63. Spectroscopy of Light Meson Resonances

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63.1 Introduction

According to the constituent quark model, a light meson consists of a color singlet quark-antiquark pair made of the u, d, or s quarks and the $\overline{u}$, $\overline{d}$, or $\overline{s}$ antiquarks, grouped into a flavor multiplet of SU(3). However, additional mesons made of bound gluons (glueballs) could exist in the same mass range, suggested by the self coupling of gluons in QCD. Multiquark structures are also possible, such as quark-antiquark pairs with an excited gluon (hybrid mesons). Tetraquarks are compact color singlets of diquark-antidiquark pairs ($qq\overline{q}\overline{q}$) or 'molecular' bound states of two mesons ($q\overline{q}q\overline{q}$). More complex systems such as $qqq\overline{q}\overline{q}\overline{q}$ (baryonia) are also predicted.

Fundamentals on the constituent quark model on light and heavy mesons and baryons (including hadrons with charm and bottom quarks), and on predictions from lattice gauge theories, are described in 'Quark Model' in this issue of the *Review of Particle Physics*, henceforth called the *Review*. In the present text we describe the experimental spectrum of light mesons and their classifications within the constituent quark model, with emphasis on states exhibiting properties incompatible with $q\bar{q}$ structures. The discussion is driven by the publications quoted in the *Listings* of the *Review*. The spectrum of kaon excitations is less well-defined and therefore deferred to a future edition, when further data might become available. More detailed discussions on exotic mesons – including those involving the c and b quarks – can be found in Refs. [1–8] and in 'Heavy Non-$q\bar{q}$ Mesons' of the *Review*. For more information on the meson (and baryon) spectrum of light (and heavy) quarks we refer to one of the textbooks [9].

Figure 63.1 shows the mass spectrum of $q\bar{q}$ mesons. The mass (vertical axis) increases with orbital excitation ℓ and radial excitation n. In the quark model one uses the notation nS, nP, nD... for the radial excitations ($n = 1$ for the ground states), in contrast to the usual $1s$, $2p$, $3d$... notation in atomic physics. Each box represents an SU(3) nonet containing multiplets of the isospin I, (*e.g.* for the pseudoscalar ground state nonet, one isotriplet (π^+, π^0, π^-), two strange

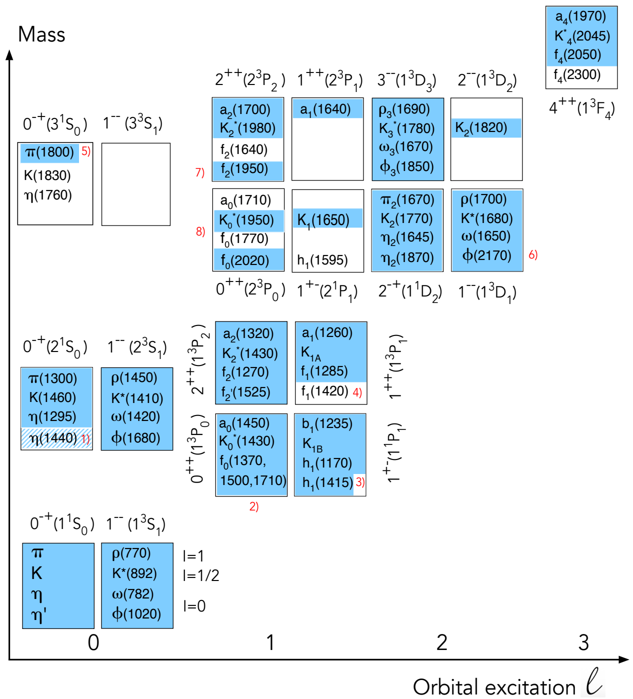

Figure 63.1: The mesons made of the u, d, and s light quarks are organised in $J^{PC}(n^{2s+1}\ell_j)$ nonets with isospin I. The established mesons (which appear in the *Summary Table* of the *Review*) are shown in the blue areas. The white areas contain those omitted in the *Summary Table* but reported in the *Listings*. States with the same J^{PC} mix, such as the 2^3S_1 and 1^3D_1 mesons. The states become broad and overlap with increasing masses, which complicates the determination of the masses, widths and spins. For a complete list of mesons see the *Listings*.
[1] The $\eta(1440)$ stands for the $\eta(1405)$ and the $\eta(1475)$ which may be manifestations of the same state, see section 63.4.
[2] The classification of the scalar nonet is controversial. In alternative schemes the 1^3P_0 nonet contains the light scalars below 1 GeV, section 63.2.
[3] Considered established, but more data would be desirable.
[4] An alternative to the $f_1(1420)$ is the $f_1(1510)$, section 63.6.
[5] The $\pi(1800)$ has also been proposed as a hybrid meson, section 63.7.
[6] The $\phi(2170)$ has also been proposed as a tetraquark state, section 63.8.
[7] The $f_2(1950)$ has also been proposed as a tensor glueball, section 63.3.2.
[8] Tentative assignment following Ref. [10].

isodoublets (K^-, $\bar{K}^0$ and K^0, K^+) and two isosinglets (η, η'). The spin, parity $[P = -(-1)^\ell]$ and C-parity $[C = (-1)^{\ell+s}]$ take the possible values $J^{PC} = 0^{++}$ (scalar), 0^{-+} (pseudoscalar), 1^{--} (vector), $1^{+\pm}$ (axial- or pseudovector), 2^{++} (tensor), etc. Therefore mesons with the *exotic* quantum numbers 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, etc. cannot be $q\bar{q}$ states. Note that the C-parity is defined only for the electrically neutral non-strange nonet members. The classification shown in fig. 63.1 is tentative for the radial excitations, in particular for those not fully established.

The two isosinglets in each nonet mix with an angle θ close to the *ideal* value of 35.3° for the 1^{--}, $1^{+\pm}$, 2^{++} and 3^{--} nonets, in which case the two isosinglets decouple to $u\bar{u} + d\bar{d}$ and $s\bar{s}$. The orbital excitations $\ell \geq 1$ consist of four nonets for each value of n, since $j = \ell$ for antiparallel quark spins and $j = \ell - 1, \ell$ or $\ell + 1$ for parallel spins. Since the C-parity is not defined for strange mesons, the K_{1A} and K_{1B} in the axial vector 1^{++} and 1^{+-} nonets of fig. 63.1 are mathematical constructs which mix to give the observed $K_1(1270)$ and $K_1(1400)$ mesons. A similar scenario occurs for the observed 2^{-+} $K_2(1770)$ and 2^{--} $K_2(1820)$ mesons, with a small mixing angle [11]. According to Ref. [12], the masses of radially excited mesons and baryons appear to grow logarithmically with quantum number n, a phenomenon that is not understood.

As described in more detail below (section 63.2), it is hard to accommodate all the known scalar mesons in the lower $q\bar{q}$ nonets: The light scalar mesons $a_0(980)$, $K_0^*(700)$ (also known as κ), the $f_0(500)$ (also known as σ) and the $f_0(980)$, not shown in the figure, could build the lightest nonet, but could also be two-meson resonances or tetraquarks (section 63.8). Three isoscalars are known in the 1500 MeV region, the $f_0(1500)$ or $f_0(1710)$ being proposed as states with a large gluon content, mixing with the $f_0(1370)$ (section 63.3.1). The pseudoscalar labeled $\eta(1440)$ may consist of two states (section 63.4). The status of vector meson excitations is described in section 63.5. The axial-vector meson $f_1(1420)$ could be replaced in the 1^{++} nonet by the $f_1(1510)$ which, however, needs confirmation (see the *Listings* and section 63.6). Mesons with exotic quantum numbers are discussed in section 63.7.

63.2 Scalar mesons

Scalar mesons decay dominantly into pairs of pseudoscalar mesons ($\pi\pi$, $K\overline{K}$, $\pi\eta$, $\eta\eta$ or $\eta\eta'$). The widths tend to be large for those decaying into $\pi\pi$, due to the absence of angular momentum barrier and the large available phase space. Broad overlapping states decaying into the same final state interfere, which complicates the determination of their masses and widths. Furthermore, the ground state glueball, a 0^{++} expected below 2 GeV, will mix with the isoscalar 0^{++} $q\bar{q}$ states. The onset of the $K\overline{K}$, $\eta\eta$ or $\eta\eta'$ thresholds also distorts the line shapes and produces cusps. This requires high statistics data from various production and decay channels, and the use of coupled channel analyses taking into account unitarity and analyticity (see 'Resonances' in the *Review*).

There is long-time evidence for the existence of two scalar nonets, one below 1 GeV, one around 1.4 GeV and one glueball below 1.7 GeV [13]. The lighter nonet consists of the $a_0(980)$, $f_0(980)$, $f_0(500)$ and $K_0^*(700)$, and the heavier one contains the mesons shown in Fig. 63.1. Two isovector scalars are established, the $a_0(980)$ and the $a_0(1450)$, together with five isoscalars: the very broad $f_0(500)$, the $f_0(980)$, the broad $f_0(1370)$, and the comparatively narrow $f_0(1500)$ and $f_0(1710)$. The strange partners are the $K_0^*(700)$ and the $K_0^*(1430)$. The $f_0(500)$ and $K_0^*(700)$ deserve a separate treatment and are discussed in 'Scalar Mesons below 1 GeV', which also contains more details on the $a_0(980)$ and $f_0(980)$ briefly reviewed in section 63.8, see also Ref. [1].

The $a_0(1450)$ was first reported by the Crystal Barrel experiment (see [14] for a review) in $\bar{p}p$ annihilation with stopped antiprotons [15, 16]. An isovector scalar, possibly the $a_0(1450)$ (albeit at a lower mass of 1317 MeV), is observed by Belle in $\gamma\gamma$ collisions leading to $\eta\pi^0$ [17]. A contribution from $a_0(1450) \to K\bar{K}$ is also found in the CLEO-c analysis of $D^\pm \to K^+K^-\pi^\pm$ [18] and $D^0 \to K_S^0 K^\pm\pi^\mp$ from LHCb [19]. The $a_0(1450)$ is reported at a lower mass in $\bar{p}p$ annihilation in flight, overlapping with the $a_2(1320)$ [20, 21].

