13. *CP* Violation in the Quark Sector

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The CP transformation combines charge conjugation C with parity P. Under C, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, *e.g.*, $Q \to -Q$ for electromagnetic charge. Under P, the handedness of space is reversed, $\vec{x} \to -\vec{x}$. Thus, for example, a left-handed electron e^-_L is transformed under CP into a right-handed positron, e^+_R.

If CP were an exact symmetry, the laws of Nature would be the same for matter and for antimatter. We observe that most phenomena are C- and P-symmetric, and therefore, also CP-symmetric. In particular, these symmetries are respected by the gravitational, electromagnetic, and strong interactions. The weak interactions, on the other hand, violate C and P in the strongest possible way. For example, the charged W bosons couple to left-handed electrons, e^-_L, and to their CP-conjugate right-handed positrons, e^+_R, but to neither their C-conjugate left-handed positrons, e^+_L, nor their P-conjugate right-handed electrons, e^-_R. While weak interactions violate C and P separately, CP is still preserved in most weak interaction processes. The CP symmetry is, however, violated in certain rare processes, as discovered in neutral K decays in 1964 [1], and observed in recent years in B decays. A K_L meson decays more often to $\pi^- e^+ \nu_e$ than to $\pi^+ e^- \overline{\nu}_e$, thus allowing electrons and positrons to be unambiguously distinguished, but the decay-rate asymmetry is only at the 0.003 level. The CP-violating effects observed in the B system are larger: the parameter describing the CP asymmetry in the decay time distribution of $B^0/\overline{B}^0$ meson transitions to CP eigenstates like $J/\psi K_S$ is about 0.7 [2,3]. These effects are related to K^0–$\overline{K}^0$ and B^0–$\overline{B}^0$ mixing, but CP violation arising solely from decay amplitudes has also been observed, first in $K \to \pi\pi$ decays [4–6], and more recently in B^0 [7,8], B^+ [9–11], and B^0_s [12] decays. Similar effects could also occur in decays of b baryons, and evidence for CP violation in Λ^0_b decays has been found at the 3.3σ level [13]. CP violation is not yet experimentally established in the D system, where the Standard Model effects are expected to be $\mathcal{O}(10^{-3})$. Moreover, CP violation has not yet been observed in processes involving the top quark, nor in flavor-conserving processes such as electric dipole moments, nor in the lepton sector; for all of these any significant observation would be a clear indication of physics beyond the Standard Model.

In addition to parity and to continuous Lorentz transformations, there is one other spacetime operation that could be a symmetry of the interactions: time reversal T, $t \to -t$. Violations of T symmetry have been observed in neutral K decays [14]. More recently, exploiting the fact that for neutral B mesons both flavor tagging and CP tagging can be used [15], T violation has been observed between states that are not CP-conjugate [16]. Moreover, T violation is expected as a corollary of CP violation if the combined CPT transformation is a fundamental symmetry of Nature [17]. All observations indicate that CPT is indeed a symmetry of Nature. Furthermore, one cannot build a locally Lorentz-invariant quantum field theory with a Hermitian Hamiltonian that violates CPT. (At several points in our discussion, we avoid assumptions about CPT, in order to identify cases where evidence for CP violation relies on assumptions about CPT.)

Within the Standard Model, CP symmetry is broken by complex phases in the Yukawa couplings (that is, the couplings of the Higgs scalar to quarks). When all manipulations to remove unphysical phases in this model are exhausted, one finds that there is a single CP-violating parameter [18]. In the basis of mass eigenstates, this single phase appears in the 3×3 unitary matrix that gives the W-boson couplings to an up-type antiquark and a down-type quark. (If the Standard Model is supplemented with Majorana mass terms for the neutrinos, the analogous mixing matrix for leptons has three CP-violating phases.) The beautifully consistent and economical Standard-Model description of CP violation in terms of Yukawa couplings, known as the Kobayashi-Maskawa (KM) mechanism [18], agrees with all measurements to date. (Some measurements are in tension with the predictions, and are discussed in more detail below. Pending verification, the results are not considered to change the overall picture of agreement with the Standard Model.) Furthermore, one can fit the data allowing new physics contributions to loop processes to compete with, or even dominate over, the Standard Model amplitudes [19,20]. Such an analysis provides model-independent proof that the KM phase is different from zero, and that the matrix of three-generation quark mixing is the dominant source of CP violation in meson decays.

The current level of experimental accuracy and the theoretical uncertainties involved in the interpretation of the various observations leave room, however, for additional subdominant sources of CP violation from new physics. Indeed, almost all extensions of the Standard Model imply that there are such additional sources. Moreover, CP violation is a necessary condition for baryogenesis, the process of dynamically generating the matter-antimatter asymmetry of the Universe [21]. Despite the phenomenological success of the KM mechanism, it fails (by several orders of magnitude) to accommodate the observed asymmetry [22]. This discrepancy strongly suggests that Nature provides additional sources of CP violation beyond the KM mechanism. The evidence for neutrino masses implies that CP can be violated also in the lepton sector. This situation makes leptogenesis [23,24], a scenario where CP-violating phases in the Yukawa couplings of the neutrinos play a crucial role in the generation of the baryon asymmetry, a very attractive possibility. The expectation of new sources motivates the large ongoing experimental effort to find deviations from the predictions of the KM mechanism.

CP violation can be experimentally searched for in a variety of processes, such as hadron decays, electric dipole moments of neutrons, electrons and nuclei, and neutrino oscillations. Hadron decays via the weak interaction probe flavor-changing CP violation. The search for electric dipole moments may find (or constrain) sources of CP violation that, unlike the KM phase, are not related to flavor-changing couplings. Following the discovery of the Higgs boson [25,26], searches for CP violation in the Higgs sector are becoming feasible. Future searches for CP violation in neutrino oscillations might provide further input on leptogenesis.

The present measurements of CP asymmetries provide some of the strongest constraints on the weak couplings of quarks. Future measurements of CP violation in K, D, B, and B_s^0 meson decays will provide additional constraints on the flavor parameters of the Standard Model, and can probe new physics. In this review, we give the formalism and basic physics that are relevant to present and near future measurements of CP violation

in the quark sector.

Before going into details, we list here the observables where CP violation has been observed at a level above 5σ [27–29]:

- Indirect CP violation in $K \to \pi\pi$ and $K \to \pi\ell\nu$ decays, and in the $K_L \to \pi^+\pi^- e^+ e^-$ decay, is given by
$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3} \,. \tag{13.1}$$

- Direct CP violation in $K \to \pi\pi$ decays is given by
$$\mathcal{R}e(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3} \,. \tag{13.2}$$

- CP violation in the interference of mixing and decay in the tree-dominated $b \to c\bar{c}s$ transitions, such as $B^0 \to \psi K^0$, is given by (we use K^0 throughout to denote results that combine K_S and K_L modes, but use the sign appropriate to K_S):
$$S_{\psi K^0} = +0.691 \pm 0.017 \,. \tag{13.3}$$

- CP violation in the interference of mixing and decay in modes governed by the tree-dominated $b \to c\bar{u}d$ transitions is given by
$$S_{D^{(*)}h^0} = +0.71 \pm 0.09 \,, \tag{13.4}$$

- CP violation in the interference of mixing and decay in various modes related to $b \to c\bar{c}d$ transitions is given by
$$S_{\psi\pi^0} = -0.93 \pm 0.15 \,, \tag{13.5}$$
$$S_{D^+D^-} = -0.84 \pm 0.12 \,. \tag{13.6}$$
$$S_{D^{*+}D^{*-}} = -0.71 \pm 0.09 \,. \tag{13.7}$$

- CP violation in the interference of mixing and decay in various modes related to $b \to q\bar{q}s$ (penguin) transitions is given by
$$S_{\phi K^0} = +0.74\,^{+0.11}_{-0.13} \,, \tag{13.8}$$
$$S_{\eta' K^0} = +0.63 \pm 0.06 \,, \tag{13.9}$$
$$S_{f_0 K^0} = +0.69\,^{+0.10}_{-0.12} \,, \tag{13.10}$$
$$S_{K^+K^-K_S} = +0.68\,^{+0.09}_{-0.10} \,, \tag{13.11}$$

- CP violation in the interference of mixing and decay in the $B^0 \to \pi^+\pi^-$ mode is given by
$$S_{\pi^+\pi^-} = -0.68 \pm 0.04 \,. \tag{13.12}$$

- Direct CP violation in the $B^0 \to \pi^+\pi^-$ mode is given by

$$C_{\pi^+\pi^-} = -0.27 \pm 0.04\,. \tag{13.13}$$

- Direct CP violation in the $\overline{B}^0 \to K^-\pi^+$ mode is given by

$$\mathcal{A}_{\overline{B}^0 \to K^-\pi^+} = -0.082 \pm 0.006\,. \tag{13.14}$$

- Direct CP violation in $B^+ \to D_+ K^+$ decays (D_+ is the CP-even neutral D state) is given by

$$\mathcal{A}_{B^+ \to D_+ K^+} = +0.129 \pm 0.012\,, \tag{13.15}$$

  while the corresponding quantity in the case that the neutral D meson is reconstructed in the suppressed $K^-\pi^+$ final state is

$$\mathcal{A}_{B^+ \to D_{K^-\pi^+} K^+} = -0.41 \pm 0.06\,, \tag{13.16}$$

- Direct CP violation in the $\overline{B}^0_s \to K^+\pi^-$ mode is given by

$$\mathcal{A}_{\overline{B}^0_s \to K^+\pi^-} = +0.26 \pm 0.04\,. \tag{13.17}$$

- Direct CP violation in $B^+ \to K^+K^-\pi^+$ decays is given by

$$\mathcal{A}_{B^+ \to K^+K^-\pi^+} = -0.118 \pm 0.022\,. \tag{13.18}$$

In addition, large CP violation effects have been observed in certain regions of the phase space of $B^+ \to K^+K^-K^+$, $\pi^+\pi^-K^+$, $\pi^+\pi^-\pi^+$ and $K^+K^-\pi^+$ decays.

13.1. Formalism

The phenomenology of CP violation for neutral flavored mesons is particularly interesting, since many of the observables can be cleanly interpreted. Although the phenomenology is superficially different for K^0, D^0, B^0, and B^0_s decays, this is primarily because each of these systems is governed by a different balance between decay rates, oscillations, and lifetime splitting. However, the general considerations presented in this section are identical for all flavored neutral pseudoscalar mesons. The phenomenology of CP violation for neutral mesons that do not carry flavor quantum numbers (such as the $\eta^{(\prime)}$ state) is quite different: such states are their own antiparticles and have definite CP eigenvalues, so the signature of CP violation is simply the decay to a final state with the opposite CP. Such decays are mediated by the electromagnetic or (OZI-suppressed) strong interaction, where CP violation is not expected and has not yet been observed. In the remainder of this review, we restrict ourselves to considerations of weakly decaying hadrons.

In this section, we present a general formalism for, and classification of, CP violation in the decay of a weakly decaying hadron, denoted M. We pay particular attention to the case that M is a K^0, D^0, B^0, or B^0_s meson. Subsequent sections describe the CP-violating phenomenology, approximations, and alternative formalisms that are specific to each system.

