79. Heavy Non-$q\overline{q}$ Mesons

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The constituent quark model describes the observed meson spectrum as bound $q\overline{q}$ states grouped into SU(N) flavor multiplets (see the 'Quark Model' in this issue of the *Review of Particle Physics*). However, the self coupling of gluons in QCD suggests that additional mesons made of bound gluons (glueballs), or $q\overline{q}$-pairs with an excited gluon (hybrids), may exist. Furthermore, multiquark color singlet states such as $qq\overline{q}\overline{q}$ (tetraquarks as compact diquark-antidiquark systems and 'molecular' bound states of two mesons) or $qqq\overline{q}\overline{q}\overline{q}$ (six-quark, 'baryonium' or quasinuclear baryon-antibaryon bound states) have also been predicted. The focus of this review is on the current understanding of exotic states that apparently do not fit into the constituent quark model and contain at least one heavy quark (charm or bottom — the lifetime of the top quark is too short to allow it to hadronise). Light non-$q\overline{q}$ candidates (made of u, d and s quarks) are discussed in 'Spectroscopy of Light Meson Resonances' in this *Review*.

The review is split into 3 parts, discussing separately heavy–light systems, heavy–heavy systems, as well as systems with more than two heavy quarks. For a more detailed discussion of the experimental and/or theoretical status on exotic meson candidates in the doubly heavy meson sector we refer to Refs. [1–7]. Reviews with main focus on tetraquarks and molecular states are presented in Ref. [8] and in Ref. [9], respectively.

79.1 Heavy-light systems

Two very narrow states, $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$, were observed at B factories [10, 11]. They lie far below the predicted masses for the two expected broad P-wave $c\overline{s}$ mesons. However, strong cusp effects, due to the nearby DK (DK^*) thresholds, could shift their masses downwards and quench the observed widths, still allowing for a conventional explanation of these states [12]. Such an effect was also claimed for the $a_0(980)$ and $f_0(980)$ mesons, which lie just below the $K\overline{K}$ threshold. In contrast to this picture, many authors favour exotic explanations for these lightest positive parity charmed states, such as four-quark states [13–16] or DK (DK^*) molecules [17–22]. Both states are isoscalars and thus the dominant open decay channels, $D_s^{(*)\pm}\pi^0$, lead to very small widths. For compact structures the hadronic decay is driven by $\eta-\pi^0$ mixing and one finds widths below 10 keV [12, 23]. In contrast to this a hadronic width of typically 100 keV would be the unequivocal signature for a prominent molecular nature of $D_{s0}^*(2317)^\pm$ [19, 21, 22, 24], since meson loops would then contribute significantly. The most refined analysis in the molecular scenario [24], where the parameters of the formalism are constrained by a large bulk of lattice QCD data on singly charmed systems, reports a prediction for the width in the range (104-116) keV. The currently measured upper bound for the width is 3.8 MeV.

It should be stressed that – akin to $q\overline{q}$ mesons – multiquark states also appear in multiplets. An important probe for the structure of hadrons is therefore the spectra, and in particular the SU(3) flavor multiplets within the different scenarios. For example the tetraquark model advocated in Refs. [16, 25] finds that the $D_{s0}^*(2317)^\pm$ should be degenerate with its non strange-partner state $D_0^*(2300)$ in line with the currently available experimental analyses. However, Ref. [26] showed that the recent LHCb data for $B \to D\pi\pi$ [27, 28] call for a significantly lighter lightest non-strange scalar charmed meson as soon as constraints from the chiral symmetry of QCD are employed in the analysis. This claim is in line with the recent lattice QCD study of Ref. [29]. In fact, the experimental as well as lattice data turn out to be consistent with the predictions from unitarised chiral perturbation theory pioneered for these systems in Ref. [18]. For more recent calculations see

Refs. [24, 30, 31]. Moreover, all these latter calculations predict that there should be in addition a heavier flavor exotic state by about 300 MeV that indeed shows up in the above mentioned LHCb data [31]. Thus, in this picture the structure that appears as $D_0^*(2300)$ in the *Listings* originates from the interplay of two poles, similar to the $\Lambda(1405)$ in the baryon sector, see 'Pole structure of the $\Lambda(1405)$ region' in this *Review*. Two poles in the non-strange scalar sector are also generated in the tetraquark picture of Ref. [16], but in this work the real parts of the poles are located at 2308 MeV and 2666 MeV, in conflict with the data mentioned above. In the refined analysis of Ref. [25] it is shown, however, that the exact pole locations and even the ordering of the higher multiplets depend on the details of the model.

