  $\Delta = m_2 - m_1$ . We cannot enter all of  $m_1$ ,  $m_2$  and  $\Delta$  into a constrained fit because they are not independent. In some cases, it is a good approximation to ignore the quantity with the largest error and put the other two into the fit. However, in some cases correlations are such that the errors on  $m_1$ ,  $m_2$  and  $\Delta$  are comparable and none of the three values can be ignored. In this case, we put all three values into the fit and invoke an automated procedure to increase the errors prior to fitting such that the three quantities can be treated as independent measurements in the constrained fit. We include a note saying that this has been done.

**5.2.2. Unconstrained averaging:** To average data, we use a standard weighted least-squares procedure and in some cases, discussed below, increase the errors with a “scale factor.” We begin by assuming that measurements of a given quantity are uncorrelated, and calculate a weighted average and error as

$$\bar{x} \pm \delta\bar{x} = \frac{\sum_i w_i x_i}{\sum_i w_i} \pm (\sum_i w_i)^{-1/2} , \quad (1)$$

where

$$w_i = 1/(\delta x_i)^2 .$$

Here  $x_i$  and  $\delta x_i$  are the value and error reported by the  $i$ th experiment, and the sums run over the  $N$  experiments. We then calculate  $\chi^2 = \sum w_i (\bar{x} - x_i)^2$  and compare it with  $N - 1$ , which is the expectation value of  $\chi^2$  if the measurements are from a Gaussian distribution.

If  $\chi^2/(N - 1)$  is less than or equal to 1, and there are no known problems with the data, we accept the results.

If  $\chi^2/(N - 1)$  is very large, we may choose not to use the average at all. Alternatively, we may quote the calculated average, but then make an educated guess of the error, a conservative estimate designed to take into account known problems with the data.

Finally, if  $\chi^2/(N - 1)$  is greater than 1, but not greatly so, we still average the data, but then also do the following:

(a) We increase our quoted error,  $\delta\bar{x}$  in Eq. (1), by a scale factor  $S$  defined as

$$S = \left[ \chi^2/(N - 1) \right]^{1/2} . \quad (2)$$

Our reasoning is as follows. The large value of the  $\chi^2$  is likely to be due to underestimation of errors in at least one of the experiments. Not knowing which of the errors are underestimated, we assume they are all underestimated by the same factor  $S$ . If we scale up all the input errors by this factor, the  $\chi^2$  becomes  $N - 1$ , and of course the output error  $\delta\bar{x}$  scales up by the same factor. See Ref. 3.

When combining data with widely varying errors, we modify this procedure slightly. We evaluate  $S$  using only the experiments with smaller errors. Our cutoff or ceiling on  $\delta x_i$  is arbitrarily chosen to be

$$\delta_0 = 3N^{1/2} \delta\bar{x} ,$$

where  $\delta\bar{x}$  is the unscaled error of the mean of all the experiments. Our reasoning is that although the low-precision experiments have little influence on the values  $\bar{x}$  and  $\delta\bar{x}$ , they can make significant contributions to the  $\chi^2$ , and the contribution of the high-precision experiments thus tends to be obscured. Note that if each experiment has the same error  $\delta x_i$ , then  $\delta\bar{x}$  is  $\delta x_i/N^{1/2}$ , so each  $\delta x_i$  is well below the cutoff. (More often, however, we simply exclude measurements with relatively large errors from averages and fits: new, precise data chase out old, imprecise data.)

Our scaling procedure has the property that if there are two values with comparable errors separated by much more than their stated errors (with or without a number of

other values of lower accuracy), the scaled-up error  $\delta\bar{x}$  is approximately half the interval between the two discrepant values.

We emphasize that our scaling procedure for *errors* in no way affects central values. And if you wish to recover the unscaled error  $\delta\bar{x}$ , simply divide the quoted error by  $S$ .

(b) If the number  $M$  of experiments with an error smaller than  $\delta_0$  is at least three, and if  $\chi^2/(M-1)$  is greater than 1.25, we show in the Particle Listings an ideogram of the data. Figure 1 is an example. Sometimes one or two data points lie apart from the main body; other times the data split into two or more groups. We extract no numbers from these ideograms; they are simply visual aids, which the reader may use as he or she sees fit.

