74. Polarization in B Decays

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We review the notation used in polarization measurements in particle production and decay, with a particular emphasis on the B decays and the CP-violating observables in polarization measurements. We look at several examples of vector-vector and vector-tensor B meson decays, while more details about the theory and experimental results in B decays can be found in a separate mini-review [1] in this *Review*.

Figure 74.1 illustrates angular observables in an example of the sequential process $ab \to X \to P_1P_2 \to (p_{11}p_{12})(p_{21}p_{22})$ [2]. The angular distributions are of particular interest because they are sensitive to spin correlations and reveal properties of particles and their interactions, such as quantum numbers and couplings. In the case of a spin-zero particle X, such as B meson or a Higgs boson, there are no spin correlations in the production mechanism and the decay chain is to be analyzed. The angular distribution of decay products can be expressed as a function of three helicity angles which describe the alignment of the particles in the decay chain. The analyzer of the B-daughter polarization is normally chosen for two-body decays, as the direction of the daughters in the center-of-mass of the parent (*e.g.*, $\rho \to 2\pi$) [3], and for three-body decays as the normal to the decay plane (*e.g.*, $\omega \to 3\pi$) [4]. An equivalent set of transversity angles is sometimes used in polarization analyses [5]. The differential decay width depends on complex amplitudes $A_{\lambda_1\lambda_2}$,

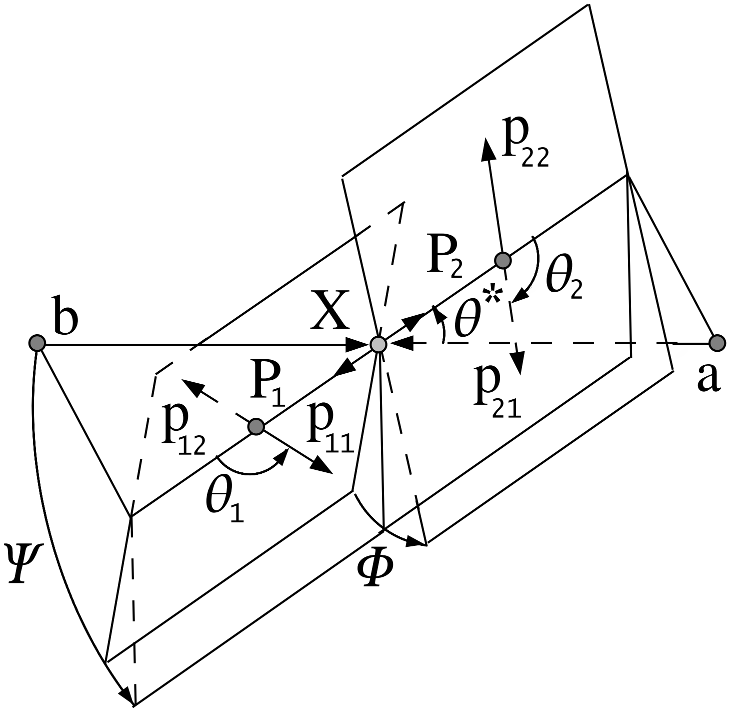

Figure 74.1: Definition of the production and helicity angles in the sequential process $ab \to X \to P_1P_2 \to (p_{11}p_{12})(p_{21}p_{22})$. The three helicity angles include θ_1 and θ_2, defined in the rest frame of the two daughters P_1 and P_2, and Φ, defined in the X frame as the angle between the two decay planes. The two production angles θ^* and Ψ are defined in the X frame, where Ψ is the angle between the production plane and the average of the two decay planes.

corresponding to the X-daughter helicity states λ_i.

In the case of a spin-zero B-meson decay, its daughter helicities are constrained to $\lambda_1 = \lambda_2 = \lambda$. Therefore we simplify amplitude notation as A_λ. Moreover, most B-decay polarization analyses are limited to the case when the spin of one of the B-meson daughters is 1. In that case, there are only three independent amplitudes corresponding to $\lambda = 0$ or ± 1 [6], where the last two can be expressed in terms of parity-even and parity-odd amplitudes $A_{\parallel,\perp} = (A_{+1} \pm A_{-1})/\sqrt{2}$. The overall decay amplitude involves three complex terms proportional to the above amplitudes and the Wigner d functions of helicity angles. The exact angular dependence would depend on the quantum numbers of the B-meson daughters and of their decay products, and can be found in the literature [6,7]. When both B-meson daughters are tensor mesons and the smaller of the two daughter spins is $J_1 > 1$, this formalism can be easily extended by introducing the parity-even and parity-odd amplitudes of higher order $A_{\parallel n,\perp n} = (A_{+n} \pm A_{-n})/\sqrt{2}$, with $1 < n \leq J_1$, while the general angular parameterization may be found in Ref. [7]. However, we limit the following discussion to $J_1 = 1$. The differential decay rate would involve six real quantities α_i, including interference terms,

$$\frac{d\Gamma}{\Gamma\, d\cos\theta_1\, d\cos\theta_2\, d\Phi} = \sum_i \alpha_i\; f_i\left(\cos\theta_1,\, \cos\theta_2,\, \Phi\right), \tag{74.1}$$

where each $f_i\left(\cos\theta_1,\, \cos\theta_2,\, \Phi\right)$ has unique angular dependence specific to particle quantum numbers, and the α_i parameters are defined as:

$$\alpha_1 = \frac{|A_0|^2}{\Sigma |A_\lambda|^2} = f_L\,, \tag{74.2}$$

$$\alpha_2 = \frac{|A_\parallel|^2 + |A_\perp|^2}{\Sigma |A_\lambda|^2} = (1 - f_L)\,, \tag{74.3}$$

$$\alpha_3 = \frac{|A_\parallel|^2 - |A_\perp|^2}{\Sigma |A_\lambda|^2} = (1 - f_L - 2\, f_\perp)\,, \tag{74.4}$$

$$\alpha_4 = \frac{\Im m(A_\perp A_\parallel^*)}{\Sigma |A_\lambda|^2} = \sqrt{f_\perp(1 - f_L - f_\perp)}\, \sin(\phi_\perp - \phi_\parallel)\,, \tag{74.5}$$

$$\alpha_5 = \frac{\Re e(A_\parallel A_0^*)}{\Sigma |A_\lambda|^2} = \sqrt{f_L\,(1 - f_L - f_\perp)}\, \cos(\phi_\parallel)\,, \tag{74.6}$$

$$\alpha_6 = \frac{\Im m(A_\perp A_0^*)}{\Sigma |A_\lambda|^2} = \sqrt{f_\perp\, f_L}\, \sin(\phi_\perp)\,, \tag{74.7}$$

where the amplitudes have been expressed with the help of polarization parameters f_L, $f_\perp$, $\phi_\parallel$, and $\phi_\perp$ defined in Table 74.1. Note that the terms proportional to $\Re e(A_\perp A_\parallel^*)$, $\Im m(A_\parallel A_0^*)$, and $\Re e(A_\perp A_0^*)$ are absent in Eqs. (74.2-74.7). However, these terms may appear for some three-body decays of a B-meson daughter, see Ref. [7].

Overall, six real parameters describe three complex amplitudes A_0, $A_\parallel$, and $A_\perp$. These could be chosen to be the four polarization parameters f_L, $f_\perp$, $\phi_\parallel$, and $\phi_\perp$, one overall size normalization, such as decay rate Γ, or branching fraction $\mathcal{B}$, and one overall phase δ_0. The phase convention is arbitrary for an isolated B decay mode. However, for several B decays, the relative phase could produce meaningful and observable effects through interference with other B decays with the same final states, such as for $B \to V K_J^*$ with $J = 0, 1, 2, 3, 4, ...$ The phase could be referenced to the single $B \to V K_0^*$ amplitude A_{00} in such a case, as shown in Table 74.1. Here V stands for any spin-one vector meson.

Moreover, *CP* violation can be tested in the angular distribution of the decay as the difference between the B and $\overline{B}$. Each of the six real parameters describing the three complex amplitudes

Table 74.1: Rate, polarization, and *CP*-asymmetry parameters defined for the B-meson decays to mesons with non-zero spin. Numerical examples are shown for the average of the $B^0 \to \varphi K^*(892)^0$ decay measurements obtained from BABAR [8], Belle [9], and LHCb [10]. The first six parameters are defined under the assumption of no *CP* violation in decay, while they are averaged between the $\overline{B}$ and B parameters in general. The last six parameters involve differences between the $\overline{B}$ and B meson decay parameters. The phase convention δ_0 is chosen with respect to a single A_{00} amplitude from a reference B decay mode, which is $B^0 \to \varphi K_0^*(1430)^0$ for numerical results.