Manifestly exotic candidates around 2.9 GeV were reported in Refs. [32, 33] by LHCb in the $\bar{D}K$ system. Since the strong interactions preserve the quark flavor and the final state contains an $\bar{s}$ as well as a $\bar{c}$, color neutrality calls for a minimum quark content of $ud\bar{c}\bar{s}$. The explanation for those states include compact tetraquarks [34–36], hadronic molecules with a prominent $D^*\bar{K}^*$ component [37–39], or kinematic effects such as cusps or triangle singularities without nearby poles [40, 41]. The different pictures are contrasted and observable differences between them are discussed in Ref. [42].

79.2 Heavy-heavy systems

With the discovery of the $\chi_{c1}(3872)$[1] in $B^\pm \to K^\pm X$ ($X \to J/\psi\,\pi^+\pi^-$) by Belle [43] in 2003, soon confirmed by BaBar [44], many searches for states beyond the standard $q\bar{q}$ quark model were initiated in the charm and bottom sectors. For an updated collection of the currently available experimental information on multiquark states we refer to 'Spectroscopy of mesons containing two heavy quarks' in this *Review*, and in particular to Table 78.2. Moreover, in the decay $\Lambda_b^0 \to J/\psi\,K^- p$ the LHCb collaboration has recently reported the observation of new baryons decaying into $J/\psi(1S)\,p$, which are candidates for heavy pentaquark states [45, 46]. They are discussed in some depth in 'Pentaquarks' in this *Review*.

When restricting ourselves to confirmed states we are faced with several states that do not seem to fit into the mass and quantum number schemes of basic $q\bar{q}$ quark models. This is clear for the five established charged states ($Z_c(3900)^\pm$, $Z_c(4020)^\pm$ [2], and $Z_c(4430)^\pm$ in the charmonium sector, and $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$ in the bottomonium sector). The neutral states ($\chi_{c1}(3872)$, $\psi(4230)$, $\psi(4360)$, and $\psi(4660)$)[3] also challenge the quark model since their masses and decay properties are in conflict with expectations.

The quantum numbers of the $\chi_{c1}(3872)$ have been determined by LHCb to be $J^{PC} = 1^{++}$, first by assuming zero angular momentum between the $J/\psi(1S)$ and the dipion [47] and then by relaxing this constraint [48]. The $\chi_{c1}(3872)$ cannot be identified with the $\chi_{c1}(2^3P_1)$ since the latter is predicted to lie about 100 MeV higher in mass [49]. Instead, the $X(3940)$ reported by Belle in $e^+e^- \to J/\psi X$, decaying into $D^*\bar{D}$ but not into $D\bar{D}$ [50] could be the $\chi_{c1}(2^3P_1)$. The $\chi_{c2}(3930)$ 2^3P_2 tensor partner was reported by Belle at 3931 MeV in $\gamma\gamma$ interactions [51].

The $\chi_{c1}(3872)$ lies within 200 keV of the $D^0\bar{D}^{*0}$ threshold and its width is very small — recent Breit-Wigner analyses by LHCb [52, 53] result in an average value of $\Gamma = (1.19 \pm 0.21)$ MeV as indicated in the *Listings*; employing the formalism of Ref. [54] under the constraint that the $D^*\bar{D}$ decay channel dominates, results in $\Gamma = 0.22^{+0.18}_{-0.19}$ MeV, where the standard convention is applied to use twice the imaginary part of the pole location as width. In the future those values can be refined more by either a direct measurement of the width that should be possible at the planned PANDA experiment [55, 56] or by exploiting the interplay of a triangle singularity and the $\chi_{c1}(3872)$

[1] The $\chi_{c1}(3872)$ is also known as the $X(3872)$. According to the PDG naming scheme, which we follow in this review, the primary name for a meson expresses its quantum numbers.

[2] While the $J^P = 1^+$ quantum numbers are plausible for this state, they are not yet established experimentally. This is why this state appears as $X(4020)$ in both the *Listings* and *Summary Tables*.

