82. D^0-$\overline{D}^0$ Mixing

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The formalism for D^0-$\overline{D}^0$ mixing is closely related to that for CP violation, which is also presented in the note "CP Violation in the Quark Sector" in this *Review.* The time evolution of the D^0–$\overline{D}^0$ system is described by the Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}, \tag{82.1}$$

where the $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance requires that $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of $\mathbf{M}$ and $\mathbf{\Gamma}$ are referred to as the dispersive and absorptive parts, respectively, of the mixing. The mass eigenstates D_1 and D_2 of the Hamiltonian $\mathbf{M}$ - $i\mathbf{\Gamma}$ are defined as

$$|D_{1,2}\rangle \;\equiv\; p|D^0\rangle \pm q|\overline{D}^0\rangle\,, \tag{82.2}$$

where normalization imposes $|p|^2 + |q|^2 = 1$. If $p = q$, then the mass eigenstates are CP eigenstates and CP is conserved. Our phase convention is $CP|D^0\rangle = -|\overline{D}^0\rangle$, which implies that, in the absence of CP violation, D_2 is CP-even and D_1 is CP-odd.

The eigenvalues of $\mathbf{M}$ - $i\mathbf{\Gamma}$ are

$$\omega_{1,2} \;=\; \left(M - \frac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right) \;\equiv\; m_{1,2} - \frac{i}{2}\Gamma_{1,2}\,, \tag{82.3}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are real and correspond to the masses and decay widths, respectively, of the $D_{1,2}$ mass eigenstates. As the trace $\Gamma_{11} + \Gamma_{22} = 2\Gamma$ is unchanged by diagonalizing $\mathbf{\Gamma}$, $\Gamma = (\Gamma_1 + \Gamma_2)/2$, i.e., the mean decay width. Solving for the eigenstates of the eigenvalues yields

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\,. \tag{82.4}$$

If CP is conserved, $(q/p) = 1$ and thus M_{12} and Γ_{12} must be real. In this case the difference in eigenvalues is $\Delta m \equiv m_2 - m_1 = 2M_{12}$ and $\Delta\Gamma \equiv \Gamma_2 - \Gamma_1 = 2\Gamma_{12}$. The signs of Δm and $\Delta\Gamma$ are difficult to predict from theory and thus must be determined experimentally.

We define dimensionless mixing parameters x and y as

$$x \;\equiv\; \frac{\Delta m}{\Gamma} \tag{82.5}$$

$$y \;\equiv\; \frac{\Delta\Gamma}{2\Gamma}\,. \tag{82.6}$$

These parameters are measured in several ways. The most precise values are obtained using the time dependence of D^0 decays. For all methods, the initial flavor of the D^0 or $\overline{D}^0$ (at the production point) must be determined. The most common method used for this is to reconstruct $D^{*+} \to D^0\pi^+$ or $D^{*-} \to \overline{D}^0\pi^-$ decays; the charge of the accompanying pion (which has low momentum in the lab frame and is often referred to as the "soft" pion) determines the flavor of the neutral D. BaBar and LHCb have also identified the flavor of the neutral D by reconstructing semileptonic $B^+ \to \overline{D}^0\ell^+\nu$, $B^0 \to D^{*-}\ell^+\nu$, $B^- \to D^0\ell^-\nu$, and $\overline{B}^0 \to D^{*+}\ell^-\nu$ decays; in this case the charge of the accompanying lepton determines the D flavor. At e^+e^- collider experiments such as Belle, BaBar, and BESIII, the D flavor can also be determined by fully reconstructing a D decay on the "opposite side" of an event, i.e., recoiling against the signal-side D decay.

At BESIII, where $D\overline{D}$ pairs are produced near their threshold via $e^+e^- \to \psi(3770) \to D^0\overline{D}^0$, there is relatively little background and the purity of opposite-side tagging is equivalent to that achieved using $D^{*\pm}$ decays. However, BESIII operates at a symmetric e^+e^- collider, and the $D\overline{D}$ pairs are produced almost at rest in the lab frame. As a consequence, the D's do not travel any appreciable distance before decaying, and time-dependent analyses are not possible. To overcome this, measurements of mixing at BESIII utilize the quantum coherence of the initial $\psi(3770) \to D^0\overline{D}^0$ state and time-integrated measurements [1–5].

82.1 Time-Dependent Analyses

We extend the formalism of this *Review*'s note on "CP Violation in Meson Decays." Our notation is as follows: Cabibbo-favored ("right-sign") decay amplitudes are denoted $\overline{A}_f \equiv \langle f|H|\overline{D}^0\rangle$ and $A_{\overline{f}} \equiv \langle \overline{f}|H|D^0\rangle$; i.e, the final state is $f = K^+\ell^-\nu$, $K^+\pi^-$, $K^+\pi^-\pi^0$, etc. Doubly-Cabibbo-suppressed ("wrong-sign") decay amplitudes are denoted $A_f \equiv \langle f|H|D^0\rangle$ and $\overline{A}_{\overline{f}} \equiv \langle \overline{f}|H|\overline{D}^0\rangle$.

Starting from a pure $|D^0\rangle$ or $|\overline{D}^0\rangle$ state at $t = 0$, the time-dependent decay rates to wrong-sign final states are

$$r(t) \;\equiv\; \left|\langle f|H|D^0(t)\rangle\right|^2 \;=\; \left|\overline{A}_f\right|^2 \left|\frac{q}{p}\right|^2 \left|g_+(t)\,\lambda_f^{-1} \;+\; g_-(t)\right|^2 \tag{82.7}$$

$$\overline{r}(t) \;\equiv\; \left|\langle \overline{f}|H|\overline{D}^0(t)\rangle\right|^2 \;=\; \left|A_{\overline{f}}\right|^2 \left|\frac{p}{q}\right|^2 \left|g_+(t)\,\lambda_{\overline{f}} \;+\; g_-(t)\right|^2 , \tag{82.8}$$

where

$$\lambda_f \;\equiv\; \frac{q}{p}\frac{\overline{A}_f}{A_f}, \quad \lambda_{\overline{f}} \;\equiv\; \frac{q}{p}\frac{\overline{A}_{\overline{f}}}{A_{\overline{f}}}, \tag{82.9}$$

and

$$g_\pm(t) \;=\; \frac{1}{2}\left(e^{-i\omega_1 t} \pm e^{-i\omega_2 t}\right) . \tag{82.10}$$

A change in convention for the relative phase of D^0 and $\overline{D}^0$ would cancel between q/p and $\overline{A}_f/A_f$ (or $\overline{A}_{\overline{f}}/A_{\overline{f}}$), leaving λ_f (or $\lambda_{\overline{f}}$) unchanged. For multibody final states, these equations apply separately to each point in phase-space. Integrating over regions of phase-space can lead to enhanced sensitivity to CP violation; see the discussion below on multibody decays and the "Review of Multibody Charm Analyses" in this *Review* [6]. As the mixing parameters x and y are very small, $r(t)$ and $\overline{r}(t)$ are usually expanded to second order in x and y.

82.2 Semileptonic decays

Consider the final state $f = K^+\ell^-\bar{\nu}_\ell$, where $A_f = \overline{A}_{\overline{f}} = 0$ is an excellent approximation in the Standard Model. The final state f is accessible from a D^0 only via mixing,[1] and the decay rate is

$$r(t) \;=\; \left|\overline{A}_f\right|^2 \left|\frac{q}{p}\right|^2 |g_-(t)|^2 \;\approx\; \left|\overline{A}_f\right|^2 \left|\frac{q}{p}\right|^2 \left(\frac{x^2+y^2}{4}\right) (\Gamma t)^2\, e^{-\Gamma t} . \tag{82.11}$$

For $\overline{r}(t)$, q/p is replaced by p/q. In the Standard Model, CP violation in charm mixing is small and $|q/p| \approx 1$. In the limit of CP conservation, $r(t) = \overline{r}(t)$, and the time-integrated mixing rate

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[1] There exists a doubly Cabibbo-suppressed amplitude in which the c and $\bar{u}$ quarks exchange a W, and then the resulting d quark (from c) decays semileptonically. We neglect this second-order process.

relative to the time-integrated right-sign decay rate for semileptonic decays is

$$\frac{\displaystyle\int_0^\infty r(t)\,dt}{\displaystyle\int_0^\infty \left|\overline{A}_f\right|^2 e^{-\Gamma t}\,dt} \;=\; \frac{x^2+y^2}{2} \;\equiv\; R_M\,. \tag{82.12}$$

Table 82.11 summarizes results for R_M from semileptonic decays; the world average from the Heavy Flavor Averaging Group (HFLAV) [7] is $R_M = (1.30 \pm 2.69) \times 10^{-4}$.

Table 82.1: Results for R_M in D^0 semileptonic decays. The HFLAV average assumes reported statistical and systematic uncertainties are uncorrelated. When a single uncertainty is listed, that corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been superseded and thus are not included in the HFLAV average.