parameter	definition	average
$\mathcal{B}$	$\Gamma/\Gamma_{\text{total}}$	$(10.1^{+0.6}_{-0.5}) \times 10^{-6}$
f_L	$\lvert A_0\rvert^2/\Sigma\lvert A_\lambda\rvert^2$	0.497 ± 0.017
$f_\perp$	$\lvert A_\perp\rvert^2/\Sigma\lvert A_\lambda\rvert^2$	0.225 ± 0.015
$\phi_\parallel - \pi$	$\arg(A_\parallel/A_0) - \pi$	-0.712 ± 0.058
$\phi_\perp - \pi$	$\arg(A_\perp/A_0) - \pi$	-0.615 ± 0.056
$\delta_0 - \pi$	$\arg(A_{00}/A_0) - \pi$	-0.26 ± 0.10
A_{CP}	$(\bar{\Gamma} - \Gamma)/(\bar{\Gamma} + \Gamma)$	-0.003 ± 0.038
A^0_{CP}	$(\bar{f}_L - f_L)/(\bar{f}_L + f_L)$	-0.007 ± 0.030
$A^\perp_{CP}$	$(\bar{f}_\perp - f_\perp)/(\bar{f}_\perp + f_\perp)$	-0.014 ± 0.057
$\Delta\phi_\parallel$	$(\bar{\phi}_\parallel - \phi_\parallel)/2$	$+0.051 \pm 0.053$
$\Delta\phi_\perp$	$(\bar{\phi}_\perp - \phi_\perp - \pi)/2$	$+0.075 \pm 0.050$
$\Delta\delta_0$	$(\bar{\delta}_0 - \delta_0)/2$	$+0.13 \pm 0.08$

would have a counterpart *CP*-asymmetry term, corresponding to three direct-*CP* asymmetries in three amplitudes, and three *CP*-violating phase differences, equivalent to the phase measurements from the mixing-induced *CP* asymmetries in the time evolution of B-decays [1]. In Table 74.1 and Ref. [11], these are chosen to be the direct-*CP* asymmetries in the overall decay rate $\mathcal{A}_{CP}$, in the f_L fraction $\mathcal{A}^0_{CP}$, and in the $f_\perp$ fraction $\mathcal{A}^\perp_{CP}$, and three weak phase differences:

$$\Delta\phi_\parallel = \frac{1}{2}\arg(\bar{A}_\parallel A_0/A_\parallel \bar{A}_0)\,, \tag{74.8}$$

$$\Delta\phi_\perp = \frac{1}{2}\arg(\bar{A}_\perp A_0/A_\perp \bar{A}_0) - \frac{\pi}{2}\,, \tag{74.9}$$

$$\Delta\delta_0 = \frac{1}{2}\arg(\bar{A}_{00} A_0/A_{00} \bar{A}_0)\,. \tag{74.10}$$

The $\frac{\pi}{2}$ term in Eq. (74.8) reflects the fact that $A_\perp$ and $\bar{A}_\perp$ differ in phase by π if *CP* is conserved. The two parameters $\Delta\phi_\parallel$ and $\Delta\phi_\perp$ are equivalent to triple-product asymmetries constructed from the vectors describing the decay angular distribution [12]. The *CP*-violating phase difference in the reference decay mode [11] is, in the Wolfenstein CKM quark-mixing phase convention,

$$\Delta\phi_{00} = \frac{1}{2}\arg(A_{00}/\bar{A}_{00})\,. \tag{74.11}$$

This can be measured only together with the mixing-induced phase difference for some of the neutral B-meson decays similar to other mixing-induced *CP* asymmetry measurements [1].

It may not always be possible to have a phase-reference decay mode which would define δ_0 and $\Delta\delta_0$ parameters. In that case, it may be possible to define the phase difference directly similarly to Eq. (74.11):

$$\Delta\phi_0 = \frac{1}{2}\arg(A_0/\bar{A}_0)\,. \tag{74.12}$$

One can measure the angles of the CKM unitarity triangle, assuming Standard Model contributions to the $\Delta\phi_0$ and B-mixing phases. Examples include measurements of $\beta = \phi_1$ with $B \to J/\psi K^*$ and $\alpha = \phi_2$ with $B \to \rho\rho$.

Most of the B decays that arise from tree-level $b \to c$ transitions have the amplitude hierarchy $|A_0| > |A_+| > |A_-|$ which is expected from analyses based on quark-helicity conservation [13]. The larger the mass of the vector-meson daughters, the weaker the inequality. The B meson decays to heavy vector particles with charm, such as $B \to J/\psi K^*$, $\psi(2S)K^*$, $\chi_{c1}K^*$, $D^*\rho$, D^*K^*, D^*D^*, and $D^*D_s^*$, show a substantial fraction of the amplitudes corresponding to transverse polarization of the vector mesons ($A_{\pm 1}$), in agreement with the factorization prediction. The detailed amplitude analysis of the $B \to J/\psi K^*$ decays has been performed by the BABAR [14], Belle [15], CDF [16], CLEO [17], D0 [18], and LHCb [19] collaborations. Most analyses are performed under the assumption of the absence of direct *CP* violation. The parameter values are given in the particle listing of this *Review*. The difference between the strong phases $\phi_\parallel$ and $\phi_\perp$ deviates significantly from zero. The measurements [14,15] of *CP*-violating terms similar to those in $B \to \varphi K^*$ [8] shown in Table 74.1 are consistent with zero.