[3] The $\psi(4230)$, $\psi(4360)$, and $\psi(4660)$ are also known as the $Y(4230)$, $Y(4360)$, and $Y(4660)$, respectively. Before improved mass measurements, the $\psi(4230)$ was originally called the $\psi(4260)$ or $Y(4260)$.

pole [57–59]. Therefore the most natural explanation for this state is a 1^{++} $D\bar{D}^*$ molecule [60], for which strong isospin breaking is predicted [60,61], since the distance of the pole of the $\chi_{c1}(3872)$ to the $D^0\bar{D}^{*0}$ threshold is significantly smaller than to the D^+D^{*-} threshold. Indeed, the comparable rates for $\omega J/\psi$ and $\rho^0 J/\psi$ are consistent with an interpretation of $\chi_{c1}(3872)$ as an isoscalar $D\bar{D}^*$ molecule, when the different widths of the ρ and ω are taken into account [62]. Similarly, a dominant molecular $D^0\bar{D}^{*0}$ structure in the $\chi_{c1}(3872)$ with further subleading hadronic components is used to explain strong and radiative decays involving $J/\psi(1S)$ and $\psi(2S)$ in the final states [63,64].

A four-quark state $cq\bar{c}\bar{q}'$ is another possible interpretation of the $\chi_{c1}(3872)$ [15] but a charged partner of the $\chi_{c1}(3872)$ has not been observed (e.g. not in $B^- \to \bar{K}^0 X^-$ nor in $B^0 \to K^+ X^-$, where $X^- \to J/\psi\,\pi^-\pi^0$ [65]) – see [66] for a possible explanation of this non-observation within the tetraquark approach assuming specific diquark correlations and the more recent discussion in Ref. [67]. The claim that $\chi_{c1}(3872)$ must be a compact (tetraquark) state, since it is also produced at very high p_T in $\bar{p}p$ collisions [68] and in high multiplicity final states [69], was challenged in [70] and [71], respectively, which in particular stress the importance of rescattering, see also Refs. [72,73].

A broad structure, originally called $Y(4260)$, decaying into $J/\psi(1S)\,\pi^+\pi^-$ was reported by BaBar in initial state radiation $e^+e^- \to (\gamma_{\rm ISR})Y(4260)$ [74]. A subsequent measurement with significantly improved precision was reported by BESIII [75], and revealed that the original $Y(4260)$ cannot be described with a simple resonant lineshape. Fitting the BESIII data with two Breit-Wigner distributions leads to a narrower and lighter structure (referred to in the *Listings* as the $\psi(4230)$), but also requires a second state at 4320 MeV. However, note that the $D_1(2420)\bar{D}$ molecular model for the $\psi(4230)$ [76] can describe the same data with just one single pole [77]. How many vector states are in the mass range between 4220 MeV and 4400 MeV is not yet settled, but knowing this is crucial to allow for further progress.

There are no charmonium states expected in this mass region with quantum numbers 1^{--} from quark models using the Cornell type of interaction, although this might not be true for some screened versions thereof – for a recent discussion we refer to Ref. [78]. In addition, a charmonium state at this mass is expected to have significant couplings to one or more of the $\bar{D}^{(*)}D^{(*)}$ channels [79, 80], a feature that is not observed for the $\psi(4230)$. This state could be a hybrid charmonium with a spin-1 $\bar{c}c$ [81, 82] or a spin-0 [83, 84] core. However, provided that the observation of $\psi(4230)$ decay into $h_c(1P)\pi\pi$ by BESIII [85] is confirmed, the hybrid hypothesis would be under pressure, since the spin of the heavy quarks (coupled to zero in the $h_c(1P)$) should be conserved in leading order in the expansion in $(\Lambda_{\rm QCD}/m_c)$. (The individual conservation of the heavy quark spin and the total angular momentum of the light quark cloud is a consequence of the heavy-quark spin symmetry, see 'Heavy-Quark and Soft-Collinear Effective Theory' in this issue of the *Review*.) The same criticism applies to the hadrocharmonium interpretation of the $\psi(4230)$, which describes this state as spin-1 quarkonium surrounded by a light quark cloud [86]. To circumvent the spin-symmetry argument, Ref. [87] argues that $\psi(4230)$ and $\psi(4360)$ could be mixtures of two hadrocharmonia with spin-triplet and spin-singlet heavy quark pairs. The same kind of mixing could also be at work for hybrid structures.

