82. D^0–$\overline{D}^0$ Mixing

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The detailed formalism for $D^0 - \overline{D}^0$ mixing is presented in the note on "*CP* Violation in the Quark Sector" in this *Review*. For completeness, we present an overview here. The time evolution of the D^0–$\overline{D}^0$ system is described by the Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}, \tag{82.1}$$

where the $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance requires that $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of these matrices describe the dispersive and absorptive parts of the mixing.

The two eigenstates D_1 and D_2 of the effective Hamiltonian matrix ($\mathbf{M}$ - $i\mathbf{\Gamma}$) are given by

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle\,, \tag{82.2}$$

where

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\,. \tag{82.3}$$

The normalization condition is $|p|^2 + |q|^2 = 1$. Our phase convention is $CP|D^0\rangle = +|\overline{D}^0\rangle$, and the sign of the square root is chosen so that D_1 is CP even, or nearly so.

The corresponding eigenvalues are

$$\omega_{1,2} \equiv m_{1,2} - \tfrac{i}{2}\Gamma_{1,2} = \left(M - \tfrac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \tfrac{i}{2}\Gamma_{12}\right)\,, \tag{82.4}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the masses and widths of the $D_{1,2}$.

We define dimensionless mixing parameters x and y by

$$x \equiv (m_1 - m_2)/\Gamma = \Delta m/\Gamma \tag{82.5}$$

and

$$y \equiv (\Gamma_1 - \Gamma_2)/2\Gamma = \Delta\Gamma/2\Gamma\,, \tag{82.6}$$

where $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$. If CP is conserved, then M_{12} and Γ_{12} are real, $\Delta m = 2M_{12}$, $\Delta\Gamma = 2\Gamma_{12}$, and $p = q = 1/\sqrt{2}$. The signs of Δm and $\Delta\Gamma$ are to be determined experimentally.

The parameters x and y are measured in several ways. The most precise values are obtained using the time dependence of D decays. Since D^0–$\overline{D}^0$ mixing is a small effect, the identifying tag of the initial particle as a D^0 or a $\overline{D}^0$ must be extremely accurate. The usual "D^*-tag" is the charge of the distinctive slow pion in the decay sequence $D^{*+} \to D^0\pi^+$ or $D^{*-} \to \overline{D}^0\pi^-$. In current experiments, the probability of mistagging is about 0.1%. The large data samples produced at the B-factories allow the production flavor to also be determined by fully reconstructing charm on the "other side" of the event—significantly reducing the mistag rate [1]. Another tag of comparable accuracy to the D^*-tag is identification of one of the D's produced from $\psi(3770) \to D^0\overline{D}^0$ decays. Although time-dependent analyses are not possible at symmetric charm-threshold facilities (the D^0 and $\overline{D}^0$ do not travel far enough), the quantum-coherent $C = -1$ $\psi(3770) \to D^0\overline{D}^0$ state provides time-integrated sensitivity [2,3].

82.1. Time-Dependent Analyses

We extend the formalism of this *Review*'s note on "CP Violation in Meson Decays." In addition to the "right-sign" instantaneous decay amplitudes $\overline{A}_f \equiv \langle f|H|\overline{D}^0\rangle$ and $A_{\overline{f}} \equiv \langle \overline{f}|H|D^0\rangle$ for final states $f = K^+\pi^-, ...$ and their CP conjugate $\overline{f} = K^-\pi^+, ...$, we include "wrong-sign" amplitudes $\overline{A}_{\overline{f}} \equiv \langle \overline{f}|H|\overline{D}^0\rangle$ and $A_f \equiv \langle f|H|D^0\rangle$.

It is conventional to normalize the wrong-sign decay distributions to the integrated rate of right-sign decays and to express time in units of the precisely measured neutral D-meson mean lifetime, $\tau_{D^0} = 1/\Gamma = 2/(\Gamma_1 + \Gamma_2)$. Starting from a pure $|D^0\rangle$ or $|\overline{D}^0\rangle$ state at $t = 0$, the time-dependent rates of decay to wrong-sign final states relative to the integrated right-sign decay rates are, to leading order:

$$r(t) \equiv \frac{\left|\langle f|H|D^0(t)\rangle\right|^2}{\left|\overline{A}_f\right|^2} = \left|\frac{q}{p}\right|^2 \left|g_+(t)\,\lambda_f^{-1} + g_-(t)\right|^2 , \tag{82.7}$$

and

$$\overline{r}(t) \equiv \frac{\left|\langle \overline{f}|H|\overline{D}^0(t)\rangle\right|^2}{\left|A_{\overline{f}}\right|^2} = \left|\frac{p}{q}\right|^2 \left|g_+(t)\,\lambda_{\overline{f}} + g_-(t)\right|^2 . \tag{82.8}$$

where

$$\lambda_f \equiv q\overline{A}_f/pA_f\,, \quad \lambda_{\overline{f}} \equiv q\overline{A}_{\overline{f}}/pA_{\overline{f}}\,, \tag{82.9}$$

and

$$g_\pm(t) = \frac{1}{2}\left(e^{-iz_1 t} \pm e^{-iz_2 t}\right), \quad z_{1,2} = \frac{\omega_{1,2}}{\Gamma}\,. \tag{82.10}$$

For multibody final states these equations apply separately to each point in phase-space. Note that a change in the convention for the relative phase of D^0 and $\overline{D}^0$ would cancel between q/p and $\overline{A}_f/A_f$ and leave λ_f unchanged. We expand $r(t)$ and $\overline{r}(t)$ to second order in x and y for modes in which the ratio of decay amplitudes, $R_D = |A_f/\overline{A}_f|^2$, is very small. Integrating over regions of phase-space leads to interesting effects. See discussion below on multibody decays and the "Review of Multibody Charm Analyses" in this *Review* [25].

82.2. Semileptonic decays

Consider the final state $f = K^+\ell^-\overline{\nu}_\ell$, where $A_f = \overline{A}_{\overline{f}} = 0$ is a very good approximation in the Standard Model. The final state f is only accessible through mixing (and tree-level second-order weak process which we neglect) and $r(t)$ is

$$r(t) = |g_-(t)|^2 \left|\frac{q}{p}\right|^2 \approx \frac{e^{-t}}{4}(x^2 + y^2)\,t^2 \left|\frac{q}{p}\right|^2 . \tag{82.11}$$

For $\overline{r}(t)$ q/p is replaced by p/q. In the Standard Model, CP violation in charm mixing is small and $|q/p| \approx 1$. In the limit of CP conservation, $r(t) = \overline{r}(t)$, and the time-integrated mixing rate relative to the time-integrated right-sign decay rate for semileptonic decays is

$$R_M = \int_0^\infty r(t)dt = \left|\frac{q}{p}\right|^2 \frac{x^2 + y^2}{2 + x^2 - y^2} \approx \frac{1}{2}(x^2 + y^2)\,. \tag{82.12}$$

Table 82.1: Results for R_M in D^0 semileptonic decays†.