13.1.1. ***Charged- and neutral-hadron decays*** :

We define decay amplitudes of M (which could be charged or neutral) and its CP conjugate $\overline{M}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$A_f = \langle f\,|\mathcal{H}|\,M\rangle\ , \quad \overline{A}_f = \langle f\,|\mathcal{H}|\,\overline{M}\rangle\ ,$$
$$A_{\overline{f}} = \langle \overline{f}\,|\mathcal{H}|\,M\rangle\ , \quad \overline{A}_{\overline{f}} = \langle \overline{f}\,|\mathcal{H}|\,\overline{M}\rangle\ , \tag{13.19}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_M and ξ_f that depend on their flavor content, according to

$$CP|M\rangle = e^{+i\xi_M}\,|\overline{M}\rangle\ , \quad CP|f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle\ , \tag{13.20}$$

with

$$CP|\overline{M}\rangle = e^{-i\xi_M}\,|M\rangle\ , \quad CP|\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \tag{13.21}$$

so that $(CP)^2 = 1$. The phases ξ_M and ξ_f are arbitrary and unobservable because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP, \mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_M)}\,A_f\ . \tag{13.22}$$

13.1.2. ***Neutral-meson mixing*** :

A state that is initially a superposition of M^0 and $\overline{M}^0$, say

$$|\psi(0)\rangle = a(0)|M^0\rangle + b(0)|\overline{M}^0\rangle\ , \tag{13.23}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|M^0\rangle + b(t)|\overline{M}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots\ . \tag{13.24}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [30]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathbf{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathbf{H}$, can be written in terms of Hermitian matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ as

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\,\mathbf{\Gamma}\ . \tag{13.25}$$

$\mathbf{M}$ and $\mathbf{\Gamma}$ are associated with $(M^0, \overline{M}^0) \leftrightarrow (M^0, \overline{M}^0)$ transitions via off-shell (dispersive), and on-shell (absorptive) intermediate states, respectively. Diagonal elements of $\mathbf{M}$ and $\mathbf{\Gamma}$ are associated with the flavor-conserving transitions $M^0 \to M^0$ and $\overline{M}^0 \to \overline{M}^0$, while off-diagonal elements are associated with flavor-changing transitions $M^0 \leftrightarrow \overline{M}^0$.

The eigenvectors of $\mathbf{H}$ have well-defined masses and decay widths. To specify the components of the strong interaction eigenstates, M^0 and $\overline{M}^0$, in the light (M_L) and heavy (M_H) mass eigenstates, we introduce three complex parameters: p, q, and, for the case that both CP and CPT are violated in mixing, z:

$$\begin{aligned} |M_L\rangle &\propto p\sqrt{1-z}\,|M^0\rangle + q\sqrt{1+z}\,|\overline{M}^0\rangle \\ |M_H\rangle &\propto p\sqrt{1+z}\,|M^0\rangle - q\sqrt{1-z}\,|\overline{M}^0\rangle \ , \end{aligned} \tag{13.26}$$

with the normalization $|q|^2 + |p|^2 = 1$ when $z = 0$. (Another possible choice, which is in standard usage for K mesons, defines the mass eigenstates according to their lifetimes: K_S for the short-lived and K_L for the long-lived state. The K_L is experimentally found to be the heavier state. Yet another choice is often used for the D mesons [31]: the eigenstates are labelled according to their dominant CP content.)

The real and imaginary parts of the eigenvalues $\omega_{L,H}$ corresponding to $|M_{L,H}\rangle$ represent their masses and decay widths, respectively. The mass and width splittings are

$$\begin{aligned} \Delta m &\equiv m_H - m_L = \mathcal{R}e(\omega_H - \omega_L) \ , \\ \Delta\Gamma &\equiv \Gamma_H - \Gamma_L = -2\,\mathcal{I}m(\omega_H - \omega_L) \ . \end{aligned} \tag{13.27}$$

Note that here Δm is positive by definition, while the sign of $\Delta\Gamma$ must be experimentally determined. The sign of $\Delta\Gamma$ has not yet been established for B^0 mesons, while $\Delta\Gamma < 0$ is established for K and B^0_s mesons. The Standard Model predicts $\Delta\Gamma < 0$ for $B^0_{(s)}$ mesons; for this reason, $\Delta\Gamma = \Gamma_L - \Gamma_H$, which is still a signed quantity, is often used in the $B^0_{(s)}$ literature and is the convention used in the PDG experimental summaries.

Solving the eigenvalue problem for $\mathbf{H}$ yields

$$\left(\frac{q}{p}\right)^2 = \frac{\mathbf{M}^*_{12} - (i/2)\mathbf{\Gamma}^*_{12}}{\mathbf{M}_{12} - (i/2)\mathbf{\Gamma}_{12}} \tag{13.28}$$

and

$$z \equiv \frac{\delta m - (i/2)\delta\Gamma}{\Delta m - (i/2)\Delta\Gamma} \ , \tag{13.29}$$

where

$$\delta m \equiv \mathbf{M}_{11} - \mathbf{M}_{22} \ , \quad \delta\Gamma \equiv \mathbf{\Gamma}_{11} - \mathbf{\Gamma}_{22} \tag{13.30}$$

are the differences in effective mass and decay-rate expectation values for the strong interaction states M^0 and $\overline{M}^0$.

If either CP or CPT is a symmetry of $\mathbf{H}$ (independently of whether T is conserved or violated), then the values of δm and $\delta\Gamma$ are both zero, and hence $z = 0$. We also find that

$$\omega_H - \omega_L = 2\sqrt{\left(\mathbf{M}_{12} - \frac{i}{2}\mathbf{\Gamma}_{12}\right)\left(\mathbf{M}^*_{12} - \frac{i}{2}\mathbf{\Gamma}^*_{12}\right)} \ . \tag{13.31}$$

If either CP or T is a symmetry of $\mathbf{H}$ (independently of whether CPT is conserved or violated), then $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_M} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 \,, \tag{13.32}$$

where ξ_M is the arbitrary unphysical phase introduced in Eq. (13.21). If, and only if, CP is a symmetry of $\mathbf{H}$ (independently of CPT and T), then both of the above conditions hold, with the result that the mass eigenstates are orthogonal

$$\langle M_H | M_L \rangle = |p|^2 - |q|^2 = 0 \,. \tag{13.33}$$

13.1.3. *CP-violating observables* :

All CP-violating observables in M and $\overline{M}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$, and $\overline{A}_{\overline{f}}$, together with, for neutral meson decays only, q/p. CP violation in charged meson and all baryon decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in flavored neutral meson decays is complicated by $M^0 \leftrightarrow \overline{M}^0$ oscillations, and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

The decay rates of the two neutral kaon mass eigenstates, K_S and K_L, are different enough ($\Gamma_S/\Gamma_L \sim 500$) that one can, in most cases, actually study their decays independently. For D^0, B^0, and B_s^0 mesons, however, values of $\Delta\Gamma/\Gamma$ (where $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$) are relatively small, and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|M^0\rangle$ or $|\overline{M}^0\rangle$ after an elapsed proper time t as $|M^0_{\rm phys}(t)\rangle$ or $|\overline{M}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, but not assuming CPT to be a good symmetry, we obtain

$$\begin{aligned}
|M^0_{\rm phys}(t)\rangle &= \left(g_+(t) + z\, g_-(t)\right)|M^0\rangle - \sqrt{1-z^2}\,\frac{q}{p}\; g_-(t)|\overline{M}^0\rangle \,, \\
|\overline{M}^0_{\rm phys}(t)\rangle &= \left(g_+(t) - z\, g_-(t)\right)|\overline{M}^0\rangle - \sqrt{1-z^2}\,\frac{p}{q}\; g_-(t)|M^0\rangle \,,
\end{aligned} \tag{13.34}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) \tag{13.35}$$

and $z = 0$ if either CPT or CP is conserved.

Defining $x \equiv \Delta m/\Gamma$ and $y \equiv \Delta\Gamma/(2\Gamma)$, and assuming $z = 0$, one obtains the following time-dependent decay rates:

$$\frac{d\Gamma\left[M^0_{\rm phys}(t) \to f\right]/dt}{e^{-\Gamma t}\mathcal{N}_f} =$$

$$\left(|A_f|^2 + |(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t) + 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\ , \tag{13.36}$$

$$\frac{d\Gamma\left[\overline{M}^0_{\rm phys}(t)\to f\right]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \left(|(p/q)A_f|^2 + |\overline{A}_f|^2\right)\cosh(y\Gamma t) - \left(|(p/q)A_f|^2 - |\overline{A}_f|^2\right)\cos(x\Gamma t) + 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\ , \tag{13.37}$$

where $\mathcal{N}_f$ is a common, time-independent, normalization factor that can be determined bearing in mind that the range of t is $0 < t < \infty$. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (13.36, 13.37). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $M^0 \leftrightarrow \overline{M}^0$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (13.36, 13.37) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of variables that describe the phase-space of the final state. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

When neutral pseudoscalar mesons are produced coherently in pairs from the decay of a vector resonance, $V \to M^0\overline{M}^0$ (for example, $\Upsilon(4S) \to B^0\overline{B}^0$ or $\phi \to K^0\overline{K}^0$), the time-dependence of their subsequent decays to final states f_1 and f_2 has a similar form to Eqs. (13.36, 13.37):

$$\frac{d\Gamma\left[V_{\rm phys}(t_1,t_2)\to f_1 f_2\right]/d(\Delta t)}{e^{-\Gamma|\Delta t|}\mathcal{N}_{f_1 f_2}} = \left(|a_+|^2 + |a_-|^2\right)\cosh(y\Gamma\Delta t) + \left(|a_+|^2 - |a_-|^2\right)\cos(x\Gamma\Delta t) - 2\,\mathcal{R}e(a_+^* a_-)\sinh(y\Gamma\Delta t) + 2\,\mathcal{I}m(a_+^* a_-)\sin(x\Gamma\Delta t)\ , \tag{13.38}$$

where $\Delta t \equiv t_2 - t_1$ is the difference in the production times, t_1 and t_2, of f_1 and f_2, respectively, and the dependence on the average decay time and on decay angles has been integrated out. The normalisation factor $\mathcal{N}_{f_1 f_2}$ can be evaluated, noting that the range of Δt is $-\infty < \Delta t < \infty$. The coefficients in Eq. (13.38) are determined by the amplitudes for no net oscillation from $t_1 \to t_2$, $\overline{A}_{f_1}A_{f_2}$, and $A_{f_1}\overline{A}_{f_2}$, and for a net oscillation, $(q/p)\overline{A}_{f_1}\overline{A}_{f_2}$ and $(p/q)A_{f_1}A_{f_2}$, via

$$a_+ \equiv \overline{A}_{f_1}A_{f_2} - A_{f_1}\overline{A}_{f_2}\ , \tag{13.39}$$

$$a_- \equiv -\sqrt{1-z^2}\left(\frac{q}{p}\overline{A}_{f_1}\overline{A}_{f_2} - \frac{p}{q}A_{f_1}A_{f_2}\right) + z\left(\overline{A}_{f_1}A_{f_2} + A_{f_1}\overline{A}_{f_2}\right)\ .$$

Assuming CPT conservation, $z = 0$, and identifying $\Delta t \to t$ and $f_2 \to f$, we find that Eqs. (13.38, 13.39) reduce essentially to Eq. (13.36) with $A_{f_1} = 0$, $\overline{A}_{f_1} = 1$, or to Eq. (13.37) with $\overline{A}_{f_1} = 0$, $A_{f_1} = 1$. Indeed, such a situation plays an important role in experiments that exploit the coherence of $V \to M^0\overline{M}^0$ (for example $\psi(3770) \to D^0\overline{D}^0$ or $\Upsilon(4S) \to B^0\overline{B}^0$) production. Final states f_1 with $A_{f_1} = 0$ or $\overline{A}_{f_1} = 0$ are called tagging states, because they identify the decaying pseudoscalar meson as, respectively, $\overline{M}^0$ or M^0. Before one of M^0 or $\overline{M}^0$ decays, they evolve in phase, so that there is always one M^0 and one $\overline{M}^0$ present. A tagging decay of one meson sets the clock for the time evolution of the other: it starts at t_1 as purely M^0 or $\overline{M}^0$, with time evolution that depends only on $t_2 - t_1$.

When f_1 is a state that both M^0 and $\overline{M}^0$ can decay into, then Eq. (13.38) contains interference terms proportional to $A_{f_1}\overline{A}_{f_1} \neq 0$ that are not present in Eqs. (13.36, 13.37). Even when f_1 is dominantly produced by M^0 decays rather than $\overline{M}^0$ decays, or vice versa, $A_{f_1}\overline{A}_{f_1}$ can be non-zero owing to doubly-CKM-suppressed decays (with amplitudes suppressed by at least two powers of λ relative to the dominant amplitude, in the language of Section 13.3), and these terms should be considered for precision studies of CP violation in coherent $V \to M^0\overline{M}^0$ decays [32]. The correlations in $V \to M^0\overline{M}^0$ decays can also be exploited to determine strong phase differences between favored and suppressed decay amplitudes [33].

