71. Leptonic Decays of Charged Pseudoscalar Mesons

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71.1 Introduction

This review updates one in Ref. [1]. Lattice QCD updates rely on the Flavor Lattice Averaging Group FLAG 24 update [2]; these updates were modest. The most significant updates involve D_s experimental results; some changes to the discussions of semileptonic decays in second-generation unitarity and inclusive-exclusive V_{ub} tension are also noted.

Charged mesons formed from a quark and antiquark can decay to a lepton-neutrino pair when these objects annihilate via a virtual W boson. Figure 71.1 illustrates this process for the purely leptonic decay of a D^+ meson.

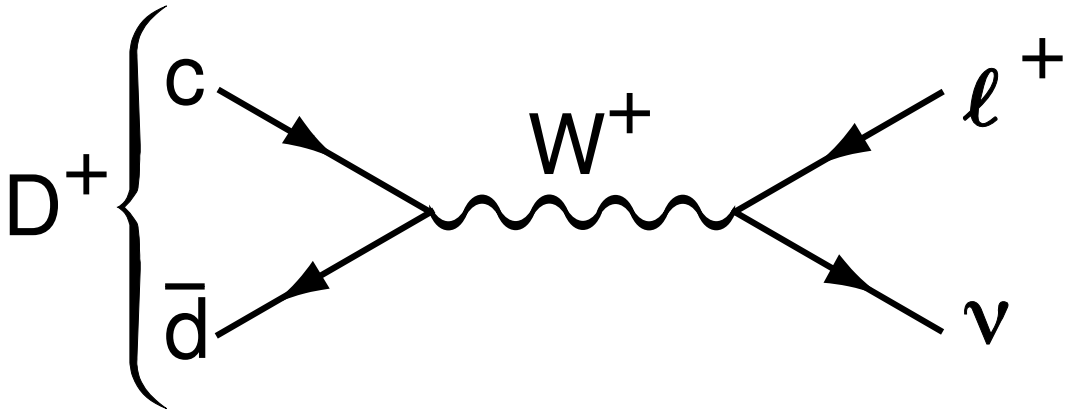

Figure 71.1: The annihilation process for pure D^+ leptonic decays in the standard model.

Similar quark-antiquark annihilations via a virtual W^+ to the $\ell^+\nu$ final states occur for the π^+, K^+, D_s^+, and B^+ mesons. (Whenever pseudoscalar-meson charges are specified in this article, use of the charge-conjugate particles and corresponding decays are also implied.) Let P be any of these pseudoscalar mesons. To lowest order, the decay width is

$$\Gamma^{(0)}(P \to \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 \; m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 |V_{q_1 q_2}|^2 \; . \tag{71.1}$$

Here M_P is the P mass, m_ℓ is the ℓ mass, $V_{q_1 q_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the quarks $q_1\bar{q}_2$ in P, and G_F is the Fermi coupling constant. The decay constant f_P is proportional to the matrix element of the axial current between the one-P-meson state and the vacuum:

$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = ip_\mu f_P \, , \tag{71.2}$$

and can be thought of as proportional to the "wave function overlap" of the quark and antiquark. In this article, we use the convention in which $f_\pi \approx 130$ MeV. For brevity, we will often denote the purely leptonic decay width in Eq. (71.1) by $\Gamma^{(0)}$.

The decay of $P^\pm$ starts with a spin-0 meson, and ends up with a left-handed neutrino or right-handed antineutrino. By angular momentum conservation, the $\ell^\pm$ must then also be left-handed or right-handed, respectively. In the $m_\ell = 0$ limit, the decay is forbidden, and can only occur as

a result of the finite ℓ mass. This helicity suppression is the origin of the m_ℓ^2 factor in the decay width.

Experimentally, it is difficult to isolate events in which there are *only* a lepton and neutrino in the final state from those with a lepton, neutrino, and soft photon. Thus, radiative contributions must be removed from the experimental measurements *a posteriori* to obtain $\Gamma^{(0)}$. The radiative contributions can be broken into three pieces: the short-distance contribution to leptonic and semileptonic decays mediated by a $W^\pm$ boson that accounts for electroweak corrections not included in the definition of G_F, the long-distance internal bremsstrahlung (IB) contribution, and the contribution from photon emission that depends upon the hadron's structure. The universal electroweak correction was calculated at $\mathcal{O}(\alpha)$ by Sirlin [3], and increases the purely leptonic decay rate by ~ 1.8–2.2% depending on the decaying meson. The $\mathcal{O}(\alpha)$ IB contribution was calculated by Kinoshita [4], and again is universal for all leptonic decays at this order. Numerically, this universal long-distance contribution lowers the purely leptonic decay rate by ~ 0.4–2.4%, where the correction is smallest for pions and largest for $D_{(s)}$ mesons. The structure-dependent contributions have been estimated within various effective theories and from lattice-QCD,QED calculations to increase the purely leptonic rate by one to a few percent [5–10].

In this review, we treat the radiative corrections differently for the light, charm, and bottom meson systems for several reasons. First, the experimental uncertainties on the decay widths vary substantially. Thus, while the inclusion of radiative corrections is essential for the pion, kaon, and $D_{(s)}$-meson decay widths, which have been measured to (sub)-percent precision, radiative corrections can be neglected (for now) for $B \to \tau\nu$ decay. Second, the photons are treated differently on the experimental side for the different decay processes. For pions and kaons, the experimental measurements of $\Gamma_{P\ell 2[\gamma]}$ are fully inclusive, while for $D_{(s)}$ mesons, the experiments impose cuts on the energy of any neutral cluster deposited in the calorimeter, which reduce the soft-photon background substantially. Some experiments also remove the QED bremsstrahlung in the leading-logarithmic approximation using the PHOTOS Monte-Carlo generator [11]. Third, the theoretical knowledge of the structure-dependent corrections varies for each meson system.

Once radiative corrections have been accounted for, measurements of purely leptonic decay branching fractions and lifetimes allow an experimental determination of the product $|V_{q_1q_2}|\, f_P$. If the decay constant f_P is known to sufficient precision from theory, one can obtain the corresponding CKM element within the standard model. If, on the other hand, one takes the value of $|V_{q_1q_2}|$ assuming CKM unitarity, one can infer an "experimental measurement" of the decay constant that can then be compared with theory.

The importance of measuring $\Gamma(P \to \ell\nu)$ depends on the particle being considered. Leptonic decays of charged pseudoscalar mesons occur at tree level within the standard model. Thus one does not expect large new-physics contributions to measurements of $\Gamma(P \to \ell\nu)$ for the lighter mesons $P = \pi^+, K^+$, and these processes, in principle, provide clean standard-model determinations of $|V_{ud}|$ and $|V_{us}|$. The situation is different for leptonic decays of charm and bottom mesons. The presence of new heavy particles such as charged Higgs bosons or leptoquarks could lead to observable effects in $\Gamma(P \to \ell\nu)$ for $P = D_{(s)}^+, B^+$ [12–16]. Thus, the determination of $|V_{ub}|$ from $B^+ \to \tau\nu$ decay, in particular, should be considered a probe of new physics. More generally, the ratio of leptonic decays to $\tau\nu$ over $\mu\nu$ final states probes lepton universality [12, 17].

The determinations of CKM elements from leptonic decays of charged pseudoscalar mesons provide complementary information to those from other decay processes. The decay $P \to \ell\nu$ proceeds in the standard model via the axial-vector current $\bar{q}_1\gamma_\mu\gamma_5 q_2$, whereas semileptonic pseudoscalar meson decays $P_1 \to P_2\ell\nu$ proceed via the vector current $\bar{q}_1\gamma_\mu q_2$. Thus the comparison of determinations of $|V_{q_1q_2}|$ from leptonic and semileptonic decays tests the $V-A$ structure of the standard-model electroweak charged-current interaction. More generally, a small right-handed ad-

mixture to the standard-model weak current would lead to discrepancies between $|V_{q_1q_2}|$ obtained from leptonic pseudoscalar-meson decays, exclusive semileptonic pseudoscalar-meson decays, exclusive semileptonic baryon decays, and inclusive semileptonic decays [18, 19].

Both measurements of the decay rates $\Gamma(P \to \ell\nu)$ and theoretical calculations of the decay constants f_P for $P = \pi^+, K^+, D^+_{(s)}$ from numerical lattice-QCD calculations are now quite precise. As a result, the elements of the first row of the CKM matrix $|V_{ud}|$ and $|V_{us}|$ can be obtained to sub-percent precision from $\pi^+ \to \ell\nu$ and $K^+ \to \ell\nu$, where the limiting error is from continuum theory radiative corrections. The elements of the second row of the CKM matrix $|V_{cd(s)}|$ can be obtained from leptonic decays of charmed pseudoscalar mesons to few-percent precision, where here the limiting error is from experiment. These enable stringent tests of the unitarity of the first and second rows of the CKM matrix.