Year	Exper.	Final state(s)	R_M ($\times 10^{-3}$)	90% C.L.
2008	Belle [4]	$K^{(*)+}e^-\overline{\nu}_e$	$0.13\pm0.22\pm0.20$	$< 0.61 \times 10^{-3}$
2007	BaBar [1]	$K^{(*)+}e^-\overline{\nu}_e$	$0.04^{+0.70}_{-0.60}$	$(-1.3, 1.2) \times 10^{-3}$
2005*	Belle [5]	$K^{(*)+}e^-\overline{\nu}_e$	$0.02\pm0.47\pm0.14$	$< 1.0 \times 10^{-3}$
2005	CLEO [6]	$K^{(*)+}e^-\overline{\nu}_e$	$1.6\pm2.9\pm2.9$	$< 7.8 \times 10^{-3}$
2004*	BaBar [7]	$K^{(*)+}e^-\overline{\nu}_e$	$2.3\pm1.2\pm0.4$	$< 4.2 \times 10^{-3}$
1996	E791 [8]	$K^+\ell^-\overline{\nu}_\ell$	$(1.1^{+3.0}_{-2.7}) \times 10^{-3}$	$< 5.0 \times 10^{-3}$
	HFLAV [9]		0.13 ± 0.27	

*These measurements are excluded from the HFLAV average. The statistical correlation of the BaBar result with Ref. 1 has not been established and the Belle result is superseded by Ref. 4. The HFLAV average of semileptonic results assumes reported statistical and systematic uncertainties are uncorrelated.

† More recently, the LHCb experiment [10] has reported the observation of charm mixing, $R_M = (9.6 \pm 3.6) \times 10^{-5}$ with 8.2σ significance, in a time dependent analysis of the ratio of $D^0 \to K^+\pi^-\pi^+\pi^-$ and $D^0 \to K^-\pi^+\pi^-\pi^+$ decay rates.

Table 82.1 summarizes results for R_M from semileptonic decays; the world average from the Heavy Flavor Averaging Group (HFLAV) [9] is $R_M = (1.30 \pm 2.69) \times 10^{-4}$.

82.3. Wrong-sign decays to hadronic non-CP eigenstates

Consider the final state $f = K^+\pi^-$, where A_f is doubly Cabibbo-suppressed. The ratio of decay amplitudes is

$$\frac{A_f}{\overline{A}_f} = -\sqrt{R_D}\, e^{-i\delta_f}, \qquad \left|\frac{A_f}{\overline{A}_f}\right| \sim O(\tan^2\theta_c)\,, \tag{82.13}$$

where R_D is the doubly Cabibbo-suppressed (DCS) decay rate relative to the Cabibbo-favored (CF) rate, δ_f is the strong phase difference between DCS and CF processes, and θ_c is the Cabibbo angle. The minus sign originates from the sign of V_{us} relative to V_{cd}.

We characterize the violation of CP with the real-valued parameters A_M, A_D, and ϕ. We adopt the parametrization (see Refs. 11 and 12)

$$\left|\frac{q}{p}\right|^2 = \sqrt{\frac{1+A_M}{1-A_M}}\,, \tag{82.14}$$

$$\lambda_f^{-1} \equiv \frac{pA_f}{q\overline{A}_f} = -\sqrt{R_D}\left(\frac{(1+A_D)(1-A_M)}{(1-A_D)(1+A_M)}\right)^{1/4} e^{-i(\delta_f+\phi)}\,, \tag{82.15}$$

$$\lambda_{\overline{f}} \equiv \frac{q\overline{A}_{\overline{f}}}{pA_{\overline{f}}} = -\sqrt{R_D}\left(\frac{(1-A_D)(1+A_M)}{(1+A_D)(1-A_M)}\right)^{1/4} e^{-i(\delta_f - \phi)} , \qquad (82.16)$$

and A_D is a measure of direct CP violation, while A_M is a measure of CP violation in mixing. From these relations, we obtain

$$\sqrt{\frac{1+A_D}{1-A_D}} = \frac{|A_f/\overline{A}_f|}{|\overline{A}_{\overline{f}}/A_{\overline{f}}|} , \qquad (82.17)$$

The angle ϕ measures CP violation in interference between mixing and decay. While A_M is independent of the decay process, A_D and ϕ, in general, depend on f. However, in the Standard Model the weak phase of $\frac{\overline{A}_{\overline{f}}}{A_{\overline{f}}}$ is negligible and ϕ is usually taken to be universal.

In general, $\lambda_{\overline{f}}$ and λ_f^{-1} are independent complex numbers. More detail on CP violation in meson decays can be found in Ref. 13. To leading order, for A_D, $A_M \ll 1$,

$$r(t) = e^{-t}\left[R_D(1+A_D) + \sqrt{R_D(1+A_M)(1+A_D)}\, y'_- t + \frac{1}{2}(1+A_M)R_M\, t^2\right] \qquad (82.18)$$

and

$$\overline{r}(t) = e^{-t}\left[R_D(1-A_D) + \sqrt{R_D(1-A_M)(1-A_D)}\, y'_+ t + \frac{1}{2}(1-A_M)R_M\, t^2\right] \qquad (82.19)$$

Here

$$y'_\pm \equiv y'\cos\phi \pm x'\sin\phi = y\cos(\delta_{K\pi} \mp \phi) - x\sin(\delta_{K\pi} \mp \phi) , \qquad (82.20)$$

where

$$x' \equiv x\cos\delta_{K\pi} + y\sin\delta_{K\pi}, \quad y' \equiv y\cos\delta_{K\pi} - x\sin\delta_{K\pi} , \qquad (82.21)$$

and $R_M = \left(x^2 + y^2\right)/2 = \left(x'^2 + y'^2\right)/2$ is the mixing rate relative to the time-integrated Cabibbo-favored rate.

