53. Plots of Cross Sections and Related Quantities

Updated in 2021. See various sections for details.

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53.1 Pseudorapidity Distributions in pp and $\overline{p}p$ Interactions

Revised August 2013 by D.R. Ward (Cavendish Lab.).

Pseudorapidity Distributions in pp and $\bar{p}p$ Interactions

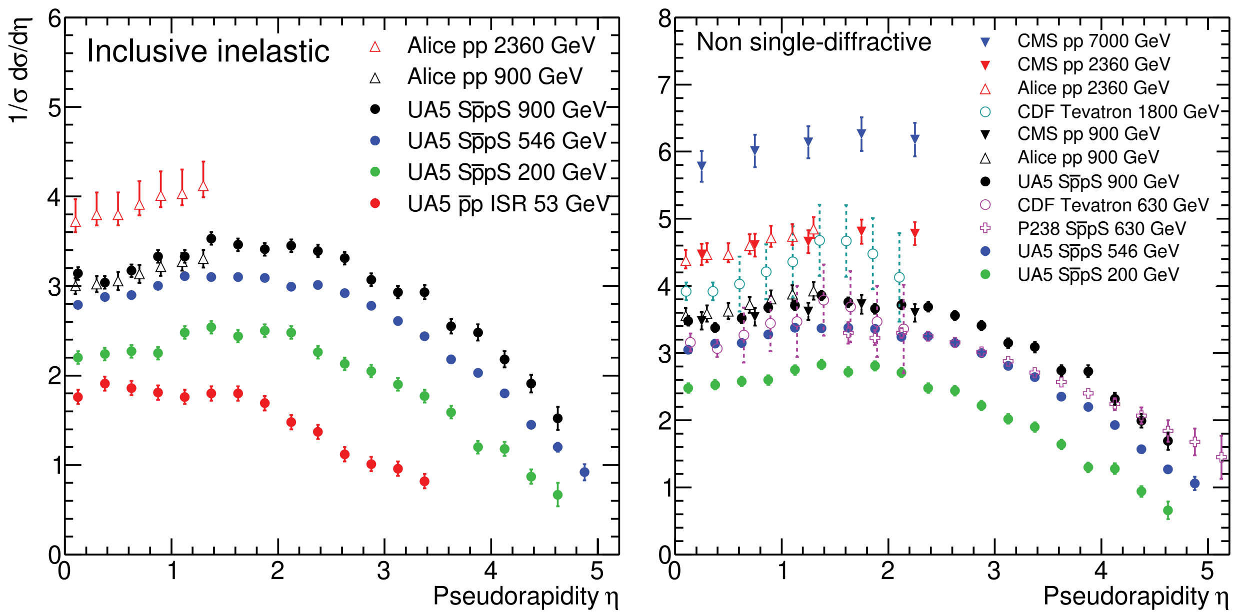

Figure 53.1: Charged particle pseudorapidity distributions in $p\overline{p}$ collisions for 53 GeV $\leq \sqrt{s} \leq$ 1800 GeV. UA5 data from the S$p\overline{p}$S are taken from [1], and from the ISR from [2]. The UA5 data are shown for both the full inelastic cross-section and with singly diffractive events excluded. Additional non single-diffractive measurements are available from CDF at the Tevatron [3] and from P238 at the S$p\overline{p}$S [4]. These may be compared with both inclusive and non single-diffractive measurements in pp collisions at the LHC from ALICE [5] and for non single-diffractive interactions from CMS [6,7]. (Courtesy of D.R. Ward, Cambridge Univ., 2013)

53.2 Average Hadron Multiplicities in Hadronic e^+e^- Annihilation Events

Revised August 2021 by O. Biebel (Ludwig-Maximilians U.).

Table 53.1: Average hadron multiplicities per hadronic e^+e^- annihilation event at $\sqrt{s} \approx 10$, 29–35, 91, and 130–200 GeV. The rates given include decay products from resonances with $c\tau < 10$ cm, and include the corresponding anti-particle state. Correlations of the systematic uncertainties were considered for the calculation of the averages. Quoted errors are not increased by scale factor S.

Particle	$\sqrt{s} \approx 10$ GeV	$\sqrt{s} = 29$–35 GeV	$\sqrt{s} = 91$ GeV	$\sqrt{s} = 130$–200 GeV	References
Pseudoscalar mesons:					
π^+	6.52 ± 0.11	0.3 ± 0.4	17.02 ± 0.19	21.24 ± 0.39	[8–17]
π^0	3.2 ± 0.3	5.83 ± 0.28	9.42 ± 0.32		[12, 18–23]
K^+	0.953 ± 0.018	1.48 ± 0.09	2.228 ± 0.059	2.82 ± 0.19	[9–17, 24, 25]
K^0	0.91 ± 0.05	1.48 ± 0.07	2.049 ± 0.026	2.10 ± 0.12	[12, 17, 20, 26–36]
η	0.20 ± 0.04	0.61 ± 0.07	1.049 ± 0.080		[12, 18, 19, 22, 23, 37–40]
$\eta\prime(958)$	0.03 ± 0.01	0.26 ± 0.10	0.152 ± 0.020		[20, 39, 41–43]
D^+	$0.194 \pm 0.019^{(a)}$	0.17 ± 0.03	0.175 ± 0.016		[12, 44–47]
D^0	$0.446 \pm 0.032^{(a)}$	0.45 ± 0.07	0.454 ± 0.030		[12, 44–47]
D_s^+	$0.063 \pm 0.014^{(a)}$	$0.45 \pm 0.20^{(b)}$	0.131 ± 0.021		[8, 39, 44, 47–49]
$B^{(c)}$	—	—	$0.134 \pm 0.016^{(d)}$		[46, 50]
B^+	—	—	$0.141 \pm 0.004^{(d)}$		[51]
B_s^0	—	—	$0.054 \pm 0.011^{(d)}$		[52, 53]
Scalar mesons:					
$f_0(980)$	0.024 ± 0.006	$0.05 \pm 0.02^{(e)}$	0.146 ± 0.012		[41, 54–56]
$a_0(980)^\pm$	—	—	$0.27 \pm 0.11^{(f)}$		[43]
Vector mesons:					
$\rho(770)^0$	0.35 ± 0.04	0.81 ± 0.08	1.231 ± 0.098		[9, 12, 55, 57, 58]
$\rho(770)^\pm$	—	—	$2.40 \pm 0.43^{(f)}$		[43]
$\omega(782)$	0.30 ± 0.08	—	1.016 ± 0.065		[40, 42, 43, 57]
$K^*(892)^+$	0.27 ± 0.03	0.64 ± 0.05	0.714 ± 0.055		[9, 12, 33, 57, 59, 60]
$K^*(892)^0$	0.29 ± 0.03	0.56 ± 0.06	0.738 ± 0.024		[9, 12, 36, 57, 58, 61, 62]
$\phi(1020)$	0.044 ± 0.003	0.085 ± 0.011	0.0963 ± 0.0032		[12, 36, 56–58, 61]
$D^*(2010)^+$	$0.177 \pm 0.022^{(a)}$	0.43 ± 0.07	$0.1937 \pm 0.0057^{(g)}$		[12, 44–46, 63, 64]
$D^*(2007)^0$	$0.168 \pm 0.019^{(a)}$	0.27 ± 0.11	—		[12, 44, 45]
$D_s^*(2112)^+$	$0.048 \pm 0.014^{(a)}$	—	$0.101 \pm 0.048^{(h)}$		[48, 65]
$B^{*\ (i)}$	—	—	0.288 ± 0.026		[66, 67]
$J/\psi(1S)$	$0.00050 \pm 0.00005^{(a)}$	—	$0.0052 \pm 0.0004^{(j)}$		[68–73]
$\psi(2S)$	—	—	$0.0023 \pm 0.0004^{(j)}$		[71, 73, 74]
$\Upsilon(1S)$	—	—	$0.00014 \pm 0.00007^{(j)}$		[75]
Pseudovector mesons:					
$f_1(1285)$	—	—	0.165 ± 0.051		[76]
$f_1(1420)$	—	—	0.056 ± 0.012		[76]
$\chi_{c1}(3510)$	—	—	$0.0041 \pm 0.0011^{(j)}$		[71, 74]
Tensor mesons:					
$f_2(1270)$	0.09 ± 0.02	0.14 ± 0.04	0.166 ± 0.020		[54–56, 77]
$f_2'(1525)$	—	—	0.012 ± 0.006		[55]
$K_2^*(1430)^+$	—	0.09 ± 0.03	—		[55, 78]
$K_2^*(1430)^0$	—	0.12 ± 0.06	0.084 ± 0.022		[54, 55, 79]