13.1.4. *Classification of CP-violating effects* :

We distinguish three types of CP-violating effects that can occur in the quark sector:

I. CP violation in decay is defined by

$$|\overline{A}_{\overline{f}}/A_f| \neq 1 \,. \tag{13.40}$$

In charged meson (and all baryon) decays, where mixing effects are absent, this is the only possible source of CP asymmetries:

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma(M^- \to f^-) - \Gamma(M^+ \to f^+)}{\Gamma(M^- \to f^-) + \Gamma(M^+ \to f^+)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2 - 1}{|\overline{A}_{f^-}/A_{f^+}|^2 + 1} \,. \tag{13.41}$$

Note that the usual sign convention for CP asymmetries of hadrons is for the difference between the rate involving the particle that contains a heavy quark and that which contains an antiquark. Hence Eq. (13.41) corresponds to the definition for $B^\pm$ mesons, but the opposite sign is used for $D^\pm_{(s)}$ decays.

II. CP (and T) violation in mixing is defined by

$$|q/p| \neq 1 \,. \tag{13.42}$$

In charged-current semileptonic neutral meson decays $M, \overline{M} \to \ell^\pm X$ (taking $|A_{\ell^+X}| = |\overline{A}_{\ell^-X}|$ and $A_{\ell^-X} = \overline{A}_{\ell^+X} = 0$, as is the case in the Standard Model, to lowest order in G_F, and in most of its reasonable extensions), this is the only

source of CP violation, and can be measured via the asymmetry of "wrong-sign" decays induced by oscillations:

$$\mathcal{A}_{\rm SL}(t) \equiv \frac{d\Gamma/dt\big[\overline{M}^0_{\rm phys}(t) \to \ell^+ X\big] - d\Gamma/dt\big[M^0_{\rm phys}(t) \to \ell^- X\big]}{d\Gamma/dt\big[\overline{M}^0_{\rm phys}(t) \to \ell^+ X\big] + d\Gamma/dt\big[M^0_{\rm phys}(t) \to \ell^- X\big]} = \frac{1-|q/p|^4}{1+|q/p|^4}. \tag{13.43}$$

Note that this asymmetry of time-dependent decay rates is actually time-independent.

III. CP violation in interference between a decay without mixing, $M^0 \to f$, and a decay with mixing, $M^0 \to \overline{M}^0 \to f$ (such an effect occurs only in decays to final states that are common to M^0 and $\overline{M}^0$, including all CP eigenstates), is defined by

$$\arg(\lambda_f) + \arg(\lambda_{\bar{f}}) \neq 0\ , \tag{13.44}$$

with

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f}\ . \tag{13.45}$$

For final CP eigenstates, f_{CP}, the condition Eq. (13.44) simplifies to

$$\mathcal{I}m(\lambda_{f_{CP}}) \neq 0\ , \tag{13.46}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into CP eigenstates

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt\big[\overline{M}^0_{\rm phys}(t) \to f_{CP}\big] - d\Gamma/dt\big[M^0_{\rm phys}(t) \to f_{CP}\big]}{d\Gamma/dt\big[\overline{M}^0_{\rm phys}(t) \to f_{CP}\big] + d\Gamma/dt\big[M^0_{\rm phys}(t) \to f_{CP}\big]}\ . \tag{13.47}$$

If $\Delta\Gamma = 0$, as expected to a good approximation for B^0 mesons, but not for K^0 and B^0_s mesons, and $|q/p| = 1$, then $\mathcal{A}_{f_{CP}}$ has a particularly simple form (see Eq. (13.92), below). If, in addition, the decay amplitudes fulfill $|\overline{A}_{f_{CP}}| = |A_{f_{CP}}|$, the interference between decays with and without mixing is the only source of asymmetry and $\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_{f_{CP}})\sin(x\Gamma t)$.

Examples of these three types of CP violation will be given in Sections 13.4, 13.5, and 13.6.

13.2. Theoretical Interpretation: General Considerations

Consider the $M \to f$ decay amplitude A_f, and the CP conjugate process, $\overline{M} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus, their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^\pm$ bosons, and hence, are often called "weak phases." The weak phase of any single term is convention-dependent. However, the difference between the weak phases in two different terms in A_f is convention-independent. A second type of phase can appear in scattering or decay amplitudes, even when the Lagrangian is real. This phase originates from the possible contribution from intermediate on-shell states in the decay process. Since such phases are generated by CP-invariant interactions, they are the same in A_f and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions; hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The "weak" and "strong" phases discussed here appear in addition to the spurious CP-transformation phases of Eq. (13.22). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution a_i to A_f in three parts: its magnitude $|a_i|$, its weak phase ϕ_i, and its strong phase δ_i. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$\begin{aligned} A_f &= |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}, \\ \overline{A}_{\overline{f}} &= |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{13.48}$$

Similarly, for neutral mesons, it is useful to write

$$\mathbf{M}_{12} = |\mathbf{M}_{12}| e^{i\phi_M} \ , \quad \mathbf{\Gamma}_{12} = |\mathbf{\Gamma}_{12}| e^{i\phi_\Gamma} \ . \tag{13.49}$$

Each of the phases appearing in Eqs. (13.48, 13.49) is convention-dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$, and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

It is now straightforward to evaluate the various asymmetries in terms of the theoretical parameters introduced here. We will do so with approximations that are often relevant to the most interesting measured asymmetries.

1. The CP asymmetry in charged meson and all baryon decays [Eq. (13.41)] is given by

$$\mathcal{A}_f = -\frac{2|a_1 a_2| \sin(\delta_2 - \delta_1) \sin(\phi_2 - \phi_1)}{|a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_2 - \delta_1) \cos(\phi_2 - \phi_1)} \, . \tag{13.50}$$

The quantity of most interest to theory is the weak phase difference $\phi_2 - \phi_1$. Its extraction from the asymmetry requires, however, that the amplitude ratio $|a_2/a_1|$ and the strong

phase difference $\delta_2 - \delta_1$ are known. Both quantities depend on non-perturbative hadronic parameters that are difficult to calculate, but in some cases can be obtained from experiment.

2. In the approximation that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}| \ll 1$ (valid for B^0 and B_s^0 mesons), the CP asymmetry in semileptonic neutral-meson decays [Eq. (13.43)] is given by

$$\mathcal{A}_{\rm SL} = -\left|\frac{\mathbf{\Gamma}_{12}}{\mathbf{M}_{12}}\right| \sin(\phi_M - \phi_\Gamma)\,. \tag{13.51}$$

The quantity of most interest to theory is the weak phase $\phi_M - \phi_\Gamma$. Its extraction from the asymmetry requires, however, that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}|$ is known. State of the art calculations of this quantity for the B^0 and B_s^0 mesons have uncertainties of around 15–20% [34].

3. In the approximations that only a single weak phase contributes to decay, $A_f = |a_f|e^{i(\delta_f+\phi_f)}$, and that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}| = 0$, we obtain $|\lambda_f| = 1$, and the CP asymmetries in decays to a final CP eigenstate f [Eq. (13.47)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f)\ \sin(\Delta m t) \ \text{ with } \ \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f)\,. \tag{13.52}$$

Note that the phase measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

The discussion above allows us to introduce another classification of CP-violating effects:

1. *Indirect CP violation* is consistent with taking $\phi_M \neq 0$ and setting all other CP violating phases to zero. CP violation in mixing (type II) belongs to this class.
2. *Direct CP violation* cannot be accounted for by just $\phi_M \neq 0$. CP violation in decay (type I) belongs to this class.

The historical significance of this classification is related to theory. In superweak models [35], CP violation appears only in diagrams that contribute to $\mathbf{M}_{12}$, hence they predict that there is no direct CP violation. In most models and, in particular, in the Standard Model, CP violation is both direct and indirect. As concerns type III CP violation, a single observation of such an effect would be consistent with indirect CP violation, but observing $\eta_{f_1}\mathcal{I}m(\lambda_{f_1}) \neq \eta_{f_2}\mathcal{I}m(\lambda_{f_2})$ (for the same decaying meson and two different final CP eigenstates f_1 and f_2) would establish direct CP violation. The experimental observation of $\epsilon' \neq 0$, which was achieved by establishing that $\mathcal{I}m(\lambda_{\pi^+\pi^-}) \neq \mathcal{I}m(\lambda_{\pi^0\pi^0})$ (see Section 13.4), excluded the superweak scenario.

13.3. Theoretical Interpretation: The KM Mechanism

Of all the Standard Model quark parameters, only the Kobayashi-Maskawa (KM) phase is CP-violating. Having a single source of CP violation, the Standard Model is very predictive for CP asymmetries: some vanish, and those that do not are correlated.

To be precise, CP could be violated also by strong interactions. The experimental upper bound on the electric-dipole moment of the neutron implies, however, that $\theta_{\rm QCD}$, the non-perturbative parameter that determines the strength of this type of CP violation, is tiny, if not zero. (The smallness of $\theta_{\rm QCD}$ constitutes a theoretical puzzle, known as "the strong CP problem.") In particular, it is irrelevant to our discussion of hadron decays.

The charged current interactions (that is, the $W^\pm$ interactions) for quarks are given by

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}}\ \overline{u_{Li}}\ \gamma^\mu\ (V_{\rm CKM})_{ij}\ d_{Lj}\ W^+_\mu + \text{h.c.} \tag{13.53}$$

Here $i,j = 1,2,3$ are generation numbers. The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix for quarks is a 3×3 unitary matrix [36]. Ordering the quarks by their masses, *i.e.*, $(u_1,u_2,u_3)\to(u,c,t)$ and $(d_1,d_2,d_3)\to(d,s,b)$, the elements of $V_{\rm CKM}$ are written as follows:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\ . \tag{13.54}$$

While a general 3×3 unitary matrix depends on three real angles and six phases, the freedom to redefine the phases of the quark mass eigenstates can be used to remove five of the phases, leaving a single physical phase, the Kobayashi-Maskawa phase, that is responsible for all CP violation in the Standard Model.

The fact that one can parametrize $V_{\rm CKM}$ by three real and only one imaginary physical parameters can be made manifest by choosing an explicit parametrization. The Wolfenstein parametrization [37,38] is particularly useful:

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3[1-(1-\frac{1}{2}\lambda^2)(\rho+i\eta)] & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\rho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}\ . \tag{13.55}$$

Here $\lambda\approx 0.23$ (not to be confused with λ_f), the sine of the Cabibbo angle, plays the role of an expansion parameter, and η represents the CP-violating phase. Terms of $\mathcal{O}(\lambda^6)$ have been neglected.

The unitarity of the CKM matrix, $(VV^\dagger)_{ij} = (V^\dagger V)_{ij} = \delta_{ij}$, leads to twelve distinct complex relations among the matrix elements. The six relations with $i\neq j$ can be represented geometrically as triangles in the complex plane. Two of these,

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$
$$V_{td}V^*_{ud} + V_{ts}V^*_{us} + V_{tb}V^*_{ub} = 0\ ,$$

have terms of equal order, $\mathcal{O}(A\lambda^3)$, and so have corresponding triangles whose interior angles are all $\mathcal{O}(1)$ physical quantities that can be independently measured. The angles of the first triangle (see Fig. 13.1) are given by

$$\begin{aligned}
\alpha \equiv \varphi_2 &\equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \simeq \arg\left(-\frac{1-\rho-i\eta}{\rho+i\eta}\right), \\
\beta \equiv \varphi_1 &\equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \simeq \arg\left(\frac{1}{1-\rho-i\eta}\right), \\
\gamma \equiv \varphi_3 &\equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \simeq \arg\left(\rho+i\eta\right).
\end{aligned} \tag{13.56}$$

The angles of the second triangle are equal to (α, β, γ) up to corrections of $\mathcal{O}(\lambda^2)$. The notations (α, β, γ) and $(\varphi_1, \varphi_2, \varphi_3)$ are both in common usage but, for convenience, we only use the first convention in the following.