In addition, the mixing-induced *CP*-violating asymmetry is measured in the $B^0 \to J/\psi K^{*0}$ decay [1,14,15] where angular analysis allows one to separate *CP*-eigenstate amplitudes. This allows one to resolve the sign ambiguity of the $\cos 2\beta$ ($\cos 2\phi_1$) term that appears in the time-dependent angular distribution due to interference of parity-even and parity-odd terms. This analysis relies on the knowledge of discrete ambiguities in the strong phases $\phi_\parallel$ and $\phi_\perp$, as discussed below. The BABAR experiment used a method based on the dependence on the $K\pi$ invariant mass of the interference between the S- and P-waves to resolve the discrete ambiguity in the determination of the strong phases $(\phi_\parallel, \phi_\perp)$ in $B \to J/\psi K^*$ decays [14]. The result is in agreement with the amplitude hierarchy expectation [13]. The CDF [20], D0 [21], and LHCb [22] experiments have studied the $B_s^0 \to J/\psi(K^+K^-)$, $J/\psi(\pi^+\pi^-)$, $\psi(K^+\pi^-)$ decays and provided the lifetime, polarization, and phase measurements.

The amplitude hierarchy $|A_0| \gg |A_+| \gg |A_-|$ was expected in B decays to light vector particles in both penguin transitions [23, 24] and tree-level transitions [13]. There is confirmation by the BABAR and Belle experiments of predominantly longitudinal polarization in the tree-level $b \to u$ transition, such as $B^0 \to \rho^+\rho^-$ [25], $B^+ \to \rho^0\rho^+$ [26], and $B^+ \to \omega\rho^+$ [27]; this is consistent with the analysis of the quark helicity conservation [13]. Because the longitudinal amplitude dominates the decay, a detailed amplitude analysis is not possible with current B samples, and limits on the transverse amplitude fraction are obtained. The small branching fractions of $B^0 \to \rho^0\rho^0, \omega\rho^0, \omega\omega$ [27–30] indicate that $b \to d$ penguin pollution is small in the charmless, strangeless vector-vector B decays. There is a measurement of large longitudinal polarization in $B^0 \to \rho^0\rho^0$ [28–30] decays. The fraction of transverse polarization is large in decays to heavier mesons such as $B^0 \to a_1(1260)^+a_1(1260)^-$ [31].

The interest in the polarization and *CP*-asymmetry measurements in penguin transition, such as $b \to s$ decays $B \to \varphi K^*$, ρK^*, ωK^*, or $B_s^0 \to \varphi\varphi$, K^*K^*, and $b \to d$ decay $B \to K^*\bar{K}^*$, is motivated by their potential sensitivity to physics beyond the Standard Model. The decay amplitudes for $B \to \varphi K^*$ have been measured by the BABAR, Belle, and LHCb experiments [9–11, 32, 33]. The fractions of longitudinal polarization are $f_L = 0.50 \pm 0.05$ for the $B^+ \to \varphi K^{*+}$ decay and $f_L = 0.497 \pm 0.017$ for the $B^0 \to \varphi K^{*0}$ decay. These indicate significant departure from the naive expectation of predominant longitudinal polarization, suggesting other contributions to the decay amplitude, previously neglected, either within the Standard Model, such as penguin annihilation [34] or QCD rescattering [35], or from physics beyond the Standard Model [36]. The complete set of twelve amplitude parameters measured in the $B^0 \to \varphi K^{*0}$ decay is given in Table 74.1. Several other parameters could be constructed from the above twelve parameters, as suggested in Ref. [37].

The discrete ambiguity in the phase $(\phi_\parallel, \phi_\perp, \Delta\phi_\parallel, \Delta\phi_\perp)$ measurements has been resolved by BABAR in favor of $|A_+| \gg |A_-|$ through interference between the S- and P-waves of $K\pi$. The search for vector-tensor and vector-axialvector $B \to \varphi K_J^{(*)}$ decays with $J = 1, 2, 3, 4$ revealed a large fraction of longitudinal polarization in the decay $B \to \varphi K_2^*(1430)$ with $f_L = 0.90^{+0.06}_{-0.07}$ [11, 38], but large contribution of transverse amplitude in $B \to \varphi K_1(1270)$ with $f_L = 0.46^{+0.13}_{-0.15}$ [39].

