Dalitz Plot Parameters for $K \rightarrow 3\pi$ Decays

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The Dalitz plot distribution for $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp$, $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$, and $K_L^0 \rightarrow \pi^+ \pi^- \pi^0$ can be parameterized by a series expansion such as that introduced by Weinberg [1]. We use the form

$$|M|^2 \propto 1 + g \frac{(s_3 - s_0)}{m^2_{\pi^+}} + h \left[ \frac{s_3 - s_0}{m^2_{\pi^+}} \right]^2$$

$$+ j \frac{(s_2 - s_1)}{m^2_{\pi^+}} + k \left[ \frac{s_2 - s_1}{m^2_{\pi^+}} \right]^2$$

$$+ f \frac{(s_2 - s_1)(s_3 - s_0)}{m^2_{\pi^+}} + \cdots,$$

(1)

where $m^2_{\pi^+}$ has been introduced to make the coefficients $g$, $h$, $j$, and $k$ dimensionless, and

$$s_i = (P_K - P_i)^2 = (m_K - m_i)^2 - 2m_K T_i, \quad i = 1, 2, 3,$$

$$s_0 = \frac{1}{3} \sum_i s_i = \frac{1}{3} (m^2_K + m^2_1 + m^2_2 + m^2_3).$$

Here the $P_i$ are four-vectors, $m_i$ and $T_i$ are the mass and kinetic energy of the $i^{th}$ pion, and the index 3 is used for the odd pion.

The coefficient $g$ is a measure of the slope in the variable $s_3$ (or $T_3$) of the Dalitz plot, while $h$ and $k$ measure the quadratic dependence on $s_3$ and $(s_2 - s_1)$, respectively. The coefficient $j$ is related to the asymmetry of the plot and must be zero if $CP$ invariance holds. Note also that if $CP$ is good, $g$, $h$, and $k$ must be the same for $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ as for $K^- \rightarrow \pi^- \pi^- \pi^+.$

Since different experiments use different forms for $|M|^2$, in order to compare the experiments we have converted to $g$, $h$, $j$, and $k$ whatever coefficients have been measured. Where such conversions have been done, the measured coefficient $a_y$, $a_t$, $a_u$, or $a_v$ is given in the comment at the right. For definitions of these coefficients, details of this conversion, and discussion of the data, see the April 1982 version of this note [2].

References: