65. Rare Kaon Decays

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65.1 Introduction

There are several useful reviews on rare kaon decays and related topics [1–13]. Activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model (SM)
2. The golden modes: $K \to \pi\nu\bar{\nu}$
3. Other constraints on SM parameters
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes the two modes that can be calculated with negligible theoretical uncertainty, $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. These modes can lead to precision determinations of CKM parameters or, in combination with other measurements of these parameters, they can constrain new interactions. They constitute the main focus of the current experimental kaon program. The search for new light particles through the reaction $K \to \pi X^0$ is a byproduct of these measurements. Category 3 is focused on decays with charged leptons, such as $K_L \to \pi^0\ell^+\ell^-$ or $K_L \to \ell^+\ell^-$ where $\ell \equiv e, \mu$. These modes are sensitive to CKM parameters, but they suffer from multiple hadronic uncertainties that can be addressed, at least in part, through a systematic study of the peripheral modes indicated in Fig. 65.1.

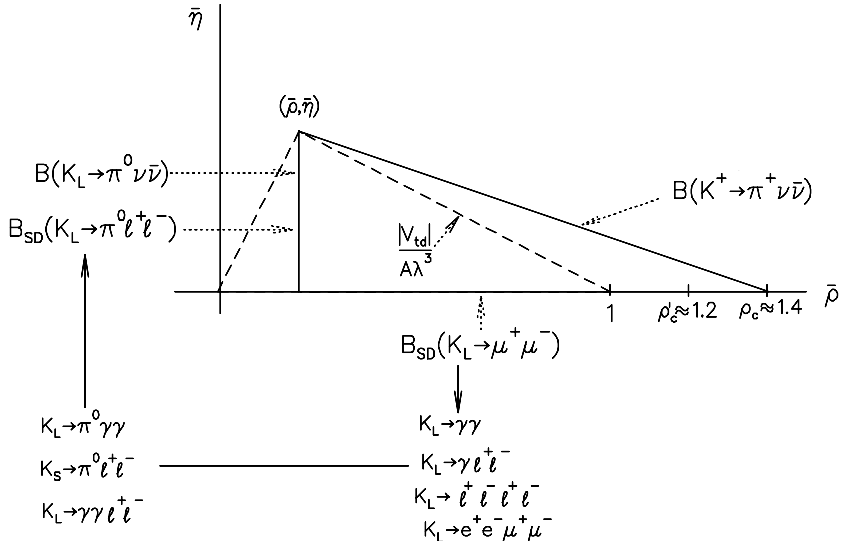

Figure 65.1: Role of rare kaon decays in determining the unitarity triangle. The solid arrows point to auxiliary modes needed to interpret the main results, or potential backgrounds to them.

The interplay between Categories 3-4 and their complementarity to Category 2 is illustrated in the figure. Category 4 includes reactions like $K^+ \to \pi^+\ell^+\ell^-$ where long distance contributions are dominant and which constitute a testing ground for the ideas of chiral perturbation theory.

Other decays in this category are $K_L \to \pi^0\gamma\gamma$ and $K_L \to \ell^+\ell^-\gamma$. The former is important in understanding a CP-conserving contribution to $K_L \to \pi^0\ell^+\ell^-$, whereas the latter could shed light on long distance contributions to $K_L \to \mu^+\mu^-$.

65.2 Explicit violations of the Standard Model

Much activity has focussed on searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 4.7 \times 10^{-12}(148\ \mathrm{TeV}/M_X)^4$ [2]. This simple dimensional analysis may be used to read from Table 65.1 that the reaction $K_L \to \mu e$ is already probing scales of over 100 TeV. Table 65.1 summarizes the present experimental situation vis-à-vis LFV. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions, since the former is sensitive to parity-odd couplings and the latter is sensitive to parity-even couplings.

Limits on certain lepton-number violating (LNV) kaon decays also have been obtained, with special interest arising from their role in constraining possible extensions of the neutrino sector [14, 15], and we list those in the table as well. Related searches in μ and τ processes are discussed in our section "Tests of Conservation Laws."