Each measurement in an ideogram is represented by a Gaussian with a central value  $x_i$ , error  $\delta x_i$ , and area proportional to  $1/\delta x_i$ . The choice of  $1/\delta x_i$  for the area is somewhat arbitrary. With this choice, the center of gravity of the ideogram corresponds to an average that uses weights  $1/\delta x_i$  rather than the  $(1/\delta x_i)^2$  actually used in the averages. This may be appropriate when some of the experiments have seriously underestimated systematic errors. However, since for this choice of area the height of the Gaussian for each measurement is proportional to  $(1/\delta x_i)^2$ , the peak position of the ideogram will often favor the high-precision measurements at least as much as does the least-squares average. See our 1986 edition [2] for a detailed discussion of the use of ideograms.

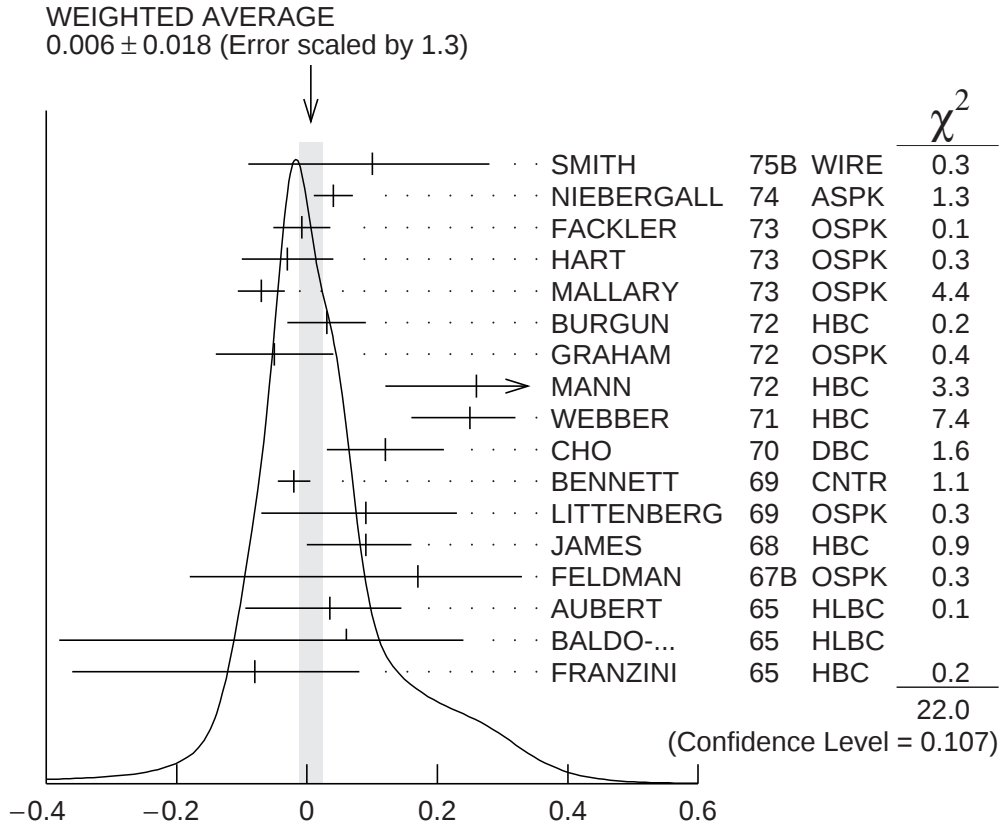
**5.2.3. Constrained fits:** In some cases, such as branching ratios or masses and mass differences, a constrained fit may be needed to obtain the best values of a set of parameters. For example, most branching ratios and rate measurements are analyzed by making a simultaneous least-squares fit to all the data and extracting the partial decay fractions  $P_i$ , the partial widths  $\Gamma_i$ , the full width  $\Gamma$  (or mean life), and the associated error matrix.

