70. $D_s^+$ Branching Fractions

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Figure 70.1 shows a partial breakdown of the $D_s^+$ branching fractions. The rest of this note is about how the figure was constructed. The values shown make heavy use of CLEO measurements of inclusive branching fractions [1]. For references to other data cited in the following, see the Listings.

70.1. Modes with leptons

The bottom ($18.0 \pm 1.0$)% of Fig. 70.1 shows the fractions for the modes that include leptons. The measured $K^0e^+\nu_e$ and $K^*0e^+\nu_e$ fractions have been doubled to take account of the corresponding $\mu^+\nu_\mu$ fractions. The sum of the exclusive $Xe^+\nu_e$ fractions is $(6.0 \pm 0.3)$%, consistent with an inclusive semileptonic measurement of $(6.5 \pm 0.4)$%. There seems to be little missing here.

70.2. Inclusive hadronic $K\bar{K}$ fractions

The Cabibbo-favored $c \to s$ decay in $D_s^+$ decay produces a final state with both an $s$ and an $\bar{s}$; and thus modes with a $K\bar{K}$ pair or with an $\eta$, $\omega$, $\eta'$, or $\phi$ predominate (as may already be seen in Fig. 70.1 in the semileptonic fractions). We consider the $K\bar{K}$ modes first. A complete picture of the exclusive $K\bar{K}$ charge modes is not yet possible, because branching fractions for many of those modes have not yet been measured. However, CLEO has measured the inclusive $K^+, K^-, K^0_S$, $K^+K^0_S$, $K^-K^0_S$, and $2K^0_S$ fractions (these include modes with leptons) [1]. And each of these inclusive fractions with a $K^0_S$ is equal to the corresponding fraction with a $K^0_L$: $f(K^+K^0_S) = f(K^0_L)$, $f(K^-K^0_S) = f(K^0_S)$, etc. Therefore, of all inclusive fractions pairing a $K^+$, $K^0_S$, or $K^0_L$ with a $K^-$, $K^0_S$, or $K^0_L$, we know all but $f(K^0_SK^0_L)$.

We can get that fraction. The total $K^0_S$ fraction is

$$f(K^0_S) = f(K^+K^0_S) + f(K^-K^0_S) + 2f(2K^0_S) + f(K^0_SK^0_L)$$

where $f(\text{single } K^0_S)$ is the sum of the branching fractions for modes such as $K^0_S\pi^+2\pi^0$ with a $K^0_S$ and no second $K$. The $K^0_S\pi^+2\pi^0$ mode is in fact the only unmeasured single-$K^0_S$ mode (throughout, we shall assume that fractions for modes with a $K$ or $K\bar{K}$ and more than three pions are negligible), and we shall take its fraction to be the same as for the $K^0_S2\pi^+\pi^-$ mode, $(0.30 \pm 0.11)$%. Any reasonable deviation from this value would be too small to matter much in the following. Adding the several small single-$K^0_S$ branching fractions, including those from semileptonic modes, we get $f(\text{single } K^0_S) = (1.7 \pm 0.2)$%.

Using this, we have:

$$f(K^0_SK^0_L) = f(K^0_S) - f(K^+K^0_S) - f(K^-K^0_S)$$

$$- 2f(2K^0_S) - f(\text{single } K^0_S)$$

$$= (19.0 \pm 1.1) - (5.8 \pm 0.5) - (1.9 \pm 0.4)$$

$$- 2 \times (1.7 \pm 0.3) - (1.7 \pm 0.2)$$

$$= (6.2 \pm 1.4)$% .

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)
Figure 70.1: A partial breakdown of $D_s^+$ branching fractions. The hadronic bins in the left column show inclusive fractions. Shading within a bin shows how much of the inclusive fraction is not yet accounted for by adding up all the relevant exclusive fractions. The inclusive hadronic $\phi$ fraction is spread over three bins, in proportion to its decay fractions into $K^+K^-$, $K^0_SK^0_L$, and $\bar{K}K$ modes.
Here and below we treat the errors as uncorrelated, although often they are not. However, our main aim is to get numbers for Fig. 70.1; errors are secondary.

There is a check on our result: The $\phi$ inclusive branching fraction is $(15.7 \pm 1.0)\%$, of which $34\%$, or $(5.34 \pm 0.34)\%$ of $D_s^+$ decays, produces a $K_S^0 K_L^0$. Our $f(K_S^0 K_L^0) = (6.2 \pm 1.4)\%$ has to be at least this large—and it is, within the sizable error.