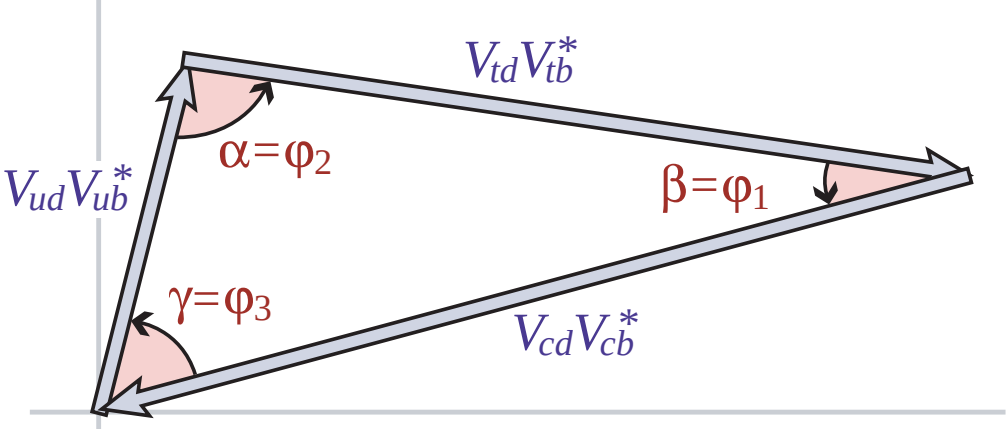

Figure 13.1: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

Another relation that can be represented as a triangle,

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0\,, \tag{13.57}$$

and, in particular, its small angle, of $\mathcal{O}(\lambda^2)$,

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right), \tag{13.58}$$

is convenient for analyzing CP violation in the B_s^0 sector.

All unitarity triangles have the same area, commonly denoted by $J/2$ [39]. If CP is violated, J is different from zero and can be taken as the single CP-violating parameter. In the Wolfenstein parametrization of Eq. (13.55), $J \simeq \lambda^6 A^2 \eta$.

13.4. Kaons

CP violation was discovered in $K \to \pi\pi$ decays in 1964 [1]. The same mode provided the first observation of direct CP violation [4–6].

The decay amplitudes actually measured in neutral K decays refer to the mass eigenstates K_L and K_S, rather than to the K and $\overline{K}$ states referred to in Eq. (13.19). The final $\pi^+\pi^-$ and $\pi^0\pi^0$ states are CP-even. In the CP conservation limit, K_S (K_L) would be CP-even (odd), and therefore would (would not) decay to two pions. We define CP-violating amplitude ratios for two-pion final states,

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle} , \quad \eta_{+-} \equiv \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle} . \tag{13.59}$$

Another important observable is the asymmetry of time-integrated semileptonic decay rates:

$$\delta_L \equiv \frac{\Gamma(K_L \to \ell^+\nu_\ell\pi^-) - \Gamma(K_L \to \ell^-\overline{\nu}_\ell\pi^+)}{\Gamma(K_L \to \ell^+\nu_\ell\pi^-) + \Gamma(K_L \to \ell^-\overline{\nu}_\ell\pi^+)} . \tag{13.60}$$

CP violation has been observed as an appearance of K_L decays to two-pion final states [27],

$$|\eta_{00}| = (2.220 \pm 0.011) \times 10^{-3} \qquad |\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3} \tag{13.61}$$

$$|\eta_{00}/\eta_{+-}| = 0.9950 \pm 0.0007 , \tag{13.62}$$

where the phase ϕ_{ij} of the amplitude ratio η_{ij} has been determined both assuming CPT invariance:

$$\phi_{00} = (43.52 \pm 0.05)^\circ , \qquad \phi_{+-} = (43.51 \pm 0.05)^\circ , \tag{13.63}$$

and without assuming CPT invariance:

$$\phi_{00} = (43.7 \pm 0.6)^\circ , \qquad \phi_{+-} = (43.4 \pm 0.5)^\circ . \tag{13.64}$$

CP violation has also been observed in semileptonic K_L decays [27]

$$\delta_L = (3.32 \pm 0.06) \times 10^{-3} , \tag{13.65}$$

where δ_L is a weighted average of muon and electron measurements, as well as in K_L decays to $\pi^+\pi^-\gamma$ and $\pi^+\pi^-e^+e^-$ [27]. CP violation in $K \to 3\pi$ decays has not yet been observed [27,40].

Historically, CP violation in neutral K decays has been described in terms of the complex parameters ϵ and ϵ'. The observables η_{00}, η_{+-}, and δ_L are related to these parameters, and to those of Section 13.1, by

$$\begin{aligned}
\eta_{00} = \frac{1 - \lambda_{\pi^0\pi^0}}{1 + \lambda_{\pi^0\pi^0}} &= \epsilon - 2\epsilon' , \\
\eta_{+-} = \frac{1 - \lambda_{\pi^+\pi^-}}{1 + \lambda_{\pi^+\pi^-}} &= \epsilon + \epsilon' , \\
\delta_L = \frac{1 - |q/p|^2}{1 + |q/p|^2} &= \frac{2\mathcal{R}e(\epsilon)}{1 + |\epsilon|^2} ,
\end{aligned} \tag{13.66}$$

where, in the last line, we have assumed that $\left|A_{\ell^+\nu_\ell\pi^-}\right| = \left|\overline{A}_{\ell^-\overline{\nu}_\ell\pi^+}\right|$ and $\left|A_{\ell^-\overline{\nu}_\ell\pi^+}\right| = \left|\overline{A}_{\ell^+\nu_\ell\pi^-}\right| = 0$. (The convention-dependent parameter $\tilde{\epsilon} \equiv (1-q/p)/(1+q/p)$, sometimes used in the literature, is, in general, different from ϵ but yields a similar expression, $\delta_L = 2\mathcal{R}e(\tilde{\epsilon})/(1+|\tilde{\epsilon}|^2)$.) A fit to the $K \to \pi\pi$ data yields [27]

$$\begin{aligned} |\epsilon| &= (2.228 \pm 0.011) \times 10^{-3} \,, \\ \mathcal{R}e(\epsilon'/\epsilon) &= (1.66 \pm 0.23) \times 10^{-3} \,. \end{aligned} \tag{13.67}$$

In discussing two-pion final states, it is useful to express the amplitudes $A_{\pi^0\pi^0}$ and $A_{\pi^+\pi^-}$ in terms of their isospin components via

$$\begin{aligned} A_{\pi^0\pi^0} &= \sqrt{\frac{1}{3}}\,|A_0|\,e^{i(\delta_0+\phi_0)} - \sqrt{\frac{2}{3}}\,|A_2|\,e^{i(\delta_2+\phi_2)}, \\ A_{\pi^+\pi^-} &= \sqrt{\frac{2}{3}}\,|A_0|\,e^{i(\delta_0+\phi_0)} + \sqrt{\frac{1}{3}}\,|A_2|\,e^{i(\delta_2+\phi_2)} \,, \end{aligned} \tag{13.68}$$

where we parameterize the amplitude $A_I(\overline{A}_I)$ for $K^0(\overline{K}^0)$ decay into two pions with total isospin $I = 0$ or 2 as

$$\begin{aligned} A_I &\equiv \langle(\pi\pi)_I\,|\mathcal{H}|\,K^0\rangle = |A_I|\,e^{i(\delta_I+\phi_I)} \,, \\ \overline{A}_I &\equiv \langle(\pi\pi)_I\,|\mathcal{H}|\,\overline{K}^0\rangle = |A_I|\,e^{i(\delta_I-\phi_I)} \,. \end{aligned} \tag{13.69}$$

The smallness of $|\eta_{00}|$ and $|\eta_{+-}|$ allows us to approximate

$$\epsilon \simeq \frac{1}{2}(1 - \lambda_{(\pi\pi)_{I=0}}) \,, \qquad \epsilon' \simeq \frac{1}{6}\left(\lambda_{\pi^0\pi^0} - \lambda_{\pi^+\pi^-}\right) \,. \tag{13.70}$$

The parameter ϵ represents indirect CP violation, while ϵ' parameterizes direct CP violation: $\mathcal{R}e(\epsilon')$ measures CP violation in decay (type I), $\mathcal{R}e(\epsilon)$ measures CP violation in mixing (type II), and $\mathcal{I}m(\epsilon)$ and $\mathcal{I}m(\epsilon')$ measure the interference between decays with and without mixing (type III).

The following expressions for ϵ and ϵ' are useful for theoretical evaluations:

$$\epsilon \simeq \frac{e^{i\pi/4}}{\sqrt{2}}\frac{\mathcal{I}m(\mathbf{M}_{12})}{\Delta m} \,, \qquad \epsilon' = \frac{i}{\sqrt{2}}\left|\frac{A_2}{A_0}\right| e^{i(\delta_2-\delta_0)}\sin(\phi_2-\phi_0) \,. \tag{13.71}$$

The expression for ϵ is only valid in a phase convention where $\phi_2 = 0$, corresponding to a real $V_{ud}V_{us}^*$, and in the approximation that also $\phi_0 = 0$. The phase of ϵ, $\arg(\epsilon) \approx \arctan(-2\Delta m/\Delta\Gamma)$, is independent of the electroweak model and is experimentally determined to be about $\pi/4$. The calculation of ϵ benefits from the fact that $\mathcal{I}m(\mathbf{M}_{12})$ is dominated by short distance physics. Consequently, the main sources of uncertainty in theoretical interpretations of ϵ are the values of matrix elements,

such as $\langle K^0 | (\overline{s}d)_{V-A} (\overline{s}d)_{V-A} | \overline{K}^0 \rangle$. The expression for ϵ' is valid to first order in $|A_2/A_0| \sim 1/20$. The phase of ϵ' is experimentally determined, $\pi/2 + \delta_2 - \delta_0 \approx \pi/4$, and is independent of the electroweak model. Note that, accidentally, ϵ'/ϵ is real to a good approximation. Determination of weak phase information from the measurement of $\mathcal{R}e(\epsilon'/\epsilon)$ given in Eq. (13.67) has until now been precluded by uncertainties in the hadronic parameters, but recent advances in lattice QCD calculations [41,42] suggest that it may become possible [43].

A future measurement of much interest is that of CP violation in the rare $K \to \pi\nu\overline{\nu}$ decays. The signal for CP violation is simply observing the $K_L \to \pi^0\nu\overline{\nu}$ decay. The effect here is that of interference between decays with and without mixing (type III) [44]:

$$\frac{\Gamma(K_L \to \pi^0\nu\overline{\nu})}{\Gamma(K^+ \to \pi^+\nu\overline{\nu})} = \frac{1}{2}\left[1 + |\lambda_{\pi\nu\overline{\nu}}|^2 - 2\,\mathcal{R}e(\lambda_{\pi\nu\overline{\nu}})\right] \simeq 1 - \mathcal{R}e(\lambda_{\pi\nu\overline{\nu}}), \tag{13.72}$$

where in the last equation we neglect CP violation in decay and in mixing (expected, model-independently, to be of order 10^{-5} and 10^{-3}, respectively). Such a measurement is experimentally very challenging but would be theoretically very rewarding [45]. Similar to the CP asymmetry in $B^0 \to J/\psi K_S$, the CP violation in $K \to \pi\nu\overline{\nu}$ decay is predicted to be large (that is, the ratio in Eq. (13.72) is neither CKM- nor loop-suppressed) and can be very cleanly interpreted.

Within the Standard Model, the $K_L \to \pi^0\nu\overline{\nu}$ decay is dominated by an intermediate top quark contribution and, consequently, can be interpreted in terms of CKM parameters [46]. (For the charged mode, $K^+ \to \pi^+\nu\overline{\nu}$, the contribution from an intermediate charm quark is not negligible, and constitutes a source of hadronic uncertainty.) In particular, $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu})$ provides a theoretically clean way to determine the Wolfenstein parameter η [47]:

$$\mathcal{B}(K_L \to \pi^0\nu\overline{\nu}) = \kappa_L [X(m_t^2/m_W^2)]^2 A^4 \eta^2\,, \tag{13.73}$$

where the hadronic parameter $\kappa_L \sim 2 \times 10^{-10}$ incorporates the value of the four-fermion matrix element which is deduced, using isospin relations, from $\mathcal{B}(K^+ \to \pi^0 e^+ \nu_e)$, and $X(m_t^2/m_W^2)$ is a known function of the top mass. An explicit calculation gives $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu}) = (3.00 \pm 0.30) \times 10^{-11}$ [49].