Assume, for example, that a state has  $m$  partial decay fractions  $P_i$ , where  $\sum P_i = 1$ . These have been measured in  $N_r$  different ratios  $R_r$ , where, e.g.,  $R_1 = P_1/P_2$ ,  $R_2 = P_1/P_3$ , *etc.* [We can handle any ratio  $R$  of the form  $\sum \alpha_i P_i / \sum \beta_i P_i$ , where  $\alpha_i$  and  $\beta_i$  are constants, usually 1 or 0. The forms  $R = P_i P_j$  and  $R = (P_i P_j)^{1/2}$  are also allowed.] Further assume that *each* ratio  $R$  has been measured by  $N_k$  experiments (we designate each experiment with a subscript  $k$ , e.g.,  $R_{1k}$ ). We then find the best values of the fractions  $P_i$  by minimizing the  $\chi^2$  as a function of the  $m-1$  independent parameters:

$$\chi^2 = \sum_{r=1}^{N_r} \sum_{k=1}^{N_k} \left( \frac{R_{rk} - R_r}{\delta R_{rk}} \right)^2, \quad (3)$$

where the  $R_{rk}$  are the measured values and  $R_r$  are the fitted values of the branching ratios.

In addition to the fitted values  $\bar{P}_i$ , we calculate an error matrix  $\langle \delta \bar{P}_i \delta \bar{P}_j \rangle$ . We tabulate the diagonal elements of  $\delta \bar{P}_i = \langle \delta \bar{P}_i \delta \bar{P}_i \rangle^{1/2}$  (except that some errors are scaled as discussed below). In the Particle Listings, we give the complete correlation matrix; we also calculate the fitted value of each ratio, for comparison with the input data, and list it above the relevant input, along with a simple unconstrained average of the same input.



**Figure 1:** A typical ideogram. The arrow at the top shows the position of the weighted average, while the width of the shaded pattern shows the error in the average after scaling by the factor  $S$ . The column on the right gives the  $\chi^2$  contribution of each of the experiments. Note that the next-to-last experiment, denoted by the incomplete error flag ( $\perp$ ), is not used in the calculation of  $S$  (see the text).

Three comments on the example above:

(1) There was no connection assumed between measurements of the full width and the branching ratios. But often we also have information on partial widths  $\Gamma_i$  as well as the total width  $\Gamma$ . In this case we must introduce  $\Gamma$  as a parameter in the fit, along with the  $P_i$ , and we give correlation matrices for the widths in the Particle Listings.

(2) We try to pick those ratios and widths that are as independent and as close to the original data as possible. When one experiment measures all the branching fractions and constrains their sum to be one, we leave one of them (usually the least well-determined one) out of the fit to make the set of input data more nearly independent. We now do allow for correlations between input data.

(3) We calculate scale factors for both the  $R_r$  and  $P_i$  when the measurements for any  $R$  give a larger-than-expected contribution to the  $\chi^2$ . According to Eq. (3), the double sum for  $\chi^2$  is first summed over experiments  $k = 1$  to  $N_k$ , leaving a single sum over ratios  $\chi^2 = \sum \chi_r^2$ . One is tempted to define a scale factor for the ratio  $r$  as  $S_r^2 = \chi_r^2 / \langle \chi_r^2 \rangle$ . However, since  $\langle \chi_r^2 \rangle$  is not a fixed quantity (it is somewhere between  $N_k$  and  $N_{k-1}$ ), we

do not know how to evaluate this expression. Instead we define

$$S_r^2 = \frac{1}{N_k} \sum_{k=1}^{N_k} \frac{(R_{rk} - \overline{R}_r)^2}{\langle (R_{rk} - \overline{R}_r)^2 \rangle}. \quad (4)$$

With this definition the expected value of  $S_r^2$  is one. We can show that

$$\langle (R_{rk} - \overline{R}_r)^2 \rangle = \langle (\delta R_{rk})^2 \rangle - (\delta \overline{R}_r)^2, \quad (5)$$

where  $\delta \overline{R}_r$  is the fitted error for ratio  $r$ .

The fit is redone using errors for the branching ratios that are scaled by the larger of  $S_r$  and unity, from which new and often larger errors  $\delta \overline{P}'_i$  are obtained. The scale factors we finally list in such cases are defined by  $S_i = \delta \overline{P}'_i / \delta \overline{P}_i$ . However, in line with our policy of not letting  $S$  affect the central values, we give the values of  $\overline{P}_i$  obtained from the original (unscaled) fit.