This review is organized as follows. Because the experimental and theoretical issues associated with measurements of pions and kaons, charmed mesons, and bottom mesons differ, we discuss each one separately. We begin with the pion and kaon system in Sec. 71.2. First, in Sec. 71.2.1, we review current measurements of the experimental decay rates. We provide tables of branching-ratio measurements and determinations of the product $|V_{ud(s)}|f_{\pi^+(K^+)}$, as well as average values for these quantities including correlations and other effects needed to combine results. Then, in Sec. 71.2.2, we summarize the status of theoretical calculations of the decay constants. We provide tables of recent lattice-QCD results for f_{π^+}, f_{K^+}, and their ratio from calculations including dynamical u, d, s, and (in some cases c) quarks, along with averages including correlations and strong SU(2)-isospin breaking corrections as needed. We next discuss the charmed meson system in Sec. 71.3, again reviewing current experimental rate measurements in Sec. 71.3.1 and theoretical decay-constant calculations in Sec. 71.3.2. Last, we discuss the bottom meson system in Sec. 71.4, following the same organization as the two previous sections. For almost all of the decay constants presented in Secs. 71.2.2, 71.3.2, and 71.4.2, we take as our preferred values the four-flavor averages from the FLAG 24 review [2], with isospin-breaking corrections as noted.

After having established the status of both experimental measurements and theoretical calculations of leptonic charged pseudoscalar-meson decays, we discuss some implications for phenomenology in Sec. 71.5. For each process discussed in Secs. 71.2–71.4, we combine the average $\mathcal{B}(P \to \ell\nu)$ with the decay constant f_P to infer the associated CKM matrix element. We then compare these results with determinations of the same CKM elements from other processes. We also use the CKM elements obtained from leptonic decays to test the unitarity of the first and second rows of the CKM matrix. Further, as in previous reviews, we combine the experimental $\mathcal{B}(P \to \ell\nu)$s with the associated CKM elements obtained from CKM unitarity to infer "experimental" values for the decay constants. The comparison of these values with theory provides a test of lattice and other QCD approaches, assuming that new-physics contributions to these processes are not significant.

71.2 Pions and kaons

71.2.1 *Experimental rate measurements*

Experimental rate measurements of pion and kaon leptonic decays are fully radiation inclusive. Following Refs. [20, 21], and references therein, we combine the $\mathcal{O}(\alpha)$ radiative corrections to the purely leptonic rate as follows:

$$\Gamma(P \to \ell\nu[\gamma]) = \Gamma^{(0)}(P \to \ell\nu)\left[1 + \frac{\alpha}{\pi}C_P\right], \tag{71.3}$$

where $P = \pi, K$. The full expressions for C_π and C_K are given in Eq. (114) of Ref. [7]. In addition to the universal short- [3] and long-distance [4] corrections, C_π and C_K include hadronic-structure

dependent contributions [5] through $\mathcal{O}(\alpha p^4)$ in chiral perturbation theory (χPT), where p is the pion or kaon momentum. The inclusion of radiative corrections to the purely leptonic rates is numerically important given the level of precision achieved on the experimental measurements of the $\pi^\pm \to \mu^\pm\nu$ and $K^\pm \to \mu^\pm\nu$ decay widths.

We evaluate $\delta_P \equiv (\alpha/\pi)C_P$ using experimentally-measured meson and lepton masses and coupling constants from the Particle Data Group [22], and taking the low-energy constants (LECs) that parametrize the hadronic contributions from Refs. [7, 23, 24]. Because the finite non-logarithmic parts of the LECs were estimated within the large-N_C approximation assuming that contributions from the lowest-lying resonances dominate, we conservatively assign a 100% uncertainty to the LECs, which leads to a ± 0.9 error in $C_{\pi,K}$.[1] We obtain the following correction factors to the individual charged pion and kaon decay widths:

$$\delta_\pi = 0.0176(21) \quad \text{and} \quad \delta_K = 0.0107(21)\,. \tag{71.4}$$

The error on the ratio of kaon-to-pion leptonic decay widths is under better theoretical control because the hadronic contributions from low-energy constants estimated within the large-N_c framework cancel at lowest order in the chiral expansion. For the ratio, we use the correction factor

$$\delta_{K/\pi} = -0.0069(17)\,, \tag{71.5}$$

where we take the estimated error due to higher-order corrections in the chiral expansion from Ref. [26].

There have been no new measurements of the pion leptonic decay rate since the review of Ref. [27]. The sum of branching fractions for $\pi^- \to \mu^-\bar\nu$ and $\pi^- \to \mu^-\bar\nu\gamma$ is 99.98770(4)% [1]. Together with the lifetime 26.033(5) ns [1] this implies $\Gamma(\pi^- \to \mu^-\bar\nu[\gamma]) = 3.8408(7) \times 10^7\ \text{s}^{-1}$. We then subtract the estimated radiative correction factor δ_π in Eq. (71.4) to obtain the purely leptonic rate $\Gamma^{(0)}(\pi^- \to \mu^-\bar\nu)$. Using this rate and the PDG masses [1] in Eq. (71.1) gives

$$f_{\pi^+}|V_{ud}| = (127.13 \pm 0.01 \pm 0.13)\ \text{MeV}\,, \tag{71.6}$$

where the errors are from the experimental rate measurement and the radiative correction factor, respectively.

The uncertainty on $f_{\pi^-}|V_{ud}|$ is dominated by that from theoretical estimate of the hadronic structure-dependent radiative corrections. The first direct lattice-QCD calculation of the radiative corrections to the pion and kaon leptonic decay rates was performed by the RM123-Soton Collaboration [10]. The results for both $\delta_\pi = 0.0153(19)$ and $\delta_K = 0.0088(9)$, which are given in the Gasser-Rusetsky-Scimemi scheme for separating strong and electromagnetic effects [28], are compatible with our chiral-perturbation-theory estimates above and have smaller quoted uncertainties, especially for δ_K. Less precise results have also been presented by RBC/UKQCD [29]. While further confirmation of these results is needed, they demonstrate a promising approach for reducing the theoretical uncertainties on the pion and kaon leptonic decay rates in the future.

The world average for the $K \to \mu\nu$ decay rate was previously obtained from a global fit of several kaon-decay branching ratios and lifetime measurements by the FlaviaNet Working Group on Kaon Decays in 2014 [42]; we have now updated to the PDG [1]. The radiation-inclusive branching ratio $\mathcal{B}(K^+ \to \mu^+\nu[\gamma]) = 63.56(11)\%$ and lifetime $\tau_{K^\pm} = 12.380(20)$ ns are now used; this is a very small change. These measurements imply $\Gamma(K^+ \to \mu^+\nu[\gamma]) = 5.134(12) \times 10^7\ \text{s}^{-1}$. As before, we

[1] This uncertainty on $C_{\pi,K}$ is smaller than the error estimated by Marciano and Sirlin in Ref. [25], which predates the calculations of the hadronic-structure contributions in Refs. [5, 7, 23, 24]. The hadronic LECs incorporate the large short-distance electroweak logarithm discussed in Ref. [25], and their dependence on the chiral renormalization scale cancels the scale-dependence induced by chiral loops, thereby removing the dominant scale uncertainty of the Marciano–Sirlin analysis [25].

Table 71.1: Recent published lattice-QCD results for f_{π^+}, f_{K^+}, and their ratio. The upper and lower panels show $(2+1+1)$-flavor and $(2+1)$-flavor determinations, respectively. When two errors are shown, they are statistical and systematic, respectively. Results for f_π and f_K in the isospin symmetric limit $m_u = m_d$ are noted with a ‡; they are corrected for isospin breaking via Eq. (71.13) before computing the averages. The 2+1+1 FLAG 24 averages are dominated by the Fermilab/MILC values.

Reference	N_f	f_{π^+}(MeV)	f_{K^+}(MeV)	f_{K^+}/f_{π^+}
ETM 21 [30]	2+1+1	–	155.92(62)(9)	1.1957(44)(7)
CalLat 20 [31]	2+1+1	–	–	1.1942(32)(31)
Fermilab/MILC 17 [32] *	2+1+1	–	–	$1.1950(15)(^{+6}_{-18})$
ETM 14 [33] *	2+1+1	–	154.4(1.5)(1.3)	1.184(12)(11)
Fermilab/MILC 14 [9] * †	2+1+1	–	$155.92(13)(^{+42}_{-34})$	$1.1956(10)(^{+26}_{-18})$ † ‡
HPQCD 13 [34] *	2+1+1	–	155.37(20)(27)	1.1916(15)(16)
FLAG 24 average [2]	2+1+1	–	155.7(3)	1.1934(19)
CLQCD 23 [35]	2+1	130.7(0.9)(2.1)	155.6(0.8)(2.7)	11907(76)(17)
QCDSF/UKQCD 16 [36]	2+1	–	–	1.190(10)(13)
BMW 16 [37]	2+1	–	–	1.178(10)(26)
RBC/UKQCD 14 [38]‡	2+1	130.19(89)	155.51(83)	1.1945(45)
MILC 10 [39]	2+1	129.2(0.4)(1.4)	$156.1(4)(^{+6}_{-9})$	$1.197(2)(^{+3}_{-7})$
BMW 10 [40]‡	2+1	–	–	1.192(7)(6)
HPQCD/UKQCD 07 [41]‡	2+1	132(2)	157(2)	1.189(2)(7)
FLAG 24 average [2]	2+1	130.2(8)	155.7(7)	1.1916(34)

*PDG 2014 value of $f_{\pi^+} = 130.41(21)$ MeV used to set absolute lattice scale.
†Superseded by f_{K^+}/f_{π^+} from Fermilab/MILC 17.

subtract δ_K in Eq. (71.4) from the radiation-inclusive decay width to obtain $\Gamma^{(0)}(K^+ \to \mu^+\nu)$. We then use Eq. (71.1) to obtain

$$f_{K^+}|V_{us}| = (35.083 \pm 0.042 \pm 0.037)\ \text{MeV}\,, \tag{71.7}$$

where the errors are from the experimental rate measurement and the radiative correction factor, respectively.

Short-distance radiative corrections cancel in the ratio of pion-to-kaon decay rates [43]:

$$\frac{\Gamma_{K_{\ell 2[\gamma]}}}{\Gamma_{\pi_{\ell 2[\gamma]}}} = \frac{|V_{us}^2| f_{K^-}^2}{|V_{ud}|^2 f_{\pi^-}^2} \frac{m_K (1 - m_\ell^2/m_K^2)^2}{m_\pi (1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{K/\pi})\,, \tag{71.8}$$

where $\delta_{K/\pi}$ is given in Eq. (71.5). The left-hand side of Eq. (71.8) is 1.3367(32), which implies

$$\frac{|V_{us}| f_{K^+}}{|V_{ud}| f_{\pi^+}} = 0.27599 \pm 0.00033 \pm 0.00024\,, \tag{71.9}$$

where the first uncertainty is from the branching fractions and the second is from $\delta_{K/\pi}$. Here, the estimated error on the hadronic structure-dependent radiative corrections is commensurate with the experimental error.

In summary, the main experimental results pertaining to charged pion and kaon leptonic decays are

$$|V_{ud}|f_{\pi^+} = (127.13 \pm 0.01 \pm 0.13)\ \text{MeV}\,, \tag{71.10}$$

$$|V_{us}|f_{K^+} = (35.083 \pm 0.042 \pm 0.037)\ \text{MeV}\,, \tag{71.11}$$

$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = 0.27599 \pm 0.00033 \pm 0.00024\,, \tag{71.12}$$

where the errors are from the experimental uncertainties in the branching fractions and the theoretical uncertainties in the radiative correction factors δ_P, respectively. These values are unchanged since the last reviews [1].

71.2.2 *Theoretical decay-constant calculations*

Table 71.1 presents recent published results for the charged pion and kaon decay constants and their ratio from numerical lattice-QCD calculations with three ($N_f = 2+1$) or four flavors ($N_f = 2+1+1$) of dynamical quarks. The uncertainties on both the individual decay constants and their ratio are at the sub-percent level. The SU(3)-breaking ratio f_{K^+}/f_{π^+} can be obtained with especially small errors because statistical errors associated with the Monte Carlo calculations are correlated between the numerator and denominator, as are some systematics. The results in Table 71.1 were obtained using several independent sets of gauge-field configurations, and a variety of lattice fermion actions that are sensitive to different systematic uncertainties.[2,3] Thus, the good agreement between them indicates that the lattice-QCD uncertainties are controlled and the associated error estimates are reliable.[4]

Table 71.1 also shows the three- and four-flavor averages for the pion and kaon decay constants and their ratio from the FLAG 24 review [2] in the lines labeled "FLAG 24 average." There is no four-flavor average for the pion decay constant in Table 71.1 because all of the four-flavor calculations use the quantity $f_{\pi^+} = 130.41(20)$ MeV [22] as an input to fix the absolute lattice scale needed to convert from lattice-spacing units to GeV [9, 33, 34].

All of the results in Table 71.1 were obtained using isospin-symmetric gauge-field configurations, *i.e.*, the dynamical up and down quarks have the same mass. Fortunately, however, the leading effect of strong-isospin breaking is easily included in lattice-QCD calculations as follows. Because the up-down mass difference $\Delta m_{ud} \equiv (m_u - m_d) \sim -2.5$ MeV [49, 51] is much less than typical hadronic scales, the strong-isospin breaking corrections to physical observables can be systematically expanded in the small parameter $\delta m_{ud} \equiv \Delta m_{ud}/\Lambda_{\text{QCD}}$. The leading strong-isospin-breaking corrections to pseudoscalar-meson decay constants arise from the light *valence* quarks in the initial- and final-state hadrons. (See, *e.g.,* the discussion in Ref. [52], for a detailed discussion of isospin-breaking effects in pion and kaon observables.) Thus, to include the effect of non-degenerate up- and down-quark masses, most recent lattice-QCD calculations of f_{π^+} and f_{K^+} evaluate the masses of the valence quarks in the pion at the physical m_u and m_d, and the mass of the valence light quark in the kaon at the physical m_u. This procedure yields a correction to the kaon decay constant below 0.5%. Consequently, strong-isospin breaking corrections from the light sea-quark masses — which are suppressed by an additional power of δm_{ud} — can be neglected given present uncertainties.

Some earlier lattice-QCD calculations, however, only provide the decay constants and their ratio in the $SU(2)$ isospin-symmetric limit [38, 40, 41]. The Flavour Lattice Averaging Group corrects these results for strong-isospin breaking using chiral perturbation theory before including them in

[2] See the PDG mini-review on "Lattice Quantum Chromodynamics" [44–46] for a general review of numerical lattice-QCD calculations. Details on the different methods used in modern lattice-QCD calculations are provided in Appendix A of the FLAG "Review[s] of lattice results concerning low energy particle physics" [47–49].

[3] Note that one must be careful concerning issues of scheme dependence, particularly in the ambiguous separation of QED and QCD effects; see, e.g., Sec. 3 of Ref. [2].

[4] See the review by Kronfeld [50] for a summary of the large body of evidence validating the methods employed in modern lattice-QCD calculations.

the averages. The leading strong-isospin-breaking corrections to the pion and kaon decay constants in χPT can be parametrized as [26, 53]

$$f_{\pi^+} = f_\pi\,, \qquad f_{K^+} = f_K\sqrt{1+\delta_{\mathrm{SU}(2)}}\,, \tag{71.13}$$

where f_π and f_K denote the values of the decay constants in the isospin-symmetric limit. The pion decay constant does not receive corrections linear in $m_u - m_d$ because of the G-parity symmetry[5] of the pion triplet, so at first order the δm_{ud} expansion, strong-isospin breaking corrections are characterized by a single parameter, $\delta_{\mathrm{SU}(2)}$. Next-to-leading order χPT yields numerical values for $\delta_{\mathrm{SU}(2)}$ of approximately -0.004. Recent direct lattice-QCD calculations of $\delta_{\mathrm{SU}(2)}$ give larger values of around -0.005 to -0.008 [10, 32–34, 52, 54], but further studies are needed. Thus, to be conservative, FLAG includes an uncertainty of 100% on the χPT estimate for $\delta_{\mathrm{SU}(2)}$ when correcting those decay-constant values that are quoted in the isospin-symmetric limit.

The errors on the decay-constant results in Table 71.1 obtained from $(2+1)$-flavor lattice-QCD calculations do not include an estimate of the systematic uncertainty from the omission of charm sea quarks in the calculation. Consequently, when the uncertainty on the $(2+1+1)$-flavor FLAG average is comparable to or better than that on the $(2+1)$-flavor FLAG average, we simply use the four-flavor average as our preferred value. This is not possible, however, for the pion decay constant. To account for this, we first estimate the systematic uncertainty on pseudoscalar-meson decay constants associated with the omission of charm sea quarks. We then add this estimate in quadrature to the quoted error on the $(2+1)$-flavor FLAG average for f_{π^+} to obtain our preferred value.

The error introduced by omitting charm sea quarks can be roughly estimated by expanding the charm-quark determinant in powers of $1/m_c$ [55]; the resulting leading contribution is of order $\alpha_s\,(\Lambda_{\mathrm{QCD}}/2m_c)^2$ [56]. Taking the $\overline{\mathrm{MS}}$ values $\overline{m}_c(\overline{m}_c) = 1.275$ GeV, $\overline{\Lambda}_{\mathrm{QCD}} \sim 340$ MeV from FLAG [47], and $\overline{\alpha}_s(\overline{m}_c) \sim 0.4$, leads to an estimate of about 0.7% for the contribution to the decay constants from charm sea quarks. We can compare this power-counting estimate of charm sea-quark contributions with the observed differences between the (2+1)- and (2+1+1)-flavor lattice-QCD averages for kaon, $D_{(s)}$-meson, and $B_{(s)}$-decay constants in Tables 71.1, 71.4, and 71.6. Looking at Table 71.1, the three- and four-flavor averages for f_{K^+} agree to much better than our simple power-counting estimate. Inspection of Tables 71.4 and 71.6 shows, however, that charm sea-quark effects of this size are still allowed for both $D_{(s)}$-meson and $B_{(s)}$-meson decay constants.