The three terms in Eq. (82.18) and Eq. (82.19) probe the three fundamental types of CP violation. In the limit of CP conservation, A_M, A_D, and ϕ are all zero. Then

$$r(t) = \overline{r}(t) = e^{-t}\left(R_D + \sqrt{R_D}\, y' t + \frac{1}{2}R_M\, t^2\right) , \qquad (82.22)$$

and the time-integrated wrong-sign rate relative to the integrated right-sign rate is

$$R = \int_0^\infty r(t)\,dt = R_D + \sqrt{R_D}\,y' + R_M\,. \tag{82.23}$$

The ratio R is the most readily accessible experimental quantity. In Table 82.2 are reported the measurements of R, R_D and A_D in $D^0 \to K^+\pi^-$, and their HFLAV average [9] from a general fit that allows for both mixing and CP violation. Typically, the fit parameters are R_D, x'^2, and y'. Table 82.3 summarizes the results for x'^2 and y'. Allowing for CP violation, the separate contributions to R can be extracted by fitting the $D^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^-\pi^+$ decay rates.

Table 82.2: Results for R, R_D, and A_D in $D^0 \to K^+\pi^-$.

Year	Exper.	$R(\times 10^{-3})$	$R_D(\times 10^{-3})$	$A_D(\%)$
2017	LHCb [14]	—	3.53 ± 0.05	-1.7 ± 1.6
2014	Belle [15]	3.86 ± 0.06	3.53 ± 0.13	—
2013*	LHCb [16]	—	3.57 ± 0.07	-0.7 ± 1.9
2013	CDF [17]	4.30 ± 0.05	3.51 ± 0.35	—
2012*	LHCb [18]	4.25 ± 0.04	3.52 ± 0.15	—
2007*	CDF [19]	4.15 ± 0.10	3.04 ± 0.55	—
2007	BaBar [20]	$3.53\pm0.08\pm0.04$	$3.03\pm0.16\pm0.10$	$-2.1\pm5.2\pm1.5$
2006*	Belle [21]	$3.77\pm0.08\pm0.05$	3.64 ± 0.18	2.3 ± 4.7
2005†	FOCUS [22]	$4.29^{+0.63}_{-0.61}\pm0.28$	$5.17^{+1.47}_{-1.58}\pm0.76$	$13^{+33}_{-25}\pm10$
2000†	CLEO [23]	$3.32^{+0.63}_{-0.65}\pm0.40$	$4.8\pm1.2\pm0.4$	$-1^{+16}_{-17}\pm1$
1998†	E791 [24]	$6.8^{+3.4}_{-3.3}\pm0.7$	—	—
	Average		3.485 ± 0.035 [9]	-0.88 ± 0.99 [9]

*These measurements are excluded from the HFLAV average of R_D. The CDF result is superseded by Ref. 17 and the LHCb results are superseded by Ref. 14. The Belle result for R and R_D is superseded by Ref. 15.

†These measurements are excluded from the HFLAV average due to poor precision.

The non trivial dependence of the efficiency as a function of decay time may explain why the values of R reported by experiments at hadron colliders are systematically larger (where online selection criteria favor D decays with longer decay times).

Extraction of the mixing parameters x and y from the results in Table 82.3 requires knowledge of the relative strong phase $\delta_{K\pi}$. An interference effect that provides useful sensitivity to $\delta_{K\pi}$ arises in the decay chain $\psi(3770) \to D^0\overline{D}^0 \to (f_{CP})(K^+\pi^-)$, where f_{CP} denotes a CP-even or -odd eigenstate from D^0 decay, such as K^+K^- or $K^0_S\pi^0$, respectively [26]. Here, the amplitude relation

$$\sqrt{2}\,A(D_\pm \to K^-\pi^+) = A(D^0 \to K^-\pi^+) \pm A(\overline{D}^0 \to K^-\pi^+). \tag{82.24}$$

Table 82.3: Results on the time-dependence of $r(t)$ in $D^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^-\pi^+$ decays. The Belle 2014 and CDF results assume no CP violation. The Belle 2006 results restrict x'^2 to the physical region. The BaBar confidence intervals are obtained from the fit, whereas Belle uses a Feldman-Cousins method, and CDF uses a Bayesian method.

Year	Exper.	y' (%)	$x'^{\,2}$ ($\times 10^{-3}$)
2017	LHCb [14]	0.52±0.08	0.036±0.043
2014*†	Belle [15]	0.46±0.34	0.09±0.22
2013*	LHCb [16]	0.48±0.10	0.055±0.049
2013	CDF [17]	0.43±0.43	0.08±0.18
2012*	LHCb [18]	0.72±0.24	−0.09±0.13
2007*	CDF [19]	0.85±0.76	−0.12±0.35
2007	BaBar [20]	0.97±0.44±0.31	−0.22±0.30±0.21
2006†	Belle [21]	$-2.8 < y' < 2.1$	< 0.72 (95% C.L.)

*These measurements are excluded from the HFLAV average. The CDF result is superseded by Ref. 17 and the LHCb results have been superseded by Ref. 14.

† This Belle result allows for CP violation. HFLAV uses this result for the CP-violation allowed fit. This result is not superseded by Ref. 15.

*† This Belle result does not allow for CP violation. HFLAV uses this result for the CP-conserving fit. This result does not supersede Ref. 21.

where $D_\pm$ denotes a CP-even or -odd eigenstate, implies that

$$\cos\delta_{K\pi} = \frac{|A(D_+ \to K^-\pi^+)|^2 - |A(D_- \to K^-\pi^+)|^2}{2\sqrt{R_D}\,|A(D^0 \to K^-\pi^+)|^2} . \tag{82.25}$$

This neglects CP violation.

The asymmetry of CP-tagged D decays rates to $K^-\pi^+$ is denoted as

$$A^{CP}_{K\pi} \equiv \frac{|A(D_- \to K^-\pi^+)|^2 - |A(D_+ \to K^-\pi^+)|^2}{|A(D_- \to K^-\pi^+)|^2 + |A(D_+ \to K^-\pi^+)|^2}. \tag{82.26}$$

To lowest order in the mixing parameters [2,3]

$$2\sqrt{R_D}\cos\delta_{K\pi} + y = (1+R)\dot{A}^{CP}_{K\pi} \tag{82.27}$$

where R is the time-integrated wrong-sign rate relative to the integrated right-sign rate from Eq. (82.23).