Table 65.1: Searches for lepton flavor and lepton number violation in K decay

LFV mode	90% CL upper limit	Experiment	Yr./Ref.	Type
$K^+ \to \pi^+ e^- \mu^+$	1.3×10^{-11}	BNL-865	2005/ [16]	LFV
$K^+ \to \pi^+ e^+ \mu^-$	6.6×10^{-11}	NA62	2021/ [17]	LFV
$K_L \to \mu e$	4.7×10^{-12}	BNL-871	1998/ [18]	LFV
$K_L \to \pi^0 e\mu$	7.6×10^{-11}	KTeV	2008/ [19]	LFV
$K_L \to \pi^0\pi^0 e\mu$	1.7×10^{-10}	KTeV	2008/ [19]	LFV
$K^+ \to \pi^- e^+ e^+$	5.3×10^{-11}	NA62	2022/ [20]	LNV
$K^+ \to \pi^-\pi^0 e^+ e^+$	8.5×10^{-10}	NA62	2022/ [20]	LNV
$K^+ \to \pi^- \mu^+ \mu^+$	4.2×10^{-11}	NA62	2019/ [21]	LNV
$K_L \to e^\pm e^\pm \mu^\mp \mu^\mp$	4.12×10^{-11}	KTeV	2003/ [22]	LNV
$K^+ \to \pi^- \mu^+ e^+$	4.2×10^{-11}	NA62	2021/ [17]	LNFV

65.3 The golden modes: $K \to \pi\nu\bar{\nu}$

In the SM, the decay $K^+ \to \pi^+\nu\overline{\nu}$ is dominated by one-loop diagrams with top-quark intermediate states while long-distance contributions are known to be quite small [23–25]. This permits a precise calculation of this rate in terms of SM parameters. Studies of this process are thus motivated by the possibility of detecting non-SM physics when comparing with the results of global fits [26, 27].

The branching ratio can be written in a compact form that exhibits the different ingredients that go into the calculation [28],

$$\mathrm{B}\left(K^+ \to \pi^+\nu\overline{\nu}(\gamma)\right) = \kappa_+(1+\varDelta_{\mathrm{EM}})\left[\left(\frac{\mathrm{Im}(V_{ts}^\star V_{td})}{\lambda^5}X_t\right)^2 + \left(\frac{\mathrm{Re}(V_{cs}^\star V_{cd})}{\lambda}P_c + \frac{\mathrm{Re}(V_{ts}^\star V_{td})}{\lambda^5}X_t\right)^2\right]. \tag{65.1}$$

The parameters in Eq. 65.1 incorporate the *a priori* unknown hadronic matrix element in terms of the very well-measured K_{e3} rate [23] in κ_+; long distance QED corrections in $\varDelta_{\mathrm{EM}}$ [29]; the Inami-Lim function for the short distance top-quark contribution [30] including NLO QCD corrections [31, 32] and the two-loop electroweak correction [28], all in X_t; and the charm-quark contributions due to short distance effects including NNLO QCD corrections [33,34] and NLO electroweak corrections via P_c [35], which also includes certain long distance dubbed $\delta P_{c,u}$ [25,36]. An interesting approximate way to cast this result in terms of the CKM parameters λ, V_{cb}, $\overline{\rho}$ and $\overline{\eta}$ (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [9] is:

$$\mathrm{B}(K^+ \to \pi^+\nu\overline{\nu}) \approx 1.6\times 10^{-5}|V_{cb}|^4[(\sigma\overline{\eta})^2 + (\rho_c - \overline{\rho})^2], \tag{65.2}$$

where $\rho_c \approx 1.45$ and $\sigma \equiv \frac{1}{(1-\frac{1}{2}\lambda^2)^2}$. Thus, $\mathrm{B}(K^+ \to \pi^+\nu\overline{\nu})$ determines an ellipse in the $\overline{\rho}$, $\overline{\eta}$ plane with center $(\rho_c, 0)$ and semiaxes $\approx \frac{1}{|V_{cb}|^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.6\times10^{-5}}}$ and $\frac{1}{\sigma|V_{cb}|^2}\sqrt{\frac{\mathrm{B}(K^+\to\pi^+\nu\overline{\nu})}{1.6\times10^{-5}}}$.

BNL-787 observed two candidate events [37, 38] in the clean high π^+ momentum and one event [39] in the low-momentum region. The successor experiment BNL-949 observed one more in the high-momentum region [40] and three more in the low-momentum region [41], yielding a branching ratio of $(1.73^{+1.15}_{-1.05})\times 10^{-10}$ [42].

