

## Width Determinations of the $\Upsilon$ States

As is the case for the  $J/\psi(1S)$  and  $\psi(2S)$ , the full widths of the  $b\bar{b}$  states  $\Upsilon(1S)$ ,  $\Upsilon(2S)$ , and  $\Upsilon(3S)$  are not directly measurable, since they are much narrower than the energy resolution of the  $e^+e^-$  storage rings where these states are produced. The common indirect method to determine  $\Gamma$  starts from

$$\Gamma = \Gamma_{\ell\ell}/B_{\ell\ell} , \quad (1)$$

where  $\Gamma_{\ell\ell}$  is one leptonic partial width and  $B_{\ell\ell}$  is the corresponding branching fraction ( $\ell = e, \mu, \text{ or } \tau$ ). One then assumes  $e$ - $\mu$ - $\tau$  universality and uses

$$\begin{aligned} \Gamma_{\ell\ell} &= \Gamma_{ee} \\ B_{\ell\ell} &= \text{average of } B_{ee}, B_{\mu\mu}, \text{ and } B_{\tau\tau} . \end{aligned} \quad (2)$$

The electronic partial width  $\Gamma_{ee}$  is also not directly measurable at  $e^+e^-$  storage rings, only in the combination  $\Gamma_{ee}\Gamma_{\text{had}}/\Gamma$ , where  $\Gamma_{\text{had}}$  is the hadronic partial width and

$$\Gamma_{\text{had}} + 3\Gamma_{ee} = \Gamma . \quad (3)$$

This combination is obtained experimentally from the energy-integrated hadronic cross section

$$\begin{aligned} &\int_{\text{resonance}} \sigma(e^+e^- \rightarrow \Upsilon \rightarrow \text{hadrons})dE \\ &= \frac{6\pi^2}{M^2} \frac{\Gamma_{ee}\Gamma_{\text{had}}}{\Gamma} C_r = \frac{6\pi^2}{M^2} \frac{\Gamma_{ee}^{(0)}\Gamma_{\text{had}}}{\Gamma} C_r^{(0)} , \end{aligned} \quad (4)$$

where  $M$  is the  $\Upsilon$  mass, and  $C_r$  and  $C_r^{(0)}$  are radiative correction factors.  $C_r$  is used for obtaining  $\Gamma_{ee}$  as defined in Eq. (1), and contains corrections from all orders of QED for describing  $(b\bar{b}) \rightarrow e^+e^-$ . The lowest order QED value  $\Gamma_{ee}^{(0)}$ , relevant for comparison with potential-model calculations, is defined by the lowest order QED graph (Born term) alone, and is about 7% lower than  $\Gamma_{ee}$ .

The Listings give experimental results on  $B_{ee}$ ,  $B_{\mu\mu}$ ,  $B_{\tau\tau}$ , and  $\Gamma_{ee}\Gamma_{\text{had}}/\Gamma$ . The entries of the last quantity have been re-evaluated consistently using the correction procedure of KURAEV 85 [1]. The partial width  $\Gamma_{ee}$  is obtained from the average values for  $\Gamma_{ee}\Gamma_{\text{had}}/\Gamma$  and  $B_{\ell\ell}$  using

$$\Gamma_{ee} = \frac{\Gamma_{ee}\Gamma_{\text{had}}}{\Gamma(1 - 3B_{\ell\ell})} . \quad (5)$$

The total width  $\Gamma$  is then obtained from Eq. (1). We do not list  $\Gamma_{ee}$  and  $\Gamma$  values of individual experiments. The  $\Gamma_{ee}$  values in the Meson Summary Table are also those defined in Eq. (1).

### References:

1. E.A. Kuraev, V.S. Fadin, Sov. J. Nucl. Phys. **41**, 466 (1985).