82.3.1. *Wrong-sign decays to multibody final states* :

For multibody final states, Eqs. (82.13)–(82.23) apply separately to each point in phase-space. Although x and y do not vary across the space, knowledge of the resonant substructure is needed to extrapolate the strong phase difference δ from point to point to determine x and y. Model-independent methods to measure D mixing parameters require input related to the relative phases of the D^0 and $\overline{D}^0$ decay amplitudes across the phase-space distribution [25]. The required phase information is accessible at the charm threshold, where CLEO-c and BESIII operate [26,27].

A time-dependent analysis of the process $D^0 \to K^+\pi^-\pi^0$ from BaBar [29,30] determines the *relative* strong phase variation across the Dalitz plot and reports $x'' = (2.61^{+0.57}_{-0.68} \pm 0.39)\%$, and $y'' = (-0.06^{+0.55}_{-0.64} \pm 0.34)\%$, where x'' and y'' are defined as

$$
\begin{aligned}
x'' &\equiv x \cos\delta_{K\pi\pi^0} + y \sin\delta_{K\pi\pi^0} \, , \\
y'' &\equiv y \cos\delta_{K\pi\pi^0} - x \sin\delta_{K\pi\pi^0} \, ,
\end{aligned}
\tag{82.28}
$$

in analogy with x', y', and $\delta_{K\pi}$ of Eq. (82.21). Here $\delta_{K\pi\pi^0}$ is the remaining strong phase difference between the DCS $D^0 \to K^+\rho^-$ and the CF $\overline{D}^0 \to K^+\rho^-$ amplitudes and does not vary across the Dalitz plot. Both strong phases, $\delta_{K\pi}$ and $\delta_{K\pi\pi^0}$, can be determined from time-integrated CP asymmetries in correlated $D^0\overline{D}^0$ produced at the $\psi(3770)$ [26,27].

For the decay modes D^0 and $\overline{D}^0 \to K^+\pi^-\pi^+\pi^-$, Belle observed $R = (0.324 \pm 0.008 \pm 0.007)\%$ [28]. Subsequently, a phase–space integrated analysis from LHCb using charm threshold data at CLEO-c has yielded the observation of charm mixing with 8.2σ significance.

Both the sign and magnitude of x and y without phase or sign ambiguity may be measured using the time-dependent resonant substructure of multibody D^0 decays [31,32]. In $D^0 \to K^0_S\pi^+\pi^-$, the DCS and CF decay amplitudes populate the same Dalitz plot, which allows direct measurement of the relative strong phases. CLEO [33], Belle [32,35], and BaBar [34] have measured the relative phase between $D^0 \to K^*(892)^-\pi^+$ and $D^0 \to K^*(892)^+\pi^-$ to be $(189 \pm 10 \pm 3^{+15}_{-5})^\circ$, $(173.9 \pm 0.7$ (stat. only)$)^\circ$, and $(177.6 \pm 1.1$ (stat. only)$)^\circ$, respectively. These results are close to the 180° expected from Cabibbo factors and a small strong phase. The LHCb [36] analysis for x, y is decay-model independent. The model of resonances in the multibody final state is replaced by strong-phase measurements from CLEO-c [38]. Table 82.4 summarizes the results of time-dependent multibody analyses.

In addition, Belle [32,35] has results for both the relative phase (statistical errors only) and ratio R (central values only) of the DCS fit fraction relative to the CF fit fractions for $K^*(892)^+\pi^-$, $K^*_0(1430)^+\pi^-$, $K^*_2(1430)^+\pi^-$, $K^*(1410)^+\pi^-$, and $K^*(1680)^+\pi^-$. Similarly, BaBar [34,39,40] has reported central values for R for $K^*(892)^+\pi^-$, $K^*_0(1430)^+\pi^-$, and $K^*_2(1430)^+\pi^-$. The systematic uncertainties on R are not evaluated. The large differences in R among these final states could point to an interesting role for hadronic effects.

Table 82.4: Results from time-dependent multibody analyses. The errors are statistical, experimental systematic, and decay-model systematic, respectively. BaBar 2016 reports a combined systematic error. The LHCb result is decay-model independent utilizing strong-phase measurements from CLEO-c [38]

No CP Violation				
Year	Exper.	Final State(s)	$x\ \times 10^{-3}$	$y\ \times 10^{-3}$
2016	BaBar [37]	$\pi^+\pi^-\pi^0$	$15 \pm 12 \pm 6$	$2 \pm 9 \pm 5$
2016	LHCb [36]	$K^0_S\pi^+\pi^-$	$-8.6 \pm 5.3 \pm 1.7$	$0.3 \pm 4.6 \pm 1.3$
2014	Belle [35]	$K^0_S\pi^+\pi^-$	$5.6 \pm 1.9\,^{+0.3}_{-0.9}\,^{+0.6}_{-0.9}$	$3.0 \pm 1.5\,^{+0.4}_{-0.5}\,^{+0.3}_{-0.6}$
2010	BaBar [34]	$K^0_S\pi^+\pi^-$, $K^0_S K^+K^-$	$1.6{\pm}2.3{\pm}1.2{\pm}0.8$	$5.7{\pm}2.0{\pm}1.3{\pm}0.7$
2007	Belle [32]	$K^0_S\pi^+\pi^-$	$8.0 \pm 2.9\,^{+0.9}_{-0.7}\,^{+1.0}_{-1.4}$	$3.3 \pm 2.4\,^{+0.8}_{-1.2}\,^{+0.6}_{-0.8}$
2005	CLEO [31]	$K^0_S\pi^+\pi^-$	$19\,^{+32}_{-33} \pm 4 \pm 4$	$-14 \pm 24 \pm 8 \pm 4$

With CP Violation				
Year	Exper.	Final State(s)	$\lvert q/p\rvert$	ϕ
2014	Belle [35]	$K^0_S\pi^+\pi^-$	$0.90\,^{+0.16}_{-0.15}\,^{+0.05}_{-0.04}\,^{+0.06}_{-0.05}$	$(-6 \pm 11 \pm 3\,^{+3}_{-4})^\circ$
2007*	Belle [32]	$K^0_S\pi^+\pi^-$	$0.86\,^{+0.30}_{-0.29}\,^{+0.06}_{-0.03} \pm 0.08$	$(-14\,^{+16}_{-18}\,^{+5}_{-3}\,^{+2}_{-4})^\circ$

* This result allows for all CP violations and is superseded by Ref. [35] that assumes no direct CP violation in DCS decays.

82.4. Decays to CP Eigenstates

When the final state f is a CP eigenstate, there is no distinction between f and $\overline{f}$, and $A_f = A_{\overline{f}}$ and $\overline{A}_{\overline{f}} = \overline{A}_f$. We denote final states with CP eigenvalues ± 1 by $f_\pm$ and write $\lambda_\pm$ for $\lambda_{f_\pm}$.

The quantity y may be measured by comparing the rate for D^0 decays to non-CP eigenstates such as $K^-\pi^+$ with decays to CP eigenstates such as K^+K^- [12]. If decays to K^+K^- have a shorter effective lifetime than those to $K^-\pi^+$, y is positive.

In the limit of slow mixing ($x, y \ll 1$) and the absence of direct CP violation ($A_D = 0$), but allowing for small indirect CP violation ($|A_M|, |\phi| \ll 1$), we can write

$$\lambda_\pm = \left|\frac{q}{p}\right| e^{\pm i\phi} \ . \tag{82.29}$$

In this scenario, to a good approximation, the decay rates for states that are initially D^0 and $\overline{D}^0$ to a CP eigenstate have exponential time dependence:

$$r_\pm(t) \propto \exp\left(-t/\tau_\pm\right) \ , \tag{82.30}$$

$$\overline{r}_\pm(t) \propto \exp\left(-t/\overline{\tau}_\pm\right) \ , \tag{82.31}$$

where τ is measured in units of $1/\Gamma$.

The effective lifetimes are given by

$$1/\tau_{\pm} = 1 \pm \left|\frac{q}{p}\right| (y\cos\phi - x\sin\phi) \ , \tag{82.32}$$

$$1/\overline{\tau}_{\pm} = 1 \pm \left|\frac{p}{q}\right| (y\cos\phi + x\sin\phi) \ . \tag{82.33}$$

The effective decay rate to a CP eigenstate combining both D^0 and $\overline{D}^0$ decays is

$$r_{\pm}(t) + \overline{r}_{\pm}(t) \propto e^{-(1 \pm y_{CP})t} \ . \tag{82.34}$$

Here

$$\begin{aligned} y_{CP} &= \frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) x\sin\phi \\ &\approx y\cos\phi - A_M x\sin\phi \ . \end{aligned} \tag{82.35}$$

If CP is conserved, $y_{CP} = y$.

All measurements of y_{CP} are relative to the $D^0 \to K^-\pi^+$ decay rate. Table 82.5 summarizes the current status of measurements. Belle [46], BaBar [47], LHCb [48], and CDF [44] have reported y_{CP} and the decay-rate asymmetry for CP even final states (assuming $A_D = 0$)

$$\begin{aligned} A_\Gamma &= \frac{\overline{\tau}_+ - \tau_+}{\overline{\tau}_+ + \tau_+} = \frac{(1/\tau_+) - (1/\overline{\tau}_+)}{(1/\tau_+) + (1/\overline{\tau}_+)} \\ &\approx \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) x\sin\phi \\ &\approx A_M y\cos\phi - x\sin\phi \ . \end{aligned} \tag{82.36}$$

Belle [50] has also reported y_{CP} for the final state $K^0_S K^+ K^-$ which is dominated by the CP odd final state $K^0_S\phi$. If CP is conserved, $A_\Gamma = 0$.

Substantial work on the time-integrated CP asymmetries in decays to CP eigenstates are summarized in this *Review* [58]. Table 82.6 summarizes the current status of measurements of the difference in time-integrated CP asymmetries, $\Delta A_{CP} = A_K - A_\pi$, between $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$. The HFLAV fit is consistent with no CP violation at the 6.5% Confidence Level [9].

Table 82.5: Results for y_{CP} from $D^0 \to K^+K^-$ and $\pi^+\pi^-$.

Year	Exper.	final state(s)	y_{CP}(%)	$A_\Gamma(\times 10^{-3})$
2016	LHCb [41]	$K^+K^-, \pi^+\pi^-$	—	$-0.13 \pm 0.28 \pm 0.10$
2016	LHCb [41]	K^+K^-	—	$-0.30 \pm 0.32 \pm 0.10$
2016	LHCb [41]	$\pi^+\pi^-$	—	$0.46 \pm 0.58 \pm 0.12$
2015	LHCb [42]	$K^+K^-, \pi^+\pi^-$	—	-1.25 ± 0.73
2015	LHCb [42]	K^+K^-	—	$-1.34 \pm 0.77\,^{+0.26}_{-0.34}$
2015	LHCb [42]	$\pi^+\pi^-$	—	$-0.92 \pm 1.45\,^{+0.25}_{-0.33}$
2015	BES III [43]	$K^0_S\pi^0, K^0_S\eta, K^0_S\omega$ $K^+K^-, \pi^+\pi^-, K^0_S\pi^0\pi^0$	$-2.0 \pm 1.3 \pm 0.7$	—
2014	CDF [44]	$K^+K^-, \pi^+\pi^-$	—	-1.2 ± 1.2
2014	CDF [44]	K^+K^-	—	$-1.9 \pm 1.5 \pm 0.4$
2014	CDF [44]	$\pi^+\pi^-$	—	$-0.1 \pm 1.8 \pm 0.3$
2013*	LHCb [45]	K^+K^-	—	$-0.35 \pm 0.62 \pm 0.12$
2013*	LHCb [45]	$\pi^+\pi^-$	—	$0.33 \pm 1.06 \pm 0.14$
2012	Belle [46]	$K^+K^-,\pi^+\pi^-$	$1.11 \pm 0.22 \pm 0.09$	$-0.3 \pm 2.0 \pm 0.7$
2012	BaBar [47]	$K^+K^-,\pi^+\pi^-$	$0.72 \pm 0.18 \pm 0.12$	$0.9 \pm 2.6 \pm 0.6$
2011	LHCb [48]	K^+K^-	$0.55 \pm 0.63 \pm 0.41$	$-5.9 \pm 5.9 \pm 2.1$
2009*	BaBar [49]	K^+K^-	$1.16 \pm 0.22 \pm 0.18$	—
2009	Belle [50]	$K^0_S K^+K^-$	$0.11 \pm 0.61 \pm 0.52$	—
2008*	BaBar [51]	$K^+K^-,\pi^+\pi^-$	$1.03 \pm 0.33 \pm 0.19$	$2.6 \pm 3.6 \pm 0.8$
2007*	Belle [52]	$K^+K^-,\pi^+\pi^-$	$1.31 \pm 0.32 \pm 0.25$	$0.1 \pm 3.0 \pm 1.5$
2003*	BaBar [53]	$K^+K^-,\pi^+\pi^-$	$0.8 \pm 0.4^{+0.5}_{-0.4}$	—
2001	CLEO [54]	$K^+K^-,\pi^+\pi^-$	$-1.2 \pm 2.5 \pm 1.4$	—
2001	Belle† [55]	K^+K^-	$-0.5 \pm 1.0^{+0.7}_{-0.8}$	—
2000	FOCUS [56]	K^+K^-	$3.42 \pm 1.39 \pm 0.74$	—
1999	E791 [57]	K^+K^-	$0.8 \pm 2.9 \pm 1.0$	—
	HFLAV [9]		0.835 ± 0.155	-0.32 ± 0.26