The $f_0(1370)$ and $f_0(1500)$ were observed by Crystal Barrel [22–24] and by WA102 in central production with 450 GeV protons [25,26]. They decay mostly into 2π and 4π. The $f_0(1500)$ was also observed to decay into $\eta\eta$ [27–29] and $\eta\eta'$ [30]. All data agree that the 4π decay mode represents about half of the $f_0(1500)$ decay width, but is dominant for the $f_0(1370)$.

The determination of the $\pi\pi$ coupling of the very broad $f_0(1370)$ is complicated by its interference with the $f_0(500)$. The existence of the $f_0(1370)$ was questioned by COMPASS from $\pi^-p \to \pi^-\pi^-\pi^+p$ data [31] and by LHCb in $\overline{B}_s^0 \to J/\psi(1S)\pi^+\pi^-$ [32]. In $\overline{B}^0 \to J/\psi(1S)\pi^+\pi^-$ the $\pi\pi$ mass spectrum does not show any significant scalar component above $\sim$1.2 GeV [33] (an analysis that is challenged in [34]). The existence of the $f_0(1370)$ has also been disputed by a reassessment of various production modes [35] (with the $K_0^*(700)$ not needed in $K\pi$ scattering). This contrasts with its $\pi\pi$ decay analyzed in Ref. [36]. However, data from CLEO-c in $D^0 \to \pi^+\pi^-\pi^+\pi^-$ require a contribution from $f_0(500)f_0(1370) \to 4\pi$ [37]. A broad 2π signal is also observed by BaBar around 1400 MeV in the decay $B^\pm \to \pi^\pm\pi^\pm\pi^\mp$ [38] which is attributed to the $f_0(1370)$ (but could also be due to the $f_0(1500)$). The existence of the $f_0(1370)$ is confirmed by a recent dispersive analysis of $\pi\pi$ scattering data, which leads to a complex pole around 1245 MeV with a very large width of about 600 MeV [39].

While the $K\bar{K}$ decay branching ratio of the $f_0(1500)$ is small [25, 40], the $f_0(1710)$ decays dominantly into $K\overline{K}$. The $f_0(1710)$ is not observed in $p\overline{p}$ annihilation at rest [40] and only weakly produced in $p\bar{p}$ annihilation in flight [41], with a rate strongly suppressed compared to that of the $f_0(1500)$ (∼7%). (For comparison, the rate for the $(s\bar{s})$ $f_2'(1525)$ production is about 9% that of the $(u\bar{u}+d\bar{d})$ $f_2(1270)$, as expected from the OZI rule [41].) On the other hand the $f_0(1370)$ does not couple strongly to $K\bar{K}$ [25]. This suggests an $n\bar{n}$ structure $(u\bar{u}+d\bar{d})$ for the $f_0(1370)$ and $s\bar{s}$ for the $f_0(1710)$. The $f_0(1500)$ would also qualify as an $n\bar{n}$ state, although it is very narrow compared to the other states. Occam's razor principle therefore suggests that the $f_0(1370)$, $a_0(1450)$, and the $f_0(1710)$ form the 1^3P_0 $q\bar{q}$ flavor nonet, the $f_0(1370)$ and $a_0(1450)$ being the two (mostly) $n\bar{n}$ states, and the $f_0(1710)$ the (dominantly) $s\bar{s}$ one. The nature of the $f_0(1500)$ is discussed along this scheme in section 63.3.1.

Other schemes have been proposed, for example a tetraquark for the $f_0(1500)$ [42] or flavor octets for the $f_0(1500)$ and $f_0(1710)$ [3]. In Refs. [43,44] the $f_0(1370)$ and $f_0(1710)$ are interpreted as bound systems of two vector mesons, a view is challenged in Ref. [45]. In Ref. [46] the $a_0(1450)$, $f_0(1370)$, $f_0(1500)$ and $K_0^*(1430)$ are radial excitations of the scalar nonet below 1 GeV. In Ref. [47] the light tetraquark scalar nonet mixes with the $q\bar{q}$ 1^3P_0 nonet, while in Ref. [48] both nonets are made of mixing tetraquarks. In the unitarized quark model with coupled $q\bar{q}$ and meson-meson channels, the scalars below 1 GeV are manifestations of bare $q\bar{q}$ confinement states, mass shifted from the 1.4 GeV region and distorted by the strong $^3P_0$ coupling to S-wave two-meson decay channels [49,50]. Thus, in these models the light scalar nonet comprising the light and 1^3P_0 scalar nonets are manifestations of the same bare input states (see also [51]). Surprisingly for a state decaying strongly into 2π and 4π, the $f_0(1370)$ is assumed to be an $s\bar{s}$ state in Ref. [52]. The $f_0(1500)$ is then a radial excitation.

There is evidence for two further 0^{++} mesons in the 1700 − 1800 MeV range, the isovector $a_0(1710)$ and the $f_0(1770)$, possibly the isoscalar partner of the $f_0(1710)$. The $f_0(1770)$, claimed by BESII/III [53–55], and also observed in $\pi^-p \to n\omega\phi$ by VES [56], could be a radial excitation [57]. Two isoscalars at 1700 and 1765 MeV are also reported by a re-analysis of BESIII radiative $J/\psi(1S)$ decay data [58,59].

The evidence for the $a_0(1710)$ was inferred from the interference of the K^+K^- and $K_S^0K_S^0$ decays of the $f_0(1710)$ in $D_s^+ \to f_0(1710)\pi^+$, leading to a relative branching ratio an order of magnitude larger than expected from isospin symmetry [60, 61]. The $a_0(1710)$ is now confirmed by LHCb in B^+ decays to $K_SK^+K^\pm\pi^\mp$ from an analysis of the $\eta(1S,2S) \to (K_SK^\pm\pi)^\mp$ Dalitz plots [62]. A signal for $a_0(1710)^\pm \to K_S^0K^\pm$ is also reported by BESIII [63], albeit at 1817 MeV, in contrast to the state observed by BaBar in $\eta\pi^\pm$ at 1704 MeV [64].

From a study of the vector-vector interaction the authors of Ref. [65], predict an isovector pole at 1780 MeV, while the $f_0(1710)$ is a $K^*\bar{K}^*$ molecule, a view also supported by Refs. [66–69]. Thus the $f_0(1710)$ and $a_0(1710)$ might be $K^*\bar{K}^*$ molecules, similar to the $f_0(980)$ and $a_0(980)$ which have been interpreted as $K\bar{K}$ molecules [70].

Scalar resonances near the D_0 mass such as the $f_0(1710)$ and the $f_0(1770)$ are also suggested to produce CP violation enhancements in the charm system [71].

63.3 Glueballs

Lattice calculations predict that the lightest glueball is a 0^{++} state with a mass in the 1600-1800 MeV range. The first and second excited states have 2^{++} and 0^{-+} with masses around 2300 and 2500 MeV, respectively, with large uncertainties. For more information and references on lattice calculations see 'Quark Model' in the *Review*, in particular fig. 15.3 for a mass spectrum. Predictions were made in the quenched approximation, neglecting $q\bar{q}$ loops. However quenched predictions and full QCD calculations lead to small mass shifts (see fig. 15.15 in the *Review*). The most recent prediction [72] quotes the more precise masses 1653 ± 26, 2376 ± 32 and 2561 ± 40 MeV for the 0^{++}, 2^{++} and 0^{-+} respectively. For an experimental review see Ref. [8].

The lightest glueballs lie in the same mass region as ordinary radial and orbital $q\bar{q}$ excitations, in the mass range of the 0^{++}, 2^{++} and 0^{-+} multiplets. Therefore, mixing of glueballs with nearby $q\bar{q}$ states of the same quantum numbers leads to supernumerary isoscalar state in the $q\bar{q}$ nonets.

Among the signatures naively expected for glueballs are (i) isoscalar states that do not fit in $q\bar{q}$ nonets, (ii) enhanced production in gluon-rich channels such as central production and radiative $J/\psi(1S)$ decay, (iii) decay branching fractions incompatible with SU(3) predictions for $q\bar{q}$ states, and (iv) small $\gamma\gamma$ couplings [73] (see however [74, 75]). Mixing effects with isoscalar $q\bar{q}$ mesons [35, 74, 76–83] and decay form factors [84] can obscure these simple signatures.

According to SU(3) the decay branching ratios for $q\bar{q}$ mesons and pure glueballs are different, and therefore useful to determine the internal structures of mesons. For pure glueballs with flavor symmetric couplings the decay ratios are $\pi\pi : K\bar{K} : \eta\eta : \eta\eta' = 3 : 4 : 1 : 0$, apart from phase space factors. The partial widths for the decay of a scalar (or a tensor) meson into a pair of pseudoscalar mesons are given in fig. 63.2 (for a derivation see e.g. [9]). The decay of a $q\bar{q}$ meson into a pair of mesons involves the creation of a $q\bar{q}$ pair, and SU(3) symmetry assumes that the matrix elements for the creation of $s\bar{s}$, $u\bar{u}$, and $d\bar{d}$ pairs are equal. (The generalization to unequal $s\bar{s}$, $u\bar{u}$, and $d\bar{d}$ couplings is given in Ref. [76].) An excellent fit to the tensor meson decay widths is obtained with $\beta \simeq 0.5$ GeV/c, $\theta_V \simeq 26°$ and $\theta_P \simeq -17°$ [76].

Isospin	Decay channel	γ^2
0	$\pi\pi$	$3\cos^2\alpha$
	$K\bar{K}$	$(\cos\alpha - \sqrt{2}\sin\alpha)^2$
	$\eta\eta$	$(\cos\alpha\cos^2\phi - \sqrt{2}\sin\alpha\sin^2\phi)^2$
	$\eta\eta'$	$\frac{1}{2}\sin^2 2\phi\ (\cos\alpha + \sqrt{2}\sin\alpha)^2$
1	$\eta\pi$	$2\cos^2\phi$
	$\eta'\pi$	$2\sin^2\phi$
	$K\bar{K}$	1
$\frac{1}{2}$	$K\pi$	$\frac{3}{2}$
	$K\eta$	$(\sin\phi - \frac{\cos\phi}{\sqrt{2}})^2$
	$K\eta'$	$(\cos\phi + \frac{\sin\phi}{\sqrt{2}})^2$

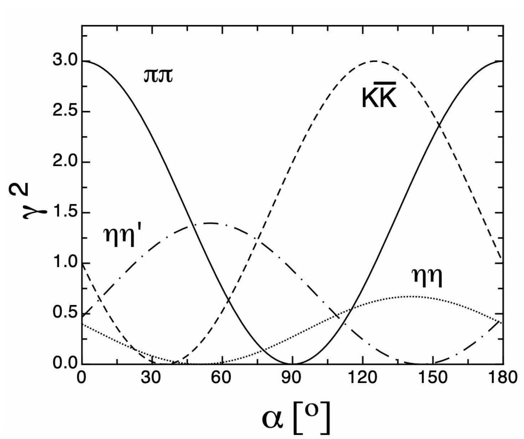

Figure 63.2: Left: SU(3) couplings γ^2 for scalar (or tensor) $q\bar{q}$ meson decays. The angles α and ϕ are defined as $\alpha = 54.7° + \theta$ and $\phi = 54.7° + \theta_P$, where θ is the mixing angle of the 0^{++} (2^{++}) isoscalars and θ_P the mixing angle in the 0^{-+} nonet. The partial decay width is given by $\Gamma = C \times \gamma^2 \times |F(q)|^2 \times q$, where C is a nonet constant, q the momentum of the decay products and $F(q)$ a form factor which may be taken as $|F(q)|^2 = q^{2\ell} \times \exp(-q^2/8\beta^2)$ with β= 0.5 GeV/c, and where ℓ is the relative angular momentum between the decay products ($\ell = 0$ for scalars and $\ell = 2$ for tensors). Right: SU(3) couplings γ^2 as a function of mixing angle α for isoscalar decays for $\theta_P = -17.3°$ (from [76]).

Note that the assumption of flavor symmetric couplings may not apply in models describing the scalar glueball by a dilaton field, which lead to mass dependent couplings [85,86] (Ref. [87] predicts that a scalar glueball above 1 GeV would be unobservably wide).

Another way to determine the flavor contents of neutral mesons is the decay $B^0 \to J/\psi(1S)X$ which filters out the $d\bar{d}$ content of X, while $B^0_s \to J/\psi(1S)X$ selects its $s\bar{s}$ component [88]. More precise data on $B^0_{(s)} \to J/\psi(1S)[f_0(1370), f_0(1500), f_0(1710)]$ would help to pin down the nature of these scalar mesons [89].

63.3.1 *Scalar glueballs*

One of the three isoscalars, the $f_0(1370)$, $f_0(1500)$ or $f_0(1710)$, appears to be supernumerary. The branching ratios in fig. 63.2 can be used to deduce their structures, assuming that they are $q\bar{q}$. The comparison of decay branching ratios from Crystal Barrel annihilation and WA102 central collision data shows that the $f_0(1500)$ is compatible with an $n\bar{n}$ structure, while the $f_0(1710)$ is mostly $s\bar{s}$ [90]. However, the close vicinity of the very broad $n\bar{n}$ $f_0(1370)$, its narrow width and enhanced production at low transverse momentum transfer in central collisions [91–93] favor the $f_0(1500)$ to be the supernumerary non-$q\bar{q}$ state. According to [76,82] the $f_0(1370)$ and $f_0(1710)$ would be dominantly $\bar{q}q$ states mixing with glue, while the $f_0(1500)$ would be dominantly a glueball mixing with $q\bar{q}$ states. Its suppressed $K\bar{K}$ decay would be due to interferences with the $f_0(1370)$ and $f_0(1710)$. In the analogous mixing scheme of Ref. [57], which uses central production data from WA102 and the hadronic $J/\psi(1S)$ decay data from BES [94,95], glue is shared between the $f_0(1370)$ (mainly $n\bar{n}$), $f_0(1500)$(mainly glue) and $f_0(1710)$ (dominantly $s\bar{s}$). This agrees with the analyses [76,82]. In the mixing scheme of Ref. [96] the QCD spectral sum rule also leads to a large gluonic component in the $f_0(1500)$ (satisfying the observed strong $\pi\pi$ and $f_0(500)f_0(500) \to 4\pi$ decays).

However, not everybody agrees that the strong $K\bar{K}$ signal is indicative of an $s\bar{s}$ structure for the $f_0(1710)$. The $f_0(1710)$ could still be the glueball, since the two-gluon coupling to $n\bar{n}$ appears to be suppressed by chiral symmetry [97], thus $K\bar{K}$ would be enhanced compared to $\pi\pi$. It was argued that chiral symmetry constraints in a multichannel analysis imply that the $f_0(1710)$ is an unmixed scalar glueball [98]. However, this view is challenged in [65].

The $K\bar{K}$ decay is also naturally enhanced in the extended linear sigma model with a dilaton as glueball [85] and in the holographic model of [86,99] which both prefer the $f_0(1710)$ as the glueball. For a scalar glueball Ref. [86] finds a strong enhancement of the decays into $K\bar{K}$ and $\eta\eta$, in fairly close agreement with the measured branching ratios of the $f_0(1710)$, while Ref. [99] predicts the (so far not measured) rate into $\eta\eta'$ to be very small.

In $\gamma\gamma$ collisions leading to K_SK_S [100] and K^+K^- [101] a spin-0 signal is observed at the $f_0(1710)$ mass (together with a dominant spin-2 component), while the $f_0(1500)$ is not observed in $\gamma\gamma \to K\bar{K}$ nor $\pi^+\pi^-$ [102]. The $f_0(1500)$ is also not observed by Belle in $\gamma\gamma \to \pi^0\pi^0$, although a shoulder is seen which could also be due to the $f_0(1370)$ [103]. The absence of $f_0(1500)$ signal in the $\pi\pi$ channel in $\gamma\gamma$ collisions does not favor an $\overline{n}n$ interpretation for the $f_0(1500)$. The upper limit for $\Gamma_{2\gamma}$ ($<$ 1.4 keV) in $\pi^+\pi^-$ [90]) excludes a large $n\overline{n}$ content, and hence points to a mainly $s\overline{s}$ content [90], which contradicts the small $K\overline{K}$ decay branching ratio of the $f_0(1500)$ [16,25,40]. Belle finds that in $\gamma\gamma \to K_SK_S$ collisions the 1500 MeV region is dominated by the $f_2'(1525)$. The $f_0(1710)$ is also observed with a production rate $\times$ branching ratio compatible with an $s\bar{s}$ state [104]. Ref. [74] predicts the glueball $\gamma\gamma$ partial width in the few keV range (comparable to that of $q\bar{q}$ mesons) due to couplings of vector mesons to γ via VDM, in agreement with holographic models [75], but this is disputed in Ref. [73] which predicts $\Gamma_{\gamma\gamma} \sim 1$ eV from lattice calculations of radiative glueball decay. Note, however, that the $\gamma\gamma$ couplings of glueballs are sensitive to glue mixing with $q\bar{q}$ [57].

Alternative assignments for the scalar glueball have been proposed:

- In Ref. [78] the gluonic signal is distributed over $f_0(1370)$, $f_0(1500)$ and another broad isoscalar around 1530 MeV, while the gluonic contribution to the $f_0(1710)$ is small.
- In Ref. [81, 85, 86] the $f_0(1710)$ is the glueball state, as in Ref. [83], where the $f_0(1500)$ is the $q\bar{q}$ octet state degenerate with the $a_0(1450)$.
- In the generalized linear sigma model [105] the $a_0(980)$ is dominantly a tetraquark and $a_0(1450)$ a $q\bar{q}$ state. The $f_0(980)$, $f_0(1370)$ and $f_0(1500)$ (or $f_0(1710)$) are dominantly tetraquark, $q\bar{q}$ and glue.
- In Ref. [80] the $f_0(500)$ and $f_0(1370)$ are signals from a single broad resonance proposed as the scalar glueball. The ground state scalar nonet then consists of the $f_0(980)$, $a_0(980)$, $K_0^*(1430)$ and $f_0(1500)$ [35]. The $f_0(980)$ and $f_0(1500)$ mix (similarly to the η and η' in the pseudoscalar nonet), while the $f_0(1500)$ mixes with a glueball in the 500 – 1000 MeV mass range, which is identified as the $f_0(500)$.
- In Ref. [106], a large K^+K^- scalar signal is reported by Belle in B decays into $KK\bar{K}$ [107], compatible with the $f_0(1500)$. It is due to constructive interference of a flavor octet with a broad glueball background. However, the Belle data are inconsistent with the BaBar measurements which show instead a broad scalar at this mass for B decays into both $K^\pm K^\pm K^\mp$ [108] and $K^+K^-\pi^0$ [109].

The $f_0(1500)$ is observed by BESII in $J/\psi(1S) \to \gamma\pi\pi$ [110] and by BESIII in $J/\psi(1S) \to \gamma\eta\eta$ [111] with a much smaller rate than for the $f_0(1710)$, which would speak against a glueball interpretation of the former, although the systematic uncertainties are large, and the $f_0(1500)$ appears at a lower mass and the $f_0(1710)$ at a higher mass than the accepted values.