A dominant $D_1(2420)\bar{D}$ component in the $\psi(4230)$ [88] explains naturally why $Z_c(3900)^\pm$ (interpreted by the authors as a $\bar{D}D^*$ bound state) is seen in $\psi(4230) \to \pi^\mp Z_c(3900)^\pm$. A similar mechanism is also found in Ref. [89] linking in addition the $J/\psi(1S)\pi^+\pi^-$ and $\psi(2S)\pi^+\pi^-$ decays of the $\psi(4230)$. Furthermore, a copious production of $\chi_{c1}(3872)$ in $\psi(4230)$ radiative decays was predicted from the prominent $D_1\bar{D}$ component of the $\psi(4230)$ [90], which was confirmed by BESIII [91]. Possible charmonia components both in $\psi(4230)$ and $\chi_{c1}(3872)$ can influence the radiative transition but they are shown to be of subleading order [92]. The $\psi(4360)$ as a $D_1\bar{D}^*$ bound state could be the spin partner of the $\psi(4230)$ [93, 94], but a detailed microscopic calculation is still lacking.

The tetraquark picture calls for four ground state vector states — once the parameters of the model are fixed from some candidate states in the negative and positive parity sector, states with other quantum numbers can be predicted. Possible scenarios are for instance discussed in Ref. [95] which builds on a tailor-made spin-spin interaction [96] to describe the $\chi_{c1}(3872)$, both $Z_c(3900)^{\pm,0}$ and $Z_c(4020)^{\pm}$ and even the $Z(4430)^{\pm}$ confirmed by Belle [97] and LHCb [98]. This model also explains the copious production of $\chi_{c1}(3872)$ in $\psi(4230)$ radiative decays mentioned above [96, 99]. However, tetraquark models (in most cases based on diquark-antidiquark configurations) tend to predict many additional charged and neutral states which have not yet been discovered. In particular, as for the conventional $\bar{q}q$ structures one should expect nearly degenerate isoscalar and isovector states in analogy to the near degeneracy of $\rho(770)$ and $\omega(782)$. The problem and possible explanations are discussed in some detail in Refs. [66, 67].

Ref. [100] found a sizeable SU(3) flavor octet contribution when analysing the $\pi\pi$ final state from $\psi(4230) \to J/\psi(1S)\pi^+\pi^-$, which is consistent with both a molecular and a tetraquark interpretation of $\psi(4230)$, but at odds with a hybrid or a $\bar{c}c$ interpretation.

In the mass range above 4600 MeV, the number of poles is also not yet settled. Experimental signals are seen in the $\psi(2S)\pi\pi$, $\Lambda_c^+\bar{\Lambda}_c^-$ and $D_s\bar{D}_{s1}$ final states. In the *Listings* the former two structures go into one node, $\psi(4660)$ due to their proximity in parameter values. Moreover, various theoretical works describe these states in a combined analysis [101–103]. The signal in the hidden strangeness mode around 4630 MeV still calls for confirmation and might be yet another realisation of the same state, but there are already speculations about its nature. While Ref. [104] argues in favor of a $D_s^{(*)}\bar{D}_{s1}(2536)$ or $D_s^{(*)}\bar{D}_{s2}(2573)$ [105] molecular nature, Ref. [106] does not confirm these claims. Ref. [107] identifies the structure as P-wave $[cs][\bar{c}\bar{s}]$ tetraquark state. Other explanations of the $\psi(4660)$ include a $\psi(2S)f_0(980)$ molecule [108] and a $\Lambda_c^+\bar{\Lambda}_c^-$ baryonium [101]. Also in this mass range studies of the partner states, driven either by spin or flavor symmetry will be very valuable — see e.g. the predictions in Ref. [109].

The isovector states $Z_c(3900)$ and $Z_c(4020)$, first observed by BESIII [110, 111], decay predominantly into $\bar{D}D^*$ and $\bar{D}^*D^*$, respectively, while the $Z_b(10610)$ and $Z_b(10650)$, first observed by Belle [112, 113], decay predominantly into $\bar{B}B^*$ and $\bar{B}^*B^*$ [114], respectively, although all four were discovered in the decay mode heavy quarkonium plus pion. This suggests that these states are close relatives and their interactions are connected via heavy quark flavor symmetry. A molecular interpretation for the bottomonium states was proposed shortly after the discovery of the two Z_b states [115] and also shortly after that of the $Z_c(3900)$ [88]. This picture is confirmed within the meson exchange model of Ref. [116]. Decay patterns of $Z_c(3900)$ and the two Z_b were also shown to be consistent with a molecular interpretation [117–119]. However, some of their properties also appear to be consistent with tetraquark structures [120]. If the molecular picture were correct for the Z_b states, spin symmetry would lead to the existence of spin partner states [121–123], which are still to be found. In Ref. [124] it was shown that the actual pole locations of these partner states would be good probes of one-pion exchange in the molecular potential, which makes the experimental search for those states even more interesting.