*These measurements are excluded from the HFLAV average. The BaBar results are superseded by Ref. 47 and the Belle result has been superseded by Ref. 46. The LHCb results Ref. 41 and Ref. 42 use different tagging methods, D^* and semimuonic, respectively, and thus are independent. Ref. 41 supersedes the 2013 LHCb results.

†This measurement is included in the result reported by Ref. 46.

82.5. Coherent $D^0\overline{D}^0$ Analyses

Measurements of R_D, $\cos\delta_{K\pi}$, $\sin\delta_{K\pi}$, x, and y can be determined simultaneously from a combined fit to the time-integrated single-tag (ST) and double-tag (DT) yields in correlated $D^0\overline{D}^0$ produced at the $\psi(3770)$ [26,27].

Table 82.6: Results for the difference in time-integrated CP asymmetries ΔA_{CP} between $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$.

Year	Exper.	$\Delta A_{CP}(\times 10^{-3})$
2016	LHCb [59]	$-1.0 \pm 0.8 \pm 0.3$
2014	LHCb [60]	$1.4 \pm 1.6 \pm 0.8$
2013	CDF [61]	$-6.2 \pm 2.1 \pm 1.0$
2012	Belle [15]	$-8.7 \pm 4.1 \pm 0.6$
2008	BaBar [62]	$2.4 \pm 6.2 \pm 2.6$

Due to quantum correlations in the $C = -1$ and $C = +1$ $D^0\overline{D}^0$ pairs produced in the reactions $e^+e^- \to D^0\overline{D}^0(\pi^0)$ and $e^+e^- \to D^0\overline{D}^0\gamma(\pi^0)$, respectively, the time-integrated $D^0\overline{D}^0$ decay rates are sensitive to interference between amplitudes for indistinguishable final states. The size of this interference is governed by the relevant amplitude ratios and can include contributions from D^0–$\overline{D}^0$ mixing.

The following categories of final states are considered:

***f* or $\overline{f}$:** Hadronic states accessed from either D^0 or $\overline{D}^0$ decay but that are not CP eigenstates. An example is $K^-\pi^+$, which results from Cabibbo-favored D^0 transitions or DCS $\overline{D}^0$ transitions.

ℓ^+ or ℓ^-: Semileptonic or purely leptonic final states, which, in the absence of mixing, tag unambiguously the flavor of the parent D^0.

f_+ or f_-: CP-even and CP-odd eigenstates, respectively.

The decay rates for $D^0\overline{D}^0$ pairs to all possible combinations of the above categories of final states are calculated in Ref. 2, for both $C = -1$ and $C = +1$, reproducing the work of Ref. 3. Such $D^0\overline{D}^0$ combinations, where both D final states are specified, are double tags. In addition, the rates for single tags, where either the D^0 or $\overline{D}^0$ is identified and the other neutral D decays generically are given in Ref. 2.

BESIII has reported results using 2.92 pb^{-1} of $e^+e^- \to \psi(3770)$ data where the quantum-coherent $D^0\overline{D}^0$ pairs are in the $C = -1$ state. The values of $y_{CP} = (-2.0 \pm 1.3 \pm 0.7)\%$ [43] and $A^{CP}_{K\pi} = (12.7 \pm 1.3 \pm 0.7)\%$ [66] are determined from DT yields including a CP eigenstate vs semileptonic and vs $K\pi$, respectively. For y_{CP}, the CP eigenstates included are K^-K^+ (f_+), $\pi^+\pi^-$ (f_+), $K^0_S\pi^0\pi^0$ (f_+), $K^0_S\pi^0$ (f_-), $K^0_S\eta$ (f_-), and $K^0_S\omega$ (f_-). For $A^{CP}_{K\pi}$, the additional CP eigenstates included are $\pi^0\pi^0$ (f_+) and $\rho^0\pi^0$ (f_+). Using the external inputs of R_D and y from HFLAV [67] and R from PDG [68]- – see Eq. (82.27) — they obtain $\cos\delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01$ [66] where the third uncertainty is due to the external inputs.

CLEO-c has reported results using 818 pb^{-1} of $e^+e^- \to \psi(3770)$ data [63–65]. The values of y, R_M, $\cos\delta_{K\pi}$, and $\sin\delta_{K\pi}$ are determined from a combined fit to the ST (hadronic only) and DT yields. The hadronic final states included are $K^-\pi^+$ (f), $K^+\pi^-$ ($\overline{f}$), K^-K^+ (f_+), $\pi^+\pi^-$ (f_+), $K^0_S\pi^0\pi^0$ (f_+), $K^0_L\pi^0$ (f_+), $K^0_L\eta$ (f_+), $K^0_L\omega$ (f_+), $K^0_S\pi^0$ (f_-), $K^0_S\eta$ (f_-), $K^0_S\omega$ (f_-), and $K^0_L\pi^0\pi^0$ (f_-), and $K^0_S\pi^+\pi^-$ (mixure of f,$\overline{f}$, f_+, and

f_-). The two flavored final states, $K^-\pi^+$ and $K^+\pi^-$, can be reached via CF or DCS transitions.

Semileptonic DT yields are also included, where one D is fully reconstructed in one of the hadronic modes listed above, and the other D is partially reconstructed in either $D \to Ke\nu$ or $D \to K\mu\nu$. When the lepton is accompanied by a flavor tag ($D \to K^-\pi^+$ or $K^+\pi^-$), both the "right-sign" and "wrong-sign" DT samples are used, where the electron and kaon charges are the same and opposite, respectively.