The authors of Ref. [58] have analyzed the high statistics data from $J/\psi(1S)$ radiative decays into S-wave $\pi^0\pi^0$, $K_S^0K_S^0$, $\eta\eta$ and $\omega\phi$ from BESIII, including data on $\pi\pi$, $\eta\eta$ and $\eta\eta'$ from the CERN SPS, BNL data on $\pi\pi \to K_S^0K_S^0$ and $\bar{p}p$ annihilation data from LEAR into various final states. The coupled channel analysis requires ten scalar mesons, the established ones in the *Review* and the $f_0(2020)$, as well as the isoscalars $f_0(2100, 2200, 2330)$ so far omitted from the *Summary Table*. Comparable contributions from $f_0(1500)$ and $f_0(1710)$ are found, with a preference for $\pi\pi$ decay over $K\bar{K}$ for the former and $K\bar{K}$ over $\pi\pi$ for the latter. As a new feature, the $f_0(1710)$ splits into two states, one at about 1700 MeV, the former $f_0(1710)$, and the new $f_0(1770)$ discussed above, being mandatory to obtain a good fit. A broad ($\sim$370 MeV) enhancement is observed in $J/\psi(1S)$ radiative decays around 1865 MeV, attributed to the contribution of glue distributed among the scalar mesons, with a very strong contribution from the 1770 MeV state [59]. In this model there are no supplementary states as the glue 'fragments' between the various isoscalars.

Ref. [112] presents a re-analysis of the BESIII $J/\psi(1S)$ radiative decay data into S- and P-wave $\pi^0\pi^0$ and $K_S^0K_S^0$ between 1.0 and 2.5 GeV. A good fit is obtained with only two scalar resonances below 2 GeV, the $f_0(1500)$ and $f_0(1710)$. The latter is more strongly produced, which points to a sizeable glueball component.

63.3.2 *Tensor glueballs*

Above the well known $f_2(1270)$ and $f_2'(1525)$ $q\bar{q}$ mesons, the $f_2(1640)$ and $f_2(1950)$ are tentatively assigned to the 2^3P_2 nonet (fig. 63.1). The broad $f_2(1950)$ has been observed by several experiments, *e.g.* in central production [26] and in $\bar{p}p$ annihilation in flight [21] and is often identified as a glueball [113], the ground state of the pomeron [114, 115].

Four further isoscalar tensors, the $f_2(2010)$, $f_2(2150)$, $f_2(2300)$ and $f_2(2340)$, which are in the range of the 1^3F_2 and 3^3P_2 nonets, exhibit a very large OZI breaking $\phi\phi$ production cross section in $\pi^-N \to \phi\phi n$ [116], in accord with earlier observations [117, 118], which could be due to the production of the 2^{++} glueball. The enhanced $\phi\phi$ production is also observed in central collisions [119] and in $\overline{p}p \to \phi\phi$ just above threshold [120]. The $f_2(2010)$, $f_2(2300)$ and $f_2(2340)$ have been reported by BESIII in $J/\psi(1S) \to \gamma\phi\phi$ [121]. The production rate of a tensor glueball in $J/\psi(1S)$ radiative decay has been calculated in quenched lattice QCD to be large (around 1%) [122] and is claimed by BESIII to be compatible with their measured rates in $J/\psi(1S) \to \gamma f_2(2340)$, $f_2(2340) \to \eta\eta$ [111], and $f_2(2340) \to \phi\phi$ [121].

The authors of Ref. [123] have re-analyzed the $J/\psi(1S)$ radiative decay into $\pi^0\pi^0$ and K_SK_S from BESIII and found as glueball candidate an enhancement described by a 360 MeV broad pole around 2210 MeV, possibly composed of the resonances observed earlier in $\phi\phi$. The relatively narrow $f_2(2300) \to \phi\phi$ (width of 150 MeV) is preferred by the holographic QCD model of Ref. [124]. However, the tensor glueball is predicted to be much broader (600-900 MeV) by the holographic model of Ref. [125].

There is no evidence for a narrow meson, $f_J(2220)$ (a tensor candidate) in $\overline{p}p$ annihilation (see the note under the $f_J(2220)$ in the 2004 issue of the *Review*). The measured partial width to $\overline{p}p$ in radiative $J/\psi(1S)$ decay [126] is too large and inconsistent with the upper limit from $\overline{p}p$ annihilation into $\pi\pi$ [127].

63.4 Pseudoscalar mesons

We now deal with the first radial excitations of the 0^{-+} nonet (fig. 63.1). The $\pi(1300)$ is a very broad resonance (200–600 MeV) decaying into 3π. The $K(1460)$ was confirmed with high statistics data from LHCb in $D^0 \to K3\pi$ decays [128] and has become an established kaon excitation.

The $\eta(1295)$ has been observed in various production mechanisms, in π^-p experiments [129–132], and possibly in $\bar{p}p$ annihilation [133, 134]. In $J/\psi(1S)$ radiative decay, a 0^{-+} signal is evident in the $\eta\pi\pi$ wave of the DM2 data, together with a 1^{++} of similar magnitude [135]. Also BaBar reports a signal around 1295 MeV in B decays into $\eta\pi\pi K$ [136]. The existence of the $\eta(1295)$ has been questioned [3, 137], being very close to the $f_1(1285)$. However, experimental evidences for a pseudoscalar in this mass region is reported by production mechanisms in which 1^{++} is suppressed, such as $\gamma\gamma$ production or $\bar{p}p$ annihilation at rest in liquid hydrogen. We therefore assume that the $\eta(1295)$ is one of the isoscalars in this nonet.

Two isoscalars were later observed in this mass region, the $\eta(1405)$ and $\eta(1475)$. The former decays mainly into $a_0(980)\pi$ (or direct $K\overline{K}\pi$) and the latter mainly into $K^*(892)\overline{K}$. The simultaneous observation of two pseudoscalars is reported in three production mechanisms: π^-p [129,138], radiative $J/\psi(1S)$ decay [135,139], and $\overline{p}p$ annihilation at rest [140–143]. All of them give values for the masses, widths, and decay modes that are in reasonable agreement (except for Ref. [135] which favors a state decaying into $K^*(892)\overline{K}$ at a lower mass than the state decaying into $a_0(980)\pi$). In $J/\psi(1S)$ radiative decay, the $\eta(1405)$ decays into $K\overline{K}\pi$ through $a_0(980)\pi$, and hence a signal is also expected in the $\eta\pi\pi$ mass spectrum. This was indeed observed by MARK III in $\eta\pi^+\pi^-$ [144], which reported a mass of 1400 MeV, in line with the existence of the $\eta(1405)$ decaying into $a_0(980)\pi$. The $\eta(1405)$ decaying into $\eta\pi\pi$ was also observed by Crystal Barrel [134]. Two states were reported by BES: Around 1452 MeV BESII observed a $K\bar{K}\pi$ enhancement in $J/\psi(1S) \to \omega K\bar{K}\pi$ but not in $J/\psi(1S) \to \phi K\bar{K}\pi$ [145], while BESIII reported a 52 MeV broad state in $\psi(2S) \to \omega K^*K$ [146], both left without J^{PC} determination.

The $K\overline{K}\pi$ and $\eta\pi\pi$ channels were studied in $\gamma\gamma$ collisions by L3 [147]. (For the 2γ couplings of glueballs and $q\bar{q}$ mesons see [74, 75, 148, 149].) The analysis led to a clear $\eta(1475)$ signal in $K\overline{K}\pi$, decaying into $K^*\overline{K}$, well identified in the untagged data sample, where spin 1 resonances are not allowed. At the same time, L3 did not observe the $\eta(1405)$, neither in $K\overline{K}\pi$ nor in $\eta\pi\pi$ [147]. On the other hand, CLEO-II did not observe any pseudoscalar signal with tagged γ's in $\gamma\gamma \to \eta(1475) \to K^0_SK^\pm\pi^\mp$ [150], with an upper limit slightly smaller than the signal observed by L3. After the CLEO-II result, L3 performed a further analysis with full statistics [151], confirming their previous evidence for the $\eta(1475)$. The CLEO upper limit for $\Gamma_{\gamma\gamma}$ [150], and the L3 results [151], are consistent with the world average for the $\eta(1475)$ width. BaBar [136] also reported the $\eta(1475)$ in B decays into $K\bar{K}^*$ recoiling against a K, while upper limits only were given for the $\eta(1405)$.

Hence, in radiative $J/\psi(1S)$ decay, $\pi^- p$ and $\bar{p}p$ annihilation at rest two isoscalar signals are observed in the 1400 – 1500 MeV mass region, while the $\eta(1405)$ is not seen in $\gamma\gamma$ interactions nor in B decays. The $\eta(1475)$ could be the first radial excitation of the η', with the $\eta(1295)$ being the first radial excitation of the η. Ideal mixing, suggested by the $\eta(1295)$ and $\pi(1300)$ mass degeneracy, would then imply that the second isoscalar in the nonet is mainly $s\bar{s}$, and hence couples to $K^*\overline{K}$, in agreement with properties of the $\eta(1475)$. In addition, its width agrees with the expected one for the radially excited $s\bar{s}$ state [84,91]. A study of radial excitations of pseudoscalar mesons [152] also favors the $s\bar{s}$ interpretation of the $\eta(1475)$. However, due to the strong kinematical suppression in $\eta(1405) \to K^*\overline{K}$ the data are not sufficient to exclude a sizeable $s\bar{s}$ admixture also in the $\eta(1405)$.