Year	Experiment	Final state(s)	R_M $(\times 10^{-3})$	90% C.L.
2008	Belle (492 fb^{-1}) [8]	$K^{(*)+}e^-\overline{\nu}_e$	$0.13\pm0.22\pm0.20$	$< 0.61 \times 10^{-3}$
2007	BaBar (344 fb^{-1}) [9]	$K^{(*)+}e^-\overline{\nu}_e$	$0.04\,^{+0.70}_{-0.60}$	$(-1.3, 1.2) \times 10^{-3}$
2005	CLEO (9.0 fb^{-1}) [10]	$K^{(*)+}e^-\overline{\nu}_e$	$1.6\pm2.9\pm2.9$	$< 7.8 \times 10^{-3}$
1996	E791 (2×10^{10} events) [11]	$K^+\ell^-\overline{\nu}_\ell$	$1.1\,^{+3.0}_{-2.7}\,^{+0.0}_{-0.1}$	$< 5.0 \times 10^{-3}$
	HFLAV Average [7]		$\mathbf{0.130 \pm 0.269}$	
2005*	Belle (253 fb^{-1}) [12]	$K^{(*)+}e^-\overline{\nu}_e$	$0.02\pm0.47\pm0.14$	$< 1.0 \times 10^{-3}$
2004*	BaBar (87 fb^{-1}) [13]	$K^{(*)+}e^-\overline{\nu}_e$	$2.3\pm1.2\pm0.4$	$< 4.2 \times 10^{-3}$

82.3 Wrong-sign decays to hadronic non-CP eigenstates

Consider the final state $f = K^+\pi^-$, i.e., A_f and $\overline{A}_{\overline{f}}$ are doubly Cabibbo-suppressed. Allowing for CP violation, the ratio of decay amplitudes can be parameterized as

$$\frac{A_f}{\overline{A}_f} \;=\; -\sqrt{R_D^+}\,e^{-i\delta_f} \qquad \frac{\overline{A}_{\overline{f}}}{A_{\overline{f}}} \;=\; -\sqrt{R_D^-}\,e^{-i\delta_f}\,, \tag{82.13}$$

where δ_f is the strong phase difference. The minus sign originates from the weak phase difference between the amplitudes, specifically, the relative signs of V_{us} and V_{cd}. The parameters R_D^+ and R_D^- are the ratios of the doubly Cabibbo-suppressed (DCS) decay rate to the Cabibbo-favored (CF) decay rate. From the relevant CKM matrix elements, one estimates R_D^+, $R_D^- \sim \tan^4\theta_c$, where θ_c is the Cabibbo angle. With this parameterization, Eq. (82.9) becomes

$$\lambda_f^{-1} \;=\; \frac{p}{q}\frac{A_f}{\overline{A}_f} \;=\; -\sqrt{R_D^+}\left|\frac{p}{q}\right| e^{-i(\delta_f+\phi)} \tag{82.14}$$

$$\lambda_{\overline{f}} \;=\; \frac{q}{p}\frac{\overline{A}_{\overline{f}}}{A_{\overline{f}}} \;=\; -\sqrt{R_D^-}\left|\frac{q}{p}\right| e^{-i(\delta_f-\phi)}\,, \tag{82.15}$$

where ϕ is a weak phase difference. In the Standard Model, the weak phase of $A_f/\overline{A}_f$ or $\overline{A}_{\overline{f}}/A_{\overline{f}}$ is, to excellent approximation, -1, which is already factored out, and thus $\phi = \mathrm{Arg}(q/p)$. As ϕ is essentially independent of the final state, it is referred to as "universal." CP violation in mixing is characterized by $|q/p| \neq |p/q| \neq 1$. CP violation in the decay amplitudes A_f, $\overline{A}_f$, $A_{\overline{f}}$, $\overline{A}_{\overline{f}}$ is referred to as *direct CP* violation and is parameterized by $A_D \equiv (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$. The mean value is denoted $R_D \equiv (R_D^+ + R_D^-)/2$.

With these definitions, we expand the decay rates Eqs. (82.7) and (82.8) to second order in the small mixing parameters x and y to obtain [14, 15]:

$$\begin{aligned} r(t) \;&=\; \left|\overline{A}_f\right|^2 e^{-\Gamma t} \;\times \\ &\left[R_D(1+A_D) \;+\; \sqrt{R_D(1+A_D)}\left|\frac{q}{p}\right| y'_+(\Gamma t) + \;\left|\frac{q}{p}\right|^2 \frac{(x'^2_+ + y'^2_+)}{4}(\Gamma t)^2 \right] \end{aligned} \tag{82.16}$$

and

$$\begin{aligned} \overline{r}(t) \;&=\; \left|A_{\overline{f}}\right|^2 e^{-\Gamma t} \;\times \\ &\left[R_D(1-A_D) \;+\; \sqrt{R_D(1-A_D)}\left|\frac{p}{q}\right| y'_-(\Gamma t) + \;\left|\frac{p}{q}\right|^2 \frac{(x'^2_- + y'^2_-)}{4}(\Gamma t)^2 \right], \end{aligned} \tag{82.17}$$

where

$$\begin{aligned} x'_\pm \;&=\; x\cos(\delta_f \pm \phi) + y\sin(\delta_f \pm \phi) \\ &\equiv\; x'\cos\phi \;\pm\; y'\sin\phi\,, \end{aligned} \tag{82.18}$$

$$\begin{aligned} y'_\pm \;&=\; y\cos(\delta_f \pm \phi) - x\sin(\delta_f \pm \phi) \\ &\equiv\; y'\cos\phi \;\mp\; x'\sin\phi\,, \end{aligned} \tag{82.19}$$

and

$$x' \;=\; x\cos\delta_f \;+\; y\sin\delta_f \tag{82.20}$$

$$y' \;=\; y\cos\delta_f \;-\; x\sin\delta_f\,. \tag{82.21}$$

In Eqs. (82.16) and (82.17), a fourth term $R_D(1 \pm A_D)(x_\pm^2 - y_\pm^2)/4 \times (\Gamma t)^2$ has been dropped, as, for the range of decay times measured by experiments, it is negligible relative to the other terms.

The parameters (x', y') are the mixing parameters (x, y) rotated by the strong phase δ_f. The parameters $(x'_\pm, y'_\pm)$ are the parameters (x', y') rotated by the weak phase $\pm\,\phi$. Note that $x'^2_+ + y'^2_+ = x'^2_- + y'^2_- = x'^2 + y'^2 = x^2 + y^2$. Comparing Eqs. (82.16) and (82.17), ones sees that $r(t) \neq \overline{r}(t)$ (*CP* is violated) if either $A_D \neq 0$, $|q/p| \neq 1$, or $\phi \neq 0$. These three inequalities correspond, respectively, to the three types of *CP* violation: in the (doubly Cabibbo-suppressed) decay amplitudes; in the mixing; and due to interference between a mixed amplitude and an unmixed decay amplitude. Whereas *CP* violation in the decay amplitudes is parameterized by A_D, *CP* violation in the mixing is parameterized by $A_M \equiv (|q/p| - |p/q|)/(|q/p| + |p/q|)$.

In the limit of *CP* conservation, $A_D = 0$, $|q/p| = 1$, and $\phi = 0$. In this case

$$r(t) \;=\; \overline{r}(t) \;=\; \left|A_{\overline{f}}\right|^2 e^{-\Gamma t}\left[R_D + \sqrt{R_D}\, y'(\Gamma t) + \frac{x'^2 + y'^2}{4}(\Gamma t)^2 \right], \tag{82.22}$$

and the number of wrong-sign decays divided by the number of right-sign decays is

$$R \;=\; \frac{\displaystyle\int_0^\infty r(t)\,dt}{\displaystyle\int_0^\infty \left|A_{\overline{f}}\right|^2 e^{-\Gamma t}\,dt} \;=\; R_D + \sqrt{R_D}\, y' + \frac{x'^2 + y'^2}{2}\,. \tag{82.23}$$

The ratio R is straightforward to measure, as there is no time-dependence. In Table 82.2 we report measurements of R, R_D, and A_D in $D^0 \to K^+\pi^-$ decays, and HFLAV world averages [7] obtained from a global fit that allows for both mixing and CP violation. Typically, the experimental fitted parameters are R_D, x'^2, and y'; the results for x'^2 and y' are summarized in Table 82.3. Allowing for CP violation, the parameters (R_D^+, x'^2_+, y'_+) and (R_D^-, x'^2_-, y'_-) [or equivalently (R_D, A_D) instead of (R_D^+, R_D^-)] are obtained by separately fitting $D^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^-\pi^+$ event samples.

Table 82.2: Results for R, R_D, and A_D as measured using $D^0 \to K^\pm\pi^\mp$ decays. When a single uncertainty is listed, that corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been superseded and thus are not included in the HFLAV average. The measurements with a dagger (†) are not included in the HFLAV average due to poorer precision.