Our final preferred theoretical values for the charged pion and kaon decay constants are

$$f_{\pi^+} = 130.2(1.2)\ \mathrm{MeV}\,, \quad f_{K^+} = 155.7(3)\ \mathrm{MeV}\,, \quad \frac{f_{K^+}}{f_{\pi^+}} = 1.193(2)\,, \tag{71.14}$$

where f_{K^+} and f_{K^+}/f_{π^+} are simply the four-flavor FLAG 2024 averages [2], and f_{π^+} is the three-flavor FLAG 2024 average with the error increased by the estimated 0.7% charm sea-quark contribution.

71.3 Charmed mesons

71.3.1 *Experimental rate measurements*

Measurements have been made of the branching fractions for D^+ and D_s^+ mesons decaying to both $\mu^+\nu$ and $\tau^+\nu$ final states. The CLEO-c and BESIII experiments have made measurements of D^+ decays using e^+e^- collisions at the $\psi(3770)$ resonant energy where D^-D^+ pairs are copiously produced. They fully reconstruct one of the D mesons; for concreteness, we will take this to be the D^-. Counting the number of these events provides the normalization for the branching fraction

[5] This is a combination of charge conjugation, C, and an $e^{i\pi I_2}$ rotation in isospin space that maps the triplet onto itself.

measurement. The experimental analyses then proceed by identifying a candidate μ^+ and forming the missing-mass squared, $MM^2 = (E_{\rm cm} - E_{D^-})^2 - \left(\vec{p}_{\rm cm} - \vec{p}_{D^-} - \vec{p}_{\mu^+}\right)^2$, where $E_{\rm cm}$ and $p_{\rm cm}$ are the center-of-mass energy (which is known) and momentum (which equals zero in e^+e^- collisions). A peak at zero MM^2 implies the existence of a missing neutrino, and hence the $\mu^+\nu$ decay of the D^+. CLEO-c does not explicitly identify the muon, so their data consists of a combination of $\mu^+\nu$ and $\tau^+\nu$, $\tau^+ \to \pi^+\nu$ events. This permits them to do two fits: in one they fit for the individual components, and in the other they fix the ratio of $\tau^+\nu/\mu^+\nu$ events to be that given by the standard-model expectation. Thus, the former measurement should be used for new-physics searches, and the latter for standard-model predictions. Our average uses the fixed-ratio value; this ratio is implied by Eq. 71.1 which is necessary to extract $|V_{cd}|f_{D^+}$.

Table 71.2: Experimental results for $\mathcal{B}(D^+ \to \mu^+\nu[\gamma])$, $\mathcal{B}(D^+ \to \tau^+\nu[\gamma])$, and $|V_{cd}|f_{D^+}$. The first two uncertainties are experimental statistics and systematics, respectively. The third uncertainty on the extracted $|V_{cd}|f_{D^+}$ is from external inputs: masses, lifetimes, and radiative corrections. The uncertainty on the latter item dominates this third value. In the second CLEO line the τ^+ decays to $\mu^+\nu_\mu\bar{\nu}_\tau$.

Experiment	Mode	$\mathcal{B}$	$\lvert V_{cd}\rvert f_{D^+}$ (MeV)
CLEO-c [57]	$\mu^+\nu$	$(3.93 \pm 0.35 \pm 0.09) \times 10^{-4}$	$46.80 \pm 2.08 \pm 0.54 \pm 0.34$
CLEO-c [57]	$\mu^+\nu + \tau^+\nu$	$(3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$	$46.14 \pm 1.93 \pm 0.54 \pm 0.34$
BESIII [58]	$\mu^+\nu$	$(4.034 \pm 0.080 \pm 0.040) \times 10^{-4}$	$47.42 \pm 0.47 \pm 0.24 \pm 0.35$
Our average	Lines 2+3	$(4.020 \pm 0.078 \pm 0.038) \times 10^{-4}$	$47.33 \pm 0.46 \pm 0.25 \pm 0.35$
CLEO-c [57]	$\tau^+\nu$ ($\pi^+\overline{\nu}$)	$< 1.2 \times 10^{-3}$	
BESIII [59]	$\tau^+\nu$ ($\pi^+\overline{\nu}$)	$(0.99 \pm 0.11 \pm 0.05) \times 10^{-3}$	$45.48 \pm 2.53 \pm 1.15 \pm 0.34$
Our average	$\mu^+\nu + \tau^+\nu$		$47.27 \pm 0.45 \pm 0.22 \pm 0.35$

Table 71.2 shows the available measurements of $D^+ \to \mu^+\nu$, an upper limit on $D^+ \to \tau^+\nu$ from CLEO-c, and the first measurement of this decay from BESIII. To extract the values of $|V_{cd}|f_{D^+}$ via Eq. (71.1), we use [1] the mass $m_{D^+} = 1.86966(5)$ GeV and the lifetime $\tau_{D^+} = 1.033(5)$ ps, and apply radiative corrections as described below. For calculating the average $\mu^+\nu$ number, we use the CLEO-c result from $\mu^+\nu + \tau^+\nu$.

To obtain the purely leptonic rates $\Gamma^{(0)}(D^+ \to \mu^+(\tau^+)\nu)$, we subtract the radiative contributions as in Sec. 71.2.1, but use numerical values for the corrections appropriate for D mesons. First, we reduce both the $\mu^+\nu$ and $\tau^+\nu$ branching fractions in Table 71.2 by 1.8%, which is the universal short-distance electroweak contribution of Sirlin [3] evaluated using the D-meson mass for the factorization scale. We do not adjust the experimental rates by the universal long-distance correction [4]. This is because QED bremsstrahlung contributions have already been subtracted at leading-log order from the measurements in Table 71.2 using Monte-Carlo estimates computed with PHOTOS [11]. The $\mu^+\nu$ rates should also be reduced by the 1% estimate of the structure-dependent contributions from Dobrescu and Kronfeld [16]. This correction accounts for tree-level radiative processes in which the D meson decays into a real photon and an off-shell vector meson, which subsequently decays weakly to a charged lepton and neutrino. It is estimated using Eq. (12) of Burdman *et al.* [6] with the CLEO-c cut on the photon energy from Ref. [72], which is typical of all the measurements. We do not need to apply the structure-dependent correction to the $\mu^+\nu$ branching fractions in Table 71.2, however, because the experiments have already included it in their quoted results. CLEO-c directly subtracts 1%, while BESIII simulates these structure- depen-

Table 71.3: Experimental results for $\mathcal{B}(D_s^+ \to \mu^+\nu[\gamma])$, $\mathcal{B}(D_s^+ \to \tau^+\nu[\gamma])$, and $|V_{cs}|f_{D_s^+}$. The uncertainties have the same meaning as in Table 71.2. The single-experiment average BFs are taken from the cited papers, for proper treatment of correlated systematic uncertainties.