The NA62 experiment, performed with in-flight decays at CERN, aims to reach a sensitivity of $\sim 10^{-12}$ per event. NA62 was commissioned in 2015 and has taken data in 2016, 2017 and 2018. They recently published results from the 2016-18 runs [43]. Twenty candidate events were observed, including an estimated background of 7.03, yielding $\mathrm{B(K^+} \to \pi^+\nu\overline{\nu}) = (10.6^{+4.0}_{-3.4}|_{\mathrm{stat}} \pm 0.9_{\mathrm{sys}})\times 10^{-11}$. NA62 will continue taking data in 2021 with improvements to the detector.

Using the latest CKMfitter input (Dec. 2019) [26], we estimate $\mathrm{B}(K^+ \to \pi^+\nu\overline{\nu})_{SM} = (8.1 \pm 0.4)\times 10^{-11}$, near the lower end of the measurements of BNL-787/949 and NA62. Current parametric uncertainty in the CKM angles can result in numbers with central values differing from this one by up to 10% [44].

The second golden mode is the neutral counterpart to our preceeding discussion: $K_L \to \pi^0\nu\overline{\nu}$. It is dominantly CP-violating and free of hadronic uncertainties [23, 45, 46]. In the Standard Model, this mode is dominated by an intermediate top-quark state and does not suffer from the small uncertainty associated with the charm-quark intermediate state that affects $K^+ \to \pi^+\nu\overline{\nu}$. The branching ratio is given by Ref. [9]:

$$\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) = \ \kappa_L\left(\frac{\mathrm{Im}(V_{ts}^\star V_{td})}{\lambda^5}X_t\right)^2 \approx 7.6\times 10^{-5}|V_{cb}|^4\overline{\eta}^2\ . \tag{65.3}$$

As with the charged mode, the hadronic matrix element can be related to that measured in $K_{\ell 3}$ decay and is parameterized in κ_L.

Our estimate for the branching ratio within the SM, using the latest CKMfitter input (Dec. 2019) [26], is $(2.8 \pm 0.2)\times 10^{-11}$. Similarly to the charged kaon case, parametric uncertainty in the CKM angles can result in a central value that differs from this one by up to almost 20% [44].

Grossman and Nir (GN) [47] pointed out that, in a nearly model-independent manner, the two golden modes satisfy the relation $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \lesssim 4.3\ \mathrm{B}(K^+ \to \pi^+\nu\overline{\nu})$. Using the BNL 787/949

90% CL bound on $K^+ \to \pi^+\nu\overline{\nu}$, GN then predict $B(K_L \to \pi^0\nu\overline{\nu}) < 1.46 \times 10^{-9}$. Using instead the latest NA62 result, the GN upper bound becomes $B(K_L \to \pi^0\nu\overline{\nu}) < 8.14 \times 10^{-10}$ [13].

The KOTO experiment at J-PARC, whose initial goal is to observe this decay, has been running since 2013 and in 2018 published a 90% CL upper limit of 3.0×10^{-9} [48], based on their 2015 data. They have run every year since, making incremental improvements to the experimental configuration between runs. In 2021 they published a result on the data taken between 2016 and 2018 in which three events were observed in the signal region [49]. Studies revealed two backgrounds previously unanticipated at the current level of sensitvity (7.2×10^{-10}/event), one from halo $K_L \to \gamma\gamma$ and a larger one from charged kaons produced by K_L interactions in the downstream collimator, followed by $K^\pm \to \pi^0 e^\pm \nu_e$ where the $e^\pm$ goes backwards and is not observed. These constituted more than 90 % of the expected background of 1.22 events. Conservatively assuming the three events were due to background, they extracted a 90% CL upper limit of 4.9×10^{-9}. Subsequent hardware and analysis improvements are expected to greatly reduce these backgrounds in future data collection. An increase of 3-5 in sensitivity is anticipated from runs planned in the next three years.