Here also, there are alternative explanations for the existence of two states:

- The isoscalar $\eta(1405)$ would be a candidate for the 0^{-+} glueball (section 63.4.1).
- In Refs. [3,137], apart from questioning the mere existence of the $\eta(1295)$, the authors propose a single pseudoscalar meson at 1440 MeV, the first radial excitation of the η. According to Refs. [84, 153] the splitting of the 1440 MeV state into $\eta(1405)$ and $\eta(1475)$ could be due to nodes in the decay amplitudes (which differ in $\eta\pi\pi$ and $K^*(892)\overline{K}$), nevertheless the following paper [154] underlines the relevance of $\gamma\gamma$ production and radiative decays to vector mesons to solve the $\eta(1440)$ riddle.
- The splitting could be due to a triangle singularity, hence the manifestation of one state only (fig. 63.3) [155–157]. The importance of the triangle singularity for the $\eta(1295)$ and $\eta(1405/1475)$ in the $\eta\pi\pi$ and $K\bar{K}\pi$ channels is stressed in Ref. [158]. In Ref. [159], using the triangle singularity approach of [155], the authors conclude that the BESIII results can be reproduced either with the $\eta(1405)$ or the $\eta(1475)$, or by a mixture of these two states.
- From a high statistics data sample in $J/\psi(1S) \to \gamma K_S K_S \pi^0$ BESIII reports the observation of both $\eta(1405)$ and $\eta(1475)$ using Breit-Wigner fits [160]. A one resonance parametrization is ruled out and triangle singularity models do not describe the data satisfactorily.
- The decay $J/\psi(1S) \to \gamma K_S K_S \pi^0$ has been studied in a three-body unitary coupled-channel analysis and compared to the BESIII data, including ratios of radiative branching ratios into $\eta\pi^+\pi^-$ and $K\bar{K}\pi$ through the $\eta(1405/1475)$ intermediate states [161]. Two poles emerge for $K^*\bar{K}$ in different Riemann sheets around 1400 MeV and only one pole around 1500 MeV.

To summarise, the experimental data on the 1400 – 1500 MeV region span several decades, various production mechanisms and decay modes, with models for data analysis evolving with time. A coherent picture of all available data is therefore difficult. We believe that there is sufficient evidence to consider the 0^{-+} nonet with the $\eta(1440)$ in fig. 63.1 as established. Whether one or two different states – $\eta(1405)$ and $\eta(1475)$ – exist is an open question, in which case the $\eta(1405)$ would be supernumerary. The most recent high statistics data and their theoretical interpretation seem to favor the two-resonance scenario.

63.4.1 *Pseudoscalar glueballs*

The first observation of an isoscalar resonance around 1425 MeV – the E-meson – was made in $p\bar{p}$ annihilation at rest into $E\pi^+\pi^-$, $E \to K\overline{K}\pi$ [162]. This state was reported to decay into $a_0(980)\pi$ and $K^*(892)\overline{K}$ with roughly equal contributions. An isoscalar state, the ι meson, was observed in radiative $J/\psi(1S)$ decay into $K\overline{K}\pi$ [163–165] and $\gamma\rho$ [166], and was considered as a

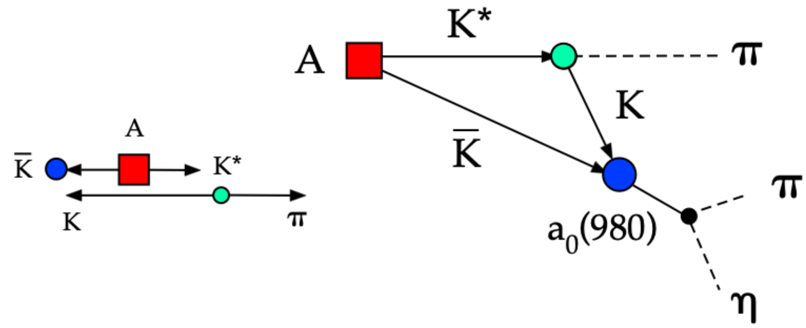

Figure 63.3: Triangle singularity: a state A decays into $K^*\overline{K}$. The K from K^* decay catches up with the $\overline{K}$ and excites the $a_0(980)$ resonance which in turn decays into $\eta\pi$. This mechanism can lead to two distinct peaks (depending on the width of A), one in $K^*\overline{K}$ and the other via rescattering in $\eta\pi\pi$. Similarly, the virtual $K^*\overline{K}$ loop can couple to $f_0(980)$ which, decaying into $\pi\pi$, leads to a peak in the 3π final state.

glueball candidate, owing to its strong signal in radiative $J/\psi(1S)$ decay. The E and ι were later assumed to be the same state, called $\eta(1440)$. For a historical survey see Ref. [167] and Ref. [168] for a review of the experimental status on the pseudoscalar glueball.

The $\eta(1405)$ would be a candidate for the 0^{-+} glueball in the fluxtube model [169], in which the 0^{++} glueball is also naturally related to a 0^{-+} glueball with mass degeneracy broken in QCD. The pseudoscalar glueball could lie at a lower mass than predicted from lattice calculation [170], see also Refs. [171–173]. The $\eta(1760)$, produced very strongly in radiative $J/\psi(1S)$ decay to $\omega\omega$ [174], has also been proposed for the 0^{-+} glueball mixed with the η and η' [175].

The holographic model of Ref. [75] predicts the pseudoscalar glueball below 2600 MeV, decaying dominantly into two vector mesons.

The $X(2370)$ has recently been studied by BESIII in radiative $J/\psi(1S)$ decay to $K_SK_S\eta'$ with $f_0(980) \to K_SK_S$ [176]. This channel is much less prone to γ background and more restrictive on the $X(2370)$ quantum numbers than $J/\psi(1S) \to \gamma K^+K^-\eta'$ and $\gamma\pi^+\pi^-\eta'$ reported earlier by BESIII. This state is determined to be 0^{-+} isoscalar (and therefore renamed $\eta(2370)$) with a (Breit-Wigner) mass of 2317 ± 9 MeV and a width of 140^{+80}_{-28} MeV (averaged over all three decay channels). According to Ref. [177] the $\eta(2370)$ would be a candidate for the ground state pseudoscalar glueball, but in a region where many 0^{-+} $q\bar{q}$ broad and overlapping radial excitations are also expected. Progress above 2000 MeV requires larger data sets and coupled channel K-matrix analyses.

63.5 Vector mesons

In this section we restrict ourselves to the isovector spectrum which contains broad and overlapping states.

63.5.1 $\rho(770)$

The determination of the parameters of the $\rho(770)$ is beset with many difficulties because of its large width. Its line shape depends on the production process and is not described by a relativistic Breit-Wigner function with a P-wave width, but requires some additional shape parameters [178]. Bose-Einstein correlations are another source of shifts in the $\rho(770)$ line shape, particularly in multiparticle final-state systems [179].

The same model dependence afflicts any other source of resonance parameters, such as the energy dependence of the phase shift δ_1^1, or the pole position. It is, therefore, not surprising that a study of $\rho(770)$ dominance in the decays of the η and η' reveals the need for specific dynamical effects, in addition to the $\rho(770)$ pole [180,181].

The cleanest determination of the $\rho(770)$ mass and width comes from e^+e^- annihilation and τ-lepton decays. ALEPH data [182] showed that the charged $\rho(770)$ parameters measured from τ-lepton decays are consistent with those of the neutral one determined from e^+e^- data [183]. This conclusion is qualitatively supported by the later studies of CLEO [184] and Belle [185]. However, a comparison of the two-pion mass spectrum in τ decays from OPAL [186], CLEO [184], and ALEPH [187,188], and the $e^+e^- \to \pi^+\pi^-$ cross section from CMD-2 [189,190], showed significant discrepancies between the two shapes which can be as high as 10% above the ρ meson mass [191,192]. This discrepancy remains after measurements of the two-pion cross section in e^+e^- annihilation at KLOE [193–196], SND [197,198], BaBar [199] and BESIII [200]. The effect is not accounted for by isospin breaking [201–204], but the accuracy of its calculation may be overestimated [205,206].

Two recent measurements of the $e^+e^- \to \pi^+\pi^-$ cross section by SND [207] and CMD-3 [208,209] show discrepancies in the ρ region: while SND is consistent with previous e^+e^- experiments, CMD-3 finds differences as high as 5% around the ρ meson mass. In addition, in the fit of the $e^+e^- \to \pi^+\pi^-$ cross section CMD-3 reports values for the mass of the ω and the mass and width of the ϕ, which differ slightly from the PDG values, showing for the first time the effect of the $\phi \to \pi^+\pi^-$ interference.

The problem of the difference in the $\pi^+\pi^-$ shapes between e^+e^- and τ decays seems to be solved after the analysis of Ref. [210], which showed that, after correcting the τ data for the missing ρ–γ mixing contribution, besides the other known isospin symmetry violating corrections, the $\pi\pi$ isospin 1 part of the hadronic vacuum polarization contribution to the muon $g-2$ is fully compatible between τ-based and e^+e^--based evaluations. The global fit of the whole set of the ρ, ω, and ϕ decays, taking into account mixing effects in the hidden local symmetry model, also showed consistency of the two-pion data in τ decay and e^+e^- annihilation [211,212]. However, because of the progress in e^+e^- data, the τ input is now less precise and less reliable due to additional theoretical uncertainties [213] decreasing the importance of τ decay for the determination of the $\rho(770)$ parameters.

63.5.2 $\rho(770)$ *excitations*

In our 1988 edition, we replaced the $\rho(1600)$ entry with two new ones, the $\rho(1450)$ and the $\rho(1700)$, because there was emerging evidence that the 1600-MeV region actually contains two ρ-like resonances. This possibility was pointed out by a theoretical analysis [214] on the consistency of the 2π and 4π electromagnetic form factors and the $\pi\pi$ scattering length. A consistent picture of $e^+e^- \to 2\pi, 4\pi$ and diffractive photoproduction is obtained with two resonances [215]. The existence of the $\rho(1450)$ was supported by the analysis of $\eta\rho^0$ mass spectra obtained in photoproduction and e^+e^- annihilation [216], as well as that of $e^+e^- \to \omega\pi$ [217].

The analysis of [215] was further extended by [218,219] to include new data on 4π-systems produced in e^+e^- annihilation, and in τ decays ($\tau \to 4\pi$ and $e^+e^- \to 4\pi$ are related by the Conserved Vector Current hypthesis). These systems were successfully analyzed using interfering contributions from two ρ-like states, and from the tail of the $\rho(770)$ two-body decay. While specific conclusions on $\rho(1450) \to 4\pi$ were obtained, little could be said about the $\rho(1700)$. Independent evidence for two 1^- states is provided by [220] in 4π electroproduction at $\langle Q^2 \rangle = 1$ (GeV/c)^2, and by [221] in a high-statistics sample of the $\eta\pi\pi$ system in π^-p charge exchange.

This scenario with two overlapping resonances is supported by other data. DM2 [222] measured the pion form factor in the interval 1.35–2.4 GeV, and observed a deep minimum around 1.6 GeV. The best fit was obtained with the hypothesis of ρ-like resonances at 1420 and 1770 MeV, with widths of about 250 MeV. DM2 [223] found that the $e^+e^- \to \eta\pi^+\pi^-$ cross section is better fitted with two fully interfering Breit-Wigners, with parameters in fair agreement with those of [215] and [222]. Recent LHCb data in $D^+ \to \pi^+\pi^+\pi^-$ also require both interfering $\rho(1450)$ and $\rho(1700)$ [224].

Decisive evidence for the $\pi\pi$ decay mode of both $\rho(1450)$ and $\rho(1700)$ comes from $\overline{p}p$ annihilation at rest [225]. It has been shown that these resonances possess a $K\overline{K}$ decay mode [16,226,227]. This decay mode has also been studied in three-body hadronic B decays [228]. High-statistics studies of the decays $\tau \to \pi\pi\nu_\tau$ [182,184], and $\tau \to 4\pi\nu_\tau$ [229] also require the $\rho(1450)$, but are not sensitive to the $\rho(1700)$, being too close to the τ mass. A high-statistics study of the $\tau \to \pi\pi\nu_\tau$ from Belle [185] reports the first observation of both $\rho(1450)$ and $\rho(1700)$ in τ decays. A clear picture of the two $\pi^+\pi^-$ resonances interfering with the $\rho(770)$ in e^+e^- annihilation was also reported by BaBar using the ISR (Initial State Radiation) method [230], and more recently by CMD-3 [208].

The structure of these ρ states is not yet completely clear. From $e^+e^- \to \omega\pi^0$ data and using the $^3P_0$ model the authors of Ref. [231] find that $\rho(1450)$ and $\rho(1700)$ mix, the former being mainly $2S$ and the latter mainly $3D$. The authors of Refs. [84,232] also claim that $\rho(1450)$ has a mass consistent with radial $2S$, but its decays show characteristics of hybrids, and suggest that this state may be a $2S$-hybrid mixture. Hybrid states could have a 4π decay mode dominated by the $a_1\pi$ [233]. Such behavior has been observed by [234] in $e^+e^- \to 4\pi$ in the energy range 1.05–1.38 GeV, and by [229] in $\tau \to 4\pi\nu_\tau$ decays. CLEO [235] and Belle [236] observe the $\rho(1450) \to \omega\pi$ decay mode in B-meson decays, however, do not find $\rho(1700) \to \omega\pi^0$. The $e^+e^- \to \omega\pi^0$ Born cross section between 1 and 2 GeV from SND (including data from BaBar and BESIII) is described by the $\rho(770)$, $\rho(1570)$, $\rho(1700)$ and $\rho(2150)$ in a vector dominance fit [237], while earlier data [238,239] did not observe a statistically significant signal of the $\rho(1700)$.

Various decay modes of the $\rho(1450)$ and $\rho(1700)$ are observed in $\overline{p}n$ and $\overline{p}p$ annihilation [240,241], but no definite conclusions can be drawn. More data should be collected to clarify the nature of the ρ states, particularly in the energy range above 1.6 GeV.

Under the $\rho(1570)$ we list the $\phi\pi$ state with $J^{PC} = 1^{--}$ earlier observed in [242] – referred to as $C(1480)$ – also reported in [243] and possibly seen in [244] which cannot exclude that the observation of the $\rho(1570)$ is due to an OZI-suppressed decay mode of the $\rho(1700)$. The $C(1480)$ has not been seen in $\overline{p}p$ [245] nor in other e^+e^- [246,247] experiments, which cannot exclude that the observation of the $\rho(1570)$ is due to an OZI-suppressed decay mode of the $\rho(1700)$.

Several observations on the $\omega\pi$ system in the 1200 MeV region [248–254] may be interpreted in terms of either $J^P = 1^-$ $\rho(770) \to \omega\pi$ production [255], or $J^P = 1^+$ $b_1(1235)$ production [253,254]. Evidence for a $\rho(1250)$ from a reanalysis of elastic scattering data is claimed in Ref. [256] and was reported earlier by LASS [257].

Ref. [258] finds a very broad 1^{--} resonance-like K^+K^- state in $J/\psi \to K^+K^-\pi^0$ decays with pole mass 1576 MeV and width 818 MeV, with possibly molecular or multiquark structure [259–261], or due to the $\rho(1450)$ and $\rho(1700)$ interference [262–265]. We quote this state as $X(1575)$ in the section 'Further States'. Another state observed in $J/\psi \to K^+K^-\pi^0$ is the $\phi(2170)$ [266] which has been interpreted as exotic, e.g. as tetraquark, see section 63.8.

Evidence for ρ-like mesons decaying into 6π was first noted in [267] in the analysis of $e^+e^- \to 6\pi$ [268,269] and diffractive photoproduction [270]. The authors of Ref. [267] argued that two states at about 2.1 and 1.8 GeV exist: while the former is a candidate for the $\rho(2150)$, the latter could be a manifestation of the $\rho(1700)$ distorted by threshold effects. BaBar reported observations of the new decay modes of the $\rho(2150)$ in the channels $\eta'(958)\pi^+\pi^-$, $f_1(1285)\pi^+\pi^-$ [271] and $\pi^+\pi^-$ [272]. The decay of the $\rho(2150)$ into K^+K^- has been observed by BESIII [273] and confirmed by BaBar [274]. The relativistic quark model [10] predicts the 2^3D_1 state with $J^{PC} = 1^{--}$ at 2.15 GeV which can be identified with the $\rho(2150)$.

Under $\rho(1900)$ we list various observations of irregular behavior of the cross sections near the $N\bar{N}$ threshold. Dips of various widths around 1.9 GeV were reported by the E687 Collaboration (a narrow one in the $3\pi^+3\pi^-$ diffractive photoproduction [275, 276]), by the FENICE experiment reporting a narrow dip [277], by BaBar in ISR (a narrow structure in $e^+e^- \to \phi\pi$ final state [244], but much broader in $e^+e^- \to 3\pi^+3\pi^-$ and $e^+e^- \to 2(\pi^+\pi^-\pi^0)$ [278]), by CMD-3 (also a rather broad dip in $e^+e^- \to 3\pi^+3\pi^-$ [279]). A dedicated scan of the $N\bar{N}$-threshold region by CMD-3 confirms this effect in the $e^+e^- \to 3\pi^+3\pi^-$ and $e^+e^- \to K^+K^-\pi^+\pi^-$ final states, but does not see it in the cross section of $e^+e^- \to 2\pi^+2\pi^-$ [280]. Most probably, these structures emerge as a threshold effect due to the opening of the $N\bar{N}$ channel [281–283]. A similar enhancement is observed by BESIII in $e^+e^- \to \Lambda\bar{\Lambda}$ near the $\Lambda\bar{\Lambda}$ threshold [284, 285] which can be described as $\Lambda\bar{\Lambda}$ final state interaction [286].

63.6 Axial-vector mesons

The 1^{++} nonet consists of the isovector $a_1(1260)$, the isoscalars $f_1(1285)$ and $f_1(1420)$, and the K_{1A}, which is a superposition of the physical states $K_1(1270)$ and $K_1(1400)$ with a mixing angle θ_{K_1} of about 35°, according to the phenomenological analysis of Ref. [287]. The nonet mixing angle $\theta_{1^{++}}$ is around 23°. The orthogonal combination – the K_{1B} – belongs to the 1^{+-} nonet with a mixing angle $\theta_{1^{+-}}$ of about 28° [287]. However, a two-pole structure for the $K_1(1270)$ emerges when using the unitary extension of chiral perturbation theory [288].

The mass region above 1400 MeV is rather complex [289–291]. The $f_1(1420)$ was first reported in π^-p reactions at 4 GeV/c, decaying into $K^*\overline{K}$ [292]. The $K\overline{K}\pi$ decay was also observed in a reanalysis of the MARK III data [139], and a C=+1 state seen in tagged $\gamma\gamma$ collisions [293]. The $K^0_S K^0_S \pi^0$ decay mode of the $f_1(1420)$ establishes unambiguously C=+1. Axial-vector mesons are not observed in $\overline{p}p$ annihilation at rest in liquid hydrogen, which proceeds dominantly through S-wave annihilation. However, in gaseous hydrogen, P-wave annihilation is enhanced and the $f_1(1420)$ is observed, decaying into $K^*\overline{K}$ [141]. The $f_1(1420) \to K\overline{K}\pi$, is also seen in pp central production, together with the $f_1(1285)$. The latter decays via $a_0(980)\pi$, and the former only via $K^*\overline{K}$ which suggests an $s\bar{s}$ structure. Neither central production [294] nor π^-p interactions at 100 GeV [131] find any evidence for a $\eta\pi\pi$ decay mode of the $f_1(1420)$. However, the $f_1(1420)$ is not seen in the K^-p data [295], which would argue against this state being the $s\bar{s}$ member of the 1^{++} nonet.

The $f_1(1285)$ has been suggested to be a $K^*\overline{K}$ molecule [296]. However, LHCb has determined the 1^{++} nonet mixing angle to be consistent with a mostly $n\bar{n}$ structure for the $f_1(1285)$ from $\overline{B}^0/\overline{B}^0 \to J/\psi(1S)f_1(1285)$, independent of the identity of its isoscalar partner [297]. The mixing angle $\theta_{1^{++}} = 24°$ agrees with that of Ref. [287] quoted above. The $n\bar{n}$ structure is also suggested by ALICE [298] using a comparison of ϕ/π and $f_1(1285)/\pi$ yields in the canonical statistical hadronization model. The ratio of $\overline{B}^0/\overline{B}^0_s$ decay rates also excludes the tetraquark interpretation of the $f_1(1285)$ [297]. This is consistent with earlier determinations assuming that the $f_1(1420)$ is the other isoscalar in the nonet [299].

The $a_1(1420)$ decaying into 3π, reported earlier by COMPASS at 1420 MeV in $\pi^-p \to \pi^-\pi^-\pi^+p$ [300, 301], originates from a triangle singularity: the decay of the $a_1(1260)$ into $K^*(\to K\pi)\bar{K}$ is followed by rescattering into the $f_0(980)\pi$ channel, $f_0(980) \to \pi\pi$ [302] (see the caption of fig. 63.3). A similar triangle singularity is proposed for the $f_1(1420)$, resulting from the $K^*\overline{K}$ and $a_0(980)\pi$ decay modes of the $f_1(1285)$ [303, 304].

The $f_1(1420)$ was also suggested to be a hybrid $q\bar{q}g$ meson [305] or a $K^*\overline{K}$ molecule, due to the proximity of the $K^*\overline{K}$ threshold [306]. Instead, the $s\bar{s}$ partner of the $f_1(1285)$ could be the $f_1(1510)$ [307] (omitted from the Summary Table). This state was seen to decay into $K\bar{K}\pi$ in K^-p interactions at 4 and 11 GeV/c, recoiling against a Λ [295, 308], which indeed points to an $s\bar{s}$ state. Evidence was reported in π^-p interactions, based on the phase motion of the 1^{++} $K^*\overline{K}$ wave [291]. The $f_1(1510)$ is also needed by CMD-3 in $e^+e^- \to f_1(1285, 1420, 1510)\rho(770)$

to describe the shoulder in the $K_S K^{\pm}\pi^{\mp}$ mass around 1500 MeV [309]. A somewhat broader 1^{++} signal is observed in $J/\psi(1S) \to \gamma\eta\pi^+\pi^-$ [310] and a small signal in $J/\psi(1S) \to \gamma\eta'\pi^+\pi^-$ [311].

However, there is no evidence for the $f_1(1510)$ in other K^-p experiments [312,313]. The $f_1(1510)$ is not seen in central collisions [314], nor in $\gamma\gamma$ collisions [315], although surprisingly for an assumed $s\bar{s}$ meson, a signal is reported in 4π decays [316]. Data from LHCb in $B \to (K_S K\pi)K$ [317] require the $f_1(1420)$ and a much weaker $f_1(1510)$ signal. Given this confusing experimental situation, the isoscalar classification in the 1^{++} nonet is not yet settled.

63.7 Hybrid mesons

Hybrids may be viewed as $q\bar{q}$ mesons with a vibrating gluon flux tube. In contrast to glueballs, they have isospin 0 or 1 and can be electrically charged. The mass spectrum of hybrids with exotic (non-$q\bar{q}$) quantum numbers was predicted in Ref. [318], while Ref. [319] also deals with non-exotic quantum numbers. The ground-state hybrids with quantum numbers (0^{-+}, 1^{-+}, 1^{--}, and 2^{-+}) are expected around 1.7 to 1.9 GeV. Lattice calculations predict that the hybrid with exotic quantum numbers 1^{-+} lies at a mass of 1.9 ± 0.2 GeV [320, 321]. Most hybrids are expected to be rather broad, but some can be as narrow as 100 MeV [322]. They prefer to decay into a pair of S- and P-wave mesons. The lattice study in [323, 324], based on full QCD with pion masses around 400 MeV, finds that several of the high-lying states observed in their spectrum show significant overlap with gluon rich source terms interpreted as hybrid states. A very broad 1^{-+} isovector structure is predicted by a recent lattice calculation [325]. The main decay mode is $b_1(1235)\pi$ (a pair of S- and P-wave mesons), while other modes, such as $\rho\pi$, $\eta\pi$ and $\eta'\pi$, are suppressed by at least an order of magnitude. For an experimental and theoretical review on hybrid mesons see Ref. [4, 326].

Two isovector 1^{-+} candidates have been reported, one around 1400 MeV (called earlier $\pi_1(1400)$) and the $\pi_1(1600)$. The ∼350–400 MeV broad $\pi_1(1400)$, was reported in $\pi^-p \to \eta\pi^-p$ [327,328] and in $\pi^-p \to \eta\pi^0 n$ [329]. It was observed as an interference between the angular momentum $L = 1$ and $L = 2$ $\eta\pi$ amplitudes, leading to a forward/backward asymmetry in the $\eta\pi$ angular distribution. This state had been reported earlier in π^-p reactions [330], but ambiguous solutions in the partial wave analysis were pointed out [331, 332]. A resonating 1^{-+} contribution to the $\eta\pi$ P-wave is also required in the Dalitz plot analysis of $\bar{p}n$ annihilation into $\pi^-\pi^0\eta$ [333], and in $\bar{p}p$ annihilation into $\pi^0\pi^0\eta$ [334]. Mass and width are consistent with the results of Ref. [327]. A coupled channel re-analysis of the $\pi^0\pi^0\eta$, $\pi^0\eta\eta$ and $K^+K^-\pi^0$ Crystal Barrel data at 900 MeV/c, supplemented with data from other collaboration in $\pi\pi \to \pi\pi, K\bar{K}, \eta\eta$ and $\eta\eta'$, leads to a single 1^{-+} ∼600 MeV broad state with a pole around 1400 MeV, decaying into $\eta\pi^0$ [20]. Ref. [335] suggested for the production in π^-p that a Deck-generated $\eta\pi$ background from final state rescattering in $\pi_1(1600)$ decay could mimic $\pi_1(1400)$, a mechanism that is, however, absent in $\bar{p}p$ annihilation.

The $\pi_1(1600)$, decaying into $\rho\pi$, was reported by COMPASS with 190 GeV pions hitting a lead target [336, 337]. It had already been observed in π^-p interactions in the decay modes $\eta'\pi$ [338], $f_1(1285)\pi$ [339], and $\omega\pi\pi$ [340], $b_1(1235)\pi$, but not $\eta\pi$ [341]. A strong enhancement in the 1^{-+} $\eta'\pi$ wave, compared to $\eta\pi$, was reported at this mass in Ref. [342]. The enhancement is also observed by COMPASS in $\pi^-p \to \eta'\pi$ but in $\eta\pi$ a peak appears at 1400 MeV and in $\eta'\pi$ at 1600 MeV [343]. A coupled channel analysis of the $\eta\pi$ and $\eta'\pi$ COMPASS data leads to a single pole at 1564 ± 89 MeV, with a width of 492 ± 115 MeV [344]. The predicted width from the lattice calculation [325] is compatible with the COMPASS result. Furthermore, a combined analysis of the COMPASS and Crystal Barrel data at 900 MeV/c leads to similar results, a single pole around 1623 MeV, with a width of about 455 MeV, although a two-pole scenario cannot be completely excluded [345]. Accordingly, the $\pi_1(1400)$ data are listed in the $\pi_1(1600)$ section.

Very recently, BESIII reported a 188 MeV broad enhancement at 1855 MeV in $J/\psi(1S)$ radiative decay into the $\eta\eta'$ P-wave [346], hence with exotic quantum numbers 1^{-+}. This isoscalar resonance is needed with very high significance, albeit the contribution to $\gamma\eta\eta'$ is weak and several other isoscalars expected to contribute are absent [347]. Hence this state needs confirmation. We have listed claims for such a state decaying into $\eta\eta'$ in this mass range under the $f_2(1910)$ (see e.g. Ref. [30] from WA102 central production, suggesting 1^{-+} as an alternative to 2^{++}). If confirmed, the $\eta(1855)$ could be one of the isospin 0 partners of the $\pi_1(1600)$, a member of the 1^{-+} hybrid nonet [348–350] or a $K\bar{K}_1(1400)$ molecule [351]. In Ref. [348] this state is interpreted as the heavy isoscalar while its light partner is much narrower, with a mass of about 1660 MeV.

Hybrid candidates with the non-exotic quantum numbers 0^{-+}, 1^{--}, and 2^{-+} have also been reported: the $\pi(1800)$ is somewhat narrow if interpreted as the second radial excitation of the pion. It decays mostly into a pair of S- and P-wave mesons [352,353], in line with expectations for 0^{-+} hybrid mesons. The evidence for 1^{--} hybrids in e^+e^- annihilation and in τ decays has been discussed in [233]. The near degeneracy of the $\eta_2(1645)$ and $\pi_2(1670)$ suggests ideal mixing in the 2^{-+} $q\overline{q}$ nonet, and hence, the second isoscalar, presumably the $\eta_2(1870)$, should be mainly $s\overline{s}$ (fig. 63.1). However, the $\eta_2(1870)$ is also observed in $\overline{p}p$ annihilation and decays mainly into $a_2(1320)\pi$ and $f_2(1270)\pi$ [354], with a relative rate compatible with a hybrid state [319].

Evidence for another exotic $\pi_1(2015)$ has been claimed in π^-p interactions [339,340]. A flavor multiplet of vector hybrids between 2200 and 2300 MeV is predicted by the constituent gluon model [350]. Decays of light hybrids in the constituent model are discussed in Ref. [355].

Summarizing, there is evidence for a very broad 1^{-+} enhancement in the 1400–1600 MeV region, which consists of one (or possibly two) exotic states. As isovectors they cannot be glueballs. The coupling to $\eta\pi$ of the lighter one points to a four-quark state [356], while the $\eta'\pi$ coupling of the heavier one is favored for hybrid states [357,358]. The mass of the latter, the $\pi_1(1600)$, is not far below the lattice and flux tube model predictions. The $\eta_1(1855)$ is an isoscalar candidate for the ground state 1^{-+} hybrid nonet.

63.8 Tetraquark states

The existence of multiquark states was suggested a long time ago, based on duality arguments [359,360]. The most prominent light tetraquark candidates are the $a_0(980)$ and $f_0(980)$ [361–363]. A remarkable prediction for the existence of low-lying four quarks states is based on color hyperfine splitting. The lowest ground state ($L = 0$) tetraquark multiplet is predicted to be a scalar nonet. The scalar nonet lies just below 1 GeV when one assumes as mass scale the hyperfine color splitting between the ρ and the π (for a simple derivation see [9]). One then gets the nonet structure

$$\begin{aligned}
|a_0(980)\rangle &= |us\bar{d}\bar{s}\rangle, \frac{1}{\sqrt{2}}|(u\bar{u} - d\bar{d})s\bar{s}\rangle, |\bar{u}\bar{s}ds\rangle, \\
|f_0(980)\rangle &= \frac{1}{\sqrt{2}}|(u\bar{u} + d\bar{d})s\bar{s}\rangle, \\
|f_0(500)\rangle &= |\bar{u}\bar{d}ud\rangle, \\
|K_0^*(700)\rangle &= |\bar{s}\bar{d}ud\rangle, |\bar{s}\bar{u}ud\rangle \quad \text{and} \quad |\bar{u}\bar{d}us\rangle, |\bar{u}\bar{d}ds\rangle.
\end{aligned} \tag{63.1}$$

The two isoscalars are expected to mix with the angle φ:

$$\begin{aligned}
|f_0(980)\rangle &= \cos\varphi|s\bar{s}\rangle + \sin\varphi|n\bar{n}\rangle, \\
|f_0(500)\rangle &= -\sin\varphi|s\bar{s}\rangle + \cos\varphi|n\bar{n}\rangle.
\end{aligned} \tag{63.2}$$

Whether these mesons are really tetraquark states or $q\bar{q}$ mesons, is still an open issue:

– The $f_0(980)$ is strongly produced in D_s^+ decay [365], which suggests a large $s\bar{s}$ component, due to the Cabibbo-favored $c \to s$ coupling. However, the mainly $n\bar{n}$ $f_0(1370)$ is also strongly produced in D_s^+ decay, hence additional graphs must contribute [366].

Table 63.1: Coupling amplitudes for $\bar{B}^0$ and $\bar{B}^0_s$ decays into $J/\psi(1S)f_0(500)/f_0(980)$, depending on the $q\bar{q}$ or tetraquark structures of the $f_0(500)$ and $f_0(980)$ (from [364] where illustrative decay diagrams can be found). The angle φ is defined in Eq. 63.2.

	$\bar{B}^0$		$\bar{B}^0_s$	
	$f_0(980)$	$f_0(500)$	$f_0(980)$	$f_0(500)$
$q\bar{q}$	$\sin\varphi/\sqrt{2}$	$\cos\varphi/\sqrt{2}$	$\cos\varphi$	$\sin\varphi$
$qq\bar{q}\bar{q}$	$1/\sqrt{2}$	1	$\sqrt{2}$	0

- The relative branching ratios for $\bar{B}^0$ and $\bar{B}^0_s \to J/\psi(1S)f_0(500)$ and $J/\psi(1S)f_0(980)$ can be used to probe the $q\bar{q}$ or tetraquark natures of the $f_0(500)$ and $f_0(980)$ [364, 367]. LHCb observes the $f_0(980)$ in $\bar{B}^0_s$ decays, but not the $f_0(500)$ [32], as would be expected for tetraquarks (see Table 63.1). In contrast, LHCb also observes the $f_0(500)$ in $\bar{B}^0$ decays, but not the $f_0(980)$ with an upper limit eight standard deviations below the predicted value for a tetraquark state [33]. However, these contradicting findings have been challenged by a dispersive analysis [368], using a model independent inclusion of hadronic final state interactions, in which a substantial $f_0(980)$ contribution is nevertheless found in $\bar{B}^0$-decays, thus still leaving the tetraquark structure as an open possibility.
- The ALICE collaboration reports a much stronger K_SK_S femtoscopic correlation than for $K_SK^{\pm}$ in high energy pp collisions, indicative of the $a_0(980)$ being a tetraquark state [369].
- The $f_0(980)$ and $a_0(980)$ could also be $K\overline{K}$ molecular states [70, 370, 371], being close to the $K\overline{K}$ threshold and decaying strongly into $K\overline{K}$. For $q\bar{q}$ states, the expected $\gamma\gamma$ widths [372, 373] are not significantly larger than for molecular states [372, 374] and both predictions are consistent with data.
- Radiative decays of the $\phi(1020)$ into $a_0(980)$ and $f_0(980)$ were proposed to disentangle compact tetraquark structures from hadronic molecules. In Ref. [13] these states are made of four-quark cores and virtual $K\bar{K}$ clouds at the periphery, a view that is challenged in Ref. [375] showing that radiative ϕ decay data are consistent with molecular structures of light scalars. However, following Refs. [376, 377], the data from KLOE [378] and SND [379] in $\phi \to \gamma\eta\pi^0$, from KLOE [380] in $\phi \to \gamma\pi^0\pi^0$ and in $\phi \to \gamma\pi^+\pi^-$ from CMD-2 [381] favor these mesons to be tetraquark states. This is also supported by a BESIII analysis of $J/\psi(1S) \to \gamma\pi^0\pi^0$ [382] and by a measurement of $a_0(980) - f_0(980)$ mixing at BESIII [383].

More information on the $a_0(980)$, $f_0(980)$, $f_0(500)$ and $K^*_0(700)$ is provided by 'Scalar Mesons below 1 GeV' in the *Review*. For the $f_0(500)$ see also Ref. [384].

As mentioned already, the $f_0(1500)$ [42] and the $f_1(1285)$ [364] have been proposed as tetraquarks, the latter also as $K^*\overline{K}$ molecule [296], together with the $f_1(1420)$ [306]. Another potential candidate for a tetraquark is the $\phi(2170)$. As an excited $q\bar{q}$ state of the $\phi(1020)$, this meson should decay strongly into K^+K^- and $K^0_SK^0_L$, and also $K^*\bar{K}^*$. However, these decays are not observed. Hence, based on its observed decay into $f_0(980)\phi(1020)$ this meson could be an $ss\bar{s}\bar{s}$ tetraquark [385]. However, Refs. [386, 387] claim that the $\phi(2170)$ may not be a good candidate for the $ss\bar{s}\bar{s}$ state, predicted to lie higher in mass, but could qualify as an $su\bar{s}\bar{u}$ tetraquark.

63.9 Baryonia

Nucleon-antinucleon ($N\bar{N}$) bound states and resonances (baryonium) were predicted a long time ago [388, 389], based on the strongly attractive $N\bar{N}$ meson exchange potential, which is obtained from the NN one by multiplying with the G-parity of the exchanged meson. Several candidates had been reported in the seventies by experiments at CERN, BNL and KEK, some of them being indisputably statistically significant (for details on the model and on experiments, see [390,391] and references therein). Most potential models predict a sequence of deeply bound isoscalar baryonia with quantum numbers $J^{PC} = 2^{++}$, 1^{--} and 0^{++}, the latter being the mostly bound [392, 393]. The $f_2(1565)$ which is observed in $\bar{p}p$ annihilation only [394, 395], is a good candidate for the 2^{++} $\bar{p}p$ bound state.

Enhancements close to the $\bar{p}p$ threshold have also been reported in B decays [396–398]. The strong signal near the $p\bar{p}$ threshold in $J/\psi(1S) \to \gamma\bar{p}p$ [399–401] could be due to a 0^{-+} baryonium [402], but could also be generated by the $N\bar{N}$ final state interaction [403–406]. The strong energy dependence of the cross section in $e^+e^- \to \bar{p}p$ [407, 408] and $\bar{p}p \to e^+e^-$ [409] is attributed to the $N\bar{N}$ final state interaction [410] (see also our comments on the $\rho(1900)$ at the end of section 63.5.2). Alternative explanations to baryonia have been proposed for the signals in $B \to \bar{p}pK$, such as the dynamics of the fragmentation mechanism [397].

63.10 Conclusions

The ground state nonets 1^1S_0, 1^3S_1, 1^1P_1, 1^3P_2, 1^1D_2 and 1^3D_3 (see fig. 63.1) are established (although more data are desirable for the $h_1(1415)$ in the 1^1P_1 nonet). There are uncertainties and ambiguities in all other nonets. In the pseudoscalar sector the $\eta(1440)$ may consist of two states, one at 1405 MeV, the other at 1475 MeV, which could also be the manifestation of a single resonance decaying into two different final states. In this case the 1475 one could be the radial excitation of the η' and the 1405 the pseudoscalar ground state glueball predicted a long time ago by the bag model, although the lattice puts it above 2 GeV. In the 1^{++} nonet two isoscalars – the $f_1(1420)$ and $f_1(1510)$ – compete for the $s\bar{s}$ slot. The first radial excitation 2^3S_1 is complete, but questions have been raised as to the nature of the $\rho(1450)$ and its relation to the $\rho(1700)$. The classification of scalar mesons is not settled, with the ones below 1 GeV being tetraquark states or building the ground state $q\bar{q}$ nonet. The nature of the scalars in the 1.5 – 1.8 GeV region is unclear, obscured by the interference with the ground state scalar glueball predicted in this mass region. For example, the $f_0(1500)$ and the $f_0(1710)$ could be $q\bar{q}$ states ($n\bar{n}$ and $s\bar{s}$, respectively) or made of gluons mixed with $q\bar{q}$. The $f_0(1500)$, or alternatively the $f_0(1710)$, have been proposed as mostly gluonic. Scalar mesons above the $f_0(1710)$ need experimental confirmation. Another pair of scalars, $a_0(1710)$ and $f_0(1770)$, has been reported. An isovector meson with exotic (1^{-+}) quantum numbers, $\pi_1(1600)$, is observed around 1.6 GeV with mass not far below theoretical predictions, although a further 1^{-+} state, decaying into $\eta\pi$, cannot be entirely excluded around 1.4 GeV. An isoscalar partner, the $\eta_1(1855)$, has been reported, needing confirmation. Above 1.8 GeV mesons are broad and overlap, which complicates the data analyses. This makes the classification difficult, in particular the identification of the 2^{++} and 0^{-+} glueballs, expected above 2000 MeV.

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