Year	Experiment	$R\,(\times 10^{-3})$	$R_D\,(\times 10^{-3})$	$A_D\,(\%)$
2018	LHCb (5.0 fb^{-1}) [16]	—	3.454 ± 0.031	-0.01 ± 0.91
2014	Belle (976 fb^{-1}) [17]	3.86 ± 0.06	3.53 ± 0.13	—
2013	CDF (9.6 fb^{-1}) [18]	4.30 ± 0.05	3.51 ± 0.35	—
2007	BaBar (384 fb^{-1}) [19]	$3.53\pm0.08\pm0.04$	$3.03\pm0.16\pm0.10$	$-2.1\pm5.2\pm1.5$
HFLAV Average [7]			$\mathbf{3.435 \pm 0.022}$	$\mathbf{-0.55\,^{+0.49}_{-0.51}}$
2013b*	LHCb (3.0 fb^{-1}) [20]	—	3.566 ± 0.067	-0.7 ± 1.9
2013a*	LHCb (1.0 fb^{-1}) [21]	4.25 ± 0.04	3.52 ± 0.15	—
2008*	CDF (1.5 fb^{-1}) [22]	4.15 ± 0.10	3.04 ± 0.55	—
2006*	Belle (400 fb^{-1}) [23]	$3.77\pm0.08\pm0.05$	3.64 ± 0.18	2.3 ± 4.7
2005†	FOCUS (234 evts) [24]	$4.29\,^{+0.63}_{-0.61}\pm0.27$	$5.17\,^{+1.47}_{-1.58}\pm0.76$	$13\,^{+33}_{-25}\pm10$
2000†	CLEO (9.0 fb^{-1}) [25]	$3.32\,^{+0.63}_{-0.65}\pm0.40$	$4.8\pm1.2\pm0.4$	$-1\,^{+16}_{-17}\pm1$
1998†	E791 (5643 evts) [26]	$6.8\,^{+3.4}_{-3.3}\pm0.7$	—	—

Extraction of the mixing parameters x and y from measurements of x' and y' requires knowledge of the strong phase difference $\delta_{K\pi}$. This can be determined from the decay rates of $D_\pm \to K^+\pi^-$, where D_+ (D_-) denotes the CP-even (CP-odd) eigenstate. Since $|D_\pm\rangle = (|D^0\rangle \mp |\overline{D}^0\rangle)/\sqrt{2}$,

$$\sqrt{2}\,A(D_\pm \to K^+\pi^-) \;=\; A(D^0 \to K^+\pi^-) \mp A(\overline{D}^0 \to K^+\pi^-)\,. \tag{82.24}$$

Squaring this amplitude and using Eq. (82.13) yields the relation

$$\cos\delta_{K\pi} \;=\; \frac{|A(D_+ \to K^+\pi^-)|^2 - |A(D_- \to K^+\pi^-)|^2}{2\,|A(D^0 \to K^+\pi^-)|\,|A(\overline{D}^0 \to K^+\pi^-)|}\,. \tag{82.25}$$

Measuring the right-hand side is possible only if one can identify pure D_+, D_-, D^0, and $\overline{D}^0$ initial states. This is accomplished at CLEOc and BESIII utilizing the processes $e^+e^- \to \psi(3770) \to \overline{D}^0 D^0 \to (f_{CP})(K^+\pi^-)$, or $\psi(3770) \to \overline{D}^0 D^0 \to (f_{\overline{D}^0})(K^+\pi^-)$. In the first case, quantum coherence and CP symmetry insures that the $K^+\pi^-$ state originates from a neutral D with CP opposite that of f_{CP}. In the second case, at the time when the $\overline{D}^0$ decays, the opposite side is D^0. However, it can potentially mix to $\overline{D}^0$ before decaying to $K^+\pi^-$, and this introduces some dependence on the mixing parameters x and y. This dependence is seen explicitly in the observable

$$A^{CP}_{K\pi} \;\equiv\; \frac{|A(D_- \to K^-\pi^+)|^2 - |A(D_+ \to K^-\pi^+)|^2}{|A(D_- \to K^-\pi^+)|^2 + |A(D_+ \to K^-\pi^+)|^2}\,. \tag{82.26}$$

Table 82.3: Results for x'^2 and y', as measured using $D^0 \to K^{\pm}\pi^{\mp}$ decays. When a single uncertainty is listed, that corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been superseded and thus are not included in the HFLAV global fit. The measurements with a dagger (†) are not included in the HFLAV global fit due to poorer precision. All confidence limits and intervals correspond to 95% C.L. The Belle 2006 results restrict x'^2 to the physical region. The BaBar confidence intervals are obtained from the fit, whereas Belle uses a Feldman-Cousins method, and CDF uses a Bayesian method.

Year	Experiment	No CP violation		Allowing for CP violation	
		$x'^2\,(\times10^{-3})$	$y'\,(\%)$	$x'^2\,(\times10^{-3})$	$y'\,(\%)$
2018	LHCb (5.0 fb^{-1}) [16]	0.039 ± 0.027	0.528 ± 0.052	D^0: 0.061 ± 0.037 $\overline{D}^0$: 0.016 ± 0.039	0.501 ± 0.074 0.554 ± 0.074
2014	Belle (976 fb^{-1}) [17]	0.09 ± 0.22	0.46 ± 0.34	—	—
2013	CDF (9.6 fb^{-1}) [18]	0.08 ± 0.18	0.43 ± 0.43	—	—
2007	BaBar (384 fb^{-1}) [19]	-0.22 ± 0.37	0.97 ± 0.54	D^0: -0.24 ± 0.52 $\overline{D}^0$: -0.20 ± 0.50	0.98 ± 0.78 0.96 ± 0.75
2006	Belle (400 fb^{-1}) [23]	$\left(0.18\,^{+0.21}_{-0.23}\right)^*$	$\left(0.06\,^{+0.40}_{-0.39}\right)^*$	< 0.72	$-2.8 < y' < 2.1$
2013b*	LHCb (3.0 fb^{-1}) [20]	0.055 ± 0.049	0.48 ± 0.09	D^0: 0.049 ± 0.070 $\overline{D}^0$: 0.060 ± 0.068	0.51 ± 0.14 0.45 ± 0.14
2013a*	LHCb (1.0 fb^{-1}) [21]	-0.09 ± 0.13	0.72 ± 0.24	—	—
2008*	CDF (1.5 fb^{-1}) [22]	-0.12 ± 0.35	0.85 ± 0.76	—	—
2005†	FOCUS (234 evts) [24]	< 8.3	$-7.2 < y' < 4.1$	< 8.0	$-11.2 < y' < 6.7$
2000†	CLEO (9.0 fb^{-1}) [25]	0.00 ± 0.23	$-2.3\,^{+1.3}_{-1.4}$	0.00 ± 0.23	$-2.5\,^{+1.4}_{-1.6}$
1998†	E791 (5643 evts) [26]	< 17	< 13	—	—

To lowest order in the mixing parameters [27],

$$A^{CP}_{K\pi} \;=\; \frac{2\sqrt{R_D}\cos\delta_{K\pi} + y}{1+R}\,, \tag{82.27}$$

where R is defined in Eq. (82.23).

82.3.1 *Wrong-sign decays to multibody final states*

For multibody final states, Eqs. (82.13)-(82.23) apply essentially to each point in phase-space. Although x and y do not vary across phase-space, knowledge of the resonant substructure is needed to extrapolate the strong phase difference δ from point to point to determine x and y. Alternatively, model-independent methods to determine x and y require knowledge of the relative phases of D^0 and $\overline{D}^0$ decay amplitudes across the phase-space distribution [6]. This required phase information can be measured at the charm threshold, where CLEO-c and BESIII took data.

A time-dependent analysis of $D^0 \to K^+\pi^-\pi^0$ decays at BaBar [28, 29] determined the *relative* strong phase variation across the Dalitz plot and reported $x'' = (2.61\,^{+0.57}_{-0.68} \pm 0.39)\%$ and $y'' = (-0.06\,^{+0.55}_{-0.64} \pm 0.34)\%$. These mixing parameters are defined as

$$\begin{aligned} x'' &= x\cos\delta_{K\pi\pi^0} + y\sin\delta_{K\pi\pi^0} \\ y'' &= y\cos\delta_{K\pi\pi^0} - x\sin\delta_{K\pi\pi^0}\,, \end{aligned} \tag{82.28}$$

in analogy with x', y', and $\delta_{K\pi}$ of Eqs. (82.20) and (82.21). Here, $\delta_{K\pi\pi^0}$ is the strong phase difference between the amplitudes $A(D^0 \to K^+\pi^-\pi^0)$ and $A(\overline{D}^0 \to K^+\pi^-\pi^0)$ at a "reference point" of the Dalitz plot. In this case the reference point chosen is $m_{\pi^-\pi^0} = m_{\rho^-}$. The strong phase difference $\delta_{K\pi\pi^0}$ can be determined in a manner similar to that for $\delta_{K\pi}$: by using Eq. (82.25) and quantum-correlated measurements of the branching fractions $B(D_+ \to K^+\rho^-)$, $B(D_- \to K^+\rho^-)$, $B(D^0 \to K^+\rho^-)$, and $B(\overline{D}^0 \to K^+\rho^-)$ in $e^+e^- \to \psi(3770)$ events.

For the decay modes D^0 and $\overline{D}^0 \to K^+\pi^-\pi^+\pi^-$, Belle measured $R = (0.324 \pm 0.008 \pm 0.007)\%$ [30]. Subsequently, a phase-space-integrated analysis from LHCb [31] measured the product of a coherence factor $R_D^{K3\pi}$ and the strong-phase-rotated mixing parameter $y''_{K3\pi}$. This measurement resulted in an observation of charm mixing with 8.2σ significance.