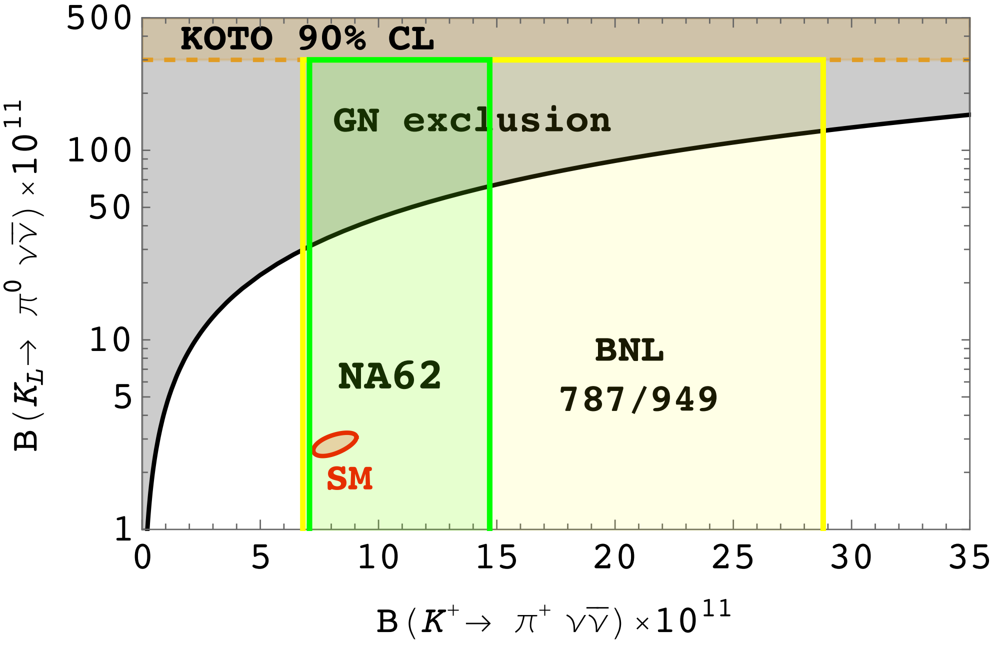

Figure 65.2: Summary of current situation for the golden modes $K \to \pi\nu\bar{\nu}$. The red ellipse shows the 1σ SM prediction with input from CKMfitter; the green (yellow) region corresponds to the NA62 (BNL787/949) 1σ measurement; and the dashed orange line marks the 90% CL KOTO upper bound. The black shaded region shows the GN exclusion.

The current theoretical and experimental situation for the golden modes is summarized in Fig. 65.2. The red area corresponds to the 1σ SM prediction we obtain with the latest input available from CKMfitter (December 2019) [26]. The yellow region shows the result established by the combined BNL-787 and BNL-949 results, whereas the green region marks the new NA62 result for $K^+ \to \pi^+\nu\bar{\nu}$. The black shaded region marks the GN exclusion, which lies significantly above the SM expectation leaving a large window for discovery of new physics contributions by experiments seeking to measure $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu})$. The 90% CL upper bound on this mode from the KOTO result published in 2018 [48] is shown as a dashed orange line, and is seen to still lie in the

GN excluded zone.

Much theoretical work has explored beyond the SM scenarios that can populate this window as well as their correlations with other rare processes outside kaon physics. Although it would be relatively straightforward to establish the existence of new physics by observing deviations from their SM values in the $K \to \pi\nu\bar{\nu}$ modes, it would take much more extensive global fits to pinpoint the origin of any such deviation. Partial summaries with references focusing on quark flavor physics can be found in Refs. [9,50–54]. Different possibilities emphasizing an interpretation of these modes as $K \to \pi + E_{\rm miss}$ can be found in Refs. [55–57].

There is a subtlety in converting the GN bound extracted from measurements of $K^+ \to \pi^+\nu\bar{\nu}$ into an upper bound for $K_L \to \pi^0\nu\bar{\nu}$ as applied to KOTO [58]. This is due to a lack of sensitivity of the charged kaon experiments to a kinematic window near the pion mass. New physics appearing through a two body mode $K^+ \to \pi^+X^0$, for an invisible X^0 with mass near the pion mass, has a much weaker constraint. The situation is illustrated in Fig. 65.3 which shows the BNL787/949 [42] and NA62 [59] 90% c.l. limits on $K^+ \to \pi^+X^0$ as a function of m_{X^0} assuming X^0 is stable. The implied GN bound for $K_L \to \pi^0X^0$, $B(K_L \to \pi^0X^0) \lesssim 4.3B(K^+ \to \pi^+X^0)$ is shown as a blue line and can be compared with the actual KOTO [48] constraint on $K_L \to \pi^0X^0$ in red. Notice that the upper bound derived from NA62 is lower than that derived from BNL787/949 except for a small mass window. For $120 \lesssim m_{X^0} \lesssim 150$ MeV, KOTO already has better sensitivity than that implied by the GN bound in this case. The possibility of new physics in this scenario has generated much theoretical speculation [60–70].

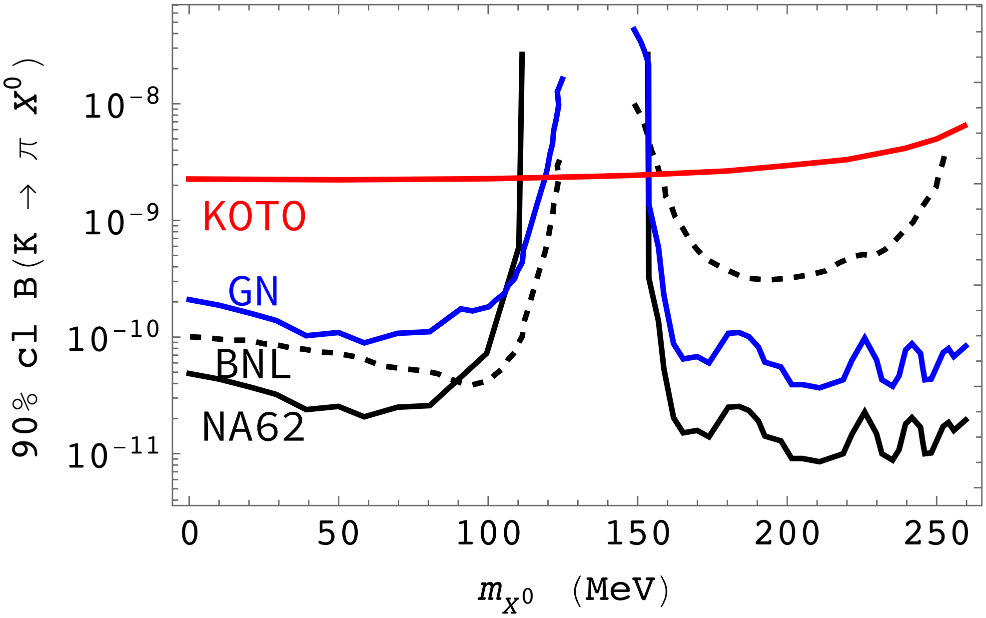

Figure 65.3: Summary of current situation for $K \to \pi X^0$. The solid (dashed) black line shows the exclusion limit for $K^+ \to \pi^+X^0$ from NA62 (BNL787/949) assuming X^0 is stable. The blue line marks the constraint implied for $K_L \to \pi^0X^0$ by the Grossman-Nir relation and the red line marks the current KOTO limit on $K^+ \to \pi^+X^0$.

The search for a new light particle X^0 in $K^+ \to \pi^+X^0$ has a long tradition covering both long-lived particles (*e.g.*, hyperphoton, axion, familon, *etc.*), and short-lived ones that decay to muon, electron, photon or neutrino pairs. The longstanding 90% CL upper limit on $K^+ \to \pi^+X^0$ from

BNL787/949 7.3×10^{-11} [40] for the case of massless X^0 has been slightly improved by NA62 to 5×10^{-11} as can be read from Fig. 65.3. These limits can also be reinterpreted in connection with a dark photon [71] or dark Z [72], and in this context NA48/2 also constrained the mode $B(K^\pm \to \pi^\pm A' \to \pi^\pm e^+ e^-)$ [73]. Complementary searches for new light particles in kaon experiments use modes with two pions and include the KTeV bound $B(K_L \to \pi^0\pi^0 X^0 \to \pi^0\pi^0\mu^+\mu^-) < 1\times 10^{-10}$ [74] and the E391a bound $B(K_L \to \pi^0\pi^0 X^0 \to \pi^0\pi^0\gamma\gamma) < 2.4 \times 10^{-7}$ [75].

Neutrino pair modes with one extra pion, $K \to \pi\pi\nu\bar\nu$, are similarly dominated by short distance contributions in the SM [76–78]. Even though they are theoretically clean, they occur with very low rates with branching ratios of order 10^{-13}. The current best bound comes from KEK-391a, $B(K_L \to \pi^0\pi^0\nu\bar\nu) < 8.1 \times 10^{-7}$ at 90% CL [79]. There is also a bound for the charged kaon mode $B(K^+ \to \pi^+\pi^0\nu\bar\nu) < 4.3 \times 10^{-5}$ at 90% CL [80] from BNL-787. New physics contributions to these modes are discussed in [81].