Like $B \to \varphi K^*$, the decays $B \to \rho K^*$ and $B \to \omega K^*$ may be sensitive to New Physics. Measurements of the longitudinal polarization fraction in $B \to \rho K^*$ [40] and in both vector-vector and vector-tensor final states of $B \to \omega K_J^*$ [27] by BABAR and Belle reveal a large fraction of transverse polarization, indicating an anomaly similar to $B \to \varphi K^*$ except for a different pattern in vector-tensor final states. An angular analysis of the $B^0 \to \rho^0 K^{*0}$ decay mode by LHCb [41] provides much higher precision and indicates remarkably small longitudinal polarization fraction and a significant direct CP asymmetry observed in angular distributions of $B \to VV$ decays for the first time. A large transverse polarization is also observed in the $B_s^0 \to \varphi\varphi$ decay by CDF [42] and LHCb [43], $B_s^0 \to K^{*0}\bar{K}^{*0}$ decays by LHCb [44], and $B_s^0 \to \varphi K^{*0}$ decays by LHCb [45]. At the same time, measurement of the polarization in the $b \to d$ penguin decays $B \to K^*\bar{K}^*$ indicates a large fraction of longitudinal polarization [44, 46]. The LHCb experiment has also provided the very first polarization results on the tensor-tensor, as well as vector-tensor, decays of the B_s^0 meson in the $(K\pi)(K\pi)$ final state [44]. The polarization pattern in penguin-dominated B-meson decays is not fully understood [34–36].

The three-body semileptonic B-meson decays, such as $B \to V\ell_1\ell_2$, share many features with the two-body $B \to VV$ decays. Their differential decay width can be parameterized with the two helicity angles defined in the V and $(\ell_1\ell_2)$ frames and with the azimuthal angle, as defined in Fig. 74.1. However, since the $(\ell_1\ell_2)$ pair does not come from an on-shell particle, the angular distribution is unique to each point in the dilepton mass $m_{\ell\ell}$ spectrum. The polarization measurements as a function of $m_{\ell\ell}$ provide complementary information on physics beyond the Standard Model, as discussed for $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays in Ref. [47]. The data in these modes have been analyzed by the BABAR, Belle, CDF, CMS, and LHCb experiments [48–53].

The examples of the angular distributions and observables in $B \to K^*\ell^+\ell^-$ are discussed in Ref. [47]. Two angular observables have been measured in this decay in certain ranges of the dilepton mass $m_{\ell\ell}$. One parameter is the fraction of longitudinal polarization F_L, which is determined by the K^* angular distribution and is similar to f_L defined for exclusive two-body decays. The other parameter is the forward-backward asymmetry of the lepton pair A_{FB}, which is the asymmetry of the decay rate with positive and negative values of $\cos\theta_1$. A complete set of observables and angular terms has been adopted by the LHCb collaboration [52] following Ref. [47] with the F_L, A_{FB}, and $S_3 - S_9$ coefficients in the angular distributions. Additional set of optimized observables $P_i^{(\prime)}$ is derived from those, for example $P_2 = 2A_{FB}/(3 - 3F_L)$ and $P_5' = S_5/\sqrt{F_L(1 - F_L)}$. These observables have the advantage that the leading form-factor uncertainties cancel. There have been hints of deviations from SM in the measurement of P_5' and lepton flavor universality [48–53].

In summary, there has been considerable interest in the polarization measurements of B-meson decays because they reveal both weak- and strong-interaction dynamics [34–36, 54]. New measurements will further elucidate the pattern of spin alignment measurements in rare B decays, and further test the Standard Model and strong interaction dynamics, including the non-factorizable contributions to the B-decay amplitudes.

References

[1] M. Kreps, Y. Kwon, and P. Eerola, “Production and Decay of b-Flavored Hadrons,” mini-review in this *Review*.

[2] Y. Gao *et al.*, Phys. Rev. D **81**, 075022 (2010), [arXiv:1001.3396].

[3] M. Jacob and G. Wick, Annals Phys. **7**, 404 (1959).

[4] S. Berman and M. Jacob, Phys. Rev. **139**, B1023 (1965).

[5] I. Dunietz *et al.*, Phys. Rev. D **43**, 2193 (1991).

[6] G. Kramer and W. Palmer, Phys. Rev. D **45**, 193 (1992).

[7] A. Datta *et al.*, Phys. Rev. D **77**, 114025 (2008), [arXiv:0711.2107].

[8] B. Aubert *et al.* (BaBar), Phys. Rev. D **78**, 092008 (2008), [arXiv:0808.3586].

[9] M. Prim *et al.* (Belle), Phys. Rev. D **88**, 7, 072004 (2013), [arXiv:1308.1830].

[10] R. Aaij *et al.* (LHCb), JHEP **05**, 069 (2014), [arXiv:1403.2888].

[11] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **93**, 231804 (2004), [hep-ex/0408017]; B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **98**, 051801 (2007), [hep-ex/0610073]; B. Aubert *et al.* (BaBar), Phys. Rev. D **78**, 092008 (2008), [arXiv:0808.3586].

[12] G. Valencia, Phys. Rev. D **39**, 3339 (1989); A. Datta and D. London, Int. J. Mod. Phys. A **19**, 2505 (2004), [hep-ph/0303159].

[13] A. Ali *et al.*, Z. Phys. C **1**, 269 (1979); M. Suzuki, Phys. Rev. D **64**, 117503 (2001), [hep-ph/0106354].

[14] B. Aubert *et al.* (BaBar), Phys. Rev. D **71**, 032005 (2005), [hep-ex/0411016]; B. Aubert *et al.* (BaBar), Phys. Rev. D **76**, 031102 (2007), [arXiv:0704.0522].

[15] R. Itoh *et al.* (Belle), Phys. Rev. Lett. **95**, 091601 (2005), [hep-ex/0504030]; M. Prim *et al.* (Belle), Phys. Rev. D **88**, 7, 072004 (2013), [arXiv:1308.1830].

[16] T. Affolder *et al.* (CDF), Phys. Rev. Lett. **85**, 4668 (2000), [hep-ex/0007034]; D. Acosta *et al.* (CDF), Phys. Rev. Lett. **94**, 101803 (2005), [hep-ex/0412057].

[17] C. Jessop *et al.* (CLEO), Phys. Rev. Lett. **79**, 4533 (1997), [hep-ex/9702013].

[18] V. Abazov *et al.* (D0), Phys. Rev. Lett. **102**, 032001 (2009), [arXiv:0810.0037].

[19] R. Aaij *et al.* (LHCb), Phys. Rev. D **88**, 052002 (2013), [arXiv:1307.2782].

[20] T. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **100**, 121803 (2008), [arXiv:0712.2348]; T. Aaltonen *et al.* (CDF), Phys. Rev. D **85**, 072002 (2012), [arXiv:1112.1726].

[21] V. Abazov *et al.* (D0), Phys. Rev. Lett. **98**, 121801 (2007), [hep-ex/0701012]; V. M. Abazov *et al.* (D0), Phys. Rev. D **85**, 032006 (2012), [arXiv:1109.3166].

[22] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **108**, 101803 (2012), [arXiv:1112.3183]; R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **114**, 4, 041801 (2015), [arXiv:1411.3104]; R. Aaij *et al.* (LHCb), Phys. Lett. B **747**, 484 (2015), [arXiv:1503.07112]; R. Aaij *et al.* (LHCb), JHEP **11**, 082 (2015), [arXiv:1509.00400]; R. Aaij *et al.* (LHCb), JHEP **08**, 037 (2017), [arXiv:1704.08217].

[23] H.-Y. Cheng and K.-C. Yang, Phys. Lett. B **511**, 40 (2001), [hep-ph/0104090]; C.-H. Chen, Y.-Y. Keum and H.-n. Li, Phys. Rev. D **66**, 054013 (2002), [hep-ph/0204166].

[24] A. L. Kagan, Phys. Lett. B **601**, 151 (2004), [hep-ph/0405134]; Y. Grossman, Int. J. Mod. Phys. A **19**, 907 (2004), [hep-ph/0310229].

[25] B. Aubert *et al.* (BaBar), Phys. Rev. D **76**, 052007 (2007), [arXiv:0705.2157]; P. Vanhoefer *et al.* (Belle), Phys. Rev. D **93**, 3, 032010 (2016), [Addendum: Phys.Rev.D 94, 099903 (2016)], [arXiv:1510.01245].

[26] J. Zhang *et al.* (Belle), Phys. Rev. Lett. **91**, 221801 (2003), [hep-ex/0306007]; B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **102**, 141802 (2009), [arXiv:0901.3522].

[27] B. Aubert *et al.* (BaBar), Phys. Rev. D **74**, 051102 (2006), [hep-ex/0605017]; B. Aubert *et al.* (BaBar), Phys. Rev. D **79**, 052005 (2009), [arXiv:0901.3703].

