

78. *CP* Violation in  $K_S \rightarrow 3\pi$ 

Written 1996 by T. Nakada (Paul Scherrer Institute) and L. Wolfenstein (Carnegie-Mellon University).

The possible final states for the decay  $K^0 \rightarrow \pi^+\pi^-\pi^0$  have isospin  $I = 0, 1, 2$ , and  $3$ . The  $I = 0$  and  $I = 2$  states have  $CP = +1$  and  $K_S$  can decay into them without violating  $CP$  symmetry, but they are expected to be strongly suppressed by centrifugal barrier effects. The  $I = 1$  and  $I = 3$  states, which have no centrifugal barrier, have  $CP = -1$  so that the  $K_S$  decay to these requires  $CP$  violation.

In order to see  $CP$  violation in  $K_S \rightarrow \pi^+\pi^-\pi^0$ , it is necessary to observe the interference between  $K_S$  and  $K_L$  decay, which determines the amplitude ratio

$$\eta_{+-0} = \frac{A(K_S \rightarrow \pi^+\pi^-\pi^0)}{A(K_L \rightarrow \pi^+\pi^-\pi^0)}. \quad (78.1)$$

If  $\eta_{+-0}$  is obtained from an integration over the whole Dalitz plot, there is no contribution from the  $I = 0$  and  $I = 2$  final states and a nonzero value of  $\eta_{+-0}$  is entirely due to  $CP$  violation.

Only  $I = 1$  and  $I = 3$  states, which are  $CP = -1$ , are allowed for  $K^0 \rightarrow \pi^0\pi^0\pi^0$  decays and the decay of  $K_S$  into  $3\pi^0$  is an unambiguous sign of  $CP$  violation. Similarly to  $\eta_{+-0}$ ,  $\eta_{000}$  is defined as

$$\eta_{000} = \frac{A(K_S \rightarrow \pi^0\pi^0\pi^0)}{A(K_L \rightarrow \pi^0\pi^0\pi^0)}. \quad (78.2)$$

If one assumes that  $CPT$  invariance holds and that there are no transitions to  $I = 3$  (or to nonsymmetric  $I = 1$  states), it can be shown that

$$\begin{aligned} \eta_{+-0} &= \eta_{000} \\ &= \epsilon + i \frac{\text{Im } a_1}{\text{Re } a_1}. \end{aligned} \quad (78.3)$$

With the Wu-Yang phase convention,  $a_1$  is the weak decay amplitude for  $K^0$  into  $I = 1$  final states;  $\epsilon$  is determined from  $CP$  violation in  $K_L \rightarrow 2\pi$  decays. The real parts of  $\eta_{+-0}$  and  $\eta_{000}$  are equal to  $\text{Re}(\epsilon)$ . Since currently-known upper limits on  $|\eta_{+-0}|$  and  $|\eta_{000}|$  are much larger than  $|\epsilon|$ , they can be interpreted as upper limits on  $\text{Im}(\eta_{+-0})$  and  $\text{Im}(\eta_{000})$  and so as limits on the  $CP$ -violating phase of the decay amplitude  $a_1$ .