Particle	$\sqrt{s} \approx 10$ **GeV**	$\sqrt{s} = 29$–35 **GeV**	$\sqrt{s} = 91$ **GeV**	$\sqrt{s} = 130$–200 **GeV**	
$B^{**\ (k)}$	—	—	0.118 ± 0.024		[80]
$D_{s1}^{\pm}$	—	—	$0.0052 \pm 0.0011^{(\ell)}$		[81]
$D_{s2}^{*\pm}$	—	—	$0.0083 \pm 0.0031^{(\ell)}$		[81]
Baryons:					
p	0.266 ± 0.008	0.640 ± 0.050	1.050 ± 0.032	1.41 ± 0.18	[10, 13–17, 24, 25, 77]
Λ	$0.093 \pm 0.006^{(a)}$	0.205 ± 0.010	0.3915 ± 0.0065	0.39 ± 0.03	[17, 20, 34, 36, 77, 82–85]
Σ^0	$0.0221 \pm 0.0018^{(a)}$	—	0.078 ± 0.010		[10,59,82,86–88]
Σ^-	—	—	0.081 ± 0.010		[88,89]
Σ^+	—	—	0.107 ± 0.011		[87,88]
$\Sigma^\pm$	—	—	0.174 ± 0.009		[84,88]
Ξ^-	$0.0055 \pm 0.0004^{(a)}$	0.0176 ± 0.0027	0.0262 ± 0.0009		[9,59,77,82–85]
$\Delta(1232)^{++}$	0.040 ± 0.010	—	0.085 ± 0.014		[90–92]
$\Sigma(1385)^-$	0.006 ± 0.002	0.017 ± 0.004	0.0240 ± 0.0017		[59,82,84,85,93]
$\Sigma(1385)^+$	$0.0062 \pm 0.0011^{(a)}$	0.017 ± 0.004	0.0239 ± 0.0015		[59,82–85,93]
$\Sigma(1385)^\pm$	0.0106 ± 0.0020	0.033 ± 0.008	0.0472 ± 0.0027		[59,82,84,85,93]
$\Xi(1530)^0$	$0.00130 \pm 0.00010^{(a)}$	—	0.00694 ± 0.00049		[59,82,83,85,94]
Ω^-	$0.00060 \pm 0.00033^{(a)}$	0.014 ± 0.007	0.00124 ± 0.00018		[59, 77, 82, 83, 85, 86]
Λ_c^+	$0.0479 \pm 0.0038^{(a,m)}$	0.110 ± 0.050	0.078 ± 0.017		[47,49,77,83,95]
Λ_b^0	—	—	0.031 ± 0.016		[96]
Σ_c^0	$0.0025 \pm 0.0004^{(a)}$	—	—		[83]
$\Lambda(1520)$	$0.0046 \pm 0.0004^{(a)}$	—	0.0222 ± 0.0027		[83,85,89,97]

(a) $\sigma_{\rm had} = 3.33 \pm 0.05 \pm 0.21$ nb (**CLEO**: [98]) has been used in converting the measured cross sections to average hadron multiplicities.

(b) B($D_s \to \eta\pi,\ \eta'\pi$) was used (RPP 1994).

(c) Comprises both charged and neutral B meson states.

(d) The Standard Model B($Z \to b\bar{b}$) = 0.217 was used.

(e) $x_p = p/p_{\rm beam} > 0.1$ only.

(f) Both charge states.

(g) B($D^*(2010)^+ \to D^0\pi^+$) × B($D^0 \to K^-\pi^+$) has been used (RPP 2000).

(h) B($D_s^* \to D_S^+\gamma$), B($D_s^+ \to \phi\pi^+$), B($\phi \to K^+K^-$) have been used (RPP 1998).

(i) Any charge state (*i.e.*, B_d^*, B_u^*, or B_s^*).

(j) B($Z \to$ hadrons) = 0.699 was used (RPP 1994).

(k) Any charge state (*i.e.*, B_d^{**}, B_u^{**}, or B_s^{**}).

(ℓ) Assumes B($D_{s1}^+ \to D^{*+}K^0 + D^{*0}K^+$) = 100% and B($D_{s2}^+ \to D^0K^+$) = 45%.

(m) The value was derived from the cross section of $\Lambda_c^+ \to p\pi K$ using (a) and assuming the branching fraction to be (5.0 ± 1.3)% (RPP 2004).

References grouped by collaboration for Table-53.1:

- **RPP**: [12]

- **ALEPH**: [13, 20, 40, 58, 59, 63, 70, 81],
- **ARGUS**: [8, 24, 37, 41, 57, 82, 90, 97],
- **BaBar**: [10, 48, 68, 95],
- **Belle**: [44, 69, 83],
- **CELLO**: [19, 26],
- **CLEO**: [9, 45, 49, 98],
- **Crystal Ball**: [38],
- **DELPHI**: [14, 17, 21, 25, 33, 46, 50–52, 55, 61, 66, 71, 76, 80, 84, 86, 89, 91, 94],
- **HRS**: [27, 54, 78, 93],
- **L3**: [22, 34, 42, 67, 72, 74, 87]
- **MARK II**: [29, 39],
- **JADE**: [18, 28],
- **OPAL**: [15, 23, 35, 43, 47, 53, 56, 60, 62, 64, 65, 73, 75, 79, 85, 88, 92, 96],
- **PLUTO**: [30]
- **SLD**: [16, 36],
- **TASSO**: [31]
- **TPC**: [32].