There is one special case in which the errors that are obtained by the preceding procedure may be changed. When a fitted branching ratio (or rate)  $\overline{P}_i$  turns out to be less than three standard deviations ( $\delta \overline{P}'_i$ ) from zero, a new smaller error  $(\delta \overline{P}''_i)^-$  is calculated on the low side by requiring the area under the Gaussian between  $\overline{P}_i - (\delta \overline{P}''_i)^-$  and  $\overline{P}_i$  to be 68.3% of the area between zero and  $\overline{P}_i$ . A similar correction is made for branching fractions that are within three standard deviations of one. This keeps the quoted errors from overlapping the boundary of the physical region.

**5.3. Rounding :** While the results shown in the Particle Listings are usually exactly those published by the experiments, the numbers that appear in the Summary Tables (means, averages and limits) are subject to a set of rounding rules.

The basic rule states that if the three highest order digits of the error lie between 100 and 354, we round to two significant digits. If they lie between 355 and 949, we round to one significant digit. Finally, if they lie between 950 and 999, we round up to 1000 and keep two significant digits. In all cases, the central value is given with a precision that matches that of the error. So, for example, the result (coming from an average)  $0.827 \pm 0.119$  would appear as  $0.83 \pm 0.12$ , while  $0.827 \pm 0.367$  would turn into  $0.8 \pm 0.4$ .

Rounding is not performed if a result in a Summary Table comes from a single measurement, without any averaging. In that case, the number of digits published in the original paper is kept, unless we feel it inappropriate. Note that, even for a single measurement, when we combine statistical and systematic errors in quadrature, rounding rules apply to the result of the combination. It should be noted also that most of the limits in the Summary Tables come from a single source (the best limit) and, therefore, are not subject to rounding.

Finally, we should point out that in several instances, when a group of results come from a single fit to a set of data, we have chosen to keep two significant digits for all the results. This happens, for instance, for several properties of the  $W$  and  $Z$  bosons and the  $\tau$  lepton.

**5.4. Discussion :** The problem of averaging data containing discrepant values is nicely discussed by Taylor in Ref. 4. He considers a number of algorithms that attempt to incorporate inconsistent data into a meaningful average. However, it is difficult to develop a procedure that handles simultaneously in a reasonable way two basic types of situations: (a) data that lie apart from the main body of the data are incorrect (contain unreported errors); and (b) the opposite—it is the main body of data that is incorrect. Unfortunately, as Taylor shows, case (b) is not infrequent. He concludes that the choice of procedure is less significant than the initial choice of data to include or exclude.

We place much emphasis on this choice of data. Often we solicit the help of outside experts (consultants). Sometimes, however, it is simply impossible to determine which of a set of discrepant measurements are correct. Our scale-factor technique is an attempt to address this ignorance by increasing the error. In effect, we are saying that present experiments do not allow a precise determination of this quantity because of unresolvable discrepancies, and one must await further measurements. The reader is warned of this situation by the size of the scale factor, and if he or she desires can go back to the literature (via the Particle Listings) and redo the average with a different choice of data.

Our situation is less severe than most of the cases Taylor considers, such as estimates of the fundamental constants like  $\hbar$ , *etc.* Most of the errors in his case are dominated by systematic effects. For our data, statistical errors are often at least as large as systematic errors, and statistical errors are usually easier to estimate. A notable exception occurs in partial-wave analyses, where different techniques applied to the same data yield different results. In this case, as stated earlier, we often do not make an average but just quote a range of values.

A brief history of early Particle Data Group averages is given in Ref. 3. Figure 2 shows some histories of our values of a few particle properties. Sometimes large changes occur. These usually reflect the introduction of significant new data or the discarding of older data. Older data are discarded in favor of newer data when it is felt that the newer data have smaller systematic errors, or have more checks on systematic errors, or have made corrections unknown at the time of the older experiments, or simply have much smaller errors. Sometimes, the scale factor becomes large near the time at which a large jump takes place, reflecting the uncertainty introduced by the new and inconsistent data. By and large, however, a full scan of our history plots shows a dull progression toward greater precision at central values quite consistent with the first data points shown.

We conclude that the reliability of the combination of experimental data and our averaging procedures is usually good, but it is important to be aware that fluctuations outside of the quoted errors can and do occur.

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