66. *CPT* Invariance Tests in Neutral Kaon Decay

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CPT theorem is based on three assumptions: quantum field theory, locality, and Lorentz invariance, and thus it is a fundamental probe of our basic understanding of particle physics. Strangeness oscillation in $K^0 - \overline{K}^0$ system, described by the equation

$$i\frac{d}{dt}\begin{bmatrix} K^0 \\ \overline{K}^0 \end{bmatrix} = [M - i\Gamma/2]\begin{bmatrix} K^0 \\ \overline{K}^0 \end{bmatrix} ,$$

where M and Γ are hermitian matrices (see PDG review [1], references [2,3], and KLOE paper [4] for notations and previous literature), allows a very accurate test of CPT symmetry; indeed since CPT requires $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, the mass and width eigenstates, $K_{S,L}$, have a CPT-violating piece, δ, in addition to the usual CPT-conserving parameter ϵ:

$$K_{S,L} = \frac{1}{\sqrt{2\left(1+|\epsilon_{S,L}|^2\right)}}\left[\left(1+\epsilon_{S,L}\right)K^0 \pm \left(1-\epsilon_{S,L}\right)\overline{K}^0\right]$$

$$\begin{aligned}\epsilon_{S,L} &= \frac{-i\Im\left(M_{12}\right) - \frac{1}{2}\Im\left(\Gamma_{12}\right) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}\left(\Gamma_{11} - \Gamma_{22}\right)\right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \\ &\equiv \epsilon \pm \delta. \end{aligned} \tag{66.1}$$

Using the phase convention $\Im(\Gamma_{12}) = 0$, we determine the phase of ϵ to be $\varphi_{SW} \equiv \arctan\frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$. Imposing unitarity to an arbitrary combination of K^0 and $\overline{K}^0$ wave functions, we obtain the Bell-Steinberger relation [5] connecting CP and CPT violation in the mass matrix to CP and CPT violation in the decay; in fact, neglecting $\mathcal{O}(\epsilon)$ corrections to the coefficient of the CPT-violating parameter, δ, we can write [4]

$$[\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i\tan\phi_{\rm SW}][\frac{\Re(\epsilon)}{1+|\epsilon|^2} - i\Im(\delta)] = \frac{1}{\Gamma_S - \Gamma_L}\sum_f A_L(f)A_S^*(f), \tag{66.2}$$

where $A_{L,S}(f) \equiv A(K_{L,S} \to f)$. We stress that this relation is phase-convention-independent. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in Eq. (66.2); in fact, defining for the hadronic modes

$$\begin{aligned}\alpha_i &\equiv \frac{1}{\Gamma_S}\langle\mathcal{A}_L(i)\mathcal{A}_S^*(i)\rangle = \eta_i\ \mathcal{B}(K_S \to i), \\ i &= \pi^0\pi^0, \pi^+\pi^-(\gamma), 3\pi^0, \pi^0\pi^+\pi^-(\gamma),\end{aligned} \tag{66.3}$$

the recent data from CPLEAR, KLOE, KTeV, and NA48 have led to the following determinations (the analysis described in Ref. [4] has been updated by using the recent measurements of K_L branching ratios from KTeV [6,7], NA48 [8,9], the results described in the CP violation in K_L decays minireview, and the KLOE result [10])

$$\begin{aligned}\alpha_{\pi^+\pi^-} &= ((1.121 \pm 0.010) + i(1.061 \pm 0.010)) \times 10^{-3}\ , \\ \alpha_{\pi^0\pi^0} &= ((0.493 \pm 0.005) + i(0.471 \pm 0.005)) \times 10^{-3}\ , \\ \alpha_{\pi^+\pi^-\pi^0} &= ((0 \pm 2) + i(0 \pm 2)) \times 10^{-6}, \\ |\alpha_{\pi^0\pi^0\pi^0}| &< 1.5 \times 10^{-6}\ \text{ at } 95\%\ \text{CL}\ . \end{aligned} \tag{66.4}$$