The currently tightest experimental limit is $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu}) < 2.6 \times 10^{-8}$ [50], which does not yet reach the bound $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu}) < 4.4 \times \mathcal{B}(K^+ \to \pi^+\nu\overline{\nu})$ [44]. Significant further progress is anticipated from experiments searching for $K \to \pi\nu\overline{\nu}$ decays in the next few years [51,52].

13.5. Charm

The existence of D^0–$\overline{D}^0$ mixing has been established in recent years [53–56]. The experimental constraints read [29,57] $x \equiv \Delta m/\Gamma = (0.32 \pm 0.14) \times 10^{-2}$ and $y \equiv \Delta\Gamma/(2\Gamma) = (0.69\,^{+0.06}_{-0.07}) \times 10^{-2}$. Thus, the data clearly show that $y \neq 0$, but improved measurements are needed to be sure of the size of x. Long-distance contributions make it difficult to calculate Standard Model predictions for the D^0–$\overline{D}^0$ mixing parameters. Therefore, the goal of the search for D^0–$\overline{D}^0$ mixing is not to constrain the CKM parameters, but rather to probe new physics. Here CP violation plays an important role. Within the Standard Model, the CP-violating effects are predicted to be small, since the mixing and the relevant decays are described, to an excellent approximation, by the physics of the first two generations only. The expectation is that the Standard Model size of CP violation in D decays is $\mathcal{O}(10^{-3})$ or less. At present, the most sensitive searches involve the $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$ and $D^0 \to K^\pm\pi^\mp$ modes.

The neutral D mesons decay via a singly-Cabibbo-suppressed transition to the CP eigenstates K^+K^- and $\pi^+\pi^-$. These decays are dominated by Standard-Model tree diagrams. Thus, we can write, for $f = K^+K^-$ or $\pi^+\pi^-$,

$$A_f = A_f^T e^{+i\phi_f^T}\left[1 + r_f e^{i(\delta_f+\phi_f)}\right] ,$$
$$\bar{A}_f = A_f^T e^{-i\phi_f^T}\left[1 + r_f e^{i(\delta_f-\phi_f)}\right] , \tag{13.74}$$

where $A_f^T e^{\pm i\phi_f^T}$ is the Standard Model tree-level contribution, ϕ_f^T and ϕ_f are weak, CP violating phases, δ_f is a strong phase difference, and r_f is the ratio between a subleading ($r_f \ll 1$) contribution with a weak phase different from ϕ_f^T and the Standard Model tree-level contribution. Neglecting r_f, λ_f is universal, and we can define an observable phase ϕ_D via

$$\lambda_f \equiv -|q/p|e^{i\phi_D} . \tag{13.75}$$

(In the limit of CP conservation, choosing $\phi_D = 0$ is equivalent to defining the mass eigenstates by their CP eigenvalue: $|D_\mp\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$, with D_- (D_+) being the CP-odd (CP-even) state; that is, the state that does not (does) decay into K^+K^-.)

We define the time integrated CP asymmetry for a final CP eigenstate f as follows:

$$a_f \equiv \frac{\int_0^\infty \Gamma(D^0_{\rm phys}(t) \to f)dt - \int_0^\infty \Gamma(\overline{D}^0_{\rm phys}(t) \to f)dt}{\int_0^\infty \Gamma(D^0_{\rm phys}(t) \to f)dt + \int_0^\infty \Gamma(\overline{D}^0_{\rm phys}(t) \to f)dt} . \tag{13.76}$$

(This expression corresponds to the D meson being tagged at production, hence the integration goes from 0 to $+\infty$; measurements are also possible with $\psi(3770) \to D^0\overline{D}^0$, in which case the integration goes from $-\infty$ to $+\infty$ giving slightly different results; see the discussion in Section 13.1.3.) We take $x, y, r_f \ll 1$ and expand to leading order in these parameters. We can then separate the contribution to a_f into three parts [58],

$$a_f = a_f^d + a_f^m + a_f^i , \tag{13.77}$$

with the following underlying mechanisms:

1. a_f^d signals CP violation in decay (similar to Eq. (13.41)):

$$a_f^d = 2r_f \sin\phi_f \sin\delta_f \,. \tag{13.78}$$

2. a_f^m signals CP violation in mixing (similar to Eq. (13.51)). With our approximations, it is universal:

$$a^m = -\frac{y}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\cos\phi_D \,. \tag{13.79}$$

3. a_f^i signals CP violation in the interference of mixing and decay (similar to Eq. (13.52)). With our approximations, it is universal:

$$a^i = \frac{x}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\sin\phi_D \,. \tag{13.80}$$

One can isolate the effects of direct CP violation by taking the difference between the CP asymmetries in the K^+K^- and $\pi^+\pi^-$ modes:

$$\Delta a_{CP} \equiv a_{K^+K^-} - a_{\pi^+\pi^-} = a^d_{K^+K^-} - a^d_{\pi^+\pi^-} \,, \tag{13.81}$$

where we neglected a residual, experiment-dependent, contribution from indirect CP violation due to the fact that there may be a decay time-dependent acceptance function that can be different for the K^+K^- and $\pi^+\pi^-$ channels. Earlier evidence for such direct CP violation [59] has become less significant when including more data, with the current average giving [29]:

$$a^d_{K^+K^-} - a^d_{\pi^+\pi^-} = (-1.3 \pm 0.7) \times 10^{-3} \,. \tag{13.82}$$

One can also isolate the effects of indirect CP violation in the following way. Consider the time-dependent decay rates in Eq. (13.36) and Eq. (13.37). The mixing processes modify the time dependence from a pure exponential. However, given the small values of x and y, the time dependences can be recast, to a good approximation, into purely exponential form, but with modified decay-rate parameters [60,61] (given here for the K^+K^- final state):

$$\begin{aligned}
\Gamma_{D^0\to K^+K^-} &= \Gamma \times [1 + |q/p|\,(y\cos\phi_D - x\sin\phi_D)] \;, \\
\Gamma_{\overline{D}{}^0\to K^+K^-} &= \Gamma \times [1 + |p/q|\,(y\cos\phi_D + x\sin\phi_D)] \;.
\end{aligned} \tag{13.83}$$

One can define CP-conserving and CP-violating combinations of these two observables (normalized to the true width Γ):

$$\begin{aligned}
y_{CP} &\equiv \frac{\Gamma_{\overline{D}{}^0\to K^+K^-} + \Gamma_{D^0\to K^+K^-}}{2\Gamma} - 1 \\
&= (y/2)\,(|q/p| + |p/q|)\cos\phi_D - (x/2)\,(|q/p| - |p/q|)\sin\phi_D \;, \\
A_\Gamma &\equiv \frac{\Gamma_{D^0\to K^+K^-} - \Gamma_{\overline{D}{}^0\to K^+K^-}}{2\Gamma} \\
&= -\,(a^m + a^i) \;.
\end{aligned} \tag{13.84}$$

In the limit of CP conservation (and, in particular, within the Standard Model), $y_{CP} = (\Gamma_+ - \Gamma_-)/2\Gamma = y$ (where $\Gamma_+(\Gamma_-)$ is the decay width of the CP-even (-odd) mass eigenstate) and $A_\Gamma = 0$. Indeed, present measurements imply that CP violation is small [29],

$$\begin{aligned} y_{CP} &= (+0.84 \pm 0.16) \times 10^{-2} \ , \\ A_\Gamma &= (-0.032 \pm 0.026) \times 10^{-2} \ . \end{aligned}$$

The $K^\pm\pi^\mp$ states are not CP eigenstates, but they are still common final states for D^0 and $\overline{D}^0$ decays. Since $D^0(\overline{D}^0) \to K^-\pi^+$ is a Cabibbo-favored (doubly-Cabibbo-suppressed) process, these processes are particularly sensitive to x and/or $y = \mathcal{O}(\lambda^2)$. Taking into account that $\left|\lambda_{K^-\pi^+}\right|, \left|\lambda^{-1}_{K^+\pi^-}\right| \ll 1$ and $x, y \ll 1$, assuming that there is no direct CP violation (these are Standard Model tree-level decays dominated by a single weak phase, and there is no contribution from penguin-like and chromomagnetic operators), and expanding the time-dependent rates for $xt, yt \lesssim \Gamma^{-1}$, one obtains

$$\begin{aligned} \Gamma[D^0_{\rm phys}(t) \to K^+\pi^-] = \ & e^{-\Gamma t}|\overline{A}_{K^-\pi^+}|^2 \\ & \times \left[r_d^2 + r_d \left|\frac{q}{p}\right| (y' \cos\phi_D - x' \sin\phi_D)\Gamma t + \left|\frac{q}{p}\right|^2 \frac{y^2+x^2}{4}(\Gamma t)^2 \right] \ , \\ \Gamma[\overline{D}^0_{\rm phys}(t) \to K^-\pi^+] = \ & e^{-\Gamma t}|\overline{A}_{K^-\pi^+}|^2 \\ & \times \left[r_d^2 + r_d \left|\frac{p}{q}\right| (y' \cos\phi_D + x' \sin\phi_D)\Gamma t + \left|\frac{p}{q}\right|^2 \frac{y^2+x^2}{4}(\Gamma t)^2 \right] \ , \end{aligned} \quad (13.85)$$

where

$$\begin{aligned} y' &\equiv y\cos\delta - x\sin\delta \ , \\ x' &\equiv x\cos\delta + y\sin\delta \ . \end{aligned} \quad (13.86)$$

The weak phase ϕ_D is the same as that of Eq. (13.75) (a consequence of neglecting direct CP violation) and $r_d = \mathcal{O}(\tan^2\theta_c)$ is the amplitude ratio, $r_d = \left|\overline{A}_{K^-\pi^+}/A_{K^-\pi^+}\right| = \left|A_{K^+\pi^-}/\overline{A}_{K^+\pi^-}\right|$, that is, $\lambda_{K^-\pi^+} = r_d|q/p|e^{-i(\delta-\phi_D)}$ and $\lambda^{-1}_{K^+\pi^-} = r_d|p/q|e^{-i(\delta+\phi_D)}$. The parameter δ is a strong-phase difference for these processes, that can be obtained from measurements of quantum correlated $\psi(3770) \to D^0\overline{D}^0$ decays [62,63]. By fitting to the six coefficients of the various time-dependences, one can determine r_d, $|q/p|$, (x^2+y^2), $y'\cos\phi_D$, and $x'\sin\phi_D$. In particular, finding CP violation ($|q/p| \neq 1$ and/or $\sin\phi_D \neq 0$) at a level much higher than 10^{-3} would constitute evidence for new physics. The most stringent constraints to date on CP violation in charm mixing have been obtained with this method [64] and from the A_Γ measurement [65].

A fit to all data [29], including also results from time-dependent analyses of $D^0 \to K_S\pi^+\pi^-$ decays, from which x, y, $|q/p|$ and ϕ_D can be determined directly, yields no evidence for indirect CP violation:

$$\begin{aligned} 1 - |q/p| &= +0.11^{+0.07}_{-0.08} \ , \\ \phi_D &= \left(-13^{+10}_{-9}\right)^\circ \ . \end{aligned}$$

With the additional assumption of no direct CP violation in doubly-Cabibbo-suppressed D decays [66–68], tighter constraints are obtained:

$$1 - |q/p| = -0.001 \pm 0.014 \ ,$$
$$\phi_D = (0.1 \pm 0.5)^\circ \ .$$

More details on various theoretical and experimental aspects of $D^0 - \overline{D}^0$ mixing can be found in Ref. [31].