Experiment	Mode	$\mathcal{B}(\%)$	$\lvert V_{cs}\rvert f_{D_s^+}$ (MeV)
CLEO-c [60]	$\mu^+\nu$	$0.565 \pm 0.045 \pm 0.017$	$248.2 \pm 9.9 \pm 3.7 \pm 1.8$
BaBar [61]	$\mu^+\nu$	$0.596 \pm 0.038 \pm 0.035$	$254.9 \pm 8.1 \pm 7.5 \pm 1.9$
Belle [62]	$\mu^+\nu$	$0.526 \pm 0.028 \pm 0.021$	$239.5 \pm 6.4 \pm 4.8 \pm 1.8$
BESIII [63]	$\mu^+\nu$	$0.517 \pm 0.075 \pm 0.021$	$237.4 \pm 17.2 \pm 4.9 \pm 1.7$
BESIII [64]	$\mu^+\nu$	$0.5294 \pm 0.0108 \pm 0.0085$	$240.3 \pm 2.5 \pm 1.9 \pm 1.8$
BESIII [65]	$\mu^+\nu$	$0.542 \pm 0.026 \pm 0.017$	$243.1 \pm 5.8 \pm 3.8 \pm 1.8$
BESIII [65]	average	$0.5304 \pm 0.0099 \pm 0.0090$*	$240.5 \pm 2.2 \pm 2.0 \pm 1.8$
Our average	$\mu^+\nu$	$0.534 \pm 0.009 \pm 0.008$	$241.6 \pm 2.0 \pm 1.7 \pm 1.8$
CLEO-c [60]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$6.42 \pm 0.81 \pm 0.18$	$268.0 \pm 16.9 \pm 3.8 \pm 2.0$
CLEO-c [66]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$5.30 \pm 0.47 \pm 0.22$	$243.5 \pm 10.8 \pm 5.1 \pm 1.8$
CLEO-c [67]	$\tau^+\nu$ $(\rho^+\overline{\nu})$	$5.52 \pm 0.57 \pm 0.21$	$248.5 \pm 12.8 \pm 4.7 \pm 1.8$
CLEO-c [67]	average	$5.58 \pm 0.33 \pm 0.13$	$249.9 \pm 7.4 \pm 2.9 \pm 1.8$
BaBar [61]	$\tau^+\nu$ $(e^+(\mu^+)\nu\overline{\nu})$	$4.96 \pm 0.37 \pm 0.57$	$235.6 \pm 8.8 \pm 13.5 \pm 1.7$
Belle [62]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$6.04 \pm 0.43^{+0.46}_{-0.40}$	$260.0 \pm 9.3^{+9.9}_{-8.6} \pm 1.9$
Belle [62]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$5.37 \pm 0.33^{+0.35}_{-0.31}$	$245.1 \pm 7.5^{+8.0}_{-7.1} \pm 1.8$
Belle [62]	$\tau^+\nu$ $(\mu^+\nu\overline{\nu})$	$5.86 \pm 0.37^{+0.34}_{-0.59}$	$256.1 \pm 8.1^{+7.4}_{-12.9} \pm 1.9$
Belle [62]	average	$5.70 \pm 0.21^{+0.31}_{-0.30}$	$252.5 \pm 4.7^{+6.9}_{-6.7} \pm 1.9$
BESIII [63]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$3.28 \pm 1.83 \pm 0.37$	$192 \pm 53 \pm 11 \pm 1.4$
BESIII [68]	$\tau^+\nu$ $(\pi^+\pi^0\overline{\nu})$	$5.29 \pm 0.25 \pm 0.20$	$243.3 \pm 5.8 \pm 4.6 \pm 1.8$
BESIII [69]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$5.27 \pm 0.10 \pm 0.12$	$242.8 \pm 2.3 \pm 3.0 \pm 1.8$
BESIII [70]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$5.44 \pm 0.17 \pm 0.13$	$246.7 \pm 3.9 \pm 3.0 \pm 1.8$
BESIII [71]	$\tau^+\nu$ $(\mu^+\nu\overline{\nu})$	$5.37 \pm 0.17 \pm 0.15$	$245.1 \pm 3.9 \pm 3.4 \pm 1.8$
BESIII [65]	$\tau^+\nu$ $(e, \mu, \pi, \pi\pi)$	$5.60 \pm 0.16 \pm 0.20$	$250.3 \pm 3.6 \pm 4.5 \pm 1.8$
BESIII [65]	average	$5.359 \pm 0.067 \pm 0.075$	$244.9 \pm 1.5 \pm 1.7 \pm 1.8$
Our average	$\tau^+\nu$	$5.388 \pm 0.063 \pm 0.068$	$245.5 \pm 1.4 \pm 1.6 \pm 1.8$
Our average	$\mu^+\nu + \tau^+\nu$		$244.0 \pm 1.2 \pm 1.2 \pm 1.8$

*An error in Ref. [65] has been corrected here; BESIII Collab., private communication.

dent radiative decays and adds a fit component describing them. Therefore, in summary, we reduce both the $D^+ \to \mu^+\nu$ and the $D^+ \to \tau^+\nu$ rates by 1.8% to account for radiative corrections. It is worth noting, however, that the universal long-distance electromagnetic contribution estimated for point-like charged mesons by Kinoshita [4], which we are not including because IB contributions are already subtracted from the measurements via PHOTOS, would *increase* both rates by about 2.5%.

We now discuss the D_s^+ decay process; there are significant updates since our previous review [1] due to new results from BESIII as well as our averaging strategy. Measurements of D_s^+ leptonic decay branching fractions by several groups are listed in Table 71.3. We exclude older values obtained by normalizing to D_s^+ decay modes that are not well-defined. Many measurements, for example, used the $\phi\pi^+$ mode. This decay is a subset of the $D_s^+ \to K^+K^-\pi^+$ channel which has interferences from other modes populating the K^+K^- mass region near the ϕ, the most prominent of which is the $f_0(980)$. Thus, the extraction of the effective $\phi\pi^+$ rate is sensitive to the mass

resolution of the experiment and the cuts used to define the ϕ mass region [73].[6]

To find D_s decays in the $\mu^+\nu$ signal channels, the experiments rely on fully reconstructing all of the final state particles except for the neutrino and use a missing-mass technique to infer the existence of the neutrino. CLEO and BESIII use $e^+e^- \to D_sD_s^*$ collisions at 4170 MeV, while BaBar and Belle use $e^+e^- \to DKn\pi D_s^*$ collisions at energies near the $\Upsilon(4S)$. CLEO and BESIII do a similar analysis as was done for the D^+ above. BaBar and Belle do a similar MM^2 calculation by using the reconstructed hadrons, the photon from the D_s^{*+} decay and a detected μ^+. To get the normalization they do a MM^2 fit without the μ^+ and use the signal at the D_s^+ mass squared to determine the total D_s^+ yield.

When selecting the $\tau^+ \to \pi^+\bar{\nu}$ and $\tau^+ \to \rho^+\bar{\nu}$ decay modes, CLEO uses both the calculation of the missing-mass and the fact that there should be no extra energy in the event beyond that deposited by the measured tagged D_s^- and the τ^+ decay products. The $\tau^+ \to e^+\nu\bar{\nu}$ mode, however, uses only extra energy. BaBar and Belle also use the extra energy to discriminate signal from background in their $\tau^+\nu$ measurements. BESIII uses $\tau^+ \to \pi^+\overline{\nu}$ decays, where they calculate the MM^2 and discriminate against μ^+ from $D_s^+ \to \mu^+\nu$ decays, and more recently $\tau^+ \to e^+\nu\overline{\nu}$ and $\tau^+ \to \pi^+\pi^0\overline{\nu}$ decays.

When extracting $|V_{cs}|f_{D_s^+}$ via Eq. (71.1), we first apply the -1.8% universal electroweak correction [3] to all of the $\mu^+\nu$ and $\tau^+\nu$ branching fractions in Table 71.3; this is the same as for D^+ mesons. We also decrease the BaBar, Belle, and third BESIII $\mu^+\nu$ branching fractions by the 1% structure-dependent correction [16]. This correction was already included in CLEO and other BESIII results for the $\mu^+\nu$ branching fractions in Table 71.3. We use the masses and lifetimes $m_{D_s^+} = 1.96835(7)$ GeV, $m_{\tau^+} = 1.77686(12)$ GeV, and $\tau_{D_s^+} = 0.5012(22)$ ps [1]. The inferred values for $f_{D_s^+}|V_{cs}|$ are in good agreement for the $\mu^+\nu$ and$\tau^+\nu$ decay modes.

It is clear from the discussion of radiative corrections in this section that they are less well understood theoretically for D^+ and D_s^+ meson decays than for pions and kaons. We therefore assign a 1.4% systematic uncertainty to the purely leptonic decay rates, which is half the size of the applied radiative corrections. This translates to a 0.7% error on the products of the decay constant times CKM matrix element. Putting everything together, the main experimental results pertaining to charmed meson leptonic decays are (see the bottom lines of Tables 71.2 and 71.3):

$$|V_{cd}|f_{D^+} = 47.27(45)(22)(35) = 47.27(0.61)\ \text{MeV}\,, \tag{71.15}$$

$$|V_{cs}|f_{D_s^+} = 244.0(1.2)(1.2)(1.8) = 244.0(2.4)\ \text{MeV}\,, \tag{71.16}$$

where the errors are the statistical and systematic uncertainties from the measured branching fractions, and the external inputs (including the applied radiative corrections), respectively.

71.3.2 *Theoretical decay-constant calculations*

Table 71.4 presents recent theoretical calculations of charmed heavy-light meson decay constants and their ratio in the isospin-symmetric limit $m_u = m_d$. (As in Sec. 71.2.2, we denote the physical D^+-meson decay constant by f_{D^+}, and use f_D for the isospin-symmetric value.) The upper two panels show results from lattice-QCD calculations with three ($N_f = 2+1$) or four flavors ($N_f = 2+1+1$) of dynamical quarks. Although there are fewer available results than for the pion and kaon sector, both f_D and f_{D_s} have been obtained using multiple sets of gauge-field configurations with different lattice fermion actions, providing independent confirmation. For comparison, the bottom panel of Table 71.4 shows QCD-model calculations of the D- and D_s-meson decay constants for which uncertainty estimates are provided. The lattice and non-lattice results agree, but numerical lattice-QCD calculations have now reached significantly greater precision than other approaches.

[6] We do not include two BaBar results [74,75] for $\mathcal{B}(D_s^+ \to \mu^+\nu)$ and $\mathcal{B}(D_s^+ \to \tau^+\nu)$ that normalize to other D_S^+ decay modes.

Table 71.4: Recent theoretical determinations of f_D, f_{D_s}, and their ratio in the isospin-symmetric limit. The upper panels show results from lattice-QCD calculations with $(2+1+1)$ and $(2+1)$ dynamical quark flavors, respectively. Statistical and systematic errors are quoted separately. The bottom panel shows estimates from QCD sum rules (QCD SR). These are not used to obtain our preferred decay-constant values. The 2+1+1 averages are dominated by the Fermilab/MILC values.