We now have all the inclusive $K\bar{K}$ fractions. We use $f(K^+\bar{K}^0) = 2 f(K^+ K_S^0)$, and likewise for $f(K^- K^0)$. For $K^+ K^-$ and $K_S^0 K_L^0$, we subtract off the contributions from $\phi\ell^+\nu$ decay to get the purely hadronic $K\bar{K}$ inclusive fractions:

$$f(K^+ K^-, \text{hadronic}) = (15.8 \pm 0.7) - (2.1 \pm 0.3) = (13.7 \pm 0.8)\%$$

$$f(K^+ \bar{K}^0, \text{hadronic}) = (11.6 \pm 1.0)\%$$

$$f(K^- K^0, \text{hadronic}) = (3.8 \pm 0.8)\%$$

$$f(2K_S^0 + 2K_L^0, \text{hadronic}) = (3.4 \pm 0.6)\%$$

$$f(K_S^0 K_L^0, \text{hadronic}) = (6.2 \pm 1.4) - (1.5 \pm 0.2) = (4.7 \pm 1.4)\% .$$

The fractions are shown in Fig. 70.1. They total $(37.2 \pm 2.2)\%$ of $D_s^+$ decays.

We can add more information to the figure by summing up measured branching fractions for exclusive modes within each bin:

$K^+ K^-$ modes—The sum of measured $K^+ K^- \pi^+$, $K^+ K^- \pi^+ \pi^0$, and $K^+ K^- 2\pi^+ \pi^-$ branching fractions is $(12.6 \pm 0.6)\%$. That leaves $(1.1 \pm 1.0)\%$ for the $K^+ K^- \pi^+ 2\pi^0$ mode, which is the only other $K^+ K^-$ mode with three or fewer pions. In Fig. 70.1, this unmeasured part of the $K^+ K^-$ bin is shaded.

$K^+ \bar{K}^0$ modes—Two times the sum of the measured $K^+ K_S^0$, $K^+ K_S^0 \pi^0$, and $K^+ K_S^0 \pi^+ \pi^-$ branching fractions is $(8.0 \pm 0.5)\%$. This leaves $(3.6 \pm 1.1)\%$ for the unmeasured $K^+ \bar{K}^0$ modes (there are three such modes with three or fewer pions). This is shaded in the figure.

$K^- K^0$ modes—Twice the $K^- K_S^0 2\pi^+$ fraction is $(3.4 \pm 0.2)\%$, which leaves about $(0.4 \pm 0.8)\%$ for $K^- K_S^0 2\pi^+ \pi^0$, the only other $K^- K^0$ mode with three or fewer pions.

$2K_S^0 + 2K_L^0$ modes—The $2K_S^0 \pi^+$ and $2K_S^0 2\pi^+ \pi^-$ fractions sum to $(0.86 \pm 0.07)\%$; this times two (for the corresponding $2K_L^0$ modes) is $(1.72 \pm 0.14)\%$. This leaves about $(1.7 \pm 0.7)\%$ for other $2K_S^0 + 2K_L^0$ modes.

$K_S^0 K_L^0$ modes—Most of the $K_S^0 K_L^0$ fraction is accounted for by $\phi$ decays (see below).
70.3. Inclusive hadronic $\eta$, $\omega$, $\eta'$, and $\phi$ fractions

These are easier. We start with the inclusive branching fractions, and then, to avoid double counting, subtract: (1) fractions for modes with leptons; (2) $\eta$ mesons that are included in the inclusive $\eta'$ fraction; and (3) $K^+K^-$ and $K_S^0K_L^0$ from $\phi$ decays:

\[
\begin{align*}
    f(\eta \text{ hadronic}) &= f(\eta \text{ inclusive}) - 0.65 f(\eta' \text{ inclusive}) \\
    -f(\eta\ell^+\nu) &= (18.5 \pm 3.0)\% \\
    f(\omega \text{ hadronic}) &= f(\omega \text{ inclusive}) - 0.026 f(\eta' \text{ inclusive}) \\
                              &= (5.8 \pm 1.4)\% \\
    f(\eta' \text{ hadronic}) &= f(\eta' \text{ inclusive}) - f(\eta'\ell^+\nu) \\
                             &= (8.5 \pm 1.5)\% \\
    f(\phi \text{ hadronic, } \not\to K\bar{K}) &= 0.17 \left[ f(\phi \text{ inclusive}) - f(\phi\ell^+\nu) \right] \\
                                   &= (1.9 \pm 0.2)\%.
\end{align*}
\]

The factors 0.65, 0.026, and 0.17 are the $\eta' \to \eta$, $\eta' \to \omega$, and $\phi \not\to K\bar{K}$ branching fractions. Figure 70.1 shows the results; the sum is $(34.7 \pm 3.6)\%$, which is about equal to the hadronic $K\bar{K}$ total.