Both the sign and magnitude of x and y without strong phases entering or sign ambiguity can be measured using the time-dependent resonant substructure of multibody D^0 decays [32, 33]. In $D^0 \to K^0_S\pi^+\pi^-$, the DCS and CF decay amplitudes populate the same Dalitz plot, which allows for direct measurement of the relative strong phases. Belle [33, 34], BaBar [35], and CLEO [36] have measured the relative strong phase between $D^0 \to K^*(892)^-\pi^+$ and $D^0 \to K^*(892)^+\pi^-$ to be $(173.9 \pm 0.7 \text{ (stat. only)})^\circ$, $(177.6 \pm 1.1 \text{ (stat. only)})^\circ$, and $(189 \pm 10 \pm 3\,^{+15}_{-5})^\circ$, respectively. These results are close to the 180° expected from Cabibbo factors and a small strong phase. Two LHCb measurements [37, 38] of x, y are decay-model independent, i.e., the model of resonances in the intermediate state is replaced by strong-phase measurements from CLEO-c [39]. Table 82.4 summarizes mixing results from time-dependent multibody analyses. World average values for the measurements listed are given later, as a result of the HFLAV global fit.

In addition, Belle [33, 34] has measured the relative strong phase (statistical errors only) and the ratio R (central values only) of the DCS fit fraction relative to the CF fit fraction for five excited K states: $K^*(892)^+\pi^-$, $K_0^*(1430)^+\pi^-$, $K_2^*(1430)^+\pi^-$, $K^*(1410)^+\pi^-$, and $K^*(1680)^+\pi^-$. Similarly, BaBar [35, 40, 41] has reported central values of R for $K^*(892)^+\pi^-$, $K_0^*(1430)^+\pi^-$, and $K_2^*(1430)^+\pi^-$. The systematic uncertainties on R are not evaluated. Large differences in R are observed among these final states, which indicates interesting hadronic effects.

82.4 Decays to *CP* Eigenstates

When the final state f is a CP eigenstate, there is no distinction between f and $\overline{f}$. Thus $A_f = A_{\overline{f}}$ and $\overline{A}_{\overline{f}} = \overline{A}_f$. We denote final states with CP eigenvalues ± 1 by $f_\pm$ and write $\lambda_\pm$ for $\lambda_{f_\pm}$.

The quantity y may be measured by comparing the rate for D^0 decays to CP eigenstates such as K^+K^- with the rate to non-CP states such as $K^-\pi^+$ [15]. If decays to K^+K^- have a shorter effective lifetime than those to $K^-\pi^+$, then $\mathit{\Gamma}_+ > \mathit{\Gamma}_-$, or, since CP violation is very small, $\mathit{\Gamma}_2 > \mathit{\Gamma}_1$ and y is positive.

In the limit of small mixing ($x, y \ll 1$) and the absence of direct CP violation in DCS decays ($A_D = 0$), one can write

$$\lambda_\pm = \left|\frac{q}{p}\right| e^{\pm i\phi} \; . \tag{82.29}$$

In this scenario, to a good approximation, the decay rates for states that are initially D^0 and $\overline{D}^0$ to a CP eigenstate have exponential time dependence:

$$r_\pm(t) \;\propto\; \exp(-\mathit{\Gamma}_\pm\, t) \tag{82.30}$$

$$\overline{r}_\pm(t) \;\propto\; \exp(-\overline{\mathit{\Gamma}}_\pm\, t) \, . \tag{82.31}$$

The effective decay widths are given by

$$\mathit{\Gamma}_\pm \;=\; 1 \pm \left|\frac{q}{p}\right| (y\cos\phi - x\sin\phi) \;, \tag{82.32}$$

$$\overline{\mathit{\Gamma}}_\pm \;=\; 1 \pm \left|\frac{p}{q}\right| (y\cos\phi + x\sin\phi) \, . \tag{82.33}$$

Table 82.4: Results from time-dependent multibody analyses. The errors are statistical, systematic, and, when a third error is listed, due to the decay-model, respectively. The measurement with an asterisk (*) has been superseded and thus is not included in the HFLAV global fit. The measurement with a dagger (†) is not included in the HFLAV global fit due to poorer precision. The 2019 LHCb result utilizes strong-phase measurements from CLEO-c [39] and thus is decay-model independent. This fit determines CP-violating parameters Δx and Δy; the translation of these parameters to $|q/p|$ and ϕ is given in Ref. [42].

No CP Violation

Year	Experiment	Final State(s)	$x \times 10^{-3}$	$y \times 10^{-3}$
2019	LHCb (3.0 fb^{-1} B tag) [38]	$K^0_S\,\pi^+\pi^-$	$2.7 \pm 1.6 \pm 0.4$	$7.4 \pm 3.6 \pm 1.1$
2016	LHCb (1.0 fb^{-1} D^* tag) [37]	$K^0_S\,\pi^+\pi^-$	$-8.6 \pm 5.3 \pm 1.7$	$0.3 \pm 4.6 \pm 1.3$
2016	BaBar (468 fb^{-1}) [43]	$\pi^+\pi^-\pi^0$	$15 \pm 12 \pm 6$	$2 \pm 9 \pm 5$
2014	Belle (921 fb^{-1}) [34]	$K^0_S\,\pi^+\pi^-$	$5.6 \pm 1.9\,^{+0.3}_{-0.9}\,^{+0.6}_{-0.9}$	$3.0 \pm 1.5\,^{+0.4}_{-0.5}\,^{+0.3}_{-0.6}$
2010	BaBar (469 fb^{-1}) [35]	$\left\{\begin{array}{c} K^0_S\,\pi^+\pi^- \\ K^0_S\,K^+K^- \end{array}\right.$	$1.6\pm2.3\pm1.2\pm0.8$	$5.7\pm2.0\pm1.3\pm0.7$
2007	Belle (540 fb^{-1}) [33]	$K^0_S\,\pi^+\pi^-$	$8.0 \pm 2.9\,^{+0.9}_{-0.7}\,^{+1.0}_{-1.4}$	$3.3 \pm 2.4\,^{+0.8}_{-1.2}\,^{+0.6}_{-0.8}$
2005†	CLEO (9.0 fb^{-1}) [32]	$K^0_S\,\pi^+\pi^-$	$19\,^{+32}_{-33} \pm 4 \pm 4$	$-14 \pm 24 \pm 8 \pm 4$

With CP Violation

Year	Experiment	Final State(s)	$\lvert q/p\rvert$	ϕ
2019	LHCb (3.0 fb^{-1}) [38]	$K^0_S\,\pi^+\pi^-$	$\Delta x \times 10^{-3} =$ $-0.53 \pm 0.70 \pm 0.22$	$\Delta y \times 10^{-3} =$ $0.6 \pm 1.6 \pm 0.3$
2014	Belle (921 fb^{-1}) [34]	$K^0_S\pi^+\pi^-$	$0.90\,^{+0.16}_{-0.15}\,^{+0.05}_{-0.04}\,^{+0.06}_{-0.05}$	$(-6 \pm 11 \pm 3\,^{+3}_{-4})^\circ$
2007*‡	Belle (540 fb^{-1}) [33]	$K^0_S\,\pi^+\pi^-$	$0.86\,^{+0.30}_{-0.29}\,^{+0.06}_{-0.03} \pm 0.08$	$(-14\,^{+16}_{-18}\,^{+5}_{-3}\,^{+2}_{-4})^\circ$

‡This result allows for all CP violations and is superseded by Ref. [34], which assumes no direct CP violation in doubly Cabibbo-suppressed decays.

Thus the effective decay rate to a CP eigenstate combining both D^0 and $\overline{D}^0$ decays is

$$r_\pm(t) \ + \ \overline{r}_\pm(t) \ \propto \ e^{-(1\pm y_{CP})t}\,, \tag{82.34}$$

where

$$y_{CP} \ = \ \frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) x\sin\phi \tag{82.35}$$

$$\approx \ y\cos\phi - A_M x\sin\phi\,. \tag{82.36}$$

If CP is conserved, $y_{CP} = y$. Almost all measurements of y_{CP} are relative to the $D^0 \to K^-\pi^+$ decay rate. Belle [44] has reported y_{CP} also for the final state $K^0_S\,K^+K^-$, which is dominated by the CP-odd final state $K^0_S\,\phi$. Table 82.5 summarizes the current status of measurements.

In addition to y_{CP}, Belle [45], BaBar [46], LHCb [47], and CDF [48] have reported measurements of the decay-rate asymmetry for CP-even final states

$$A_\Gamma \equiv \frac{\Gamma_+ - \overline{\Gamma}_+}{\Gamma_+ + \overline{\Gamma}_+} = \frac{(1/\tau_+) - (1/\overline{\tau}_+)}{(1/\tau_+) + (1/\overline{\tau}_+)} = \frac{\overline{\tau}_+ - \tau_+}{\overline{\tau}_+ + \tau_+} \tag{82.37}$$

$$\approx \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) y\cos\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) x\sin\phi \tag{82.38}$$

$$\approx A_M\, y\cos\phi - x\sin\phi\,. \tag{82.39}$$

If CP is conserved, $A_\Gamma = 0$.