65.4 Other constraints on Standard Model parameters

The decay $K_L \to \mu^+\mu^-$ has a short distance contribution sensitive to the CKM parameter $\bar\rho$, given by [5]:

$$\mathrm{B_{SD}}(K_L \to \mu^+\mu^-) \approx 2.7 \times 10^{-4} |V_{cb}|^4 (\rho_c' - \bar\rho)^2 \tag{65.4}$$

where ρ_c' depends on the charm quark mass and is approximately 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is determined by the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B_{abs}}(K_L \to \mu^+\mu^-) = (6.64 \pm 0.07) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$ [82]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. The latter cannot be derived directly from experiment [83], but can be estimated with certain assumptions [84, 85].

By contrast, the decay $K_L \to e^+e^-$ is completely dominated by long distance physics and is easier to estimate. The result, $B(K_L \to e^+e^-) \sim 9 \times 10^{-12}$ [83, 86], is in good agreement with the BNL-871 measurement, $(8.7^{+5.7}_{-4.1}) \times 10^{-12}$ [87].

The mode $K_S \to \mu^+\mu^-$ has a short distance contribution proportional to the square of the CKM parameter $\bar\eta$ entering at the 10^{-13} level [10]. It also has long distance contributions arising from the two photon intermediate state which result in a rate $B(K_S \to \mu^+\mu^-)_{LD} = 5.1 \times 10^{-12}$ [10]. There is a 90% CL limit $B(K_S \to \mu^+\mu^-) < 2.1 \times 10^{-10}$ from LHCb [88, 89]. The interplay between $K_L \to \mu^+\mu^-$ and $K_S \to \mu^+\mu^-$ has been the subject of [90, 91], and it has been pointed out that a measurement of time-dependent interference effects could be used to extract information on CKM angles from $K \to \mu^+\mu^-$ measurements [92].

The decay $K_L \to \pi^0 e^+ e^-$ is sensitive to the CKM parameter η through its CP-violating component. There are both direct and indirect CP-violating amplitudes that can interfere. The direct CP-violating amplitude is short distance dominated and has been calculated in detail within the SM [5]. The indirect CP-violating amplitude can be inferred from a measurement of $K_S \to \pi^0 e^+ e^-$. The complete CP-violating contribution to the rate can be written as [93–95]:

$$\mathrm{B_{CPV}} \approx \left[15.7|a_S|^2 \pm 1.4 \left(\frac{|V_{cb}|^2\bar\eta}{10^{-4}}\right)|a_S| + 0.12\left(\frac{|V_{cb}|^2\bar\eta}{10^{-4}}\right)^2\right] \times 10^{-12} \tag{65.5}$$

where the three terms correspond to the indirect CP violation, the interference, and the direct CP violation, respectively. The parameter a_S has been extracted by NA48/1 from a measurement of $K_S \to \pi^0 e^+ e^-$ with the result $|a_S| = 1.06^{+0.26}_{-0.21} \pm 0.07$ [96], as well as from a measurement of $K_S \to \pi^0\mu^+\mu^-$ with the result $|a_S| = 1.54^{+0.40}_{-0.32} \pm 0.06$ [97]. With current constraints on the CKM parameters, and assuming a positive sign for the interference term [95, 98], this implies that $\mathrm{B}_{\mathrm{CPV}}(K_L \to \pi^0e^+e^-) \approx (3.1 \pm 0.9) \times 10^{-11}$, where the three contributions to the central value from indirect, interference and direct CP violation are $(1.76, 0.9, 0.45) \times 10^{-11}$ respectively. It should be noted that more recent studies suggest a much larger uncertainty in the value of a_S [99].

$K_L \to \pi^0e^+e^-$ also has a CP-conserving component dominated by a two-photon intermediate state. This component can be decomposed into an absorptive and a dispersive part. The absorptive part can be extracted from the measurement of the low $m_{\gamma\gamma}$ region of the $K_L \to \pi^0\gamma\gamma$ spectrum. The rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$ in $K_L \to \pi^0\gamma\gamma$ are well described in chiral perturbation theory in terms of three (*a priori*) unknown parameters [100–103].