Searches for CP violation in charged $D_{(s)}$ decays have been performed in many modes. Searches in decays mediated by Cabibbo-suppressed amplitudes are particularly interesting, since in other channels effects are likely to be too small to be observable in current experiments. Examples of relevant two-body modes are $D^+ \to \pi^+\pi^0$, $K_S K^+$, $\phi\pi^+$ and $D_s^+ \to K^+\pi^0$, $K_S\pi^+$, ϕK^+. The most precise results are $\mathcal{A}_{D^+\to K_S K^+} = -0.0003 \pm 0.0017$ and $\mathcal{A}_{D_s^+\to K_S\pi^+} = +0.0063 \pm 0.0047$ [29]. The precision of experiments is now sufficient that the effect from CP violation in the neutral kaon system can be seen in $D^+ \to K_S\pi^+$ decays [69,70].

Three- and four-body final states provide additional possibilities to search for CP violation, since effects may vary over the phase-space [71]. A number of methods have been proposed to exploit this feature and search for CP violation in ways that do not require modelling of the decay distribution [72–74]. Such methods are useful for analysis of charm decays since they are less sensitive to biases from production asymmetries, and are well suited to address the issue of whether or not CP violation effects are present. They can also be applied to tagged neutral D meson as well as to charged $D_{(s)}$ decays (flavour tagging is typically achieved from the charge of the pion producted in $D^{*+} \to D^0\pi^+$ decays). The results of all searches to date are consistent with the absence of CP violation, with the most significant hint at the level of 2.7σ [75].

13.6. Beauty

13.6.1. *CP violation in mixing of B^0 and B_s^0 mesons* :

The upper bound on the CP asymmetry in semileptonic B decays [28] implies that CP violation in $B^0 - \overline{B}^0$ mixing is a small effect (we use $\mathcal{A}_{\rm SL}/2 \approx 1 - |q/p|$, see Eq. (13.43)):

$$\mathcal{A}_{\rm SL}^d = (-2.1 \pm 1.7) \times 10^{-3} \implies |q/p| = 1.0010 \pm 0.0008 \ . \tag{13.87}$$

The Standard Model prediction is

$$\mathcal{A}_{\rm SL}^d = \mathcal{O}\left[(m_c^2/m_t^2)\sin\beta\right] \lesssim 0.001 \ . \tag{13.88}$$

An explicit calculation gives $(-4.7 \pm 0.6) \times 10^{-4}$ [34].

The experimental constraint on CP violation in $B_s^0 - \overline{B}_s^0$ mixing is somewhat weaker than that in the $B^0 - \overline{B}^0$ system [28]

$$\mathcal{A}_{\rm SL}^s = (-0.6 \pm 2.8) \times 10^{-3} \implies |q/p| = 1.0003 \pm 0.0014 \ . \tag{13.89}$$

The Standard Model prediction is $\mathcal{A}_{\rm SL}^s = \mathcal{O}\left[(m_c^2/m_t^2)\sin\beta_s\right] \lesssim 10^{-4}$, with an explicit calculation giving $(2.22 \pm 0.27) \times 10^{-5}$ [34].

The fit to experimental data that results in the averages quoted above has a χ^2 probability of 4.5% indicating some tension between the different measurements [29]. This originates in part from a result from the D0 collaboration for the inclusive same-sign dimuon asymmetry that deviates from the Standard Model prediction by 3.6σ [76]. As yet, this has not been confirmed by independent studies.

In models where $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is approximately real, such as the Standard Model, an upper bound on $\Delta\Gamma/\Delta m \approx \mathcal{R}e(\mathbf{\Gamma}_{12}/\mathbf{M}_{12})$ provides yet another upper bound on the deviation of $|q/p|$ from one. This constraint does not hold if $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is approximately imaginary. (An alternative parameterization uses $q/p = (1-\tilde{\epsilon}_B)/(1+\tilde{\epsilon}_B)$, leading to $\mathcal{A}_{\rm SL} \simeq 4\mathcal{R}e(\tilde{\epsilon}_B)$.)

13.6.2. *CP violation in interference of B^0 decays with and without mixing* :

The small deviation (less than one percent) of $|q/p|$ from 1 implies that, at the present level of experimental precision, CP violation in B^0 mixing is a negligible effect. Thus, for the purpose of analyzing CP asymmetries in hadronic B^0 decays, we can use

$$\lambda_f = e^{-i\phi_{M(B^0)}}(\overline{A}_f/A_f)\ , \tag{13.90}$$

where $\phi_{M(B^0)}$ refers to the phase of $\mathbf{M}_{12}$ appearing in Eq. (13.49) that is appropriate for $B^0 - \overline{B}^0$ oscillations. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_{M(B^0)}} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*)\ . \tag{13.91}$$

The class of CP violation effects in interference between mixing and decay is studied with final states that are common to B^0 and $\overline{B}^0$ decays [77,78]. It is convenient to rewrite Eq. (13.47) for B^0 decays as [79–81]

$$\mathcal{A}_f(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)\ ,$$

$$S_f \equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1+\left|\lambda_f\right|^2}\ , \quad C_f \equiv \frac{1-\left|\lambda_f\right|^2}{1+\left|\lambda_f\right|^2}\ , \tag{13.92}$$

where we assume that $\Delta\Gamma = 0$ and $|q/p| = 1$. An alternative notation in use is $A_f \equiv -C_f$ – this A_f should not be confused with the A_f of Eq. (13.19), but in the limit that $|q/p| = 1$ is equivalent with the $\mathcal{A}_f$ of Eq. (13.41).

A large class of interesting processes proceed via quark transitions of the form $\overline{b} \to \overline{q}q\overline{q}'$ with $q' = s$ or d. For $q = c$ or u, there are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 13.2) which carry different weak phases:

$$A_f = \left(V_{qb}^* V_{qq'}\right) t_f + \sum_{q_u=u,c,t} \left(V_{q_u b}^* V_{q_u q'}\right) p_f^{q_u}\ . \tag{13.93}$$

(The distinction between tree and penguin contributions is a heuristic one; the separation by the operator that enters is more precise. A detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, can be found in Ref. [82] for example.) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations. For example, for $f = \pi\pi$, which proceeds via a $\overline{b} \to \overline{u}u\overline{d}$ transition, we can write

$$A_{\pi\pi} = (V_{ub}^* V_{ud})\, T_{\pi\pi} + (V_{tb}^* V_{td})\, P_{\pi\pi}^t \,, \tag{13.94}$$

where $T_{\pi\pi} = t_{\pi\pi} + p_{\pi\pi}^u - p_{\pi\pi}^c$ and $P_{\pi\pi}^t = p_{\pi\pi}^t - p_{\pi\pi}^c$. CP-violating phases in Eq. (13.94) appear only in the CKM elements, so that

$$\frac{\overline{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{\left(V_{ub}V_{ud}^*\right) T_{\pi\pi} + \left(V_{tb}V_{td}^*\right) P_{\pi\pi}^t}{\left(V_{ub}^* V_{ud}\right) T_{\pi\pi} + \left(V_{tb}^* V_{td}\right) P_{\pi\pi}^t} \,. \tag{13.95}$$

For $f = J/\psi K$, which proceeds via a $\overline{b} \to \overline{c}c\overline{s}$ transition, we can write

$$A_{\psi K} = (V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u \,, \tag{13.96}$$

where $T_{\psi K} = t_{\psi K} + p_{\psi K}^c - p_{\psi K}^t$ and $P_{\psi K}^u = p_{\psi K}^u - p_{\psi K}^t$. A subtlety arises in this decay that is related to the fact that B^0 decays into a final $J/\psi K^0$ state while $\overline{B}^0$ decays into a final $J/\psi \overline{K}^0$ state. A common final state, *e.g.*, $J/\psi K_S$, is reached only via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor (defined in Eq. (13.49)) corresponding to neutral K mixing, $e^{-i\phi_{M(K)}} = (V_{cd}^* V_{cs})/(V_{cd} V_{cs}^*)$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{\left(V_{cb}V_{cs}^*\right) T_{\psi K} + \left(V_{ub}V_{us}^*\right) P_{\psi K}^u}{\left(V_{cb}^* V_{cs}\right) T_{\psi K} + \left(V_{ub}^* V_{us}\right) P_{\psi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \,. \tag{13.97}$$

For $q = s$ or d, there are only penguin contributions to A_f, that is, $t_f = 0$ in Eq. (13.93). (The tree $\overline{b} \to \overline{u}u\overline{q}'$ transition followed by $\overline{u}u \to \overline{q}q$ rescattering is included below in the P^u terms.) Again, CKM unitarity allows us to write A_f in terms of two CKM combinations. For example, for $f = \phi K_S$, which proceeds via a $\overline{b} \to \overline{s}s\overline{s}$ transition, we can write

$$\frac{\overline{A}_{\phi K_S}}{A_{\phi K_S}} = -\frac{\left(V_{cb}V_{cs}^*\right) P_{\phi K}^c + \left(V_{ub}V_{us}^*\right) P_{\phi K}^u}{\left(V_{cb}^* V_{cs}\right) P_{\phi K}^c + \left(V_{ub}^* V_{us}\right) P_{\phi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \,, \tag{13.98}$$

where $P_{\phi K}^c = p_{\phi K}^c - p_{\phi K}^t$ and $P_{\phi K}^u = p_{\phi K}^u - p_{\phi K}^t$.

Since in general the amplitude A_f involves two different weak phases, the corresponding decays can exhibit both CP violation in the interference of decays with and without mixing, $S_f \neq 0$, and CP violation in decay, $C_f \neq 0$. (At the present level of experimental precision, the contribution to C_f from CP violation in mixing is negligible, see Eq. (13.87).) If the contribution from a second weak phase is suppressed, then the interpretation of S_f in terms of Lagrangian CP-violating parameters is clean, while

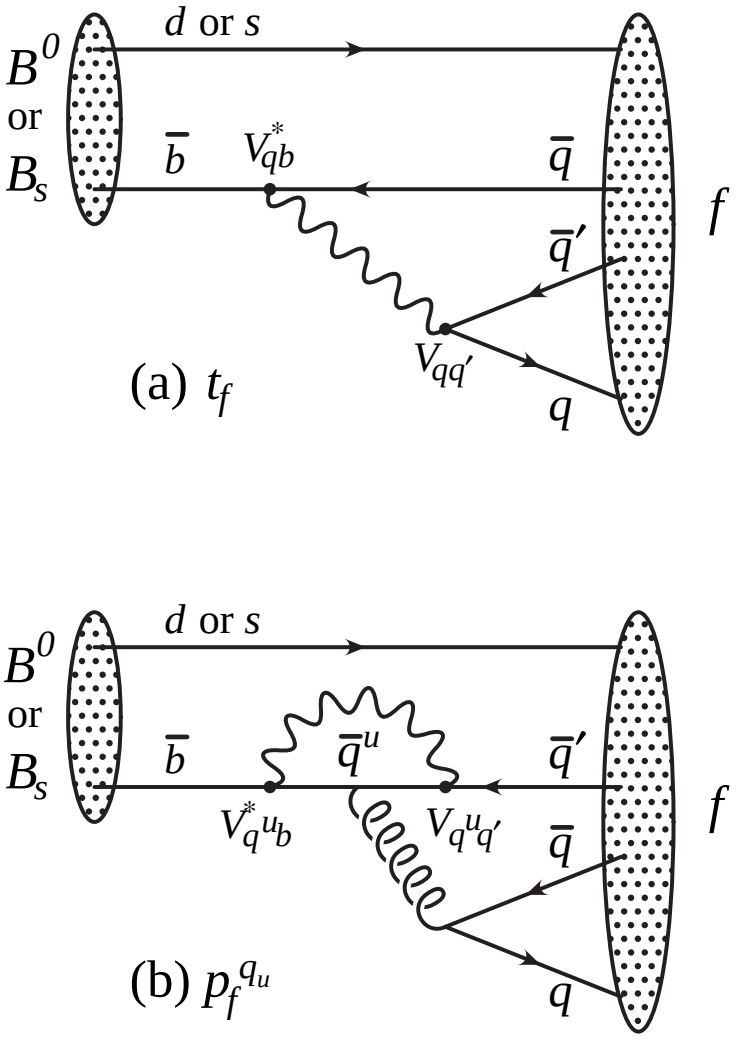

Figure 13.2: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s^0 \to f$ via a $\overline{b} \to \overline{q}q\overline{q}'$ quark-level process.

C_f is small. If such a second contribution is not suppressed, S_f depends on hadronic parameters and, if the relevant strong phase difference is large, C_f is large.

A summary of $\overline{b} \to \overline{q}q\overline{q}'$ modes with $q' = s$ or d is given in Table 13.1. The $\overline{b} \to \overline{d}d\overline{q}$ transitions lead to final states that are similar to those from $\overline{b} \to \overline{u}u\overline{q}$ transitions and have similar phase dependence. Final states that consist of two vector mesons ($\psi\phi$ and $\phi\phi$) are not CP eigenstates, and angular analysis is needed to separate the CP-even from the CP-odd contributions.

The cleanliness of the theoretical interpretation of S_f can be assessed from the

Table 13.1: Summary of $\overline{b} \to \overline{q}q\overline{q}'$ modes with $q' = s$ or d. The second and third columns give examples of hadronic final states (usually those which are experimentally most convenient to study). The fourth column gives the CKM dependence of the amplitude A_f, using the notation of Eqs. (13.94, 13.96, 13.98), with the dominant term first and the subdominant second. The suppression factor of the second term compared to the first is given in the last column. "Loop" refers to a penguin versus tree-suppression factor (it is mode-dependent and roughly $\mathcal{O}(0.2-0.3)$) and $\lambda \simeq 0.23$ is the expansion parameter of Eq. (13.55).

$\overline{b} \to \overline{q}q\overline{q}'$	$B^0 \to f$	$B_s^0 \to f$	CKM dependence of A_f	Suppression
$\bar{b} \to \bar{c}c\bar{s}$	ψK_S	$\psi\phi$	$(V_{cb}^* V_{cs})T + (V_{ub}^* V_{us})P^u$	loop $\times\, \lambda^2$
$\bar{b} \to \bar{s}s\bar{s}$	ϕK_S	$\phi\phi$	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})P^u$	λ^2
$\bar{b} \to \bar{u}u\bar{s}$	$\pi^0 K_S$	K^+K^-	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})T$	λ^2/loop
$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-	ψK_S	$(V_{cb}^* V_{cd})T + (V_{tb}^* V_{td})P^t$	loop
$\bar{b} \to \bar{s}s\bar{d}$	$K_S K_S$	ϕK_S	$(V_{tb}^* V_{td})P^t + (V_{cb}^* V_{cd})P^c$	$\lesssim 1$
$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\rho^0 K_S$	$(V_{ub}^* V_{ud})T + (V_{tb}^* V_{td})P^t$	loop
$\bar{b} \to \bar{c}u\bar{d}$	$D_{CP}\pi^0$	$D_{CP}K_S$	$(V_{cb}^* V_{ud})T + (V_{ub}^* V_{cd})T'$	λ^2
$\bar{b} \to \bar{c}u\bar{s}$	$D_{CP}K_S$	$D_{CP}\phi$	$(V_{cb}^* V_{us})T + (V_{ub}^* V_{cs})T'$	$\lesssim 1$

information in the last column of Table 13.1. In case of small uncertainties, the expression for S_f in terms of CKM phases can be deduced from the fourth column of Table 13.1 in combination with Eq. (13.91) (and, for $b \to q\overline{q}s$ decays, the example in Eq. (13.97)). Here we consider several interesting examples.

For $B^0 \to J/\psi K_S$ and other $\overline{b} \to \overline{c}c\overline{s}$ processes, we can neglect the P^u contribution to A_f, in the Standard Model, to an approximation that is better than one percent, giving:

$$\lambda_{\psi K_S} = -e^{-2i\beta} \;\Rightarrow\; S_{\psi K_S} = \sin 2\beta\,, \quad C_{\psi K_S} = 0\,. \tag{13.99}$$

It is important to verify experimentally the level of suppression of the penguin contribution. Methods based on flavor symmetries [83–86] allow limits to be obtained. All are currently consistent with the P^u term being negligible.

In the presence of new physics, A_f is still likely to be dominated by the T term, but the mixing amplitude might be modified. We learn that, model-independently, $C_f \approx 0$ while S_f cleanly determines the mixing phase $(\phi_M - 2\arg(V_{cb}V_{cd}^*))$. The experimental measurement [29], $S_{\psi K} = +0.691 \pm 0.017$, gave the first precision test of the Kobayashi-Maskawa mechanism, and its consistency with the predictions for $\sin 2\beta$ makes it very likely that this mechanism is indeed the dominant source of CP violation in the quark sector.

For $B^0 \to \phi K_S$ and other $\overline{b} \to \overline{s}s\overline{s}$ processes (as well as some $\overline{b} \to \overline{u}u\overline{s}$ processes), we can neglect the subdominant contributions, in the Standard Model, to an approximation

that is good to the order of a few percent:

$$\lambda_{\phi K_S} = -e^{-2i\beta} \;\Rightarrow\; S_{\phi K_S} = \sin 2\beta \;, \quad C_{\phi K_S} = 0 \;. \tag{13.100}$$

A review of explicit calculations of the effects of subleading amplitudes can be found in Ref. [87]. In the presence of new physics, both A_f and $\mathbf{M}_{12}$ can have contributions that are comparable in size to those of the Standard Model and carry new weak phases. Such a situation gives several interesting consequences for penguin-dominated $b \to \overline{q}q\overline{s}$ decays ($q = u, d, s$) to a final state f:

1. The value of $-\eta_f S_f$ may be different from $S_{\psi K_S}$ by more than a few percent, where η_f is the CP eigenvalue of the final state.
2. The values of $\eta_f S_f$ for different final states f may be different from each other by more than a few percent (for example, $S_{\phi K_S} \neq S_{\eta' K_S}$).
3. The value of C_f may be different from zero by more than a few percent.

While a clear interpretation of such signals in terms of Lagrangian parameters will be difficult because, under these circumstances, hadronic parameters play a role, any of the above three options will clearly signal new physics. Fig. 13.3 summarizes the present experimental results: none of the possible signatures listed above is unambiguously established, but there is definitely still room for new physics.

For the $\overline{b} \to \overline{u}u\overline{d}$ process $B \to \pi\pi$ and other related channels, the penguin-to-tree ratio can be estimated using SU(3) relations and experimental data on related $B \to K\pi$ decays. The result (for $\pi\pi$) is that the suppression is at the level of $0.2 - 0.3$ and so cannot be neglected. The expressions for $S_{\pi\pi}$ and $C_{\pi\pi}$ to leading order in $R_{PT} \equiv (|V_{tb}V_{td}|\, P^t_{\pi\pi})/(|V_{ub}V_{ud}|\, T_{\pi\pi})$ are:

$$\lambda_{\pi\pi} = e^{2i\alpha}\left[(1 - R_{PT}e^{-i\alpha})/(1 - R_{PT}e^{+i\alpha})\right] \;\Rightarrow$$

$$S_{\pi\pi} \approx \sin 2\alpha + 2\,\mathcal{R}e(R_{PT})\cos 2\alpha \sin\alpha \;, \quad C_{\pi\pi} \approx 2\,\mathcal{I}m(R_{PT})\sin\alpha \;. \tag{13.101}$$

Note that R_{PT} is mode-dependent and, in particular, could be different for $\pi^+\pi^-$ and $\pi^0\pi^0$. If strong phases can be neglected, then R_{PT} is real, resulting in $C_{\pi\pi} = 0$. The size of $C_{\pi\pi}$ is an indicator of how large the strong phase is. The present experimental average is $C_{\pi^+\pi^-} = -0.27 \pm 0.04$ [29]. As concerns $S_{\pi\pi}$, it is clear from Eq. (13.101) that the relative size or strong phase of the penguin contribution must be known to extract α. This is the problem of penguin pollution.

The cleanest solution involves isospin relations among the $B \to \pi\pi$ amplitudes [88]:

$$\frac{1}{\sqrt{2}} A_{\pi^+\pi^-} + A_{\pi^0\pi^0} = A_{\pi^+\pi^0} \;. \tag{13.102}$$

The method exploits the fact that the penguin contribution to $P^t_{\pi\pi}$ is pure $\Delta I = 1/2$ (this is not true for the electroweak penguins which, however, are expected to be small), while the tree contribution to $T_{\pi\pi}$ contains pieces that are both $\Delta I = 1/2$ and $\Delta I = 3/2$.

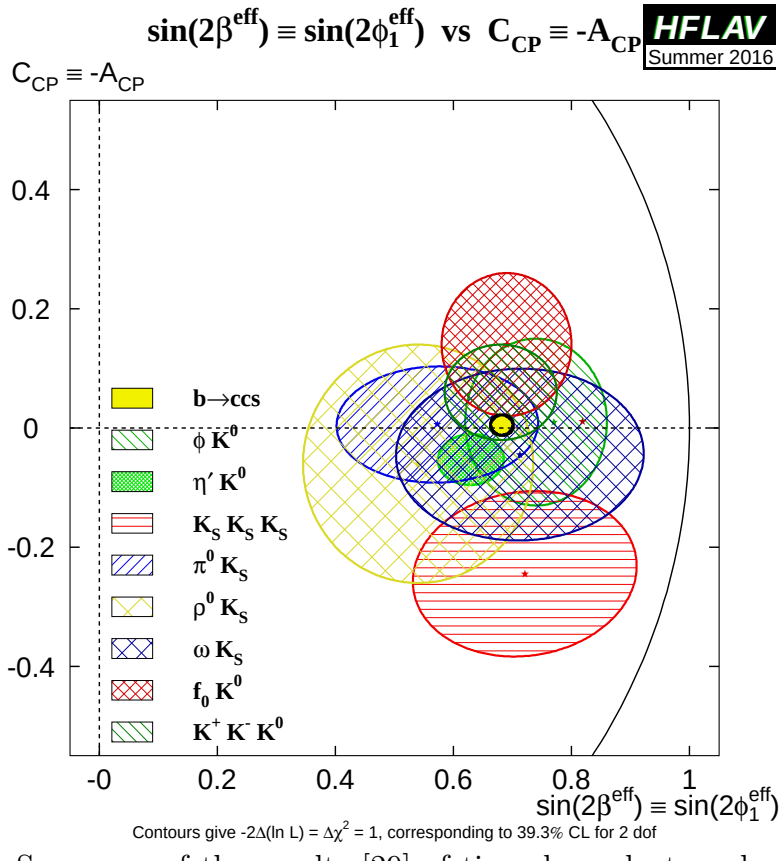

Figure 13.3: Summary of the results [29] of time-dependent analyses of $b \to q\overline{q}s$ decays, which are potentially sensitive to new physics.

A simple geometric construction then allows one to find R_{PT} and extract α cleanly from $S_{\pi^+\pi^-}$. The key experimental difficulty is that one must measure accurately the separate rates for B^0 and $\overline{B}^0 \to \pi^0\pi^0$.

CP asymmetries in $B \to \rho\pi$ and $B \to \rho\rho$ can also be used to determine α. In particular, the $B \to \rho\rho$ measurements are presently very significant in constraining α. The extraction proceeds via isospin analysis similar to that of $B \to \pi\pi$. There are, however, several important differences. First, due to the finite width of the ρ mesons, a final $(\rho\rho)_{I=1}$ state is possible [89]. The effect is, however, of the order of $(\Gamma_\rho/m_\rho)^2 \sim 0.04$. Second, due to the presence of three helicity states for the two vector mesons, angular analysis is needed to separate the CP-even and CP-odd components. The theoretical expectation is that the CP-odd component is small, which is supported by experiments which find that the $\rho^+\rho^-$ and $\rho^\pm\rho^0$ modes are dominantly longitudinally polarized. Third, an important advantage of the $\rho\rho$ modes is that the penguin contribution is expected to be small due to different hadronic dynamics. This expectation is confirmed by the smallness of $\mathcal{B}(B^0 \to \rho^0\rho^0) = (0.95 \pm 0.16) \times 10^{-6}$ [29,90] compared to $\mathcal{B}(B^0 \to \rho^+\rho^-) = (24.2 \pm 3.1) \times 10^{-6}$ [29]. Thus, $S_{\rho^+\rho^-}$ is not far from $\sin 2\alpha$. Finally, both $S_{\rho^0\rho^0}$ and $C_{\rho^0\rho^0}$ are experimentally accessible, which may allow

a precision determination of α. However, a full isospin analysis should allow that the fractions of longitudinal polarisation in B and $\overline{B}$ decays may differ, which has not yet been done by the experiments.