[28] B. Aubert *et al.* (BaBar), Phys. Rev. D **78**, 071104 (2008), [arXiv:0807.4977].

[29] I. Adachi *et al.* (Belle), Phys. Rev. D **89**, 7, 072008 (2014), [Addendum: Phys.Rev.D 89, 119903 (2014)], [arXiv:1212.4015].

[30] R. Aaij *et al.* (LHCb), Phys. Lett. B **747**, 468 (2015), [arXiv:1503.07770].

[31] B. Aubert *et al.* (BaBar), Phys. Rev. D **80**, 092007 (2009), [arXiv:0907.1776].

[32] K.-F. Chen *et al.* (Belle), Phys. Rev. Lett. **94**, 221804 (2005), [hep-ex/0503013].

[33] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **99**, 201802 (2007), [arXiv:0705.1798].

[34] A. L. Kagan, Phys. Lett. B **601**, 151 (2004), [hep-ph/0405134]; H.-n. Li and S. Mishima, Phys. Rev. D **71**, 054025 (2005), [hep-ph/0411146]; C.-H. Chen *et al.*, Phys. Rev. D **72**, 054011 (2005), [hep-ph/0507012]; M. Beneke, J. Rohrer and D. Yang, Phys. Rev. Lett. **96**, 141801 (2006), [hep-ph/0512258]; C.-H. Chen and C.-Q. Geng, Phys. Rev. D **75**, 054010 (2007), [hep-ph/0701023]; A. Datta *et al.*, Phys. Rev. D **76**, 034015 (2007), [arXiv:0705.3915]; M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B **774**, 64 (2007), [hep-ph/0612290]; H.-Y. Cheng and K.-C. Yang, Phys. Rev. D **78**, 094001 (2008), [Erratum: Phys.Rev.D 79, 039903 (2009)], [arXiv:0805.0329].

[35] C. W. Bauer *et al.*, Phys. Rev. D **70**, 054015 (2004), [hep-ph/0401188]; P. Colangelo, F. De Fazio and T. Pham, Phys. Lett. B **597**, 291 (2004), [hep-ph/0406162]; M. Ladisa *et al.*, Phys. Rev. D **70**, 114025 (2004), [hep-ph/0409286]; H.-Y. Cheng, C.-K. Chua and A. Soni, Phys. Rev. D **71**, 014030 (2005), [hep-ph/0409317]; H.-Y. Cheng and K.-C. Yang, Phys. Rev. D **83**, 034001 (2011), [arXiv:1010.3309].

[36] Y. Grossman, Int. J. Mod. Phys. A **19**, 907 (2004), [hep-ph/0310229]; E. Alvarez *et al.*, Phys. Rev. D **70**, 115014 (2004), [hep-ph/0410096]; P. K. Das and K.-C. Yang, Phys. Rev. D **71**, 094002 (2005), [hep-ph/0412313]; C.-H. Chen and C.-Q. Geng, Phys. Rev. D **71**, 115004 (2005), [hep-ph/0504145]; Y.-D. Yang, R.-M. Wang and G.-R. Lu, Phys. Rev. D **72**, 015009 (2005), [hep-ph/0411211]; K.-C. Yang, Phys. Rev. D **72**, 034009 (2005), [Erratum: Phys.Rev.D 72, 059901 (2005)], [hep-ph/0506040]; S. Baek *et al.*, Phys. Rev. D **72**, 094008 (2005), [hep-ph/0508149]; C.-S. Huang *et al.*, Phys. Rev. D **73**, 034026 (2006), [hep-ph/0511129]; C.-H. Chen and H. Hatanaka, Phys. Rev. D **73**, 075003 (2006), [hep-ph/0602140]; A. Faessler *et al.*, Phys. Rev. D **75**, 074029 (2007), [hep-ph/0702020].

[37] D. London, N. Sinha and R. Sinha, Phys. Rev. D **69**, 114013 (2004), [hep-ph/0402214].

[38] B. Aubert *et al.* (BaBar), Phys. Rev. D **76**, 051103 (2007), [arXiv:0705.0398].

[39] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **101**, 161801 (2008), [arXiv:0806.4419].

[40] J. Zhang *et al.* (Belle), Phys. Rev. Lett. **95**, 141801 (2005), [hep-ex/0408102]; B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **97**, 201801 (2006), [hep-ex/0607057]; P. del Amo Sanchez *et al.* (BaBar), Phys. Rev. D **83**, 051101 (2011), [arXiv:1012.4044]; J. Lees *et al.* (BaBar), Phys. Rev. D **85**, 072005 (2012), [arXiv:1112.3896].