Both KTeV and NA48 have studied the mode $K_L \to \pi^0\gamma\gamma$, reporting similar results. KTeV finds $B(K_L \to \pi^0\gamma\gamma) = (1.29 \pm 0.03_{\mathrm{stat}} \pm 0.05_{\mathrm{sys}}) \times 10^{-6}$ [104], while NA48 finds $B(K_L \to \pi^0\gamma\gamma) = (1.36 \pm 0.03_{\mathrm{stat}} \pm 0.03_{\mathrm{sys}} \pm 0.03_{\mathrm{norm}}) \times 10^{-6}$ [105]. Both experiments are consistent with a negligible rate in the low $m_{\gamma\gamma}$ region, suggesting a very small CP-conserving component $B_{\mathrm{CP}}(K_L \to \pi^0e^+e^-) \sim \mathcal{O}(10^{-13})$ [95, 103, 105]. There remains some model dependence in the estimate of the dispersive part of the CP-conserving $K_L \to \pi^0e^+e^-$ [95].

The related process, $K_L \to \pi^0\gamma e^+e^-$, is potentially an additional background to $K_L \to \pi^0e^+e^-$ in some region of phase space [106]. This process has been observed with a branching ratio of $(1.62 \pm 0.14_{\mathrm{stat}} \pm 0.09_{\mathrm{sys}}) \times 10^{-8}$ [107].

The decay $K_L \to \gamma\gamma e^+e^-$ constitutes the dominant background to $K_L \to \pi^0e^+e^-$. It was first observed by BNL-845 [108], and subsequently confirmed with a much larger sample by KTeV [109]. It has been estimated that this background will enter at about the 10^{-10} level [110, 111], comparable to or larger than the signal level. Because of this, the observation of $K_L \to \pi^0e^+e^-$ at the SM level will depend on background subtraction with good statistics. Possible alternative strategies are discussed in Ref. [95] and references cited therein.

The 90% CL upper bound for the process $K_L \to \pi^0e^+e^-$ is 2.8×10^{-10} [111]. For the closely related muonic process, the published upper bound is $\mathrm{B}(K_L \to \pi^0\mu^+\mu^-) \leq 3.8 \times 10^{-10}$ [112], compared with the SM prediction of $(1.9 \pm 0.5) \times 10^{-11}$. The latter assumes positive interference between the direct- and indirect-CP violating components and includes a CP-conserving component which contributes about 30% of the total [10, 113].

A study of $K_L \to \pi^0\mu^+\mu^-$ has indicated that it might be possible to extract the direct CP-violating contribution by a joint study of the Dalitz plot variables and the components of the μ^+ polarization [114]. The latter tends to be quite substantial so that large statistics may not be necessary.

Combined information from $K_L \to \pi^0\ell^+\ell^-$ as well as $K_L \to \mu^+\mu^-$ complements the $K \to \pi\nu\overline{\nu}$ measurements in constraining physics beyond the SM [115].

65.5 Other long distance dominated modes

The decays $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e$ or μ) have received considerable attention. The rate and spectrum have been measured for both the electron and muon modes [116–121]. A review of the theoretical status of these modes can be found in [99, 122].

Ref. [93, 94] has proposed a parameterization inspired by chiral perturbation theory, which provides a successful description of data but indicates the presence of large corrections beyond leading order. More work is needed to fully understand the origin of these large corrections. The mode $K^+ \to \pi^+\pi^0e^+e^-$, analyzed by NA48/2 [123], is also dominated by long distance physics but it has been argued that measuring asymmetries can provide information on the short distance components [124]. The current status of these modes is discussed in [125].

The decay $K^+ \to \pi^+\gamma\gamma$ can be predicted in terms of one unknown parameter to leading order in χPT resulting in a correlation between the rate and the diphoton mass spectrum [126]. Certain important corrections at the next order are also known [127]. The rate was first measured by E787 [128], and NA48/2 [129] has obtained a more precise result with a 6% error, as well as the corresponding spectrum fits. The most recent, and precise, result is from NA62 based on a sample of 232 events [130] but is still insufficient to distinguish between the leading order and next order χPT parameterizations. The NA48 and NA62 results have been combined in [131].

Much information has been recorded by KTeV and NA48 on the rates and spectrum for the Dalitz pair conversion modes $K_L \to \ell^+\ell^-\gamma$ [132,133], and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ [22,134]. All these results are used to test hadronic models and should eventually help unravel the underlying physics in $K_L \to \mu^+\mu^-$ [85,90,135].

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