The main results of the CLEO-c analysis are the determination of $\cos\delta_{K\pi} = 0.81^{+0.22+0.07}_{-0.18-0.05}$, $\sin\delta_{K\pi} = -0.01 \pm 0.49 \pm 0.04$, and world averages for the mixing parameters from an "extended" fit that combines the CLEO-c data with previous mixing and branching-ratio measurements [65]. These fits allow $\cos\delta_{K\pi}$, $\sin\delta_{K\pi}$ and x^2 to be unphysical. Constraining $\cos\delta_{K\pi}$ and $\sin\delta_{K\pi}$ to $[-1, +1]$ — that is interpreting $\delta_{K\pi}$ as an angle — yields $\delta_{K\pi} = (18\,^{+11}_{-17} \pm 7)^\circ$. Note that measurements of y (Table 82.4 and Table 82.5) and y' (Table 82.3) dominate the determination of $\delta_{K\pi} = 15.2\,^{+7.6}_{-10.0}$ [9].

82.6. Summary of Experimental Results

Several recent results indicate that charm mixing is at the upper end of the range of Standard Model estimates.

For $D^0 \to K^+\pi^-$, LHCb [16,18], CDF [17], and Belle [15] each exclude the no-mixing hypothesis by more than 5 standard deviations.

For y_{CP} in $D^0 \to K^+K^-$ and $\pi^+\pi^-$, Belle [46] and BaBar [47] find 4.5σ and 3.3σ effects. The most sensitive measurement of x and y is in $D^0 \to K^0_S\pi^+\pi^-$ from Belle [35] and the no mixing solution is only excluded at 2.5σ. In a similar analysis using $D^0 \to K^0_S\pi^+\pi^-$ and $D^0 \to K^0_S K^+K^-$ BaBar [34] also finds the no mixing solution excluded at 1.9σ. LHCb [10] has reported the observation of charm mixing in $D^0 \to K^+\pi^-\pi^+\pi^-$ with 8.2σ significance.

The current situation would benefit from better knowledge of the strong phase difference $\delta_{K\pi}$ than provided by the current CLEO-c [65] and BESIII [66] results. This would allow one to unfold x and y from the $D^0 \to K^+\pi^-$ measurements of x'^2 and y', and directly compare them to the $D^0 \to K^0_S\pi^+\pi^-$ results.

The experimental data consistently indicate that the D^0 and $\overline{D}^0$ do mix. The mixing is presumably dominated by long-range processes. Under the assumption that the observed mixing is due entirely to non-Standard Model processes, significant constraints on a variety of new physics models are obtained [69]. A serious limitation to the interpretation of charm oscillations in terms of New Physics is the theoretical uncertainty of the Standard Model prediction [70,71].

82.7. HFLAV Averaging of Charm Mixing Results

The Heavy Flavor Averaging Group (HFLAV) has made a global fit to all mixing measurements to obtain values of x, y, $\delta_{K\pi}$, $\delta_{K\pi\pi^0}$, R_D, $A_D \equiv (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$, $|q/p|$, $\mathrm{Arg}(q/p) \equiv \phi$, and the time-integrated CP asymmetries A_K and A_π. Correlations among observables are taken into account by using the error matrices from the experiments. The measurements of $D^0 \to K^{(*)+}\ell^-\overline{\nu}$, K^+K^-, $\pi^+\pi^-$, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, $K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$, and $\pi^+\pi^-\pi^0$ decays, as well as CLEO-c and BESIII results for double-tagged branching fractions measured at the $\psi(3770)$ are used.

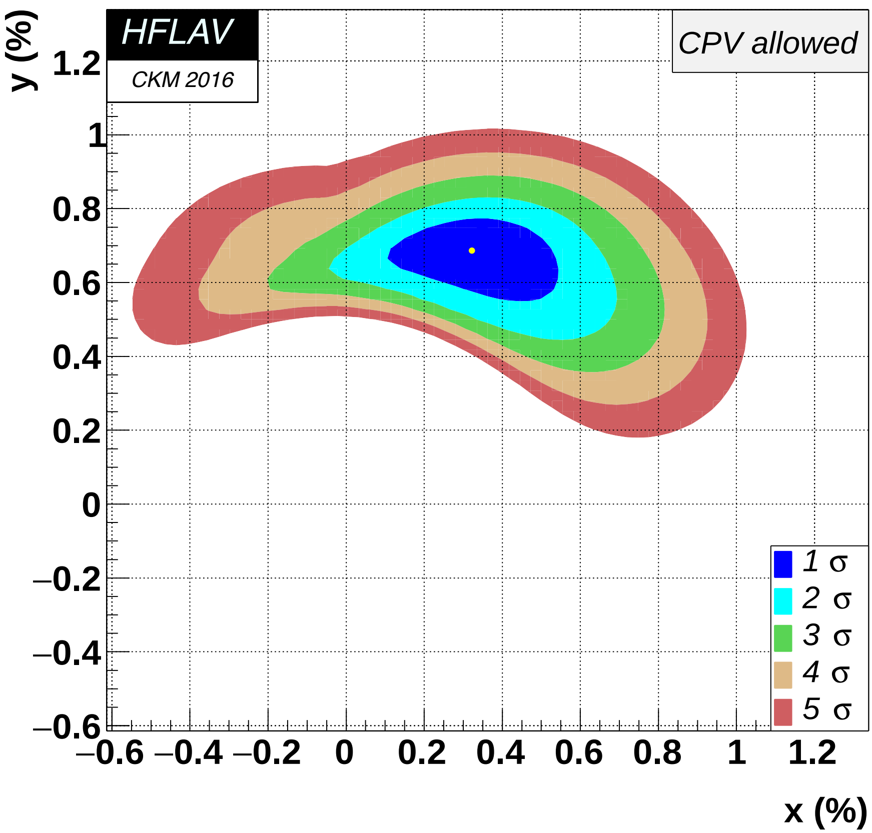

Figure 82.1: Two-dimensional 1σ-5σ contours for (x, y) from measurements of $D^0 \to K^{(*)+}\ell\nu$, h^+h^-, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, $K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$, and $\pi^+\pi^-\pi^0$ decays, and double-tagged branching fractions measured at the $\psi(3770)$ resonance (from HFLAV [9]) .

For the global fit, confidence contours in the two dimensions (x, y) and $(|q/p|, \phi)$ are obtained by letting, for any point in the two-dimensional plane, all other fit parameters take their preferred values. Figures 1 and 2 show the resulting 1 to 5 σ contours. The fits exclude the no-mixing point $(x = y = 0)$ at more than 11.5σ, when CP violation is

Table 82.7: HFLAV Charm Mixing Averages [9].