53.3 σ and R in e^+e^- Collisions

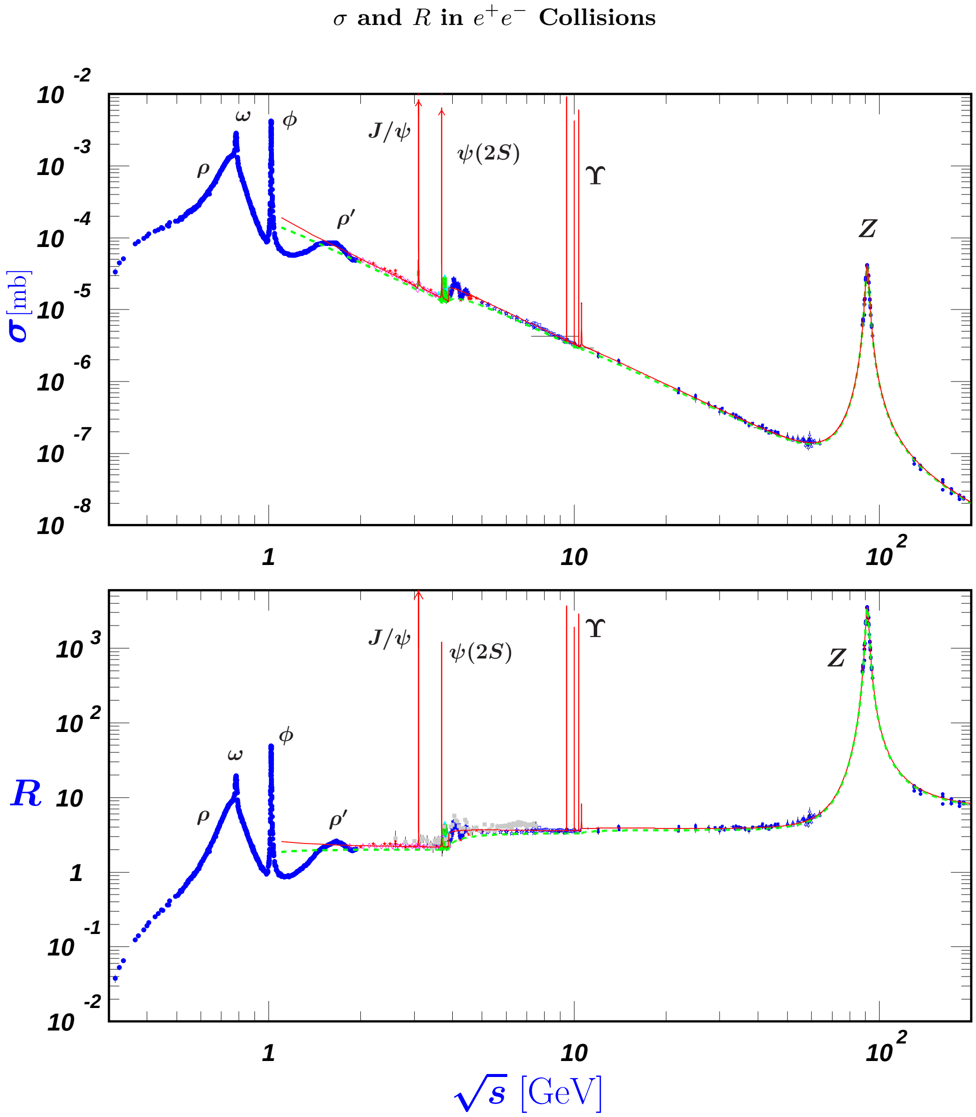

Figure 53.2: World data on the total cross section of $e^+e^- \to$ *hadrons* and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.7) or, for more details [99], Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in [100]. Corresponding computer-readable data files are available at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, August 2021. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.))

R in Light-Flavor, Charm, and Beauty Threshold Regions

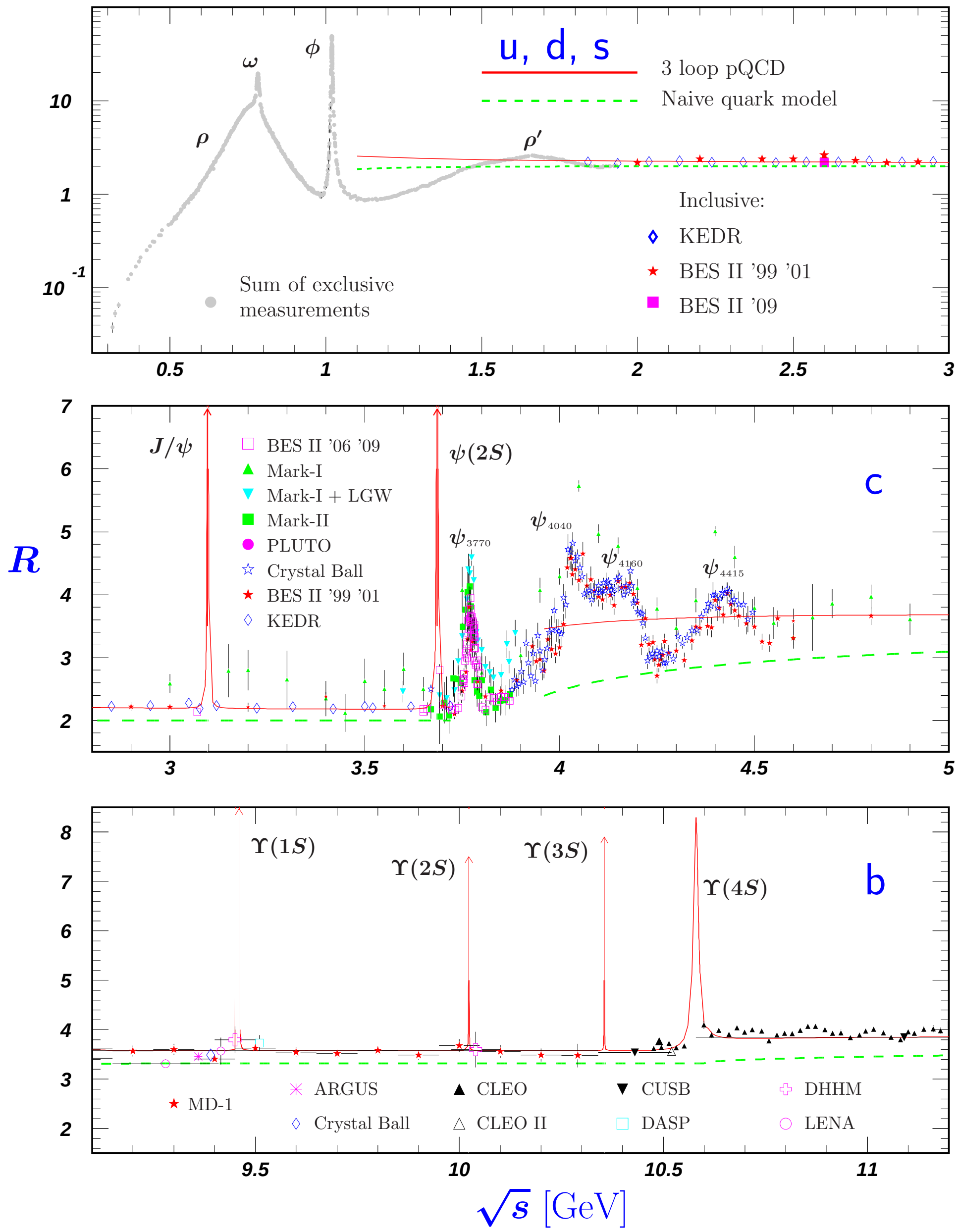

Figure 53.3: R in the light-flavor, charm, and beauty threshold regions. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are the same as in Fig. 53.2. **Note:** CLEO data above $\Upsilon(4S)$ were not fully corrected for radiative effects, and we retain them on the plot only for illustrative purposes with a normalization factor of 0.8. The full list of references to the original data and the details of the R ratio extraction from them can be found in [100]. The computer-readable data are available at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, August 2021.)

53.4 Annihilation Cross Section Near M_Z

Courtesy of M. Grünewald and the LEP Electroweak Working Group, 2007.

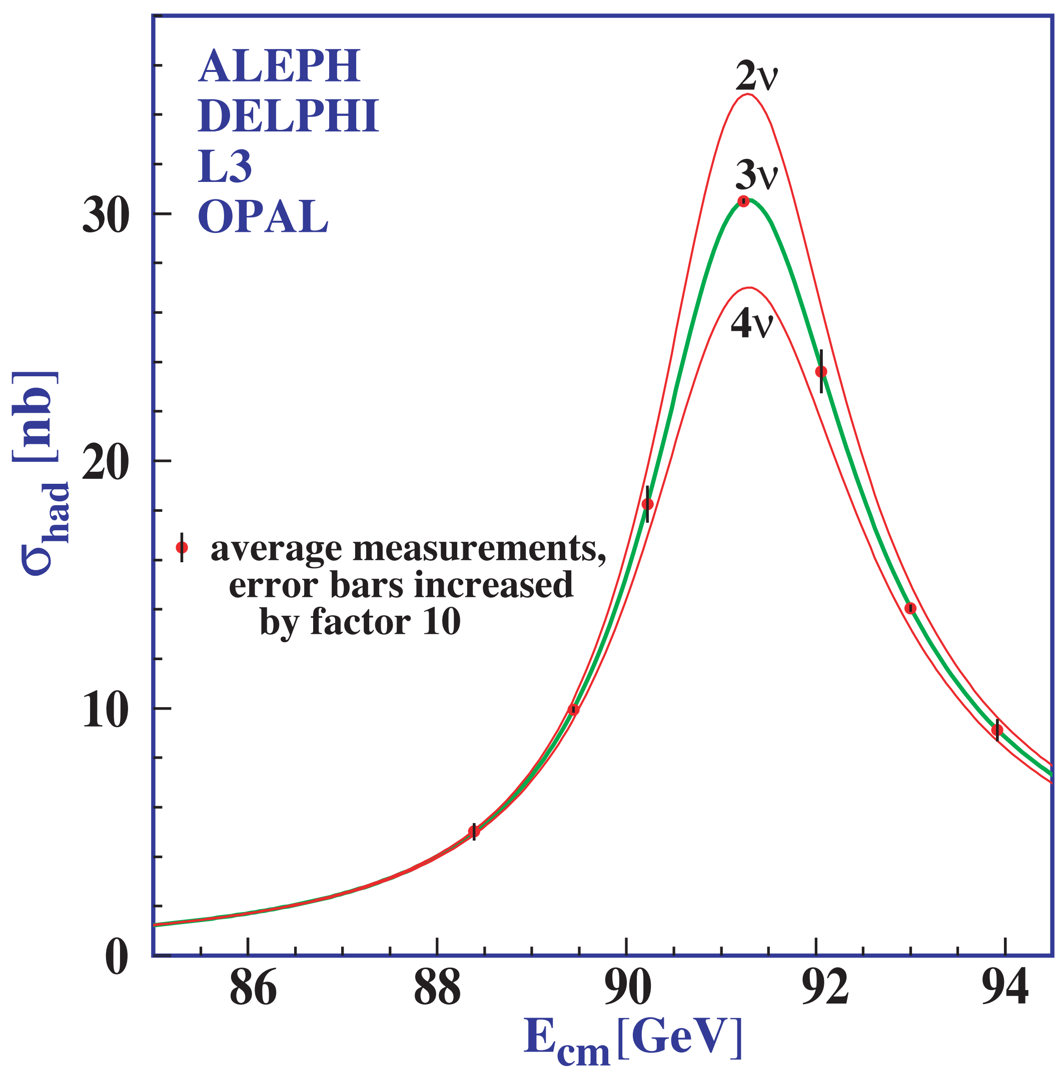

Figure 53.4: Combined data from the ALEPH, DELPHI, L3, and OPAL Collaborations for the cross section in e^+e^- annihilation into hadronic final states as a function of the center-of-mass energy near the Z pole. The curves show the predictions of the Standard Model with two, three, and four species of light neutrinos. The asymmetry of the curve is produced by initial-state radiation. Note that the error bars have been increased by a factor ten for display purposes. References: ALEPH [101], DELPHI [102], L3 [103], OPAL [104], Combination [105],

53.5 Total Hadronic Cross Sections

Revised August 2021 by COMPAS Group (NRC KI – IHEP, Protvino).

There are a significant number of physical quantities extracted from experimental data that are related to the physics of hadronic interactions. Depending on the magnitude of the transverse momenta of particles in the final states, the interactions are either classified as "hard" or "soft". In hard processes, the transverse momenta are comparable in magnitude with the momenta of the initial particles, while in soft processes they are vanishingly small compared to the initial momenta. Soft processes reveal characteristic interference pattern of the scattered states, and are often referred to as diffractive processes. The set of experimentally measured quantities related to diffractive processes include total and elastic cross sections, differential cross sections of elastic scattering and diffraction dissociation processes, as well as secondary quantities such as the slopes of differential cross sections and the ratio of real parts of the scattering amplitude to its imaginary parts (ρ parameter). In this section, plots of total and elastic cross section for various processes are presented. They include the data from purely hadronic collisions such as pp and πp, as well as γp, γd, and $\gamma\gamma$[a]

One of the most important basic notions and tools in general theoretical framework related to the diffractive processes is the notion of the Regge poles, or Reggeons, generalizing a simple one-particle exchange (of Yukawa type) by virtual particles of fixed spin to exchanges by states with "running spin" dependent on the transferred momenta [106, 107]. The simplest case of the one-Reggeon exchange amplitude is given by the amplitude (at high c.m. energy $\sqrt{s}$ and fixed (small) transferred momentum squared, t): $T(s,t) = \beta(t)s^{\alpha(t)}$ which qualitatively exhibits many typical features of generic diffractive processes (e.g. the growth of the interaction radius with energy). In practice the single-pole Reggeon model is insufficient for many diffractive processes but serves as a building block for more sophisticated schemes. Up to now no firm results concerning Regge trajectories $\alpha(t)$ and Regge residues $\beta(t)$ are obtained from the first principles of QCD. Therefore one has to rely on general principles of quantum field theory as a guide. These imply, in particular, that both $\alpha(t)$ and $\beta(t)$ are analytic functions with right cuts from some $t_0 > 0$ to positive infinity.

The theoretical requisite for analyzing diffractive phenomena is therefore represented by various model approaches. The more commonly discussed models in the literatures are:

- **Regge-eikonal approach** [108–115]: this approach automatically satisfies the s-channel unitarity condition and generalizes the impact parameter approximation to the relativistic case. The main element of the model is the Born amplitude defined by one or several leading "Pomeronic" Regge trajectories with intercepts higher than 1. To take into account the difference between C-conjugated processes (such as pp and $\bar{p}p$), a term with an Odderon Reggeon is also added, as well as, if one wishes to describe low (5 to 10 GeV) energies, a secondary Reggeon with low intercepts.
- **Regge pole models with minimal corrections due to two-Reggeon exchanges** [116–118]: in this model, contribution of the leading trajectory in the scattering amplitude is supplemented by a two-Reggeon exchange with an arbitrary coefficient chosen from the fitting details.
- **U-matrix (or "resonance") approach** [119, 120]: the unitarity respecting approach with the scattering amplitude defined by a "U-matrix".
- **Direct functional modelling of the amplitudes with limited Regge trajectories** [121, 122]: this approach appeals to only very general (analytic) properties of the amplitudes leaving aside all dynamical assumptions and mostly aiming at the best phenomenological description of the data.
- **Quasi-classical approach** [123–125]: this model is based on the observation that diffractive processes deal with high quantum numbers, in particular, with large number of virtual quanta.

The ratio $\rho(s) = \mathrm{Re}T(s,0)/\mathrm{Im}T(s,0) = \cot \Phi(s,0)$, where $\Phi(s,0)$ is the phase of the forward scattering amplitude, $T(s,0) = |T(s,0)| \exp(i\Phi(s,0))$, is accessible due to Coulomb-nuclear interference (CNI). There are several different ways to deal with CNI in the literature [126–130]. The value ρ is important for measurements of the total cross-sections as well as for discrimination of different models [131, 132]. The phase $\Phi(s,0)$ starts from π at low energies, evolves further down to $\pi/2$, achieves a shallow minimum and then slowly rise up to $\pi/2$ at asymptotic energies.

The total cross-sections results shown in this review exhibit a common feature: starting from a certain energy (unique for each initial state) the cross-sections begin a slow but steady rise which continue up to several tens of TeV. The current data show the upper bound for total hadronic cross-section is well below the Froissart-Martin asymptotic bound [133, 134], more than 20000 mb at the LHC energies [135].

For readers who are interested in examples of both total and elastic cross section parametrizations and fits, see previous edition of the *Plots of Cross Sections and Related Quantities* review [136]. For the cross section plots shown in the following pages, the example fits are using parametrizations as described in [136] with the fit range starting at about $\sqrt{s} = 5$ GeV.

[a] In spite of its electromagnetic nature the photon interacts with hadron targets as an effective hadron (via transmutation of γ into virtual vector mesons $\rho^0, \omega, \phi, \ldots$).

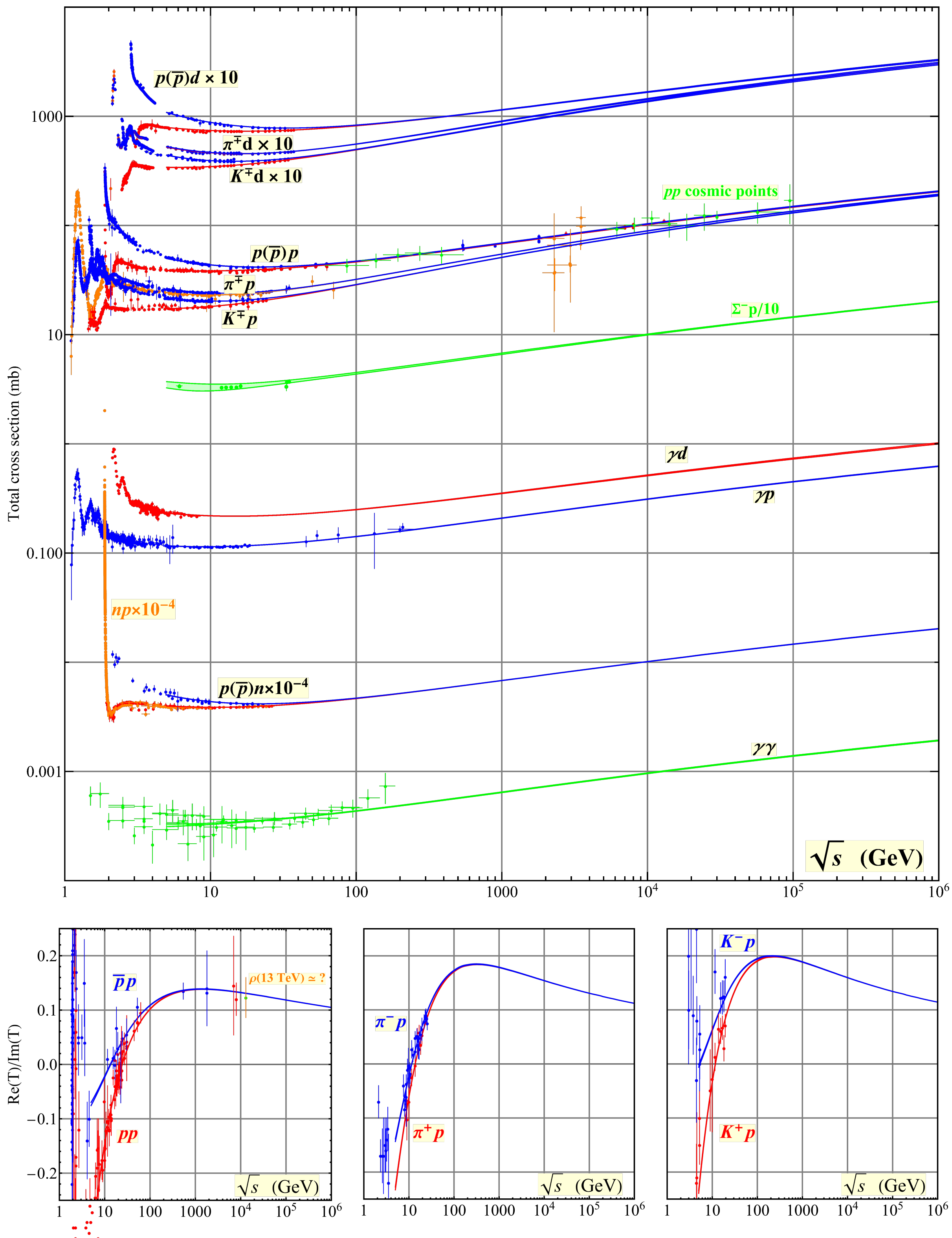

Figure 53.5: Summary of hadronic, γp, γd, and $\gamma\gamma$ total cross sections (top), and ratio of the real to imaginary parts of the forward hadronic amplitudes (bottom). Corresponding computer-readable data files can be found at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS group, NRC KI – IHEP, Protvino, August 2021.)

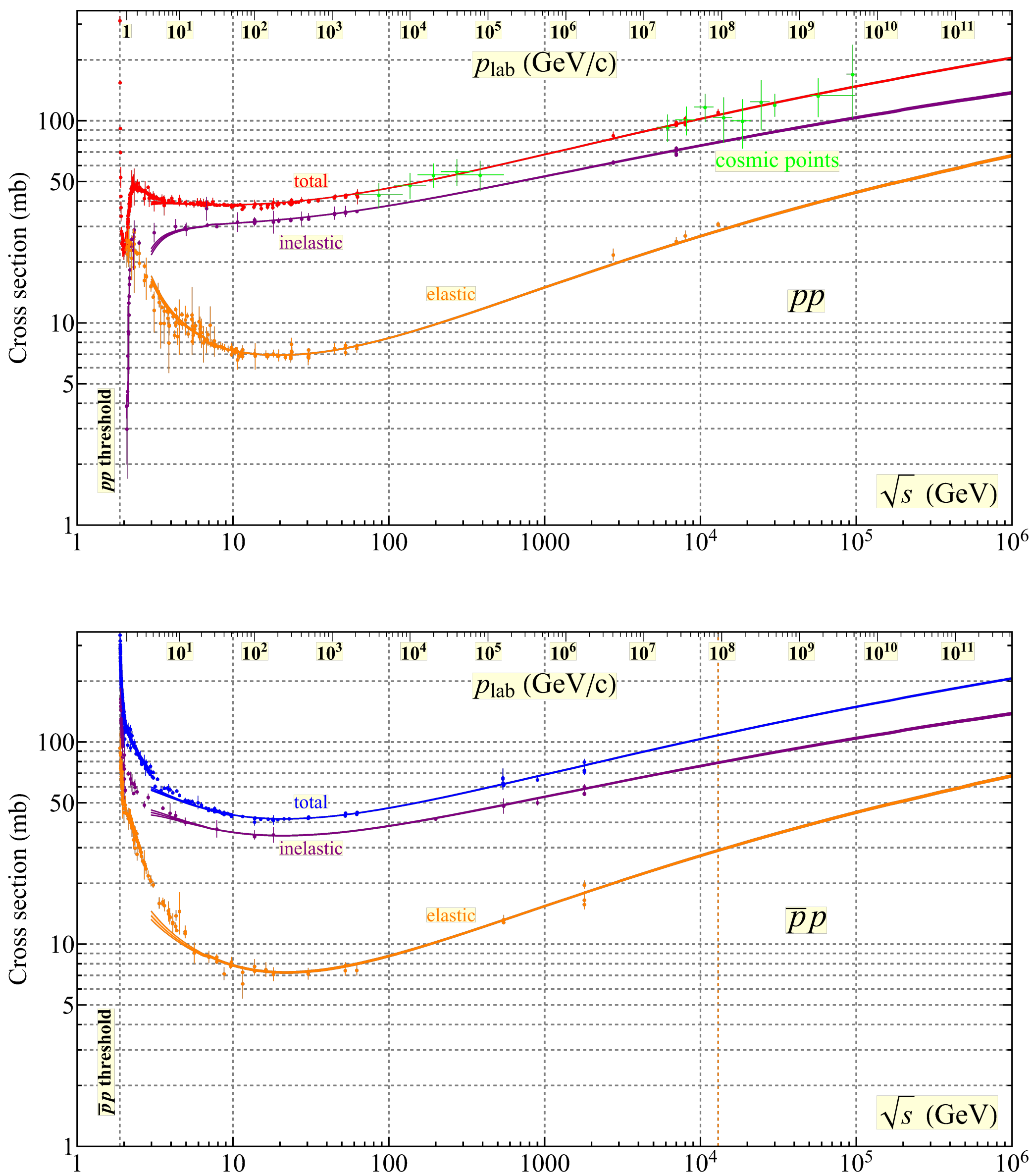

Figure 53.6: Total and elastic cross sections for pp and $\overline{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. $\sigma_{\rm el}$ is computed using the nuclear part of the elastic scattering amplitude [137]. Corresponding computer-readable data files may be found at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS group, NRC KI – IHEP, Protvino, August 2021.)

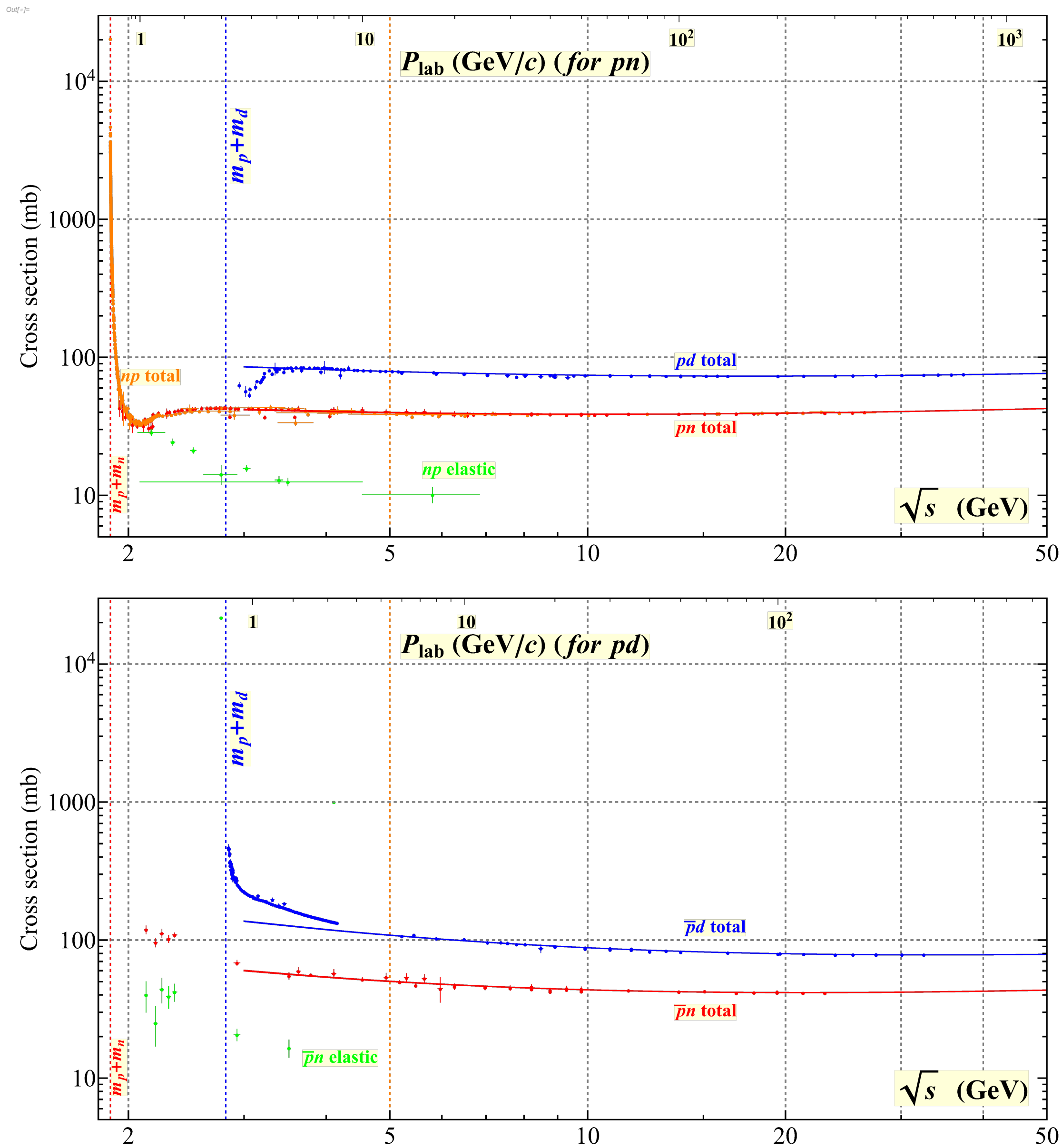

Figure 53.7: Total and elastic cross sections for pd (total only), np, $\overline{p}d$ (total only), and $\overline{p}n$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS Group, NRC KI – IHEP, Protvino, August 2021. Corrections by N. Otsuka (IAEA))

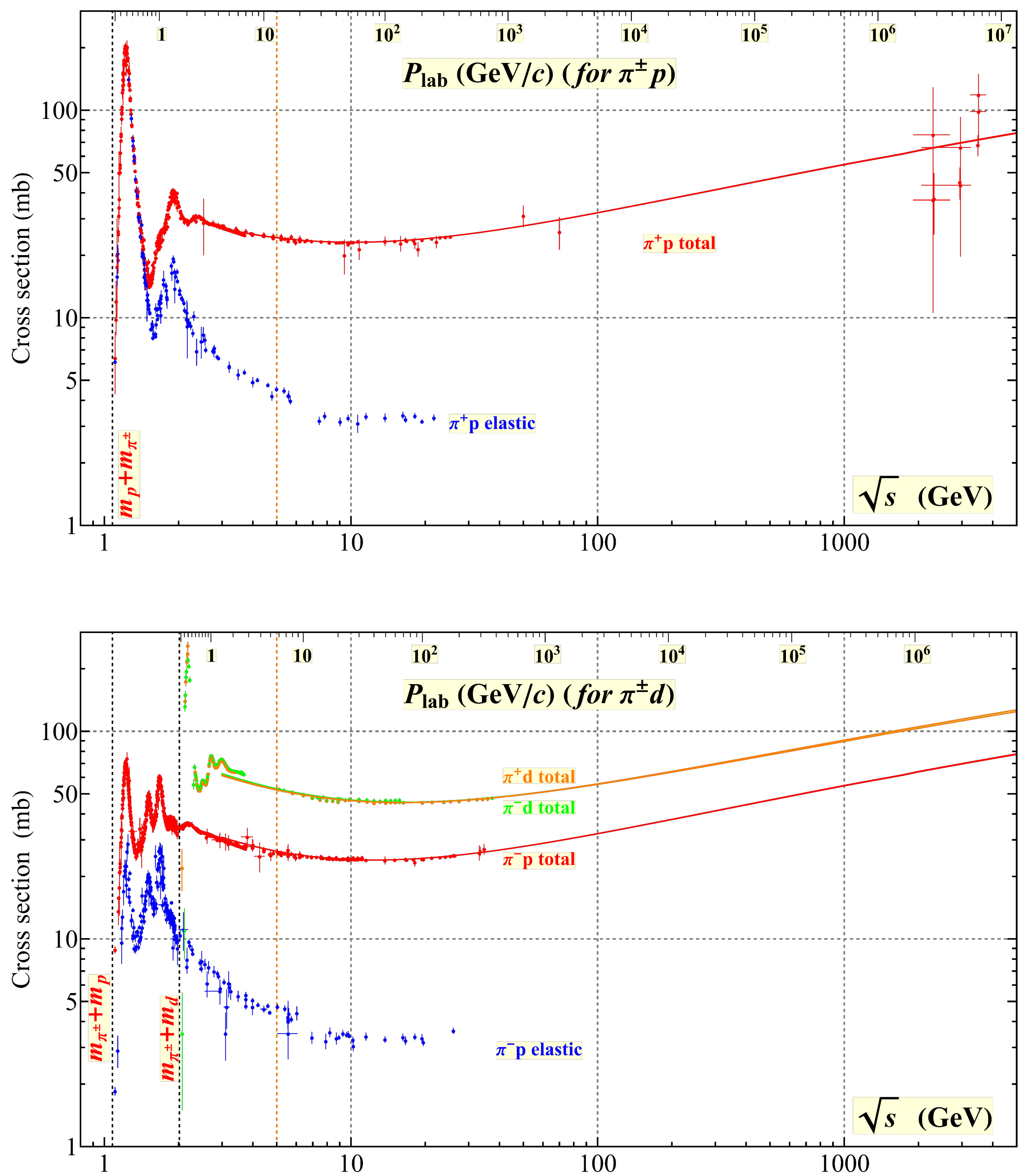

Figure 53.8: Total and elastic cross sections for $\pi^{\pm}p$ and $\pi^{\pm}d$ (total only) collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files can be found at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS Group, NRC KI – IHEP, Protvino, August 2021.)

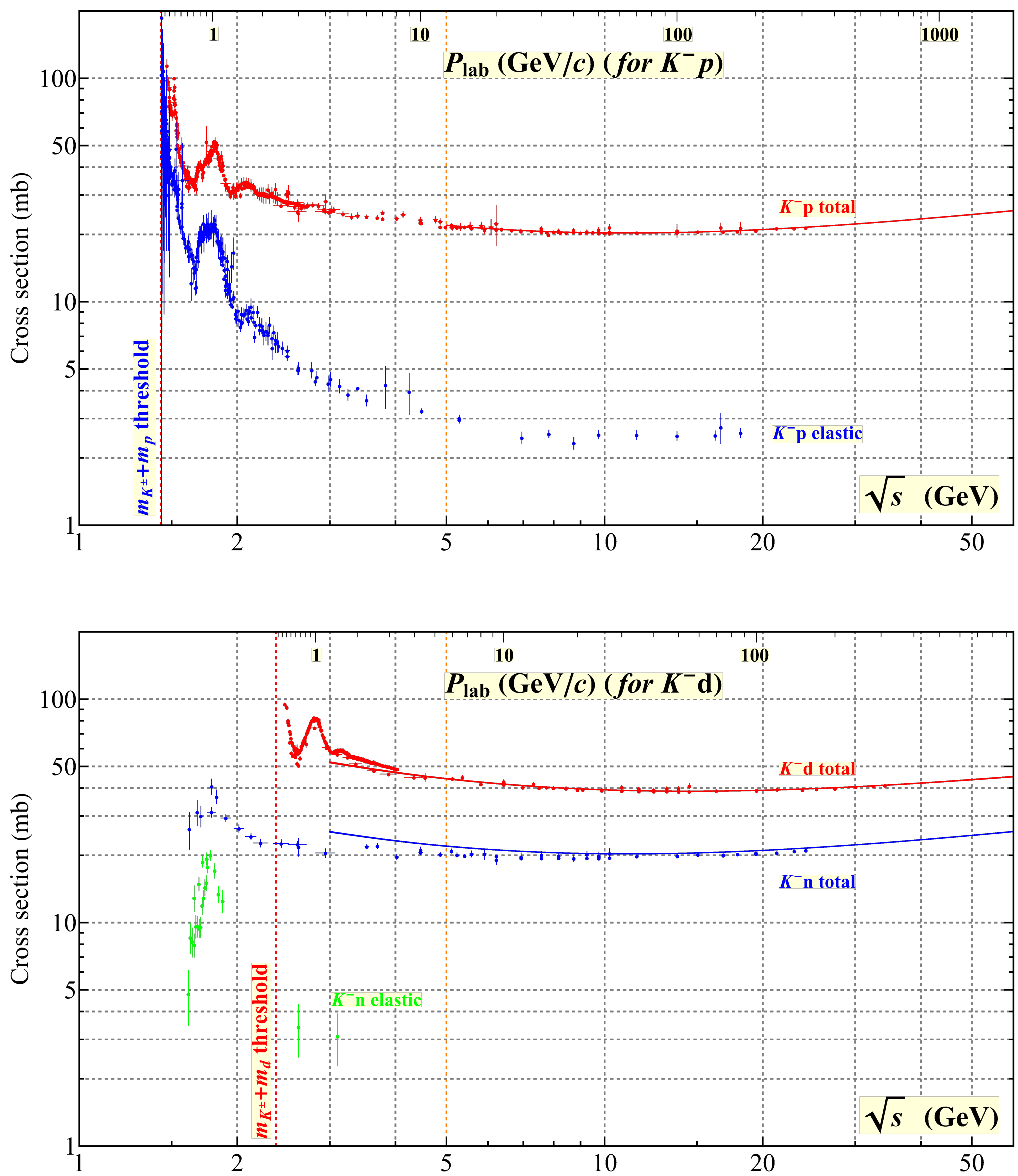

Figure 53.9: Total and elastic cross sections for K^-p and K^-d (total only), and K^-n collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files can be found at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS Group, NRC KI – IHEP, Protvino, August 2021.)

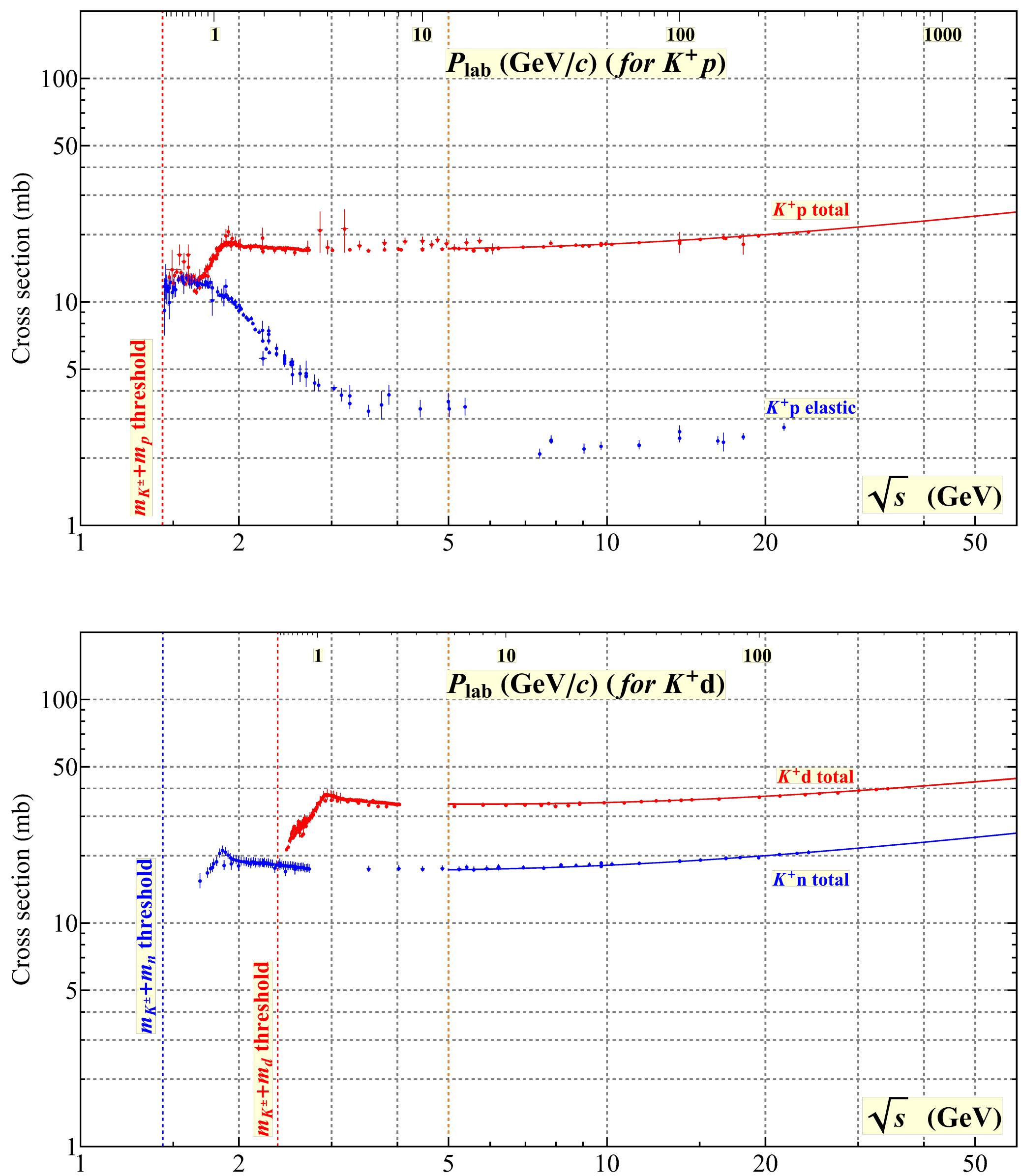

Figure 53.10: Total and elastic cross sections for K^+p and total cross sections for K^+d and K^+n collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files can be found at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS Group, NRC KI – IHEP, Protvino, August 2021.)

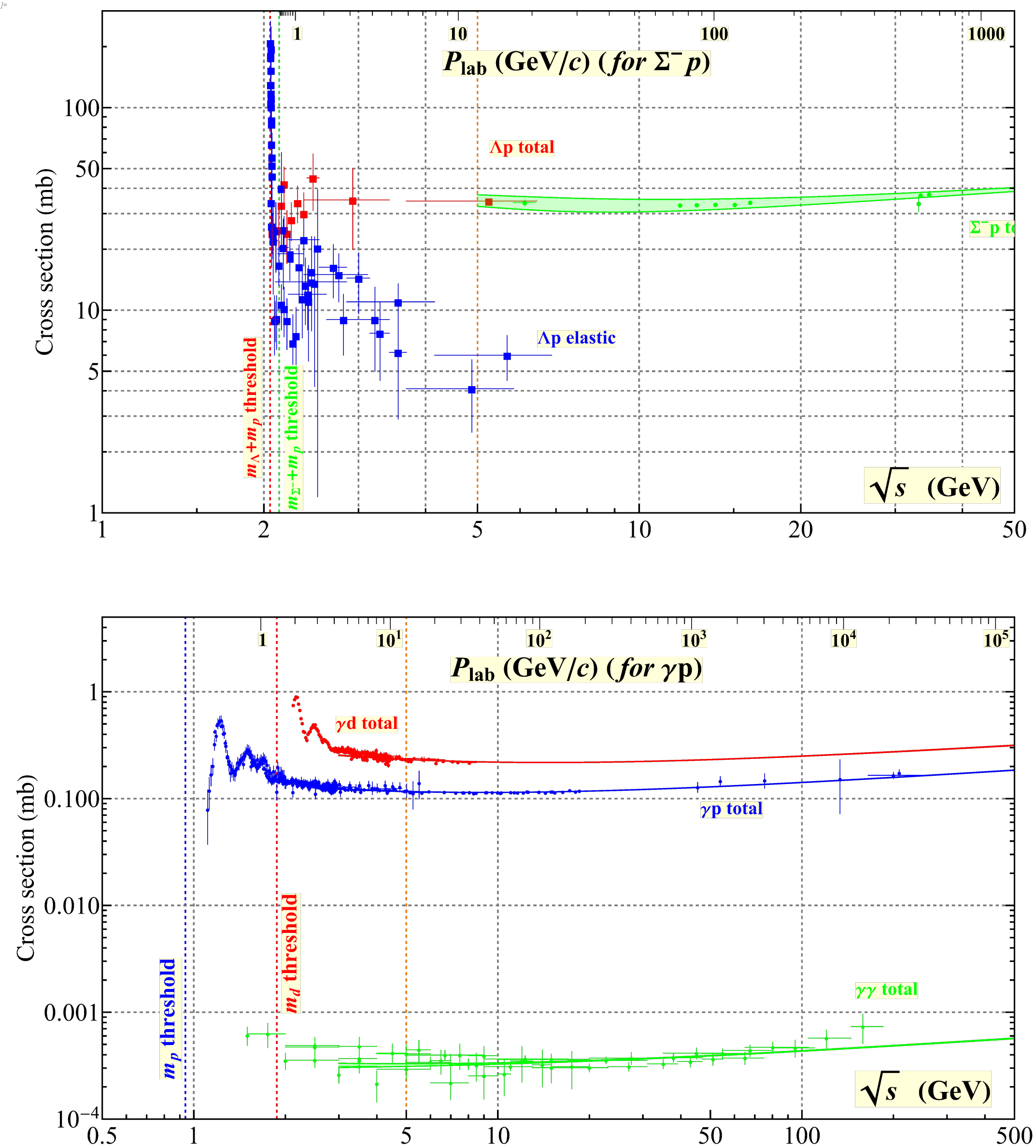

Figure 53.11: Total and elastic cross sections for Λp, total cross section for $\Sigma^- p$, and total hadronic cross sections for γd, γp, and $\gamma\gamma$ collisions as a function of laboratory beam momentum and the total center-of-mass energy. Corresponding computer-readable data files can be found at https://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS group, NRC KI – IHEP, Protvino, August 2021.)

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