Detailed discussion of the determination of α with these methods, and the latest world average, can be found in Refs. [36,91]. The consistency between the range of α determined by the $B \to \pi\pi$, $\rho\pi$ and $\rho\rho$ measurements and the range allowed by CKM fits (excluding these direct determinations) provides further support to the Kobayashi-Maskawa mechanism.

All modes discussed in this Section so far have possible contributions from penguin amplitudes. As shown in Table 13.1, CP violation can also be studied with final states, typically containing charmed mesons, where no such contribution is possible. The neutral charmed meson must be reconstructed in a final state, such as a CP eigenstate, common to D^0 and $\overline{D}^0$ so that the amplitudes for the B and $\overline{B}$ meson decays interfere. Although there is a second tree amplitude with a different weak phase, the contributions of the different diagrams can in many cases be separated experimentally (for example by exploiting different decays of the $\overline{D}^0$ mesons) making these channels very clean theoretically. The first determination of $\sin(2\beta)$, with significance of CP violation over 5σ, with this method has recently been reported [92]. Moreover, the interference between the two tree diagrams gives sensitivity to γ, as will be discussed in Section 13.6.4.

13.6.3. *CP violation in interference of B_s^0 decays with and without mixing* :

As discussed in Section 13.6.1, the world average for $|q/p|$ in the B_s^0 system currently deviates from the Standard Model expectation due to an anomalous value of the dimuon asymmetry. Attributing the dimuon asymmetry result to a fluctuation, we again neglect the deviation of $|q/p|$ from 1, and use

$$\lambda_f = e^{-i\phi_M(B_s^0)}(\overline{A}_f/A_f) \; . \tag{13.103}$$

Within the Standard Model,

$$e^{-i\phi_{M(B_s^0)}} = (V_{tb}^* V_{ts})/(V_{tb} V_{ts}^*) \; . \tag{13.104}$$

Note that $\Delta\Gamma/\Gamma = 0.122 \pm 0.009$ [29] and therefore y should not be put to zero in Eqs. (13.36, 13.37). However, $|q/p| = 1$ is expected to hold to an even better approximation than for B^0 mesons. One therefore obtains

$$\mathcal{A}_f(t) = \frac{S_f \sin(\Delta m t) - C_f \cos(\Delta m t)}{\cosh\left(\Delta\Gamma t/2\right) - A_f^{\Delta\Gamma} \sinh\left(\Delta\Gamma t/2\right)} \; ,$$

$$A_f^{\Delta\Gamma} \equiv \frac{-2\,\mathcal{R}e(\lambda_f)}{1 + \left|\lambda_f\right|^2} \; . \tag{13.105}$$

The presence of the $A_f^{\Delta\Gamma}$ term implies that information on λ_f can be obtained from analyses that do not use tagging of the initial flavor, through so-called effective lifetime measurements [93].

The $B_s^0 \to J/\psi\phi$ decay proceeds via the $\overline{b} \to \overline{c}c\overline{s}$ transition. The CP asymmetry in this mode thus determines (with angular analysis to disentangle the CP-even and CP-odd components of the final state) $\sin 2\beta_s$, where β_s is defined in Eq. (13.58) [94]. The $B_s^0 \to J/\psi\pi^+\pi^-$ decay, which has a large contributions from $J/\psi f_0(980)$ and is assumed to also proceed dominantly via the $\overline{b} \to \overline{c}c\overline{s}$ transition, has also been used to determine β_s. In this case no angular analysis is necessary, since the final state has been shown to be dominated by the CP-even component [95]. The combination of measurements yields [29]

$$-2\beta_s = 0.021 \pm 0.031\,, \tag{13.106}$$

consistent with the Standard Model prediction, $\beta_s = 0.0185 \pm 0.0003$ [19].

The experimental investigation of CP violation in the B_s^0 sector is still at a relatively early stage, and far fewer modes have been studied than in the B^0 system. First results on the $\overline{b} \to \overline{q}q\overline{s}$ decays $B_s^0 \to \phi\phi$ and K^+K^- have been reported. More channels are expected to be studied in the near future.

13.6.4. *Direct CP violation in the B system* :

An interesting class of decay modes is that of the tree-level decays $B^\pm \to D^{(*)}K^\pm$. These decays provide golden methods for a clean determination of the angle γ [97–101]. The method uses the decays $B^+ \to D^0K^+$, which proceeds via the quark transition $\overline{b} \to \overline{u}c\overline{s}$, and $B^+ \to \overline{D}^0K^+$, which proceeds via the quark transition $\overline{b} \to \overline{c}u\overline{s}$, with the D^0 and $\overline{D}^0$ decaying into a common final state. The decays into common final states, such $(\pi^0K_S)_DK^+$, involve interference effects between the two amplitudes, with sensitivity to the relative phase, $\delta + \gamma$ (δ is the relevant strong phase difference). The CP-conjugate processes are sensitive to $\delta - \gamma$. Measurements of branching ratios and CP asymmetries allow the determination of γ and δ from amplitude triangle relations. The method suffers from discrete ambiguities but, since all hadronic parameters can be determined from the data, has negligible theoretical uncertainty [102].

Unfortunately, the smallness of the CKM-suppressed $b \to u$ transitions makes it difficult to use the simplest methods alone [97–99] to determine γ. These difficulties are overcome (and the discrete ambiguities are removed) by performing a Dalitz plot analysis for multi-body D decays [100,101]. Detailed discussion of the determination of γ with these methods can be found in Ref. [36].

Constraints on γ from combinations of results on various $B \to D^{(*)}K^{(*)}$ processes have been obtained by experiments [103,104]. The latest world average is [29]:

$$\gamma = \left(76.2\,^{+4.7}_{-5.0}\right)^\circ\,.$$

The consistency between the range of γ determined by the $B \to DK$ measurements and the range allowed by CKM fits (excluding these direct determinations) provides further support to the Kobayashi-Maskawa mechanism. As more data become available, determinations of γ from $B_s^0 \to D_s^\mp K^\pm$ [105,106] and $B^0 \to DK^{*0}$ [107–110] are expected to also give competitive measurements.

Decays to the final state $K^\mp\pi^\pm$ provided the first observations of direct CP violation in both B^0 and B_s^0 systems. The asymmetry arises due to interference between tree and

penguin diagrams [111], similar to the effect discussed in Section 13.6.2. In principle, measurements of $\mathcal{A}_{\overline{B}^0\to K^-\pi^+}$ and $\mathcal{A}_{\overline{B}^0_s\to K^+\pi^-}$ could be used to determine the weak phase difference γ, but lack of knowledge of the relative magnitude and strong phase of the contributing amplitudes limits the achievable precision. The uncertainties on these hadronic parameters can be reduced by exploiting flavor symmetries, which predict a number of relations between asymmetries in different modes. One such relation is that the partial rate differences for B^0 and B^0_s decays to $K^\mp\pi^\pm$ are expected to be approximately equal and opposite [112], which is consistent with current data. It is also expected that the partial rate asymmetries for $\overline{B}^0\to K^-\pi^+$ and $B^-\to K^-\pi^0$ should be approximately equal; however, the experimental results currently show a significant discrepancy [29]:

$$\mathcal{A}_{\overline{B}^0\to K^-\pi^+} = -0.082\pm 0.006\ , \qquad \mathcal{A}_{B^-\to K^-\pi^0} = 0.040\pm 0.021\ .$$

It is therefore of great interest to understand whether this originates from Standard Model QCD corrections, or whether it is a signature of new dynamics. Improved tests of a more precise relation between the partial rate differences of all four $K\pi$ final states [113–116], currently limited by knowledge of the CP asymmetry in $\overline{B}^0\to K_S\pi^0$ decays, may help to resolve the situation.

It is also of interest to investigate whether similar patterns appear among the CP violating asymmetries in B meson decays to final states containing one pseudoscalar and one vector meson. Since the vector resonance decays to two particles, such channels can be studied through Dalitz plot analysis of the three-body final state. Model-independent analyses of $B^+\to K^+K^-K^+$, $\pi^+\pi^-K^+$, $\pi^+\pi^-\pi^+$ and $K^+K^-\pi^+$ decays have revealed large CP violation effects in certain regions of phase space [117]. It remains to be seen whether these are associated to particular resonances or to interference effects, which will be necessary to understand the underlying dynamics.

13.7. Summary and Outlook

CP violation has been experimentally established in K and B meson decays. A full list of CP asymmetries that have been measured at a level higher than 5σ is given in the introduction to this review. In Section 13.1.4 we introduced three types of CP-violating effects. Examples of these three types include the following:

1. All three types of CP violation have been observed in $K\to\pi\pi$ decays:

$$\begin{aligned}
\mathcal{R}e(\epsilon') &= \frac{1}{6}\left(\left|\frac{\overline{A}_{\pi^0\pi^0}}{A_{\pi^0\pi^0}}\right| - \left|\frac{\overline{A}_{\pi^+\pi^-}}{A_{\pi^+\pi^-}}\right|\right) = (2.5\pm 0.4)\times 10^{-6} \quad \text{(I)}\\
\mathcal{R}e(\epsilon) &= \frac{1}{2}\left(1-\left|\frac{q}{p}\right|\right) = (1.66\pm 0.02)\times 10^{-3} \quad \text{(II)}\\
\mathcal{I}m(\epsilon) &= -\frac{1}{2}\mathcal{I}m(\lambda_{(\pi\pi)_{I=0}}) = (1.57\pm 0.02)\times 10^{-3}\ . \quad \text{(III)}
\end{aligned} \tag{13.107}$$

2. CP violation in decay has been observed in, for example, $B^0\to K^+\pi^-$ transitions, while CP violation in interference of decays with and without mixing has been

observed in, for example, the $B^0 \to J/\psi K_S$ channel:

$$\begin{aligned} \mathcal{A}_{K^+\pi^-} &= \frac{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 - 1}{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 + 1} = -0.082 \pm 0.006 \quad \text{(I)} \\ S_{\psi K} &= \mathcal{I}m(\lambda_{\psi K}) = +0.691 \pm 0.017\,. \qquad \text{(III)} \end{aligned} \tag{13.108}$$

Based on Standard Model predictions, further observations of CP violation in B^0, B^+ and B_s^0 decays seem likely in the near future, at both LHCb and its upgrades [118–120] as well as the Belle II experiment [121]. The first observation of CP violation in b baryons is also likely to be within reach of LHCb. The same experiments have great potential to improve the sensitivity to CP violation effects in the charm sector, though uncertainty in the Standard Model predictions makes it difficult to forecast whether or not discoveries will be forthcoming. A number of upcoming experiments have potential to make significant progress on rare kaon decays. Observables that are subject to clean theoretical interpretation, such as β from $S_{\psi K_S}$, β_s from $B_s^0 \to J/\psi\phi$, $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu})$ and γ from CP violation in $B \to DK$ decays, are of particular value for constraining the values of the CKM parameters and probing the flavor sector of extensions to the Standard Model. Progress in lattice QCD calculations is also needed to complement the anticipated experimental results. Other probes of CP violation now being pursued experimentally include the electric dipole moments of the neutron and electron, and the decays of tau leptons. Additional processes that are likely to play an important role in future CP studies include top-quark production and decay, Higgs boson decays and neutrino oscillations.

All measurements of CP violation to date are consistent with the predictions of the Kobayashi-Maskawa mechanism of the Standard Model. In fact, it is now established that the KM mechanism plays a major role in the CP violation measured in the quark sector. However, a dynamically-generated matter-antimatter asymmetry of the universe requires additional sources of CP violation, and such sources are naturally generated by extensions to the Standard Model. New sources might eventually reveal themselves as small deviations from the predictions of the KM mechanism, or else might not be observable in the quark sector at all, but observable with future probes such as neutrino oscillations or electric dipole moments. The fundamental nature of CP violation demands a vigorous search.

A number of excellent reviews of CP violation are available [122–129], where the interested reader may find a detailed discussion of the various topics that are briefly reviewed here.

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