Reference	Method	N_f	f_D(MeV)	f_{D_s}(MeV)	f_{D_s}/f_D
Fermilab/MILC 17 [32]	LQCD	2+1+1	212.1(0.3)(0.5)	249.9(0.3)(0.3)	1.1782(06)(15)*
ETM 14 [33]	LQCD	2+1+1	207.4(3.7)(0.9)	247.2(3.9)(1.4)	1.192(19)(11)
FLAG 24 average [76]	LQCD	2+1+1	212.0(0.7)	249.9(0.5)	1.1783(16)
ALPHA 23 [77]	211.3(1.9)(0.6)	247.0(1.9)(0.7)	1.177(15)(5)		
χQCD 20A [78]	LQCD	2+1	213(5)	249(7)	1.16(3)
RBC/UKQCD 18A [79]†	LQCD	2+1	–	–	1.1740(51)(68)
RBC/UKQCD 17 [80]	LQCD	2+1	$208.7(2.8)(^{+2.1}_{-1.8})$	$246.4(1.3)(^{+1.3}_{-1.9})$	$1.1667(77)(^{+57}_{-43})$
χQCD 14 [81]	LQCD	2+1	–	254(2)(4)	–
HPQCD 12 [82]	LQCD	2+1	208.3(1.0)(3.3)	–	1.187(4)(12)
Fermilab/MILC 11 [83]	LQCD	2+1	218.9(9.2)(6.6)	260.1(8.9)(6.1)	1.188(14)(21)
HPQCD 10 [84]	LQCD	2+1	–	248.0(1.4)(2.1)	–
FLAG 24 average [76]	LQCD	2+1	210.4(1.5)	247.7(1.2)	1.174(7)
Pullin 21 [85]	QCD SR		190(15)	226(17)	1.19(7)
Wang 15 [86]‡	QCD SR		208(10)	240(10)	1.15(6)
Gelhausen 13 [87]	QCD SR		$201\binom{+12}{-13}$	$238\binom{+13}{-23}$	$1.15\binom{+0.04}{-0.05}$
Narison 12 [88]	QCD SR		204(6)	246(6)	1.21(4)
Lucha 11 [89]	QCD SR		206.2(8.9)	245.3(16.3)	1.193(26)

*Ref. [32] provides values for f_D and f_{D_s} in the isopsin-symmetric limit, but not for their ratio. Here we infer the central value from those of the individual decay constants, and take the statistical and systematic errors to be the same as for the physical ratio f_{D_s}/f_{D^+}.

†Slight difference from preliminary value quoted in 2019 FLAG review.

‡Obtained using $m_c^{\overline{\rm MS}}$; results using $m_c^{\rm pole}$ are also given in the paper.

The lattice-QCD decay-constant results in Table 71.4 were all obtained using isospin-symmetric gauge-field configurations. As discussed in Sec. 71.2.2, however, the leading strong-isospin breaking corrections to heavy-light pseudoscalar-meson decay constants can be accounted for by using the physical down (or up) quark in the D (or B) meson. Strong-isospin breaking corrections to heavy-strange meson decay constants are roughly an order-of-magnitude smaller because there are no light valence quarks involved. Recently, the Fermilab Lattice and MILC Collaborations used this approach to calculate directly the leading strong-isospin breaking corrections to both f_D and f_B, finding [32]

$$\begin{aligned} f_{D^+} - f_D &= +0.58(1)(7)(1)\ \text{MeV}\,, \\ f_{B^+} - f_B &= -0.53(5)(7)(0)\ \text{MeV}\,, \end{aligned} \tag{71.17}$$

where the uncertainties are from statistics, systematics, and the EM scheme, respectively. These results agree with independent estimates of the strong-isospin-breaking corrections to heavy-light meson decay constants from QCD sum rules [90]. Combined with the determinations of f_D and f_B from the same work, Eq. (71.17) implies that the corrections to the SU(3)-flavor breaking ratios are

$$\frac{f_{D_s}}{f_{D^+}} = \frac{f_{D_s}}{f_D}(1 - 0.0027(3))\,, \qquad \frac{f_{B_s}}{f_{B^+}} = \frac{f_{B_s}}{f_B}(1 + 0.0028(5))\,. \tag{71.18}$$

These estimated strong-isospin-breaking corrections to f_D and f_{D_s}/f_D above are commensurate with the uncertainties on the (2+1+1)-flavor FLAG averages in Table 71.4. Consequently, it is important to account for isospin-breaking effects before combining the theoretical decay constants with the corresponding experimental decay rates.

To obtain the charged D^+-meson decay constant, we apply the correction in Eq. (71.17) to the (2+1+1)-flavor FLAG 2024 average for the D-meson decay constant in the isospin-symmetric limit. Similarly we use Eq. (71.18) to correct the (2+1+1)-flavor FLAG 2024 average for f_{D_s}/f_D. We take the four-flavor FLAG 2024 average for f_{D_s} directly. Our final preferred theoretical values for the charmed pseudoscalar-meson decay constants are

$$f_{D^+} = 212.6(7)\ \text{MeV}\,, \quad f_{D_s} = 249.9(5)\ \text{MeV}\,, \quad \frac{f_{D_s}}{f_{D^+}} = 1.175(2)\,. \tag{71.19}$$

71.4 Bottom mesons

71.4.1 *Experimental rate measurements*

The Belle and BaBar collaborations have found evidence for $B^- \to \tau^- \overline{\nu}$ decay in $e^+e^- \to B^-B^+$ collisions at the $\Upsilon(4S)$ energy. The analysis relies on reconstructing a hadronic or semi-leptonic B decay tag, finding a τ candidate in the remaining track and photon candidates, and examining the extra energy in the event which should be close to zero for a real τ^- decay to $e^-\nu\bar{\nu}$ or $\mu^-\nu\bar{\nu}$ opposite a B^+ tag. The results are listed in Table 71.5. These are unchanged since the previous review [1]. But we note a very recent result from Belle II using hadronic tagging [91] which gives $\mathcal{B}(B^+ \to \tau^+\nu_\tau) = (1.24 \pm 0.41 \pm 0.19) \times 10^{-4}$; this is not yet included in our averages pending an evaluation of correlated systematic uncertainties.

Table 71.5: Experimental results for $\mathcal{B}(B^- \to \tau^-\overline{\nu})$ and $|V_{ub}|f_{B^+}$. External systematic uncertainties from masses and lifetimes are negligible compared to current experimental uncertainties.

Experiment	Tag	$\mathcal{B}$ (units of 10^{-4})	$\lvert V_{ub}\rvert f_{B^+}$ (MeV)
Belle [92]	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$	
Belle [93]	Semileptonic	$1.25 \pm 0.28 \pm 0.27$	
Belle [93]	Average	0.91 ± 0.22	0.72 ± 0.09
BaBar [94]	Hadronic	$1.83\,^{+0.53}_{-0.49} \pm 0.24$	
BaBar [95]	Semileptonic	$1.7 \pm 0.8 \pm 0.2$	
BaBar [94]	Average	1.79 ± 0.48	1.01 ± 0.14
Our average		1.06 ± 0.20	0.77 ± 0.07

Because there are large backgrounds under the signals for these measurements, as well as substantial systematic errors, the significances of the individual results are still below the 5σ discovery threshold. Belle quotes 4.6σ for their combined hadronic and semileptonic tags, while BaBar quotes 3.3σ and 2.3σ, for hadronic and semileptonic tags. Greater precision is necessary to determine if any effects beyond the standard model are present.

We do not correct the measured branching ratios in Table 71.5 for radiative corrections because the experimental uncertainties are so large. The relative radiative corrections are expected to be larger, however, for $B \to \mu\nu$ leptonic decays because the corrections are no longer helicity suppressed [8], and may be a significant fraction of the purely leptonic rate. More theoretical work is needed to understand radiative corrections to leptonic B decays in anticipation of future measurements with greater precision, and of new decay channels.

71.4.2 *Theoretical decay-constant calculations*

Table 71.6 presents recent theoretical calculations of bottom heavy-light meson decay constants and their ratio in the isospin-symmetric limit $m_u = m_d$. The upper two panels show results from lattice-QCD calculations with three ($N_f = 2 + 1$) or four flavors ($N_f = 2 + 1 + 1$) of dynamical quarks. For all decay constants, calculations using different gauge-field configurations, light-quark actions, and b-quark actions provide independent confirmation. For comparison, the bottom panel of Table 71.6 shows QCD-model calculations of the B- and B_s-meson decay constants for which uncertainty estimates are provided. These are consistent with the lattice values, but with much larger uncertainties.

Table 71.6: Recent theoretical determinations of f_B, f_{B_s}, and their ratio in the isospin-symmetric limit. The upper panels show results from lattice-QCD calculations with $(2 + 1 + 1)$ and $(2 + 1)$ dynamical quark flavors, respectively. When available, statistical and systematic errors are quoted separately. The bottom panel shows estimates from the relativistic potential model (RPM) and QCD sum rules (QCD SR), which are not used to obtain our preferred decay-constant values. The 2+1+1 FLAG 24 averages are dominated by the Fermilab/MILC values.

