# 78. *CP* Violation in $K_S \to 3\pi$

Written 1996 by T. Nakada (Paul Scherrer Institute) and L. Wolfenstein (Carnegie-Mellon University).

The possible final states for the decay $K^0 \to \pi^+\pi^-\pi^0$ have isospin $I = 0$, 1, 2, and 3. The $I = 0$ and $I = 2$ states have $CP = +1$ and $K_S$ can decay into them without violating $CP$ symmetry, but they are expected to be strongly suppressed by centrifugal barrier effects. The $I = 1$ and $I = 3$ states, which have no centrifugal barrier, have $CP = -1$ so that the $K_S$ decay to these requires $CP$ violation.

In order to see $CP$ violation in $K_S \to \pi^+\pi^-\pi^0$, it is necessary to observe the interference between $K_S$ and $K_L$ decay, which determines the amplitude ratio

$$\eta_{+-0} = \frac{A(K_S \to \pi^+\pi^-\pi^0)}{A(K_L \to \pi^+\pi^-\pi^0)} \ . \tag{78.1}$$

If $\eta_{+-0}$ is obtained from an integration over the whole Dalitz plot, there is no contribution from the $I = 0$ and $I = 2$ final states and a nonzero value of $\eta_{+-0}$ is entirely due to $CP$ violation.

Only $I = 1$ and $I = 3$ states, which are $CP = -1$, are allowed for $K^0 \to \pi^0\pi^0\pi^0$ decays and the decay of $K_S$ into $3\pi^0$ is an unambiguous sign of $CP$ violation. Similarly to $\eta_{+-0}$, $\eta_{000}$ is defined as

$$\eta_{000} = \frac{A(K_S \to \pi^0\pi^0\pi^0)}{A(K_L \to \pi^0\pi^0\pi^0)} \ . \tag{78.2}$$

If one assumes that $CPT$ invariance holds and that there are no transitions to $I = 3$ (or to nonsymmetric $I = 1$ states), it can be shown that

$$\begin{aligned} \eta_{+-0} &= \eta_{000} \\ &= \epsilon + i\frac{\text{Im } a_1}{\text{Re } a_1} \ . \end{aligned} \tag{78.3}$$

With the Wu-Yang phase convention, $a_1$ is the weak decay amplitude for $K^0$ into $I =$ 1 final states; $\epsilon$ is determined from $CP$ violation in $K_L \to 2\pi$ decays. The real parts of $\eta_{+-0}$ and $\eta_{000}$ are equal to $\text{Re}(\epsilon)$. Since currently-known upper limits on $|\eta_{+-0}|$ and $|\eta_{000}|$ are much larger than $|\epsilon|$, they can be interpreted as upper limits on $\text{Im}(\eta_{+-0})$ and $\text{Im}(\eta_{000})$ and so as limits on the $CP$-violating phase of the decay amplitude $a_1$.