The semileptonic contribution to the right-handed side of Eq. (66.2) requires the determination of several observables: we define [2,3]

$$\begin{aligned}
\mathcal{A}(K^0 \to \pi^- l^+ \nu) &= \mathcal{A}_0(1-y) \ , \\
\mathcal{A}(K^0 \to \pi^+ l^- \nu) &= \mathcal{A}_0^*(1+y^*)(x_+ - x_-)^* \ , \\
\mathcal{A}(\overline{K}^0 \to \pi^+ l^- \nu) &= \mathcal{A}_0^*(1+y^*) \ , \\
\mathcal{A}(\overline{K}^0 \to \pi^- l^+ \nu) &= \mathcal{A}_0(1-y)(x_+ + x_-) \ ,
\end{aligned} \tag{66.5}$$

where x_+ (x_-) describes the violation of the $\Delta S = \Delta Q$ rule in CPT-conserving (violating) decay amplitudes, and y parameterizes CPT violation for $\Delta S = \Delta Q$ transitions. Taking advantage of their tagged $K^0(\overline{K}^0)$ beams, CPLEAR has measured $\Im(x_+)$, $\Re(x_-)$, $\Im(\delta)$, and $\Re(\delta)$ [11]. These determinations have been improved in Ref. [4] by including the information $A_S - A_L = 4[\Re(\delta) + \Re(x_-)]$ (valid at first order in the small parameters), where $A_{L,S}$ are the K_L and K_S semileptonic charge asymmetries, respectively, from the PDG [12] and the new KLOE semileptonic measurement [13]. Here we are also including the T-violating asymmetry measurement from CPLEAR [14] with a finer binning than appearing in the published article.

Table 66.1: Values, errors, and correlation coefficients for $\Re(\delta)$, $\Im(\delta)$, $\Re(x_-)$, $\Im(x_+)$, and $A_S + A_L$ obtained from a combined fit, including KLOE [4,13] and CPLEAR [14].

	value	Correlations coefficients				
$\Re(\delta)$	$(4.3 \pm 2.7) \times 10^{-4}$	1				
$\Im(\delta)$	$(-0.9 \pm 0.6) \times 10^{-2}$	-0.40	1			
$\Re(x_-)$	$(-0.22 \pm 0.10) \times 10^{-2}$	-0.14	-0.30	1		
$\Im(x_+)$	$(0.06 \pm 0.19) \times 10^{-2}$	-0.12	-0.02	0.34	1	
$A_S + A_L$	$(-0.23 \pm 0.38) \times 10^{-2}$	-0.12	-0.29	0.94	0.18	1

The value $A_S + A_L$ in Table 66.1 can be directely included in the semileptonic contributions to the Bell Steinberger relations in Eq. (66.2)

$$\begin{aligned}
&\sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu) \mathcal{A}_S^*(\pi\ell\nu) \rangle \\
&\quad = 2\Gamma(K_L \to \pi\ell\nu)(\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\delta))) \\
&\quad = 2\Gamma(K_L \to \pi\ell\nu)((A_S + A_L)/4 - i(\Im(x_+) + \Im(\delta))) \ .
\end{aligned} \tag{66.6}$$

Defining

$$\alpha_{\pi\ell\nu} \equiv \frac{1}{\Gamma_S} \sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu) \mathcal{A}_S^*(\pi\ell\nu) \rangle + 2i \frac{\tau_{K_S}}{\tau_{K_L}} \mathcal{B}(K_L \to \pi\ell\nu) \Im(\delta) \ , \tag{66.7}$$

we find:

$$\alpha_{\pi\ell\nu} = ((-0.1 \pm 0.2) + i(-0.1 \pm 0.5)) \times 10^{-5} \, . \tag{66.8}$$

The analysis of semileptonic decay asymmetries implicitly assumes Lepton Flavour Universality (LFU) of any effect violating CPT or $\Delta S = \Delta Q$, through the use of a single set of x, y parameters, consistently with the availability of experimental information on A_S for the electron mode only. The