Table 82.5: Results for y_{CP} and A_Γ from $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays. When a single uncertainty is listed, that corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been superseded and thus are not included in the HFLAV average. Results from LHCb labeled "B tag" have the D^0 or $\overline{D}^0$ flavor identified via $B^\pm \to D\,\mu^\pm\nu_\mu\,X$ decays.

Year	Experiment	final state(s)	y_{CP} (%)	A_Γ ($\times 10^{-3}$)
2019	LHCb (3 fb^{-1} B tag) [49]	K^+K^-, $\pi^+\pi^-$	$0.57 \pm 0.13 \pm 0.09$	—
2017	LHCb (3 fb^{-1} D^* tag) [50]	K^+K^-, $\pi^+\pi^-$	—	$-0.13 \pm 0.28 \pm 0.10$
2017	LHCb (3 fb^{-1} D^* tag) [50]	K^+K^-	—	$-0.30 \pm 0.32 \pm 0.10$
2017	LHCb (3 fb^{-1} D^* tag) [50]	$\pi^+\pi^-$	—	$0.46 \pm 0.58 \pm 0.12$
2016	Belle (976 fb^{-1}) [45]	K^+K^-, $\pi^+\pi^-$	$1.11 \pm 0.22 \pm 0.09$	$-0.3 \pm 2.0 \pm 0.7$
2015	LHCb (3 fb^{-1} B tag) [51]	K^+K^-, $\pi^+\pi^-$	—	-1.25 ± 0.73
2015	LHCb (3 fb^{-1} B tag) [51]	K^+K^-	—	$-1.34 \pm 0.77\,^{+0.26}_{-0.34}$
2015	LHCb (3 fb^{-1} B tag) [51]	$\pi^+\pi^-$	—	$-0.92 \pm 1.45\,^{+0.25}_{-0.33}$
2015	BES III (2.9 fb^{-1}) [52]	K^+K^-, $\pi^+\pi^-$ $K^0_S\,\pi^0$, $K^0_S\,\pi^0\pi^0$ $K^0_S\,\eta$, $K^0_S\,\omega$	$-2.0 \pm 1.3 \pm 0.7$	—
2014	CDF (9.7 fb^{-1}) [48]	K^+K^-, $\pi^+\pi^-$	—	-1.2 ± 1.2
2014	CDF (9.7 fb^{-1}) [48]	K^+K^-	—	$-1.9 \pm 1.5 \pm 0.4$
2014	CDF (9.7 fb^{-1}) [48]	$\pi^+\pi^-$	—	$-0.1 \pm 1.8 \pm 0.3$
2012	BaBar (468 fb^{-1}) [46]	K^+K^-, $\pi^+\pi^-$	$0.72 \pm 0.18 \pm 0.12$	$0.9 \pm 2.6 \pm 0.6$
2009	Belle (673 fb^{-1}) [44]	$K^0_S\,K^+K^-$	$0.11 \pm 0.61 \pm 0.52$	—
2002	CLEO (9.0 fb^{-1}) [53]	K^+K^-, $\pi^+\pi^-$	$-1.2 \pm 2.5 \pm 1.4$	—
2000	FOCUS (1×10^6 events) [54]	K^+K^-	$3.42 \pm 1.39 \pm 0.74$	—
1999	E791 (2×10^{10} events) [55]	K^+K^-	$0.73 \pm 2.89 \pm 1.03$	—
	HFLAV Average [7]		$\mathbf{0.715 \pm 0.111}$	$\mathbf{-0.32 \pm 0.26}$
2013*	LHCb (1.0 fb^{-1} D^* tag) [56]	K^+K^-	—	$-0.35 \pm 0.62 \pm 0.12$
2013*	LHCb (1.0 fb^{-1} D^* tag) [56]	$\pi^+\pi^-$	—	$0.33 \pm 1.06 \pm 0.14$
2011*‡	LHCb (29 pb^{-1} D^* tag) [47]	K^+K^-	$0.55 \pm 0.63 \pm 0.41$	$-5.9 \pm 5.9 \pm 2.1$
2009*	BaBar (384 fb^{-1}) [57]	K^+K^-	$1.16 \pm 0.22 \pm 0.18$	—
2008*	BaBar (384 fb^{-1}) [58]	K^+K^-, $\pi^+\pi^-$	$1.03 \pm 0.33 \pm 0.19$	$2.6 \pm 3.6 \pm 0.8$
2007*	Belle (540 fb^{-1}) [59]	K^+K^-, $\pi^+\pi^-$	$1.31 \pm 0.32 \pm 0.25$	$0.1 \pm 3.0 \pm 1.5$
2003*	BaBar (91 fb^{-1}) [60]	K^+K^-, $\pi^+\pi^-$	$0.8 \pm 0.4\,^{+0.5}_{-0.4}$	—
2001*	Belle (23.4 fb^{-1}) [61]	K^+K^-	$-0.5 \pm 1.0\,^{+0.7}_{-0.8}$	—

‡This result for y_{CP} is not superseded but is not included in the HFLAV average due to having some correlations with, and much poorer precision than, the result of Ref. [49].

The asymmetry A_Γ relates to the full decay width. An asymmetry in partial widths is referred to as A_{CP} and is final-state dependent:

$$A_{CP} \equiv \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D}^0 \to \overline{f})}{\Gamma(D^0 \to f) + \Gamma(\overline{D}^0 \to \overline{f})} . \qquad (82.40)$$

Unlike A_Γ, A_{CP} is a time-integrated quantity, i.e., it does not require measuring decay times. For neutral D decays, A_{CP} receives contributions from both direct (in the decay amplitudes) and indirect (due to mixing) processes: $A_{CP}(D^0 \to f) = A^f_{CP} + A^{\text{indirect}}_{CP}$. The latter indirect contribution depends on the mixing parameters x and y:

$$A^{\text{indirect}}_{CP} = \frac{1}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) x \sin\phi - \frac{1}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) y \cos\phi = -A_\Gamma . \qquad (82.41)$$

Numerous measurements of A_{CP} for decays to CP eigenstates are listed in this *Review* [62]. Table 82.6 summarizes the current status of measurements of the *difference* in A_{CP} for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays: $\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$. Measuring the difference is advantagous, as numerous systematic uncertainties cancel. As A^{indirect}_{CP} is universal (independent of final state), it subtracts out of the difference ΔA_{CP}. However, at hadron experiments such as LHCb, there are differences in efficiencies between K^+K^- and $\pi^+\pi^-$ final states, and a small contribution to ΔA_{CP} remains. The most recent result from LHCb [63], based on 8.9 fb^{-1} of data, differs from zero with a statistical significance of 5.3σ. Thus, this measurement constitutes the first observation of CP violation in charm meson or charm baryon decays. These CP asymmetries are included in HFLAV's global fit for charm mixing parameters discussed below; the fit shows that the CP violation observed is due to the direct contributions A^{KK}_{CP} and $A^{\pi\pi}_{CP}$.

Table 82.6: Results for the difference in time-integrated CP asymmetries ΔA_{CP} between $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays. When a single uncertainty is listed, that corresponds to statistical and systematic uncertainties combined. The measurements with an asterisk (*) have been either superseded or combined with subsequent results and thus are not included in the HFLAV global fit.

Year	Experiment	$\Delta A_{CP}\,(\times 10^{-3})$
2019	LHCb (8.9 fb^{-1} D^* tag + B tag) [63]	-1.54 ± 0.29
2013	CDF (9.7 fb^{-1} D^* tag) [64]	$-6.2 \pm 2.1 \pm 1.0$
2008	BaBar (386 fb^{-1}) [65]	$2.4 \pm 6.2 \pm 2.6$
2008	Belle (540 fb^{-1}) [66]	$-8.6 \pm 6.0 \pm 0.7$
2016*	LHCb (3.0 fb^{-1} D^* tag) [67]	$-1.0 \pm 0.8 \pm 0.3$
2014*	LHCb (3.0 fb^{-1} B tag) [68]	$1.4 \pm 1.6 \pm 0.8$
2013*	LHCb (1.0 fb^{-1} B tag) [69]	$4.9 \pm 3.0 \pm 1.4$
2012*	LHCb (0.62 fb^{-1} D^* tag) [70]	$-8.2 \pm 2.1 \pm 1.1$
2012‡	Belle (976 fb^{-1}) [71]	$-8.7 \pm 4.1 \pm 0.6$

‡This preliminary result was not published and thus is not included in the HFLAV global fit.