Parameter	No CP Violation	CP Violation Allowed	95% C.L. Interval CPV Allowed
$x(\%)$	$0.46\,^{+0.14}_{-0.15}$	0.32 ± 0.14	$[0.04, 0.62]$
$y(\%)$	0.62 ± 0.08	$0.69^{+0.06}_{-0.07}$	$[0.50, 0.80]$
$R_D(\%)$	$0.348\,^{+0.004}_{-0.003}$	$0.349\,^{+0.004}_{-0.003}$	$[0.342, 0.356]$
$\delta_{K\pi}(^\circ)$	$8.0\,^{+9.7}_{-11.2}$	$15.2\,^{+7.6}_{-10.0}$	$[-16.8, 30.1]$
$\delta_{K\pi\pi^0}(^\circ)$	$20.4\,^{+23.3}_{-23.8}$	$31.7\,^{+23.5}_{-24.2}$	$[-16.4, 77.7]$
$A_D(\%)$	—	-0.88 ± 0.99	$[-2.8, 1.0]$
$\lvert q/p\rvert$	—	$0.89\,^{+0.08}_{-0.07}$	$[0.77, 1.12]$
$\phi(^\circ)$	—	$-12.9\,^{+9.9}_{-8.7}$	$[-30.2, 10.6]$
A_K	—	-0.11 ± 0.13	$[-0.37, 0.14]$
A_π	—	0.01 ± 0.14	$[-0.25, 0.28]$

allowed. The fits are consistent with no CP violation at the 40% Confidence Level. The parameters x and y differ from zero by 1.9σ and 9.1σ, respectively. One-dimensional likelihood functions for parameters are obtained by allowing, for any value of the parameter, all other fit parameters to take their preferred values. The resulting likelihood functions give central values, 68.3% C.L. intervals, and 95% C.L. intervals as listed in Table 82.7. The χ^2 for the HFLAV fit is 77 for 50 degrees of freedom, indicating some disagreement among the measurements included in the combination.

From the results of the HFLAV averaging, the following can be concluded: (1) Since CP violation is small and y_{CP} is positive, the CP-even state is shorter-lived, as in the $K^0\overline{K}^0$ system; (2) However, since x appears to be positive, the CP-even state is heavier, unlike in the $K^0\overline{K}^0$ system; (3) The strong phase difference $\delta_{K\pi}$ is consistent with the SU(3) expectation of zero but large values are not excluded; (4) There is no evidence yet for CP-violation in $D^0\overline{D}^0$ mixing. Observing CP-violation in mixing ($|q/p| \neq 1$) at the current level of sensitivity would indicate new physics.

82.8. Future Prospects

Current results are based primarily upon CLEO-c (818 pb^{-1} of $e^+e^- \to \psi(3770)$), B-factories ($\sim$1 ab^{-1} of $e^+e^- \to \Upsilon(4S)$), and LHCb Run 1 (3 fb^{-1} of pp collisions at 3.5-4.0 TeV). Only a subset of the LHCb results reported use the full Run 1 data sample. Order of magnitude or more increases in data analyzed from each of these data types are expected.

BESIII has accumulated 2.9 fb^{-1} of $e^+e^- \to \psi(3770)$ and may integrate up to 10 fb^{-1} in the next few years. These data will provide strong phase difference measurements that enable improved model-independent determination of mixing parameters from Belle II and LHCb. In 2018, Belle II will begin to accumulate $e^+e^- \to \Upsilon(4S)$ data, 50 ab^{-1}

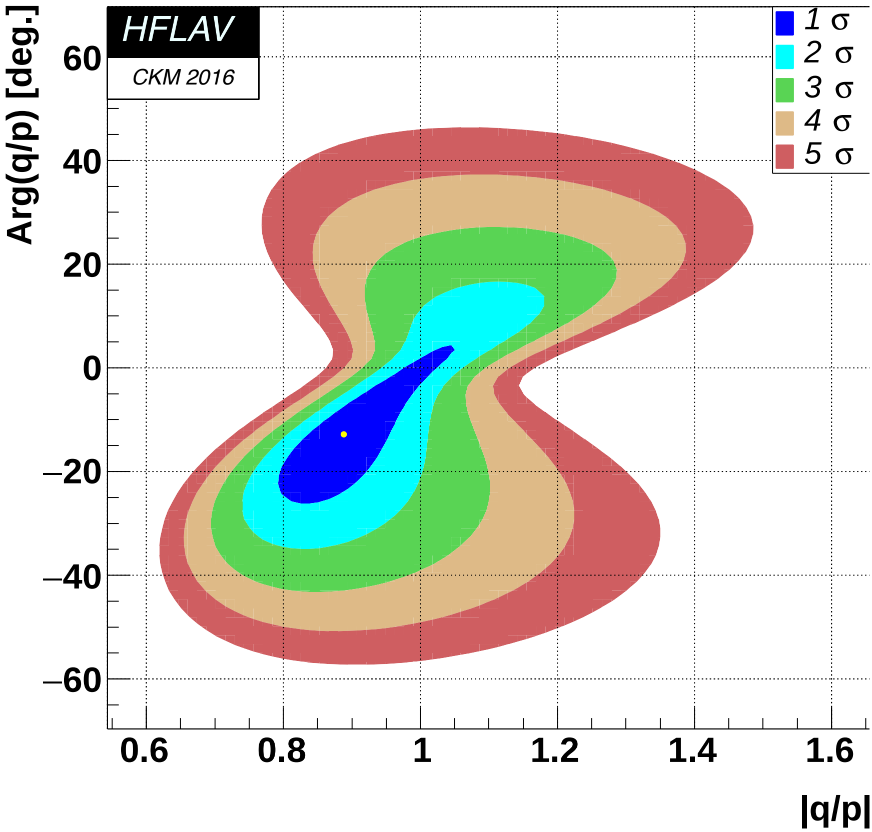

Figure 82.2: Two-dimensional 1σ-5σ contours for ($|q/p|$,Arg(q/p)) from measurements of $D^0 \to K^{(*)+}\ell\nu$, h^+h^-, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, $K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$, and $\pi^+\pi^-\pi^0$ decays, and double-tagged branching fractions measured at the $\psi(3770)$ resonance (from HFLAV [9]) .

is anticipated by 2024. The sensitivity of these data to charm mixing parameters is expected to be comparable to LHCb Run 2 [72]. LHCb Run 2 will complete in 2018 and Run 3 is planned for 2021-23, concurrent with Belle II.

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