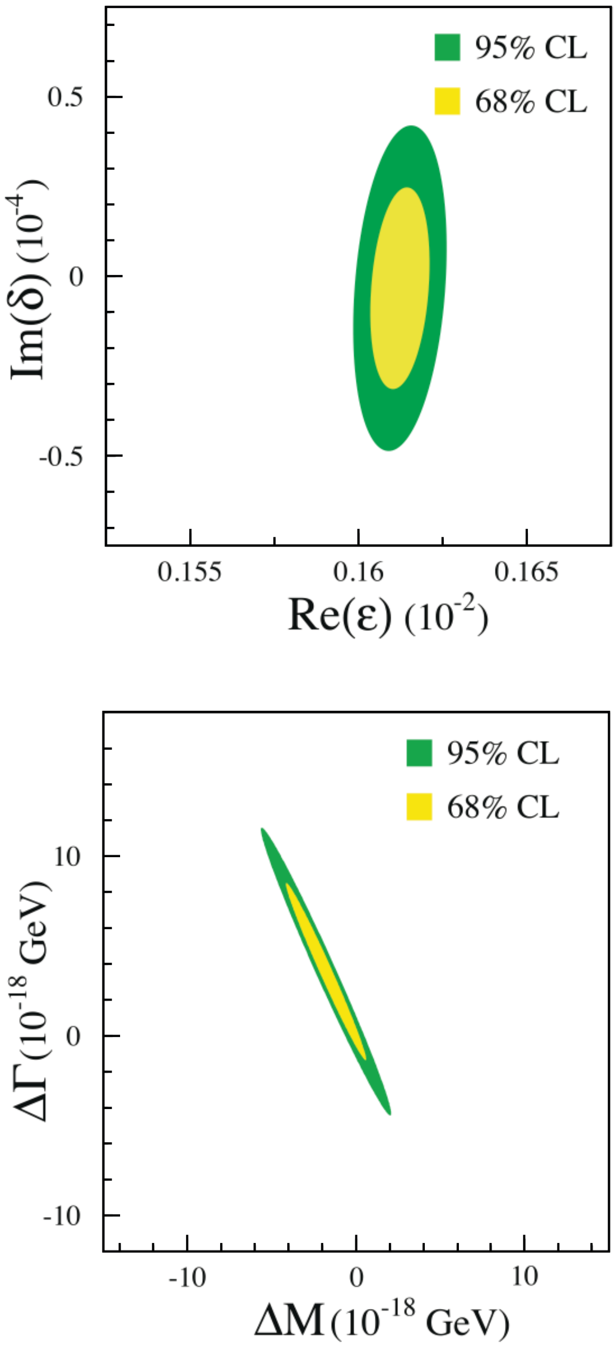

Figure 66.1: Top: allowed region at 68% and 95% C.L. in the $\Re(\epsilon)$, $\Im(\delta)$ plane. Bottom: allowed region at 68% and 95% C.L. in the $\Delta M, \Delta\Gamma$ plane.

explicit LFU assumption in the input BR measurements has been lifted exploiting the measurement of $BR(K_S \to \pi\mu\nu)$ Ref. [15], with no effect on the numerical results.

Inserting the values of the α parameters into Eq. (66.2), we find

$$\begin{aligned} \Re(\epsilon) &= (161.2 \pm 0.5) \times 10^{-5}, \\ \Im(\delta) &= (-0.3 \pm 1.4) \times 10^{-5}\,. \end{aligned} \tag{66.9}$$

Table 66.2: Summary of results: values, errors, and correlation coefficients for $\Re(\epsilon)$, $\Im(\delta)$, $\Re(\delta)$, and $\Re(x_-)$.

	value	Correlations coefficients			
$\Re(\epsilon)$	$(161.2 \pm 0.5) \times 10^{-5}$	$+1$			
$\Im(\delta)$	$(-0.3 \pm 1.4) \times 10^{-5}$	$+0.08$	1		
$\Re(\delta)$	$(2.6 \pm 2.5) \times 10^{-4}$	$+0.00$	-0.05	1	
$\Re(x_-)$	$(-2.7 \pm 1.0) \times 10^{-3}$	$+0.05$	0.13	-0.30	1

The complete information on Eq. (66.9) is given in Table 66.2.

Now the agreement with CPT conservation, $\Im(\delta) = \Re(\delta) = \Re(x_-) = 0$, is at 18% C.L.

The allowed region in the $\Re(\epsilon) - \Im(\delta)$ plane at 68% CL and 95% C.L. is shown in the top panel of Fig. 66.1.

The process giving the largest contribution to the size of the allowed region is $K_L \to \pi^+\pi^-$, through the uncertainty on ϕ_{+-}.

The limits on $\Im(\delta)$ and $\Re(\delta)$ can be used to constrain the $K^0 - \overline{K}^0$ mass and width difference

$$\delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos\phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)] \,.$$

The allowed region in the $\Delta M = (m_{K^0} - m_{\overline{K}^0})$, $\Delta\Gamma = (\Gamma_{K^0} - \Gamma_{\overline{K}^0})$ plane is shown in the bottom panel of Fig. 66.1. As a result, we improve on the previous limits (see for instance, P. Bloch in Ref. [12]) and in the limit $\Gamma_{K^0} - \Gamma_{\overline{K}^0} = 0$ we obtain

$$-4.0 \times 10^{-19}\ \text{GeV} < m_{K^0} - m_{\overline{K}^0} < 4.0 \times 10^{-19}\ \text{GeV} \quad \text{at } 95\ \%\ \text{C.L}\,.$$

References

[1] See the "CP Violation in Meson Decays," in this *Review.*

[2] L. Maiani, "CP And CPT Violation in Neutral Kaon Decays," L. Maiani, G. Pancheri, and N. Paver, *The Second DAΦNE Physics Handbook,* Vol. 1,2.

[3] G. D'Ambrosio, G. Isidori and A. Pugliese, in "2nd DAΦNE Physics Handbook:63-96," 63–96 (1994), [hep-ph/9411389], URL http://preprints.cern.ch/cgi-bin/setlink?base=preprint&categ=cern&id=th-7504-94.

[4] G. D'Ambrosio and G. Isidori (KLOE), JHEP **12**, 011 (2006), [hep-ex/0610034].

[5] J. S. Bell and J. Steinberger, in "Wolfenstein, L. (eq.): CP violation, 42-57. (In Oxford International Symposium Conference on Elementary Particles) ," 195–208, 221–222 (1966), (See Book Index).

[6] T. Alexopoulos *et al.* (KTeV), Phys. Rev. **D70**, 092006 (2004), [hep-ex/0406002].

[7] E. Abouzaid *et al.* (KTeV), Phys. Rev. **D83**, 092001 (2011), [arXiv:1011.0127].

[8] A. Lai *et al.* (NA48), Phys. Lett. **B645**, 26 (2007), [hep-ex/0611052]; A. Lai *et al.* (NA48), Phys. Lett. **B602**, 41 (2004), [hep-ex/0410059].

[9] We thanks G. Isidori and M. Palutan for their contribution to the original analysis [4] performed with KLOE data.

[10] D. Babusci *et al.* (KLOE), Phys. Lett. **B723**, 54 (2013), [arXiv:1301.7623].

[11] A. Angelopoulos *et al.* (CPLEAR), Phys. Lett. **B444**, 52 (1998).

[12] W. M. Yao *et al.* (Particle Data Group), J. Phys. **G33**, 1 (2006).

[13] A. Anastasi *et al.* (KLOE-2), JHEP **09**, 021 (2018), [arXiv:1806.08654].

[14] P. Bloch, M. Fidecaro, private communication of the data in a finer binning format; A. Angelopoulos *et al.* (CPLEAR), Phys. Lett. **B444**, 43 (1998).

[15] D. Babusci *et al.* (KLOE-2), Phys. Lett. B **804**, 135378 (2020), [arXiv:1912.05990].