The heaviest confirmed charged state in the charmonium sector is the $Z(4430)^{\pm}$ observed by Belle [97]. It is interpreted as hadrocharmonium [86], $\bar{D}_1D^*$ molecule [125], as well as tetraquark [96]. Alternatively, in [126, 127] the $Z(4430)^{\pm}$ is explained as a cross-channel effect enhanced by a triangle singularity from open charm states. These works were criticised in Ref. [128] where an alternative triangle consisting of a K^*, a π, and the $\psi(4230)$, is proposed to generate the $Z_c(4430)$. The Argand diagram shows an anticlockwise circle, in line with the experimental analysis [98], while the one of Ref. [127] shows a clockwise motion. By replacing the $\psi(4230)$ by the $\psi(3770)$ and changing the K^* one can also interpret the $Z_c(4200)$ as a kinematic effect [128]. A possible interpretation of $Z_c(3900)$ and $Z_c(4020)$ as crossed channel effects is put forward in Ref. [129]. It remains to be seen, however, if this kind of explanation is also capable of explaining the observations of these lowest Z_c states, also at other total energies.

There is recent evidence for a charged charmonium-like state with strangeness, $Z_{cs}(3985)$, from BESIII [130]. The possible existence of a strange partner to the Z_c near the $D_s\bar{D}^*$ thresholds has been predicted in molecular models [131, 132], for tetraquarks [133, 134], for hadrocharmonium structures [134, 135], and as a coupled-channel effect [136]. Later on this state is interpreted in Refs. [137, 138] as a member of the same multiquark octet and in Refs. [139, 140] as a member of the same molecular one as the $Z_c(3900)$. Ref. [138] is also able to describe the recent LHCb data in the $J/\psi(1S)K^-$ system [141], although the extracted total width for their lowest Z_{cs} state is an order of magnitude larger than that found by BESIII. Refs. [142, 143] claim that both the molecular components and the compact tetraquark core are relevant to describe the $Z_c(3900)$ and $Z_{cs}(3985)$ resonances. Again, in [144] the $Z_{cs}(3985)$ can be explained as a coupled-channel effect producing the enhancement close to threshold.

It should be stressed that the various scenarios, while describing much of the available data, also make decisive predictions, *e.g.* yet unobserved quantum numbers [95, 145]. The forthcoming data on heavy meson spectroscopy from various facilities should provide a much deeper understanding of how QCD forms matter out of quarks and gluons.

There is a very recent report by LHCb [146] on the observation of a narrow peak in the $D^0D^0\pi^+$ invariant mass distribution just below the $D^{*+}D^0$ threshold. This structure would possess a minimal $cc\bar{u}\bar{d}$ quark content. An assessment of this state will be discussed in the next issue of this review.

79.3 Systems containing four heavy quarks

Recently LHCb reported the observation of pronounced structures in a double-$J/\psi(1S)$ invariant mass distribution [147] thus pointing at states with $cc\bar{c}\bar{c}$ quarks contents. From quark models the possible existence of bound states like this was discussed already long ago [148, 149]. There are now also many model calculations available in the literature. For discussions of those data from the compact tetraquark perspective see Refs. [150–156]. Most of these quark model calculations assign the reported structure to a $cc\bar{c}\bar{c}$ state in the $2S$ multiplet with near-degenerate $J^P = 0^+$, 1^+ and 2^+ configurations. The dominance of a compact tetraquark component in the apparently exotic structure is also supported by the coupled multichannel study of Ref. [157]. For direct analyses of the data within the tetraquark approach, see, e.g., Ref. [158]. Also QCD sum rule studies of the system were published, but do not give consistent results: For example Ref. [159] interprets the structure at 6900 MeV as a hybrid state, Ref. [160] as a tetraquark, and Ref. [161] states that both molecular and tetraquark interpretations are possible. An alternative view on the LHCb double-$J/\psi(1S)$ data is provided in Refs. [162–164] where the analyses are performed using coupled channel T-matrices. In both works the structures in the data emerge from the interplay of thresholds and resonances with the non-trivial prediction that there should exist, if this dynamical picture were correct, a state located very near the double-$J/\psi(1S)$ threshold which can be searched for experimentally.

In contrast to the works above, Ref. [165, 166] explains the double-$J/\psi(1S)$ data as cusps without nearby poles. This claim can be tested experimentally: if this explanation were correct, there should not be any narrow near threshold structure in the channel that generates the cusp, for that would call for a non-perturbative interaction in that channel as pointed out in Ref. [167].

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