65. Three-Neutrino Mixing Parameters

Updated July 2019 by M. Goodman (ANL).

65.1. Introduction and Notation

With the exception of possible short-baseline anomalies (such as LSND), current accelerator, reactor, solar and atmospheric neutrino data can be described within the framework of a 3×3 mixing matrix between the flavor states ν_e, ν_μ and ν_τ and mass eigenstates ν_1, ν_2 and ν_3. (See equation 14.34 of the review "Neutrino Mass, Mixing and Oscillations" by M.C. Gonzalez-Garcia and M. Yokoyama.) Whether or not this is the ultimately correct framework, it is currently widely used to parametrize neutrino mixing data and to plan new experiments.

The mass differences are called $\Delta m^2_{21} \equiv m^2_2 - m^2_1$ and $\Delta m^2_{32} \equiv m^2_3 - m^2_2$. Until recently, we assumed

$$\Delta m^2_{32} \sim \Delta m^2_{31}. \tag{65.1}$$

But the experimental error is comparable to the difference $\Delta m^2_{31} - \Delta m^2_{32} = \Delta m^2_{21}$, so we quote them separately when appropriate. The measurements made by ν_μ disappearance at accelerators and by ν_e disappearance at reactors are slightly different mixtures of Δm^2_{32} and Δm^2_{31}. The angles are labeled θ_{12}, θ_{23} and θ_{13}. The CP violating phase is called δ_{CP}.

The familiar two neutrino form for oscillations is

$$P(\nu_a \to \nu_b; a \neq b) = \sin^2(2\theta)\sin^2(\Delta m^2 L/4E). \tag{65.2}$$

Despite the fact that the mixing angles have been measured to be much larger than in the quark sector, the two neutrino form is often a very good approximation and is used in many situations.

The angles appear in the equations below in many forms. They often appear as $\sin^2(2\theta)$. The listings currently now use $\sin^2(\theta)$ because this distinguishes the octant, i.e. whether θ_{23} is larger or smaller than 45°.

65.2. Accelerator neutrino experiments

Ignoring Δm^2_{21}, *CP* violation, and matter effects, the equations for the probability of appearance in an accelerator oscillation experiment are:

$$P(\nu_\mu \to \nu_\tau) = \sin^2(2\theta_{23})\cos^4(\theta_{13})\sin^2(\Delta m^2_{32} L/4E) \tag{65.3}$$

$$P(\nu_\mu \to \nu_e) = \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2(\Delta m^2_{32} L/4E) \tag{65.4}$$

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2(\Delta m^2_{32} L/4E) \tag{65.5}$$

$$P(\nu_e \to \nu_\tau) = \sin^2(2\theta_{13})\cos^2(\theta_{23})\sin^2(\Delta m^2_{32} L/4E)\ . \tag{65.6}$$

Current and future long-baseline accelerator experiments are studying non-zero θ_{13} through $P(\nu_\mu \to \nu_e)$. Including the CP terms and low mass scale, the equation for neutrino oscillation in vacuum is:

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= P1 + P2 + P3 + P4 \\ P1 &= \sin^2(\theta_{23})\sin^2(2\theta_{13})\sin^2(\Delta m^2_{32}L/4E) \\ P2 &= \cos^2(\theta_{23})\sin^2(2\theta_{13})\sin^2(\Delta m^2_{21}L/4E) \\ P3 &= -/+\, J\sin(\delta_{CP})\sin(\Delta m^2_{32}L/4E) \\ P4 &= J\cos(\delta_{CP})\cos(\Delta m^2_{32}L/4E) \end{aligned} \tag{65.7}$$

where

$$\begin{aligned} J = &\cos(\theta_{13})\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\times \\ &\sin(\Delta m^2_{32}L/4E)\sin(\Delta m^2_{21}L/4E) \end{aligned} \tag{65.8}$$

and the sign in P3 is negative for neutrinos and positive for anti-neutrinos respectively. For most new long-baseline accelerator experiments, P2 can safely be neglected. Also, depending on the distance and the mass order, matter effects need to be included.

65.3. Reactor neutrino experiments

Nuclear reactors are prolific sources of $\bar{\nu}_e$ with an energy near 4 MeV. The oscillation probability can be expressed

$$\begin{aligned} P(\bar{\nu}_e \to \bar{\nu}_e) = 1 &- \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta m^2_{21}L/4E) \\ &- \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta m^2_{31}L/4E) \\ &- \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta m^2_{32}L/4E) \end{aligned} \tag{65.9}$$

not using the approximation in Eq. (65.1). For short distances (L<5 km) we can ignore the second term on the right and can reimpose approximation Eq. (65.1). This takes the familiar two neutrino form with θ_{13} and Δm^2_{32}:

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2(2\theta_{13})\sin^2(\Delta m^2_{32}L/4E). \tag{65.10}$$

65.4. Solar and Atmospheric neutrino experiments

Solar neutrino experiments are sensitive to ν_e disappearance and have allowed the measurement of θ_{12} and Δm^2_{21}. They are also sensitive to θ_{13}. We identify $\Delta m^2_{\odot} = \Delta m^2_{21}$ and $\theta_{\odot} = \theta_{12}$.

Atmospheric neutrino experiments are primarily sensitive to ν_μ disappearance through $\nu_\mu \to \nu_\tau$ oscillations, and have allowed the measurement of θ_{23} and Δm^2_{32}. We identify $\Delta m^2_A = \Delta m^2_{32}$ and $\theta_A = \theta_{23}$. Despite the large ν_e component of the atmospheric neutrino flux, it is difficult to measure Δm^2_{21} effects. This is because of a cancellation between $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\mu$ together with the fact that the ratio of ν_μ and ν_e atmospheric fluxes, which arise from sequential π and μ decay, is near 2.

65.5. Oscillation Parameter Listings

In Section (B) we encode the three mixing angles θ_{12}, θ_{23}, θ_{13}, δ_{CP}, and two mass squared differences Δm^2_{21} and Δm^2_{32}. Our knowledge of θ_{12} and Δm^2_{21} comes from the KamLAND reactor neutrino experiment together with solar neutrino experiments. Our knowledge of θ_{23} and Δm^2_{32} comes from atmospheric, reactor and long-baseline accelerator neutrino experiments. For the earlier experiments, we identified the large mass splitting as Δm^2_{32}. Now that $\sigma(\Delta m^2_{32}) \approx \Delta m^2_{21}$, some experiments report separate values for the two mass orders. Results on θ_{13} come from reactor antineutrino disappearance experiments. There are also results from long-baseline accelerator experiments looking for ν_e appearance. The interpretation of both kinds of results depends on Δm^2_{32}, and the accelerator results also depend on the mass order, θ_{23} and the CP violating phase δ_{CP}.

Accelerator and atmospheric experiments have some sensitivity to the CP violation phase δ_{CP} through Eq. (65.7). Note that P3 depends on the sign of Δm^2_{32} so the sensitivity depends on the mass order. For non-maximal θ_{23} mixing, it also depends on the octant of θ_{23}, i.e. whether $\theta_{23} > \pi/4$ or $\theta_{23} < \pi/4$.