Reference	Method	N_f	f_B(MeV)	f_{B_s}(MeV)	f_{B_s}/f_B
Frezzotti 24 [96]	LQCD	2+1+1	–	224.5(5.0)	–
Fermilab/MILC 17 [32]	LQCD	2+1+1	189.9(1.4)	230.7(1.2)	1.2146(49)
HPQCD 17 [97]*	LQCD	2+1+1	190(4)	229(5)	1.206(5)
ETM 16 [98]	LQCD	2+1+1	193(6)	229(5)	1.184(25)
HPQCD 13 [99]	LQCD	2+1+1	186(4)	224(5)	1.205(7)
FLAG 24 average [76]	LQCD	2+1+1	190.0(1.3)	230.3(1.3)	1.209(5)
Aoki 14 [100]†	LQCD	2+1	218.8(6.5)(30.8)	263.5(4.8)(36.7)	1.193(20)(44)
RBC/UKQCD 14 [101]‡	LQCD	2+1	195.6(6.4)(13.3)	235.4(5.2)(11.1)	1.223(14)(70)
HPQCD 12 [102]	LQCD	2+1	191(1)(8)	228(3)(10)	1.188(12)(13)
HPQCD 12 [102]	LQCD	2+1	189(3)(3)*	–	–
HPQCD 11 [103]	LQCD	2+1	–	225(3)(3)	–
Fermilab/MILC 11 [83]	LQCD	2+1	196.9(5.5)(7.0)	242.0(5.1)(8.0)	1.229(13)(23)
FLAG 24 average [76]	LQCD	2+1	192.0(4.3)	228.4(3.7)	1.201(16)
Pullin 21 [85]	QCD SR		192^{+20}_{-19}	225^{+21}_{-20}	1.17(7)
Sun 16 [104]§	RPM		219(15)	266(19)	1.21(9)
Wang 15 [86]§	QCD SR		194(15)	231(16)	1.19(10)
Baker 13 [105]	QCD SR		186(14)	222 (12)	1.19(4)
Lucha 13 [106]	QCD SR		192.0(14.6)	228.0(19.8)	1.184(24)
Gelhausen 13 [87]	QCD SR		$207\binom{+17}{-9}$	$242\binom{+17}{-12}$	$1.17\binom{+3}{-4}$
Narison 12 [88]	QCD SR		206(7)	234(5)	1.14(3)

*Average of new HPQCD 17 and previous HPQCD 13 results allowing for correlations between the two sets of results.
†Obtained with static b quarks (*i.e.*, $m_b \to \infty$).
‡Ref. [101] does not provide results in the isospin-symmetric limit, so we show f_{B^+} and f_{B_s}/f_{B^+} for this work.
§Obtained using $m_b^{\overline{\rm MS}}$; results using $m_b^{\rm pole}$ are also given in the paper.

The lattice-QCD decay-constant results in Table 71.6 were all obtained using isospin-symmetric gauge-field configurations. Some calculations, however, account for the leading effect of strong-isospin-breaking by using the correct value for the valence light-quark mass in the B meson (m_u for

f_{B^+} and m_d for f_{B^0}). Early estimates of the strong-isospin-breaking correction obtained $f_{B^+} - f_B \sim -2$ MeV [99,101], which would be significant given the present lattice-QCD uncertainties. It turns out, however, that these calculations inadvertently introduced a spurious sea-quark contribution, and therefore overestimated the size of the effect. A more recent calculation by the Fermilab/MILC Collaboration finds very little evidence for isospin violation ($f_{B^+} - f_B \sim -0.5$ MeV) [32], which is more than two times smaller than the total uncertainties on present lattice-QCD calculations. For this reason, we quote isospin averages in the current review. Our preferred theoretical values for the bottom pseudoscalar-meson decay constants are simply the $N_f = 2+1+1$ FLAG 2024 averages [2], after inclusion of the isospin-breaking corrections from Eq. (71.17) and Eq. (71.18):

$$f_{B^+} = 189.5(1.3)\ \text{MeV}\,, \quad f_{B_s} = 230.3(1.3)\ \text{MeV}\,, \quad \frac{f_{B_s}}{f_{B^+}} = 1.212(5)\,. \tag{71.20}$$

Because the uncertainties on the three-flavor results in Table 71.6 are substantially larger than those on the four-flavor results, including them in the average leaves the central values almost unchanged, and decreases the errors only slightly. Note that Ref. [96] is not included in the f_{B_s} average since it was published after the FLAG 24 deadline; including it would decrease f_{B_s} by about 0.3 MeV with little effect on the uncertainty.

71.5 Phenomenological implications

71.5.1 $|V_{ud}|$, $|V_{us}|$, and status of first-row unitarity

Using the average values for $f_{\pi^+}|V_{ud}|$, $f_{K^+}|V_{us}|$, and their ratio from Eqs. (71.10)–(71.12) and for f_{π^+}, f_{K^+}, and their ratio from Eq. (71.14), we obtain the following determinations of the CKM matrix elements $|V_{ud}|$, $|V_{us}|$, and their ratio from leptonic decays within the standard model:

$$|V_{ud}| = 0.9763(1)(90)(10)\,, \quad |V_{us}| = 0.2253(3)(4)(2), \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2313(3)(4)(2)\,, \tag{71.21}$$

where the errors are from the experimental branching fraction(s), the pseudoscalar decay constant(s), and radiative corrections, respectively. These results enable a test of the unitarity of the first row of the CKM matrix from leptonic decays alone (the contribution from $|V_{ub}|$ is negligible). Using the values of $|V_{ud}|$ and $|V_{us}|$ from Eq. (71.21), we find

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.004(18)\,, \tag{71.22}$$

which is consistent with three-generation unitarity at the few-percent level.

The determinations of $|V_{ud}|$ and $|V_{us}|$ from leptonic decays in Eq. (71.21) can be compared to those obtained from other processes. The result above for $|V_{ud}|$ agrees with the determination from super-allowed β-decay (a previous value of $|V_{ud}| = 0.97420(21)$ [107] has been updated to $|V_{ud}| = 0.97373(31)$ [108]) but has a considerably larger error, primarily due to the uncertainty in the theoretical determination of f_{π^+}.

The CKM element $|V_{us}|$ can be determined from semileptonic $K^+ \to \pi^0 \ell^+ \nu$ decay. Here experimental measurements provide a value for the product $f_+^{K\pi}(0)|V_{us}|$, where $f_+^{K\pi}(0)$ is the form factor at zero four-momentum transfer between the initial state kaon and the final state pion. We quote a recent analysis by Seng *et al.* [109] which finds $|V_{us}| f_+^{K\pi}(0) = 0.21635(39)(3)$, with the first uncertainty from experiment and the second reflecting uncertainty in higher order corrections. This average involves detailed considerations of radiative corrections; these are taken from chiral perturbation theory with lattice-QCD inputs for some needed low-energy constants. Choosing the FLAG 2024 2+1+1-flavor average for $f_+(0)^{K\pi} = 0.9698(17)$ [2] based on the calculations of

ETM [110] and Fermilab/MILC [111] gives $|V_{us}| = 0.22309(39)_{\rm exp}(39)_{\rm LQCD}(3)_{\rm HO}$ from semileptonic decay. The determinations of $|V_{us}|$ from leptonic and semileptonic kaon decays are both quite precise, with comparable errors, but the central values differ by 2.8σ.

Finally, the combination of the ratio $|V_{us}|/|V_{ud}|$ from leptonic decays [Eq. (71.21)] with $|V_{ud}|$ from β decay implies an alternative determination of $|V_{us}| = 0.2252(5)$ which agrees with the value from leptonic kaon decay, but again disagrees with the semileptonic-decay result at the 2.8σ level.

Given the roughly 3σ tension between $|V_{us}|$ from leptonic and semileptonic kaon decays, it is important to scrutinize the uncertainties on the theoretical and experimental inputs to $|V_{us}|$ and other elements of the first row of the CKM matrix. Seng *et al.* have introduced a new approach for calculating radiative corrections to neutron and nuclear beta decays using dispersion relations [112–115]. These calculations imply a value of $|V_{ud}| = 0.97395(23)$ [113], consistent with the updated analysis of Ref. [108]. An independent calculation of the radiative corrections by Czarnecki and Marciano using QCD sum rules yields similar results [116]. Using this value of $|V_{ud}|$ with the determination of $|V_{us}|$ from leptonic kaon decays in Eq. (71.21), we obtain $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0007(5)$, again consistent with first-row unitarity.

Last, we combine the experimental measurement of $f_{\pi^+}|V_{ud}|$ in Eq. (71.10) with $|V_{ud}|$ from super-allowed β-decay [108] to infer an "experimental" value for the pion decay constant:

$$f_{\pi^-}^{\text{“exp”}} = 130.56(2)(4)(13)\ \text{MeV}\,, \tag{71.23}$$

where the uncertainties are from the errors on Γ, $|V_{ud}|$, and higher-order corrections, respectively. Many recent (2+1+1)-flavor lattice-QCD calculations use this quantity to set the overall physical scale in their simulations, *e.g.*, Refs. [9, 32–34]. Conversely, comparing $f_{\pi^-}^{\text{“exp”}}$ with the FLAG 2024 (2+1)-flavor average [2] $f_{\pi^+} = 130.2(8)$ MeV, which only includes lattice-QCD results that employ other observables to set the scale [36–41], provides a test of lattice-QCD methods. The values are in good agreement within present uncertainties. We do not quote an "experimental" value for the kaon decay constant because the value of $|V_{us}|$ is less clear given the approximately 3σ tension between the values of $|V_{us}|$ obtained from leptonic and semileptonic kaon decays.

71.5.2 $|V_{cd}|$, $|V_{cs}|$, *and status of second-row unitarity*

Using the average values for $|V_{cd}|f_{D^+}$ and $|V_{cs}|f_{D_s^+}$ from Eqs. (71.15) and (71.16), and for f_{D^+} and $f_{D_s^+}$ from Eq. (71.19), we obtain the following determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ from leptonic decays within the standard model:

$$|V_{cd}| = 0.2223(24)(7)(17) \quad \text{and} \quad |V_{cs}| = 0.9764(66)(20)(72)\,, \tag{71.24}$$

where the errors are from the measured branching fractions, decay constants, and external inputs (radiative corrections and masses and lifetimes), respectively. These results enable a test of the unitarity of the second row of the CKM matrix. Taking $|V_{cb}| = 41.8(0.8)) \times 10^{-3}$ [1], we obtain

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = +0.005(20)\,, \tag{71.25}$$

in agreement with three-generation unitarity. Note that correlations among the external uncertainties in the $f|V|$ values have a negligible effect.

The uncertainty on $|V_{cd}|$ in Eq. (71.24) is limited by the measurement error on the $D^+ \to \mu^+\nu$ decay rate. For $|V_{cs}|$, however, the experimental and radiative-correction errors are commensurate. The value of $|V_{cs}|$ from leptonic D_s decays decreased substantially from the value of 1.007(17) in an earlier version of this review [27, 117], and is now below unity as expected in the three-generation CKM framework. This change is due to our more consistent treatment of the radiative corrections, which lower the purely leptonic decay rates for $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$. We emphasize, however,

that we have taken a generous uncertainty on these estimates, and that more theoretical work is needed to really pin down the sizes of the radiative corrections to $D_{(s)}$-meson leptonic decays.

The CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ can also be obtained from semileptonic $D^+ \to \pi^0 \ell^+ \nu$ and $D_s^+ \to K^0 \ell^+ \nu$ decays, respectively. Here experimental measurements determine the product of the form factor times the CKM element, and theory provides the value for the form factor at zero four-momentum transfer between the initial $D_{(s)}$ meson and the final pion or kaon.

The latest experimental averages (2023, with data as of 2021) from the Heavy Flavor Averaging Group (HFLAV) are $f_+^{D\pi}(0)|V_{cd}| = 0.1426(18)$ and $f_+^{DK}(0)|V_{cs}| = 0.7180(33)$ [118]. The FLAG 2024 $2+1+1$ lattice-QCD averages for the form-factors at $q^2 = 0$ are $f_+^{D\pi}(0) = 0.6295(50)$ and $f_+^{DK}(0) = 0.7430(27)$. The $D\pi$ number is an average of FNAL/MILC Collaborations [119] and ETM Collaboration [120] results, while the DK number includes these two plus the HPQCD Collaboration [121]. The older ETM results have lower precision than the others. Combining of the HFLAV $f(0)|V|$ products and the FLAG average for $f(0)$ gives results for $|V_{cd}|$ and $|V_{cs}|$ consistent with the leptonic decays in Eq. (71.24):

$$|V_{cd}| = 0.2265(29)(18) \quad \text{and} \quad |V_{cs}| = 0.9664(44)(35)\,, \tag{71.26}$$

However, as with the leptonic case, one must exercise some care with corrections; this is beyond the scope of the present review. We do note that Ref. [119] presents a detailed discussion of CKM element extractions using their lattice-QCD results, including consideration of QED, electroweak and isospin corrections; a comparison of leptonic and semileptonic results and unitarity tests is also included.

We can combine the experimental measurements of $f_{D^+}|V_{cd}|$ and $f_{D_s^+}|V_{cs}|$ from Tables 71.2 and 71.3 with $|V_{cd}| = 0.22487(68)$ and $|V_{cs}| = 0.97349(16)$ from the PDG 2024 global unitarity-triangle analysis [1] to infer "experimental" values for the decay constants within the standard model. We take the CKM elements from the global fit because they are based on many input quantities, thereby reducing the sensitivity to any one outlying measurement or calculation. We obtain for the decay constants

$$\begin{aligned} f_{D^+}^{\text{"exp"}} &= 210.2(2.2)(0.6)(1.6)\ \text{MeV}\,, \\ f_{D_s^+}^{\text{"exp"}} &= 250.6(1.7)(0.04)(1.8)\ \text{MeV}\,, \\ (f_{D_s^+}/f_{D^+})^{\text{"exp"}} &= 1.191(16)(4)(9)\,. \end{aligned} \tag{71.27}$$

where the uncertainties are from the errors on $f|V|$, CKM matrix elements, and external inputs (radiative corrections and masses and lifetimes), respectively. Here, the 0.7% radiative correction uncertainty is taken as the the same size on the ratio as on the first two results (as opposed to a quadrature sum), due to presumed partial cancellation. Recent improvements in experimental precision make an improved understanding of the degree of cancellation desirable. The "experimental" values f_{D^+} $(f_{D_s^+}/f_{D^+})$ are consistent within 1σ with the (2+1+1)-flavor lattice-QCD averages in Eq. (71.19). The CKM matrix element $|V_{cd}|$ is approximately equal to $|V_{us}|$, as first shown by Wolfenstein [122, 123]. Thus, resolving the inconsistencies between determinations of $|V_{us}|$ from leptonic and semileptonic decays discussed in Sec. 71.5.1 may benefit from the constraints observed here.

Last, we can test lepton-flavor universality in charm meson decays by checking the following relationship derived from Eq. (71.1):

$$\frac{\Gamma(D_s^+ \to \tau^+ \nu)}{\Gamma(D_s^+ \to \mu^+ \nu)} = \frac{m_\tau^2 \left(1 - m_\tau^2/M_{D_s}^2\right)^2}{m_\mu^2 \left(1 - m_\mu^2/M_{D_s}^2\right)^2} = 9.75\,, \tag{71.28}$$

where the uncertainties from the masses are negligible to the number of digits quoted. The measured ratio of $\tau^+\nu$ to $\mu^+\nu$ rates is 10.06 ± 0.28 which, while 1.1σ high, is consistent with the standard-model expectation.

71.5.3 $|V_{ub}|$ and other applications

Using the average value for $|V_{ub}|f_{B^+}$ from Table 71.5, and for f_{B^+} from Eq. (71.20), we obtain the following determination of the CKM matrix element $|V_{ub}|$ from leptonic decays within the standard model:

$$|V_{ub}| = (4.06 \pm 0.37 \pm 0.003) \times 10^{-3}\,, \tag{71.29}$$

where the errors are from experiment and theory, respectively. One should bear in mind when interpreting Eq. (71.29) that none of the experimental measurements of the branching fractions that enter the average for $|V_{ub}|f_{B^+}$ have individually reached the 5σ discovery level (see Sec. 71.4.1). Further, decays involving the third generation of quarks and leptons may be particularly sensitive to new physics associated with electroweak symmetry breaking due to their larger masses [12,14], so Eq. (71.29) is more likely to be influenced by new physics than the determinations of the elements of the first and second rows of the CKM matrix in the previous sections.

The CKM element $|V_{ub}|$ can also be obtained from semileptonic B-meson decays. A long-standing 2-3σ tension between the determinations of $|V_{ub}|$ from exclusive $B \to \pi\ell\nu$ decay and from inclusive $B \to X_u\ell\nu$ decay has diminished somewhat; here, X_u denotes all hadronic states populated by the $b \to u$ weak transition [22, 124–128]. The semileptonic review in the last PDG compilation [129] gives $|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$ (exclusive) and $|V_{ub}| = (4.13 \pm 0.12\,^{+0.13}_{-0.14} \pm 0.18) \times 10^{-3}$ (inclusive). These agree now at the 1.4σ level; the difference is still only 1.8σ when dropping the third model uncertainty added to the inclusive value by the authors of Ref. [129]. The value of $|V_{ub}|$ from leptonic $B \to \tau\nu$ decay in Eq. (71.29) lies between these inclusive and exclusive determinations, and is compatible (within large uncertainties) with both.

Given the large uncertainties on the experimental measurements of $\mathcal{B}(B^- \to \tau^-\overline{\nu})$, and the evolving values of $|V_{ub}|$ obtained from inclusive and exclusive semileptonic B decays, we do not present an "experimental" value of the decay constant f_{B^+}.

71.6 Concluding remarks

The increasing precision of measurements of leptonic decays of charged pseudoscalar mesons has begun to require control over radiative corrections at a level superior to that presented here. Improvements will be called for both in theory and in experiment (for example, by BESIII and Belle II).

Unitarity of the CKM matrix is in reasonable shape. However, there is a lingering almost 3σ tension between the values of V_{us} determined by semileptonic and leptonic kaon decays. It will be interesting to follow experimental and theoretical developments on this quantity.

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