82.5 Quantum-correlated $D^0\overline{D}^0$ Analyses

Measurements of R_D, $\cos\delta_{K\pi}$, $\sin\delta_{K\pi}$, x, and y can be obtained from a combined fit to time-integrated yields of single-tagged (ST) and double-tagged (DT) $D^0\overline{D}^0$ events produced at the $\psi(3770)$ resonance [72, 73]. Single-tagged events are those in which either the D^0 or $\overline{D}^0$ decay is reconstructed (identified), and the other neutral D decays generically. Double-tagged events are those in which both D^0 and $\overline{D}^0$ decays are identified. Due to quantum correlations, the decay of a D^0, $\overline{D}^0$, D_+, or D_- projects the other neutral D into a state $\overline{D}^0$, D^0, D_-, and D_+, respectively. The CP-specific D_- and D_+ decays (or, neglecting CP violation, D_1 and D_2 decays) include interference between D^0 and $\overline{D}^0$ amplitudes, and this provides sensitivity to R_D and $\cos\delta_{K\pi}$. The flavor-specific D^0 and $\overline{D}^0$ decays include interference between D_1 and D_2 amplitudes, and this provides sensitivity to x and y. For details of this method, see Refs. [1–5].

BESIII has reported results using 2.92 fb^{-1} of $e^+e^- \to \psi(3770)$ data, where the quantum-correlated $D^0\overline{D}^0$ pairs are produced in a $C=-1$ state. They measure $y_{CP} = (-2.0\pm1.3\pm0.7)\%$ [52] from DT yields using a CP eigenstate tag for one D and a semileptonic tag for the other; and they measure $A^{CP}_{K\pi} = (12.7\pm1.3\pm0.7)\%$ [27] from DT yields using a CP tag for one D and a $K^\pm\pi^\mp$ tag for the other. For y_{CP}, the CP eigenstates used are K^-K^+ (f_+), $\pi^+\pi^-$ (f_+), $K^0_S\pi^0\pi^0$ (f_+), $K^0_S\pi^0$ (f_-), $K^0_S\eta$ (f_-), and $K^0_S\omega$ (f_-). For $A^{CP}_{K\pi}$, additional CP eigenstates included are $\pi^0\pi^0$ (f_+) and $\rho^0\pi^0$ (f_+). Using external inputs for R_D and y from HFLAV [74], and R from the PDG [75] (see Eq. (82.27)), BESIII obtains $\cos\delta_{K\pi} = 1.02\pm0.11\pm0.06\pm0.01$ [27], where the third uncertainty is due to the external inputs.

CLEO-c has reported results using 0.82 fb^{-1} of $e^+e^- \to \psi(3770)$ data [76–78]. The values of y, R_M, $\cos\delta_{K\pi}$, and $\sin\delta_{K\pi}$ are determined from a combined fit to the ST (hadronic only) and DT yields. The DT yields include events in which one D is reconstructed in a hadronic mode and the other D is partially reconstructed in either $D \to K^\mp e^\pm\nu$ or $D \to K^\mp\mu^\pm\nu$. The CLEO-c analysis obtains $\cos\delta_{K\pi} = 0.81\,^{+0.22}_{-0.18}\,^{+0.07}_{-0.05}$ and $\sin\delta_{K\pi} = -0.01 \pm 0.41 \pm 0.04$. These fits allow $\cos\delta_{K\pi}$, $\sin\delta_{K\pi}$, and x^2 to be unphysical. Constraining $\cos\delta_{K\pi}$ and $\sin\delta_{K\pi}$ to the physical range $[-1,+1]$ (i.e., interpreting $\delta_{K\pi}$ as an angle) and also using external inputs for x, y and y_{CP} from HFLAV [79], CLEO-c obtains $\delta_{K\pi} = (18\,^{+11}_{-17})^\circ$ [78].

82.6 Summary of Experimental Results

Several recent results indicate that charm mixing is at the upper end of the range of Standard Model predictions. For $D^0 \to K^+\pi^-$, LHCb [20, 21], CDF [18], and Belle [17] each exclude the no-mixing hypothesis by more than 5 standard deviations. For y_{CP} in $D^0 \to K^+K^-$ and $\pi^+\pi^-$, BaBar [46], Belle [45], and LHCb [49] measure 3.3σ, 4.7σ, and 3.6σ effects, respectively. The most sensitive measurements of x and y are from $D^0 \to K^0_S\pi^+\pi^-$ decays at Belle [34] and LHCb [38]. In a similar analysis using $D^0 \to K^0_S\pi^+\pi^-$ and $D^0 \to K^0_S K^+K^-$ decays, BaBar [35] finds the no-mixing solution excluded at 1.9σ. LHCb [31] has reported the observation of charm mixing in $D^0 \to K^+\pi^-\pi^+\pi^-$ with 8.2σ significance. However, the strong phase difference for this decay is not known, and thus x and y cannot be extracted. The current situation would benefit from better knowledge of the strong phase difference $\delta_{K\pi}$ than provided by the current CLEO-c [78] and BESIII [27] results, and knowledge of the strong phase difference $\delta_{K\pi\pi\pi}$. Such knowledge would allow one to extract x and y from $D^0 \to K^+\pi^-$ measurements of (x'^2, y'), and from $D^0 \to K^+\pi^-\pi^+\pi^-$ measurements of (x''^2, y''). In fact, the most precise knowledge of $\delta_{K\pi}$ comes from combining measurements of y' (Table 82.3) with measurements of y (Table 82.4) and y_{CP} (Table 82.5), as done in the HFLAV global fit described below.

The experimental data consistently indicate that the D^0 and $\overline{D}^0$ mesons mix. The mixing is presumably dominated by long-range processes. Under the assumption that the observed mixing is due entirely to non-Standard Model processes, significant constraints on New Physics models can be obtained [80]. A serious limitation to the interpretation of charm mixing in terms of New Physics is the theoretical uncertainty on the Standard Model predictions [81, 82].

82.7 HFLAV Global Fit for Charm Mixing Parameters

The Heavy Flavor Averaging Group (HFLAV) performs a global fit to all relevant mixing measurements to obtain world average values for 10 fitted parameters: x, y, $\delta_{K\pi}$, $\delta_{K\pi\pi^0}$, $R_D(K^+\pi^-)$, $A_D(K^+\pi^-)$, $|q/p|$, $\mathrm{Arg}(q/p) \equiv \phi$, and the direct CP-violating asymmetries A_{CP}^{KK} and $A_{CP}^{\pi\pi}$. Correlations among observables are taken into account by using the error matrices provided by the experiments. Measurements of $D^0 \to K^{(*)+}\ell^-\overline{\nu}$, K^+K^-, $\pi^+\pi^-$, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, $K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$, and $\pi^+\pi^-\pi^0$ decays are used, as well as CLEO-c and BESIII results for double-tagged branching fractions measured at the $\psi(3770)$. There are three observables input to the fit that are themselves world average values calculated by HFLAV: R_M from $D^0 \to K^{(*)+}\ell^-\overline{\nu}$ decays (Table 82.1), and y_{CP} and A_Γ from $D^0 \to f_{CP}$ decays (Table 82.5). A measurement by LHCb of R_M using $D^0 \to K^+\pi^-\pi^+\pi^-$ decays is input separately. Details of the fitting procedure are given in Ref. [7].

The results of the fit are listed in Table 82.7. Three separate fits are performed: *(a)* assuming no CP violation; *(b)* assuming no CP violation in doubly Cabibbo-suppressed decays; and *(c)* allowing all CP violation. The second fit *(b)* corresponds to the theory expectation [83,84]; in this case four fitted parameters are reduced to three using the relationship $\tan\phi = (x/y)\cdot(1-|q/p|^2)/(1+|q/p|^2)$ [83–85]. Alternatively, one can fit for the three parameters $x_{12} \equiv 2|M_{12}|/\Gamma$, $y_{12} \equiv |\Gamma_{12}|/\Gamma$, and $\phi_{12} \equiv \mathrm{Arg}(M_{12}/\Gamma_{12})$, from which x, y, $|q/p|$, and ϕ can be derived.

Table 82.7: HFLAV global fit results (see text) [7].

Parameter	No CP Violation	No CP Violation in DCS Decays	All CP Violation Allowed	95% C.L. Interval CPV Allowed
x (%)	$0.50\,^{+0.13}_{-0.14}$	$0.43\,^{+0.10}_{-0.11}$	$0.39\,^{+0.11}_{-0.12}$	[0.16, 0.61]
y (%)	0.62 ± 0.07	0.63 ± 0.06	$0.651\,^{+0.063}_{-0.069}$	[0.51, 0.77]
$\delta_{K\pi}$ (°)	$8.9\,^{+8.2}_{-8.9}$	$9.3\,^{+8.3}_{-9.2}$	$12.1\,^{+8.6}_{-10.2}$	[−10.4, 28.2]
R_D (%)	0.344 ± 0.002	0.344 ± 0.002	0.344 ± 0.002	[0.339, 0.348]
A_D (%)	–	–	$-0.55\,^{+0.49}_{-0.51}$	[−1.5, 0.4]
$\|q/p\|$	–	0.998 ± 0.008	$0.969\,^{+0.050}_{-0.045}$	[0.89, 1.07]
ϕ (°)	–	0.08 ± 0.31	$-3.9\,^{+4.5}_{-4.6}$	[−13.2, 5.1]
$\delta_{K\pi\pi}$ (°)	$18.5\,^{+22.7}_{-23.4}$	$22.1\,^{+22.6}_{-23.4}$	$25.8\,^{+23.0}_{-23.8}$	[−21.3, 70.3]
$A_{CP}^{\pi\pi}$(%)	–	0.05 ± 0.16	0.06 ± 0.16	[−0.25, 0.38]
A_{CP}^{KK}(%)	–	-0.11 ± 0.16	-0.09 ± 0.16	[−0.40, 0.22]
x_{12} (%)	–	$0.43\,^{+0.10}_{-0.11}$		[0.22, 0.63]
y_{12} (%)	–	0.63 ± 0.06		[0.50, 0.75]
ϕ_{12}(°)	–	$-0.25\,^{+0.96}_{-0.99}$		[−2.5, 1.8]

Confidence contours in the two dimensions (x, y) and $(|q/p|, \phi)$ from the fit are plotted in Figs. 82.1 and 82.2, respectively. These contours are obtained by letting, for any point in the two-dimensional plane, all other fit parameters take their preferred values. The 1σ–5σ boundaries drawn are the loci of points in which the χ^2 has risen above the minimum by 2.30, 6.18, 11.83, 19.33, and 28.67 units. The fit excludes the no-mixing point $x = y = 0$ at more than 11.5σ, when CP violation is allowed. The fit is consistent with no CP violation ($|q/p| = 1$, $\phi = 0$) at the 63% confidence level.

One-dimensional likelihood functions for parameters are obtained by allowing, for any value of the parameter, all other fit parameters to take their preferred values. The resulting likelihood functions give central values, 68.3% C.L. intervals, and 95% C.L. intervals as listed in Table 82.7. The parameter ranges $x \leq 0$ and $y \leq 0$ are excluded at 3.1σ and $> 11.4\sigma$ significance, respectively.

The χ^2 of the fit is 60.7 for $49 - 10 = 39$ degrees of freedom, indicating some disagreement among the measurements.

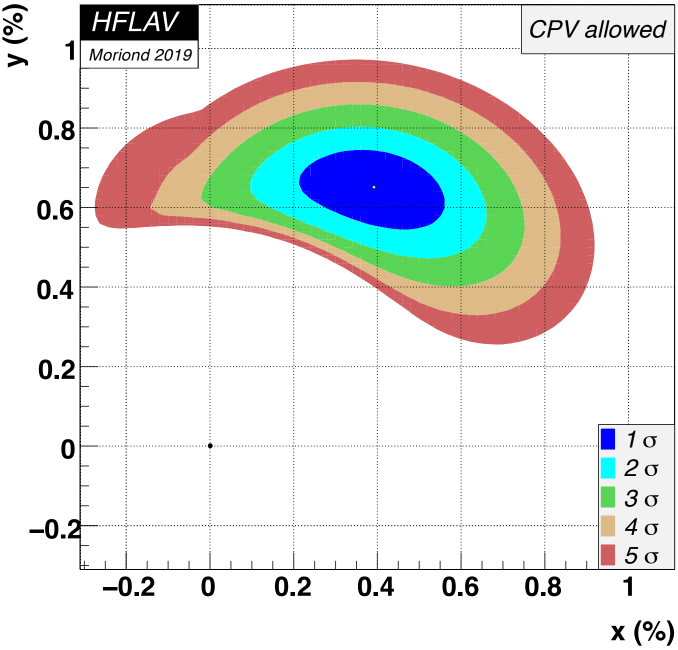

Figure 82.1: Two-dimensional 1σ-5σ contours for (x, y) as obtained by HFLAV [7], from measurements of $D^0 \to K^{(*)+}\ell\nu$, h^+h^-, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, $K^0_S\pi^+\pi^-$, $K^0_S K^+K^-$, and $\pi^+\pi^-\pi^0$ decays, and double-tagged branching fractions measured at the $\psi(3770)$ resonance.

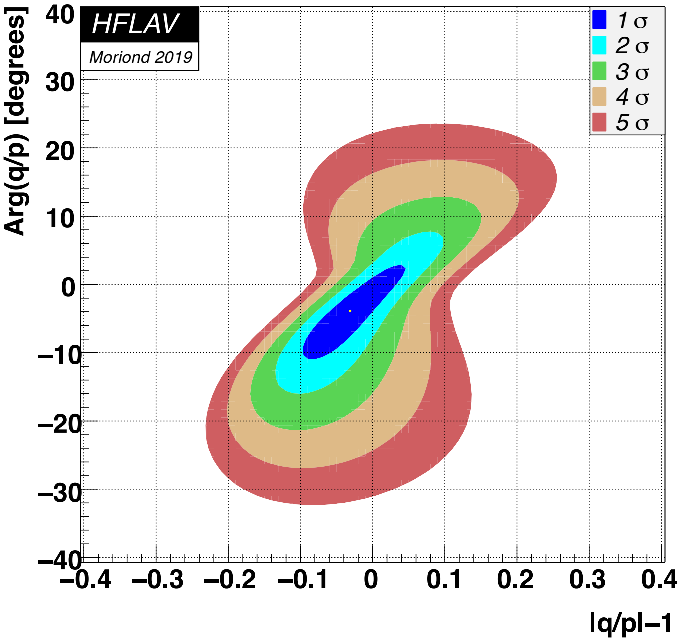

Figure 82.2: Two-dimensional 1σ-5σ contours for ($|q/p|$,Arg(q/p)) as obtained by HFLAV [7], from measurements of $D^0 \to K^{(*)+}\ell\nu$, h^+h^-, $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, $K^0_S\pi^+\pi^-$, $K^0_S K^+K^-$, and $\pi^+\pi^-\pi^0$ decays, and double-tagged branching fractions measured at the $\psi(3770)$ resonance.

From the results of the HFLAV averaging, the following can be concluded: (1) Since CP violation is small and y_{CP} is positive, the CP-even state is shorter-lived, as in the $K^0\overline{K}^0$ system. (2) However, since x appears to be positive, the CP-even state is heavier, unlike in the $K^0\overline{K}^0$ system. (3) The strong phase difference $\delta_{K\pi}$ is consistent with the $SU(3)$ expectation of zero, and large values are unlikely (its magnitude is $< 30°$ at 95% C.L.) (4) While direct CP violation has now been observed in D decays, there is no evidence for indirect CP violation, i.e., $|q/p| \neq 1$ or $\phi \neq 0$. Observing such CP violation at the current level of sensitivity would indicate new physics.

82.8 Future Data

Current results are based primarily upon CLEO-c (0.82 fb^{-1} of $e^+e^- \to \psi(3770)$ data), Belle and BaBar ($\sim$1.4 ab^{-1} of $e^+e^- \to \Upsilon(4S)$ data), and LHCb Runs 1 and 2 ($3.0\ \mathrm{fb}^{-1} + 5.9\ \mathrm{fb}^{-1}$ of pp collision data at $\sqrt{s} = 7$, 8, 13 TeV).

BESIII has accumulated 2.9 fb^{-1} of $e^+e^- \to \psi(3770)$ data and plans to collect up to 20 fb^{-1} in the next few years. These data should provide strong phase measurements that enable improved model-independent determinations of mixing parameters from Belle II and LHCb. In 2019, Belle II began accumulating 50 ab^{-1} of $e^+e^- \to \Upsilon(4S)$ data [86], which is expected to take approximately ten years to collect. At LHCb, Run 2 was completed in 2018, Run 3 is planned for 2021-23, and Run 4 is planned for 2026-29 [87]. The goal for Runs $3+4$ is to accumulate an additional 50 fb^{-1} of pp data at $\sqrt{s} = 14$ TeV. These data, along with the large e^+e^- dataset from Belle II, should provide significantly greater sensitivity to D^0-$\overline{D}^0$ mixing and direct and indirect CP violation in D^0 decays.

References

[1] D. M. Asner and W. M. Sun, Phys. Rev. **D73**, 034024 (2006), [Erratum: Phys. Rev.D77,019901(2008)], [hep-ph/0507238].

[2] D. Atwood and A. A. Petrov, Phys. Rev. **D71**, 054032 (2005), [hep-ph/0207165].

[3] M. Gronau, Y. Grossman and J. L. Rosner, Phys. Lett. **B508**, 37 (2001), [hep-ph/0103110].

[4] Z.-z. Xing, Phys. Rev. **D55**, 196 (1997), [hep-ph/9606422].

[5] M. Goldhaber and J. L. Rosner, Phys. Rev. **D15**, 1254 (1977).

[6] See "Review of Multibody Charm Analyses" in this *Review.*

[7] Y. Amhis *et al.* (Heavy Flavor Averaging Group) (2019), [arXiv:1909.12524].

[8] U. Bitenc *et al.* (Belle), Phys. Rev. **D77**, 112003 (2008), [arXiv:0802.2952].

[9] B. Aubert *et al.* (BaBar), Phys. Rev. **D76**, 014018 (2007), [arXiv:0705.0704].

[10] C. Cawlfield *et al.* (CLEO), Phys. Rev. **D71**, 077101 (2005), [hep-ex/0502012].

[11] E. M. Aitala *et al.* (E791), Phys. Rev. Lett. **77**, 2384 (1996), [hep-ex/9606016].

[12] U. Bitenc *et al.* (Belle), Phys. Rev. **D72**, 071101 (2005), [hep-ex/0507020].

[13] B. Aubert *et al.* (BaBar), Phys. Rev. **D70**, 091102 (2004), [hep-ex/0408066].

[14] Y. Nir, Lectures given at 27th SLAC Summer Institute on Particle Physics: "CP Violation in and Beyond the Standard Model (SSI 99)," Stanford, California, 7-16 July 1999. Published in Trieste 1999, *Particle Physics*, pp. 165-243.

[15] S. Bergmann *et al.*, Phys. Lett. **B486**, 418 (2000), [hep-ph/0005181].

[16] R. Aaij *et al.* (LHCb), Phys. Rev. **D95**, 5, 052004 (2017), [Erratum: Phys. Rev.D96,no.9,099907(2017)], [arXiv:1611.06143].

[17] B. R. Ko *et al.* (Belle), Phys. Rev. Lett. **112**, 11, 111801 (2014), [Addendum: Phys. Rev. Lett.112,no.13,139903(2014)], [arXiv:1401.3402].

[18] T. A. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **111**, 23, 231802 (2013), [arXiv:1309.4078].

[19] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **98**, 211802 (2007), [hep-ex/0703020].

[20] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **111**, 25, 251801 (2013), [arXiv:1309.6534].

[21] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **110**, 10, 101802 (2013), [arXiv:1211.1230].

[22] T. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **100**, 121802 (2008), [arXiv:0712.1567].

[23] L. M. Zhang *et al.* (Belle), Phys. Rev. Lett. **96**, 151801 (2006), [hep-ex/0601029].

[24] J. M. Link *et al.* (FOCUS), Phys. Lett. **B618**, 23 (2005), [hep-ex/0412034].

[25] R. Godang *et al.* (CLEO), Phys. Rev. Lett. **84**, 5038 (2000), [hep-ex/0001060].

[26] E. M. Aitala *et al.* (E791), Phys. Rev. **D57**, 13 (1998), [hep-ex/9608018].

[27] M. Ablikim *et al.* (BESIII), Phys. Lett. **B734**, 227 (2014), [arXiv:1404.4691].

[28] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **97**, 221803 (2006), [hep-ex/0608006].

[29] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **103**, 211801 (2009), [arXiv:0807.4544].

[30] E. White *et al.* (Belle), Phys. Rev. **D88**, 5, 051101 (2013), [arXiv:1307.5935].

[31] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **116**, 24, 241801 (2016), [arXiv:1602.07224].

[32] D. M. Asner *et al.* (CLEO), Phys. Rev. **D72**, 012001 (2005), [hep-ex/0503045].

[33] K. Abe *et al.* (Belle), Phys. Rev. Lett. **99**, 131803 (2007), [arXiv:0704.1000].

[34] T. Peng *et al.* (Belle), Phys. Rev. **D89**, 9, 091103 (2014), [arXiv:1404.2412].

[35] P. del Amo Sanchez *et al.* (BaBar), Phys. Rev. Lett. **105**, 081803 (2010), [arXiv:1004.5053].

[36] H. Muramatsu *et al.* (CLEO), Phys. Rev. Lett. **89**, 251802 (2002), [Erratum: Phys. Rev. Lett.90,059901(2003)], [hep-ex/0207067].

[37] R. Aaij *et al.* (LHCb), JHEP **04**, 033 (2016), [arXiv:1510.01664].

[38] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **122**, 23, 231802 (2019), [arXiv:1903.03074].

[39] J. Libby *et al.* (CLEO), Phys. Rev. **D82**, 112006 (2010), [arXiv:1010.2817].

[40] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **95**, 121802 (2005), [hep-ex/0504039].

[41] B. Aubert *et al.* (BaBar), Phys. Rev. **D78**, 034023 (2008), [arXiv:0804.2089].

[42] A. Di Canto *et al.*, Phys. Rev. **D99**, 1, 012007 (2019), [arXiv:1811.01032].

[43] J. P. Lees *et al.* (BaBar), Phys. Rev. **D93**, 11, 112014 (2016), [arXiv:1604.00857].

[44] A. Zupanc *et al.* (Belle), Phys. Rev. **D80**, 052006 (2009), [arXiv:0905.4185].

[45] M. Starič *et al.* (Belle), Phys. Lett. **B753**, 412 (2016), [arXiv:1509.08266].

[46] J. P. Lees *et al.* (BaBar), Phys. Rev. **D87**, 1, 012004 (2013), [arXiv:1209.3896].

[47] R. Aaij *et al.* (LHCb), JHEP **04**, 129 (2012), [arXiv:1112.4698].

[48] T. A. Aaltonen *et al.* (CDF), Phys. Rev. **D90**, 11, 111103 (2014), [arXiv:1410.5435].

[49] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **122**, 1, 011802 (2019), [arXiv:1810.06874].

[50] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **118**, 26, 261803 (2017), [arXiv:1702.06490].

[51] R. Aaij *et al.* (LHCb), JHEP **04**, 043 (2015), [arXiv:1501.06777].

[52] M. Ablikim *et al.* (BESIII), Phys. Lett. **B744**, 339 (2015), [arXiv:1501.01378].

[53] S. E. Csorna *et al.* (CLEO), Phys. Rev. **D65**, 092001 (2002), [hep-ex/0111024].

[54] J. M. Link *et al.* (FOCUS), Phys. Lett. **B485**, 62 (2000), [hep-ex/0004034].

[55] E. M. Aitala *et al.* (E791), Phys. Rev. Lett. **83**, 32 (1999), [hep-ex/9903012].

[56] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **112**, 4, 041801 (2014), [arXiv:1310.7201].

[57] B. Aubert *et al.* (BaBar), Phys. Rev. **D80**, 071103 (2009), [arXiv:0908.0761].

[58] B. Aubert *et al.* (BaBar), Phys. Rev. **D78**, 011105 (2008), [arXiv:0712.2249].

[59] M. Staric *et al.* (BELLE), Phys. Rev. Lett. **98**, 211803 (2007), [,65(2007)], [hep-ex/0703036].

[60] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **91**, 121801 (2003), [hep-ex/0306003].

[61] K. Abe *et al.* (Belle), Phys. Rev. Lett. **88**, 162001 (2002), [hep-ex/0111026].

[62] See the tabulation of A_{CP} results in the D^0 and D^+ Listings in this *Review.*

[63] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **122**, 21, 211803 (2019), [arXiv:1903.08726].

[64] T. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **109**, 111801 (2012), [arXiv:1207.2158].

[65] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **100**, 061803 (2008), [arXiv:0709.2715].

[66] M. Staric *et al.* (Belle), Phys. Lett. **B670**, 190 (2008), [arXiv:0807.0148].

[67] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **116**, 19, 191601 (2016), [arXiv:1602.03160].

[68] R. Aaij *et al.* (LHCb), JHEP **07**, 041 (2014), [arXiv:1405.2797].

[69] R. Aaij *et al.* (LHCb), Phys. Lett. **B723**, 33 (2013), [arXiv:1303.2614].

[70] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **108**, 111602 (2012), [arXiv:1112.0938].

[71] B. R. Ko (Belle), in "7th International Workshop on the CKM Unitarity Triangle (CKM 2012) Cincinnati, Ohio, USA, September 28-October 2, 2012," (2012), [arXiv:1212.5320].

[72] R.A. Briere *et al.*, (CLEO Collab.), CLNS 01-1742, (2001).

[73] D. M. Asner *et al.*, (BES-III Collab.), Int. J. Mod. Phys. **A**, 24 (2009).

[74] Heavy Flavor Averaging Group, https://hflav-eos.web.cern.ch/hflav-eos/charm/CHARM13/results_mix_cpv.html.

[75] J. Beringer *et al.* (Particle Data Group), Phys. Rev. **D86**, 010001 (2012).

[76] J. L. Rosner *et al.* (CLEO), Phys. Rev. Lett. **100**, 221801 (2008), [arXiv:0802.2264].

[77] D. M. Asner *et al.* (CLEO), Phys. Rev. **D78**, 012001 (2008), [arXiv:0802.2268].

[78] D. M. Asner *et al.* (CLEO), Phys. Rev. **D86**, 112001 (2012), [arXiv:1210.0939].

[79] Heavy Flavor Averaging Group, https://hflav-eos.web.cern.ch/hflav-eos/charm/March12/results_mix_cpv.html.

[80] E. Golowich *et al.*, Phys. Rev. **D76**, 095009 (2007), [arXiv:0705.3650].

[81] G. Isidori *et al.*, Phys. Lett. **B711**, 46 (2012), [arXiv:1111.4987].

[82] E. Franco, S. Mishima and L. Silvestrini, JHEP **05**, 140 (2012), [arXiv:1203.3131].

[83] Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. **103**, 071602 (2009), [arXiv:0904.0305].

[84] A. L. Kagan and M. D. Sokoloff, Phys. Rev. **D80**, 076008 (2009), [arXiv:0907.3917].

[85] M. Ciuchini *et al.*, Phys. Lett. **B655**, 162 (2007), [hep-ph/0703204].

[86] E. Kou *et al.* (Belle-II) (2018), [arXiv:1808.10567].

[87] R. Aaij *et al.* (LHCb) (2018), [arXiv:1808.08865].