[41] R. Aaij *et al.* (LHCb), JHEP **05**, 026 (2019), [arXiv:1812.07008].

[42] T. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **107**, 261802 (2011), [arXiv:1107.4999].

[43] R. Aaij *et al.* (LHCb), Phys. Lett. B **713**, 369 (2012), [arXiv:1204.2813]; R. Aaij *et al.* (LHCb), Phys. Rev. D **90**, 5, 052011 (2014), [arXiv:1407.2222]; R. Aaij *et al.* (LHCb), JHEP **12**, 155 (2019), [arXiv:1907.10003].

[44] R. Aaij *et al.* (LHCb), Phys. Lett. B **709**, 50 (2012), [arXiv:1111.4183]; R. Aaij *et al.* (LHCb), JHEP **07**, 166 (2015), [arXiv:1503.05362]; R. Aaij *et al.* (LHCb), JHEP **07**, 032 (2019), [arXiv:1905.06662].

[45] R. Aaij *et al.* (LHCb), JHEP **11**, 092 (2013), [arXiv:1306.2239].

[46] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **100**, 081801 (2008), [arXiv:0708.2248]; B. Aubert *et al.* (BaBar), Phys. Rev. D **79**, 051102 (2009), [arXiv:0901.1223].

[47] G. Burdman, Phys. Rev. D **52**, 6400 (1995), [hep-ph/9505352]; F. Kruger and J. Matias, Phys. Rev. D **71**, 094009 (2005), [hep-ph/0502060]; E. Lunghi and J. Matias, JHEP **04**, 058 (2007), [hep-ph/0612166]; W. Altmannshofer *et al.*, JHEP **01**, 019 (2009), [arXiv:0811.1214]; J. Matias *et al.*, JHEP **04**, 104 (2012), [arXiv:1202.4266]; S. Descotes-Genon *et al.*, JHEP **01**, 048 (2013), [arXiv:1207.2753].

[48] B. Aubert *et al.* (BaBar), Phys. Rev. D **79**, 031102 (2009), [arXiv:0804.4412]; J. Lees *et al.* (BaBar), Phys. Rev. D **93**, 5, 052015 (2016), [arXiv:1508.07960].

[49] J.-T. Wei *et al.* (Belle), Phys. Rev. Lett. **103**, 171801 (2009), [arXiv:0904.0770]; S. Wehle *et al.* (Belle), Phys. Rev. Lett. **118**, 11, 111801 (2017), [arXiv:1612.05014]; A. Abdesselam *et al.* (Belle), Phys. Rev. Lett. **126**, 16, 161801 (2021), [arXiv:1904.02440].

[50] T. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **108**, 081807 (2012), [arXiv:1108.0695].

[51] S. Chatrchyan *et al.* (CMS), Phys. Lett. B **727**, 77 (2013), [arXiv:1308.3409]; V. Khachatryan *et al.* (CMS), Phys. Lett. B **753**, 424 (2016), [arXiv:1507.08126]; A. M. Sirunyan *et al.* (CMS), Phys. Lett. B **781**, 517 (2018), [arXiv:1710.02846]; A. M. Sirunyan *et al.* (CMS), JHEP **04**, 124 (2021), [arXiv:2010.13968].

[52] R. Aaij *et al.* (LHCb), JHEP **08**, 131 (2013), [arXiv:1304.6325]; R. Aaij *et al.* (LHCb), JHEP **04**, 064 (2015), [arXiv:1501.03038]; R. Aaij *et al.* (LHCb), JHEP **02**, 104 (2016), [arXiv:1512.04442]; R. Aaij *et al.* (LHCb), JHEP **11**, 047 (2016), [Erratum: JHEP 04, 142 (2017)], [arXiv:1606.04731]; R. Aaij *et al.* (LHCb), JHEP **12**, 065 (2016), [arXiv:1609.04736]; R. Aaij *et al.* (LHCb), JHEP **08**, 055 (2017), [arXiv:1705.05802]; R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **125**, 1, 011802 (2020), [arXiv:2003.04831]; R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **126**, 16, 161802 (2021), [arXiv:2012.13241].

[53] R. Aaij *et al.* (LHCb), JHEP **09**, 179 (2015), [arXiv:1506.08777]; R. Aaij *et al.* (LHCb) (2021), [arXiv:2107.13428].

[54] C.-H. Chen and H.-N. Li, Phys. Rev. D **71**, 114008 (2005), [hep-ph/0504020].