Note that the bin marked $\phi$ near the top of Fig. 70.1 includes neither the $\phi\ell^+\nu$ decays nor the 83% of other $\phi$ decays that produce a $K\bar{K}$ pair. There is twice as much $\phi$ in the $K_S^0K_L^0$ bin, and nearly three times as much in the $K^+K^-$ bin. These contributions are indicated in those bins.

Again, we can show how much of each bin is accounted for by measured exclusive branching fractions:

$\eta$ modes—The sum of $\eta\pi^+$, $\eta\pi^+\pi^0$ (nearly all $\eta\rho^+$), and $\eta K^+$ branching fractions is $(11.1 \pm 1.2)\%$, which leaves a good part of the inclusive hadronic $\eta$ fraction, $(18.5 \pm 3.0)\%$, to be accounted for. This is shaded in the figure.

$\omega$ modes—The sum of $\omega\pi^+$, $\omega\pi^+\pi^0$, and $\omega 2\pi^+\pi^-$ fractions is $(4.6 \pm 0.9)\%$, which is nearly as large as the inclusive hadronic $\omega$ fraction, $(5.8 \pm 1.4)\%$.

$\eta'$ modes—The sum of $\eta'\pi^+$, $\eta'\rho^+$, and $\eta' K^+$ fractions is $(9.9 \pm 1.5)\%$, which is larger than but not in serious disagreement with the inclusive hadronic $\eta'$ fraction, $(8.5 \pm 1.5)\%$.

70.4. Cabibbo-suppressed modes

The sum of the fractions for modes with a $K\bar{K}$, $\eta$, $\omega$, $\eta'$, or leptons is $(89.9 \pm 4.4)\%$. The remaining $(10.1 \pm 4.4)\%$ is to Cabibbo-suppressed modes, mainly single-$K^+$ pions and multiple-pion modes (see below). However, it should be noted that some small parts of the modes already discussed are Cabibbo-suppressed. For example, the $(1.1 \pm 0.2)\%$ of $D^+_s$ decays to $K^0\ell\nu$ or $K^{*0}\ell\nu$ is already in the $X\ell\nu$ bin in Fig. 70.1. And the inclusive measurements of $\eta$, $\omega$, and $\eta'$ fractions do not distinguish between (and therefore include
both) Cabibbo-allowed and -suppressed modes. We shall not try to make a separation here.

\[ K^0 + \text{pions} \]—Above, we found that \( f(\text{single } K_S^0) = (1.7 \pm 0.2)\% \). Subtracting semileptonic fractions with a \( K_S^0 \) leaves \( (1.3 \pm 0.2)\% \). The hadronic single-\( K^0 \) fraction is twice this, about \( (2.6 \pm 0.4)\% \). The sum of measured \( K^0 \pi^+, K^0 \pi^+ \pi^0 \), and \( K^0 2\pi^+ \pi^- \) fractions is \( (1.8 \pm 0.3)\% \), about two-thirds as much.

\[ K^+ + \text{pions} \]—The \( K^+ \pi^0 \) and \( K^+ \pi^+ \pi^- \) fractions sum to \( (0.72 \pm 0.05)\% \). The total \( K^+ \) fraction wanted here is probably in the 1-to-2% range.

\[ \text{Multi-pions} \]—The \( 2\pi^+ \pi^- \), \( \pi^+ 2\pi^0 \), and \( 3\pi^+ 2\pi^- \) fractions total \( (2.5 \pm 0.2)\% \). Modes not measured might double this.

The sum of the actually measured fractions is, including the semileptons, \( (4.9 \pm 0.3)\% \). The error on our Cabibbo-suppressed total, \( (10.1 \pm 4.4)\% \) is too large to know how much we might be missing.

References: