92. Grand Unified Theories

Revised August 2025 by N. Nagata (UTokyo); written with A. Hebecker (Heidelberg U.) and J. Hisano (Nagoya U.; KMI, Nagoya U).

92.1 The Standard Model

The Standard Model (SM) can be defined as a renormalizable quantum field theory with the gauge group $G_{\rm SM} = {\rm SU}(3)_C \times {\rm SU}(2)_L \times {\rm U}(1)_Y$ and three generations of fermions transforming under the representations

$$(\mathbf{3},\mathbf{2})_{1/3} + (\overline{\mathbf{3}},\mathbf{1})_{-4/3} + (\overline{\mathbf{3}},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{1})_{2}\ , \tag{92.1}$$

along with a scalar Higgs doublet H transforming as $(\mathbf{1},\mathbf{2})_1$. Throughout this review, we use bold numbers to indicate the dimension of representations of non-Abelian gauge groups (fundamental and antifundamental in the present case), while the subscript denotes the ${\rm U}(1)_Y$ hypercharge.[1] The fermion fields in Eq. (92.1) correspond to the familiar set $[Q, u^c, d^c, L, e^c]$, where $Q = (u, d)$ and $L = (\nu, e)$ are the quark and lepton ${\rm SU}(2)_L$ doublets, and u^c, d^c, e^c are the charge-conjugate ${\rm SU}(2)_L$ singlets. Especially after the discovery of the Higgs boson, the SM stands out as a remarkably complete and experimentally consistent framework.

A notable exception to the success of the SM lies in the neutrino sector: although neutrino masses are now known to be non-zero, no such masses arise in the SM, even after the Higgs field acquires a vacuum expectation value (VEV). A minimal extension to account for these non-zero neutrino masses involves the inclusion of a dimension-five operator of the form $(HL)^2$ [1], which induces Majorana masses for neutrinos after electroweak symmetry breaking. This operator can be generated by integrating out heavy gauge-singlet fermions which couple to L and H—commonly referred to as right-handed neutrinos; this is known as the seesaw mechanism [2–8]. Alternatively, if light singlet neutrinos are introduced, neutrinos may instead acquire Dirac masses.

The SM also faces a variety of theoretical and phenomenological challenges. Among the most pressing is the electroweak hierarchy problem, which concerns the stability of the Higgs mass against large quantum corrections. Moreover, we still lack an understanding of the underlying dynamics responsible for spontaneous symmetry breaking in the electroweak sector. The SM also fails to provide a viable dark matter candidate, nor does it offer a mechanism for generating the observed baryon asymmetry of the Universe. In addition, the smallness of the QCD θ parameter, $\theta_{\rm QCD}$—the so-called strong CP problem—remains unexplained. The seemingly intricate pattern of quantum number assignments in Eq. (92.1) is also left without theoretical justification. Taken together with the large number of apparently arbitrary coupling constants, these shortcomings cast doubt on the plausibility of the SM as a truly fundamental theory, even leaving aside the issues expected to arise from quantum gravity at energies near the Planck scale, M_P.

To be precise, the SM contains 19 parameters that must be determined from experimental data. These include the three gauge couplings, g_3, g_2, and g_1;[2] 13 parameters associated with the Yukawa sector (comprising nine charged fermion masses, three mixing angles, and one CP phase in the CKM matrix); the Higgs mass and its quartic coupling; and $\theta_{\rm QCD}$. Majorana neutrinos add 3 mass parameters and 6 mixing and phase parameters. As we will see, grand unified theories (GUTs) can address the values of the three gauge couplings, in addition to the group-theoretic structure of Eq. (92.1). In many concrete realizations, GUTs can also shed light on other features of the SM, such as the family structure and the hierarchical values of fermion masses.

[1] We adopt the convention $Q = T_3 + Y/2$ for electric charge. All spinor fields are taken to be left-handed.

[2] Equivalently, the ${\rm SU}(2)_L$ and ${\rm U}(1)_Y$ gauge couplings are denoted by $g = g_2$ and $g' = \sqrt{3/5}\, g_1$. One often uses $\alpha_s = \alpha_3 = g_3^2/(4\pi)$ and $\alpha_{\rm EM} = e^2/(4\pi)$, with $e = g \sin\theta_W$ and $\sin^2\theta_W = (g')^2/(g^2 + (g')^2)$.

92.2 Basic group theory and charge quantization

92.2.1 $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

Historically, the first attempt at unification was the Pati–Salam model, based on the gauge group $G_{\mathrm{PS}} = \mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ [9].[3] In this framework, the SM fermions of one generation, together with an additional SM singlet, are embedded in the representations $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ of G_{PS}. The quantum numbers in Eq. (92.1) can be reproduced by considering the symmetry breaking pattern $\mathrm{SU}(4)_C \to \mathrm{SU}(3)_C \times \mathrm{U}(1)_{B-L}$, together with the identification of the SM hypercharge as a linear combination of $B - L$ (baryon minus lepton number) and the T_3 generator of $\mathrm{SU}(2)_R$: $Y/2 = T_{3R} + (B-L)/2$. The model naturally explains charge quantization—that is, why all electric charges in the SM are integer multiples of the smallest one. Specifically, the $\mathbf{4}$ and $\overline{\mathbf{4}}$ representations of $\mathrm{SU}(4)_C$ treat lepton number as a fourth color, and the traceless nature of the diagonal generator implies that quark charges are fixed as fractional multiples of lepton charges, proportional to $1/N_c$ with $N_c = 3$ the number of colors. However, since G_{PS} is not a simple group—it consists of three simple factors—it does not predict gauge coupling unification.

92.2.2 *SU(5)*

Since G_{SM} has rank four—two from $\mathrm{SU}(3)_C$, and one each from $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$—the rank-four group SU(5) is the minimal choice for a simple group into which G_{SM} can be embedded [13]. In this framework, the three SM gauge couplings originate from a single unified coupling α_G at the grand unification scale M_G. The embedding of G_{SM} into SU(5) is straightforward. The $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ subgroups can be identified with the upper-left 3×3 and lower-right 2×2 blocks, respectively, within the traceless 5×5 matrices representing the generators of SU(5) in its fundamental representation. The $\mathrm{U}(1)_Y$ generator corresponds to the diagonal matrix $\mathrm{diag}(-2/3, -2/3, -2/3, 1, 1)$, which commutes with the generators of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \subset \mathrm{SU}(5)$. From this structure, one finds that a single generation of SM fermions is elegantly unified into the $\mathbf{10} \oplus \overline{\mathbf{5}}$ representations of SU(5), where the $\mathbf{10}$ denotes the rank-2 antisymmetric tensor representation:

$$\mathbf{10} : \begin{pmatrix} 0 & u_b^c & -u_g^c & u^r & d^r \\ -u_b^c & 0 & u_r^c & u^g & d^g \\ u_g^c & -u_r^c & 0 & u^b & d^b \\ -u^r & -u^g & -u^b & 0 & e^c \\ -d^r & -d^g & -d^b & -e^c & 0 \end{pmatrix} \quad \text{and} \quad \overline{\mathbf{5}} : \begin{pmatrix} d_r^c \\ d_g^c \\ d_b^c \\ e \\ -\nu_e \end{pmatrix} . \tag{92.2}$$

In addition to explaining charge quantization, this structure accounts for why the left-handed quark and lepton states form $\mathrm{SU}(2)_L$ doublets, while their right-handed counterparts are $\mathrm{SU}(2)_L$ singlets.

Since SU(5) has 24 generators, SU(5) GUTs predict 12 additional gauge bosons—known as X bosons (or X/Y bosons)—beyond those of the SM. These X bosons transform as an $\mathrm{SU}(3)_C$ triplet and an $\mathrm{SU}(2)_L$ doublet. Their interactions connect quarks and leptons, leading to the violation of both baryon and lepton number, and consequently predicting nucleon decay. Furthermore, the $\mathrm{U}(1)_Y$ hypercharge is automatically quantized, as it corresponds to a generator of SU(5).

In the minimal SU(5) model, the Higgs field is embedded in either a $\mathbf{5}_H$ or a $\overline{\mathbf{5}}_H$ representation. Each multiplet contains three extra states known as color-triplet Higgs bosons. These triplets couple to matter in a way that violates both baryon and lepton number, thereby inducing nucleon decay. To remain consistent with the experimental non-observation of such decay, the triplet mass must be $\gtrsim 10^{11}$ GeV [14–16].[4]

Supersymmetry (SUSY) is frequently invoked in the context of GUTs. It is a symmetry relating bosons and fermions, and it requires the existence of superpartners for all SM particles.

[3] See also Refs. [10–12].

[4] This bound can be relaxed through a fine-tuned cancellation between the renormalizable Yukawa interactions and higher-dimensional operators involving the GUT-breaking Higgs field [17].

SUSY was originally introduced as a solution to the electroweak hierarchy problem [18–21]. In addition, as discussed in Sec. 92.5, SUSY theories have the appealing feature that gauge coupling unification is realized with high precision. In SUSY GUTs [22–27], both $\mathbf{5}_H$ and $\overline{\mathbf{5}}_H$ Higgs multiplets are required—not only to cancel gauge anomalies, but also to provide masses for both up- and down-type quarks. As we will discuss in Sec. 92.6, nucleon decay tends to be more severe in the minimal SUSY SU(5) model, since integrating out the color-triplet Higgs multiplets generates baryon-number-violating dimension-five operators [28, 29], whereas only dimension-six operators are generated in the non-SUSY case. The lower bound on the triplet mass from nucleon decay experiments depends on the SUSY scale but is typically above the unification scale M_G. The mass splitting between the Higgs doublet and triplet components arises from their interactions with the SU(5)-breaking sector, as discussed in Sec. 92.3.

92.2.3 *SO(10)*

While SU(5) provides the minimal framework for grand unification, it does not fully unify the matter content: each SM generation must be split across two distinct representations, **10** and $\overline{\mathbf{5}}$. To incorporate the right-handed neutrinos required for the seesaw mechanism, additional SU(5) singlets must be introduced. In this case, the right-handed neutrino masses are not necessarily tied to the grand unification scale.

By contrast, a single **16**-dimensional spinor representation of SO(10) accommodates an entire SM generation along with a singlet that naturally plays the role of a right-handed neutrino [30,31]. This structure can be understood via the breaking pattern $\mathrm{SO}(10) \to \mathrm{SU}(5) \times \mathrm{U}(1)_X$ and the associated branching rule: $\mathbf{16} = \mathbf{10}_{-1} + \overline{\mathbf{5}}_3 + \mathbf{1}_{-5}$,[5] where the subscripts denote charges under the $\mathrm{U}(1)_X$ subgroup, which can be written as $2Y - 5(B - L)$. This $\mathrm{U}(1)_X$ is orthogonal to SU(5) and reflects the fact that SO(10) has rank five. Remarkably, all representations of SO(10) are anomaly-free in four dimensions (4d). As a result, the anomaly cancellation observed in SU(5) GUTs—or within each SM generation—can be viewed as a consequence of this broader property of SO(10).

We now describe in more detail how a single family of quarks and leptons is embedded in the **16**. To understand this, recall that the Γ-matrices of the 10-dimensional (10d) Clifford algebra give rise to five independent, anticommuting 'creation-annihilation' operators $\Gamma^{a\pm} = (\Gamma^{2a-1} \pm i\Gamma^{2a})/2$ for $a = 1, \ldots, 5$. These correspond to five fermionic harmonic oscillators, or "spin" 1/2 systems. The 32-dimensional spinor space, constructed as a tensor product of these five "spin" states, is reducible because the 10d rotation generators $M_{mn} = [\Gamma^m, \Gamma^n]/(4i)$ (for $m, n = 1, \ldots, 10$) always flip an even number of "spin" directions. This reducibility gives rise to the **16**-dimensional irreducible chiral spinor representation, as shown in Table 92.1. Next, one also recalls that the natural embedding of SU(5) in SO(10) relies on 'pairing up' the 10 real dimensions to produce 5 complex dimensions, $\mathbb{R}^{10} \equiv \mathbb{C}^5$, similarly to the paring up of Γ^ms used above. Specifically, each $|\pm\rangle$ system corresponds to a complex dimension of SU(5), as indicated by the labeling of the "spin" columns in Table 92.1: The first three and last two "spins" correspond to $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$, respectively. An $\mathrm{SU}(3)_C$ rotation just raises one color index and lowers another, changing colors $\{r, g, b\}$, or changes relative phases between the three spin states. Similarly, an $\mathrm{SU}(2)_L$ rotation raises one weak index and lowers another, thereby flipping the weak isospin from up to down or vice versa, or changes the relative phase between the two spin states. In this representation $\mathrm{U}(1)_Y$ hypercharge is simply given by $Y = -2/3(\sum \text{color spins}) + (\sum \text{weak spins})$.[6] SU(5) rotations that are orthogonal to G_{SM} act by raising (or lowering) a color index while simultaneously lowering (or raising) a weak index. Such transformations can mix the states within the sets $\{Q, u^c, e^c\}$ and $\{d^c, L\}$, respectively, while ν^c remains a singlet under these rotations. Since SO(10) has 45 generators, it introduces 21 additional

[5] Useful references on group theory in this context include [32–35], and references therein.

[6] The spin sum is calculated as $(N_+ - N_-)/2$, where N_+ and N_- are the numbers of "+" and "−" entries in the color or weak column of Table 92.1.

generators beyond those of SU(5), including $U(1)_X$ mentioned above. The $U(1)_X$ charge, associated with the SU(5)-invariant generator, is given by $X = 2\sum(\text{spins})$. The remaining 20 generators correspond to SO(10) transformations outside of SU(5), and can be realized as operations that simultaneously raise or lower two spin directions. These rotations relate **1** and $\overline{\mathbf{5}}$ to **10**.

Table 92.1: Quantum numbers in the **16** representation of SO(10)

State	$U(1)_Y$	Color	Weak	SU(5)	$U(1)_X$	SO(10)
ν^c	0	$---$	$--$	**1**	-5	
e^c	2	$---$	$++$			
u^r	1/3	$+--$	$-+$			
d^r	1/3	$+--$	$+-$			
u^g	1/3	$-+-$	$-+$			
d^g	1/3	$-+-$	$+-$	**10**	-1	
u^b	1/3	$--+$	$-+$			
d^b	1/3	$--+$	$+-$			**16**
u^c_r	$-4/3$	$-++$	$--$			
u^c_g	$-4/3$	$+-+$	$--$			
u^c_b	$-4/3$	$++-$	$--$			
d^c_r	2/3	$-++$	$++$			
d^c_g	2/3	$+-+$	$++$			
d^c_b	2/3	$++-$	$++$	$\overline{\mathbf{5}}$	3	
ν	-1	$+++$	$-+$			
e	-1	$+++$	$+-$			

SO(10) has two inequivalent maximal subgroups—and hence two distinct symmetry breaking patterns: $SO(10) \to SU(5) \times U(1)_X$ and $SO(10) \to SU(4)_C \times SU(2)_L \times SU(2)_R$. In the first case, one can proceed with symmetry breaking down to $G_{\rm SM} \subset SU(5)$ exactly as in the minimal SU(5). Alternatively, $U(1)_Y$ can be identified as a linear combination of $U(1)_X$ and the U(1) subgroup of SU(5), leading to the flipped SU(5) model [36–39] as an intermediate stage in the breaking of SO(10) to $G_{\rm SM}$. The second path features the Pati–Salam model at the intermediate scale, with the decomposition $\mathbf{16} = (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$. Alternatively, SO(10) may break directly to the SM at M_G. In such "direct breaking" scenarios, as well as in those following the chain $SO(10) \to SU(5) \to G_{\rm SM}$ with SU(5) broken at M_G, gauge coupling unification remains intact. In contrast, gauge couplings need not unify at the intermediate scale in flipped SU(5) or Pati–Salam models.

In the minimal SO(10) model, the Higgs fields reside in the fundamental representation $\mathbf{10}_H$, which is decomposed as $\mathbf{10}_H = \mathbf{5}_H \oplus \overline{\mathbf{5}}_H$ in SU(5). Notably, SO(10) is unique among the discussed GUTs in that the representation itself distinguishes SM matter from Higgs fields (see Sec. 92.6).

92.2.4 *Beyond SO(10)*

Finally, one may consider larger symmetry groups. A notable example is the exceptional group E_6, which has a maximal subgroup $SO(10) \times U(1)$ [40]. Its fundamental representation decomposes as $\mathbf{27} = \mathbf{16}_1 + \mathbf{10}_{-2} + \mathbf{1}_4$. Another maximal subgroup is $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \subset \mathrm{E}_6$, with the branching rule $\mathbf{27} = (\mathbf{3}, \mathbf{3}, \mathbf{1}) + (\overline{\mathbf{3}}, \mathbf{1}, \overline{\mathbf{3}}) + (\mathbf{1}, \overline{\mathbf{3}}, \mathbf{3})$. Even independently of an underlying E_6, the gauge group $[\mathrm{SU}(3)]^3$—augmented by a $\mathbb{Z}_3$ permutation symmetry that interchanges the three SU(3) factors—can be considered. This framework is known as "trinification" [41, 42], where the $\mathbb{Z}_3$ symmetry guarantees gauge coupling unification. The breaking pattern $\mathrm{E}_6 \to [\mathrm{SU}(3)]^3$ has been employed in phenomenological studies of heterotic string theory [43–46]. However, in models based on larger symmetry groups such as E_6, SU(6), etc., the presence of many additional states poses a challenge: these extra states must be decoupled from the low-energy EFT.

Intriguingly, the hierarchical embedding of gauge groups—where G_{SM} is a maximal subgroup of SU(5), which itself, together with $\mathrm{U}(1)_X$, forms a maximal subgroup of SO(10)—continues in a remarkably elegant and systematic fashion, culminating in the largest exceptional group. This leads to the well-known symmetry breaking chain $\mathrm{E}_8 \to \mathrm{E}_7 \to \mathrm{E}_6 \to \mathrm{SO}(10) \to \mathrm{SU}(5) \to G_{\mathrm{SM}}$ and, together with the distinguished role of E_8 in both group theory and string theory, offers a tantalizing hint at a deeper unifying structure. However, since all representations of E_8 and E_7 are real and therefore incapable of accommodating chiral fermions in 4d, such constructions necessarily lie outside the scope of conventional 4d GUTs.

92.2.5 *Global structures of the Standard Model gauge group*

While our focus so far has been on the Lie algebra of the gauge symmetry, it is worth recalling that distinct Lie groups can share the same Lie algebra. In the case of the SM, it is known that there are four possible gauge groups that correspond to the same SM Lie algebra [47, 48] (see also Ref. [49] for a recent discussion): $G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y / \Gamma$ with $\Gamma = 1, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6$.[7] While these gauge groups describe the same perturbative theory, they can differ in their non-perturbative properties, such as the types of topological defects or instanton effects they allow.

If the SM gauge symmetry originates from the spontaneous breaking of a GUT gauge symmetry, which of the above four gauge groups is realized depends on the choice of the GUT gauge group. For example, in the case of SU(5), the SM gauge group can be embedded as

$$\begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} \in \mathrm{SU}(5) \;, \tag{92.3}$$

where $U \in \mathrm{U}(3)$, $V \in \mathrm{U}(2)$, and $\det U \det V = 1$. Thus, the resulting subgroup is $\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))$. Each element can be parametrized as $U = e^{i\theta/3}\widetilde{U}$ and $V = e^{-i\theta/2}\widetilde{V}$ with $\theta \in \mathbb{R}$, $\widetilde{U} \in \mathrm{SU}(3)$, and $\widetilde{V} \in \mathrm{SU}(2)$. Then, one finds that the parameters $\theta' = 2\pi n + \theta$, $\widetilde{U}' = e^{-2\pi i n/3}\widetilde{U}$, and $\widetilde{V}' = e^{2\pi i n/2}\widetilde{V}$ with $n = 0, 1, 2, 3, 4, 5$ all correspond to the same group element, indicating that the gauge group is equivalent to $G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y / \mathbb{Z}_6$. Similarly, embeddings into other simple GUT groups such as SO(10) and E_6 lead to the same SM gauge group. In contrast, partial unification scenarios yield different global structures: the Pati–Salam group G_{PS} leads to $\Gamma = \mathbb{Z}_3$, whereas the trinification group $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ gives $\Gamma = \mathbb{Z}_2$.

92.3 GUT breaking and doublet-triplet splitting

In the standard 4d field-theoretic approach to GUTs, the unified gauge group is spontaneously broken by an appropriately chosen GUT Higgs sector. Scalar potentials (or superpotentials in the case of SUSY GUTs) can be constructed whose vacua break the GUT gauge group. While these potentials are somewhat ad hoc—much like the Higgs potential in the SM—the simplest expectation is that all dimensionful parameters in the theory are of $\mathcal{O}(M_G)$. In the minimal SU(5), the GUT-breaking Higgs field is the adjoint representation **24** and acquires a VEV along the G_{SM}-singlet direction, $\langle \Phi \rangle \propto \mathrm{diag}(-2/3, -2/3, -2/3, 1, 1)$. For SO(10) to break to SU(5), a Higgs field in the

[7] The cases with $\Gamma = \mathbb{Z}_2$, $\mathbb{Z}_3$, and $\mathbb{Z}_6$ are also referred to as $\mathrm{SU}(3) \times \mathrm{U}(2)$, $\mathrm{U}(3) \times \mathrm{SU}(2)$, and $\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))$, respectively. The group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ serves as the covering group of these.

16 or **126** representation must acquire a VEV along a direction that is an SU(5) singlet but carries nonzero $U(1)_X$ charge.

The masses of the doublet and triplet components in the $\mathbf{5}_H$ (and $\overline{\mathbf{5}}_H$) are generically split due to their couplings to the GUT-breaking Higgs field. In addition, both components receive an equal contribution from an SU(5)-invariant GUT-scale mass term. Without further structure, achieving a light doublet at the electroweak scale requires an extreme fine-tuning between two large contributions. SUSY helps stabilize the doublet mass against large radiative corrections, thanks to the non-renormalization theorem [50]. However, even in SUSY models, fine-tuning is still required at tree level. This constitutes the well-known doublet-triplet splitting problem, which is closely related—within the SUSY context—to the μ-problem, *i.e.*, the puzzle of why the coefficient of the superpotential term $\mu H_u H_d$ is much smaller than the GUT scale.

Several mechanisms for realizing natural doublet-triplet splitting have been proposed within the framework of SUSY. These include the sliding singlet mechanism [51], the missing partner mechanism [52,53], the missing VEV mechanism [54,55], and the pseudo-Nambu–Goldstone boson mechanism [56]. Concrete implementations of these ideas have been studied in various GUT models: the missing partner mechanism in SU(5) [57–61] and in the flipped SU(5) [38,62], the missing VEV mechanism in SO(10) [63–75], and the pseudo-Nambu–Goldstone boson mechanism [76–84]. These models have been shown to be compatible with gauge coupling unification and nucleon decay constraints. Once a model with vanishing fundamental μ term has been realized, an *effective* μ term of order the SUSY breaking scale can then be induced by, for example, higher-mass-dimension terms in the Kähler potential [85] or superpotential [86]. For a review of the μ problem and some suggested solutions in SUSY GUTs and string theory, see Refs. [87–95] and references therein.

In general, GUT-breaking sectors that resolve the doublet-triplet splitting problem, stabilize all GUT-scale VEVs, and yield realistic neutrino masses and Yukawa couplings—including controlled violations of GUT relations in the latter—require a number of nontrivial ingredients. However, the introduction of large or numerous representations is constrained, as it can drive the theory to a Landau pole below the Planck scale. Moreover, in the 4d field-theoretic approach, GUTs are regarded as EFTs valid only below the Planck scale. Since the GUT scale M_G lies relatively close to this cutoff, the effects of higher-dimensional operators cannot be neglected in general. In particular, operators involving the GUT-breaking Higgs fields can influence low-energy observables, such as quark and lepton masses, and must therefore be taken into account in realistic model building.

Thus, especially in the context of GUT breaking and doublet-triplet splitting, models beyond 4d field theory appear attractive. While this is mainly the subject of the next section, some advantages can already be noted: In models with extra dimensions, in particular string constructions, GUT breaking may occur due to boundary conditions in the compactified dimensions [96–110]. No complicated GUT breaking sector is then required. Moreover, boundary conditions can give mass only to the triplet, leaving the doublet massless. This is similar to the missing partner mechanism since the effective mass term does not 'pair up' the triplets from $\mathbf{5}_H$ and $\overline{\mathbf{5}}_H$ but rather each of them with further fields which are automatically present in the higher-dimensional theory. This can eliminate dimension-five nucleon decay (cf. Sec. 92.6).

92.4 String-theoretic and higher-dimensional unified models

As noted earlier, the GUT scale is dangerously close to the scale of quantum gravity. It may hence be necessary to discuss unified models of particle physics in the latter, more ambitious context. Among the models of quantum gravity, superstring or M-theory stands out as the best-studied and technically most developed proposal, possessing in particular a high level of internal, mathematical consistency. For our purposes, it is sufficient to know that five 10d and one 11d low-energy effective supergravity theories arise in this setting (see Refs. [111–114] and references therein).

Grand unification is realized most naturally in the context of the two heterotic string theories with gauge groups $E_8 \times E_8$ and SO(32) [101, 115] (see Refs. [116–118] for some of the more recent results). Justified in part by the intriguing breaking path $E_8 \to \cdots \to G_{SM}$ mentioned above, the focus has historically largely been on $E_8 \times E_8$. To describe particle physics, solutions of the 10d theory with geometry $\mathbb{R}^{1,3} \times M_6$ are considered, where M_6 is a Calabi–Yau (CY) 3-fold (with 6 real dimensions) [101]. The background solution involves expectation values of higher-dimensional components of the $E_8 \times E_8$ gauge fields. This includes both Wilson lines [96] and non-vanishing field-strength and leads, in general, to a reduced gauge symmetry and to chirality in the resulting 4d effective theory. The 4d fermions arise from 10d gauginos.

Given an appropriate embedding[8] of G_{SM} in $E_8 \times E_8$, gauge coupling unification is automatic at leading order. Corrections arise mainly through (string)-loop effects and are similar to the familiar field-theory thresholds of 4d GUTs [119] discussed in Sec. 92.5. Thus, one may say that coupling unification is a generic prediction in spite of the complete absence[9] of a 4d GUT at any energy scale. This absence is both an advantage and a weakness. On the up side, GUT breaking and doublet-triplet splitting [122] are more naturally realized and dimension-five nucleon decay is relatively easy to avoid. On the down side, there is no reason to expect full GUT representations in the matter sector and flavor model building is much less tied to the GUT structure than in 4d.

Let us pause to explain the beautiful idea behind the advertised solution of the doublet-triplet splitting problem: One starts with a simply connected CY X and mods out the action of a discrete group G (say $\mathbb{Z}_2$). In the absence of fixed points, X/G is smooth and has a non-contractible 1-cycle. Furthermore, let G also act on the gauge bundle, according to an embedding $G \to E_8$. Now the parallel transport around the 1-cycle is tied to a gauge rotation (one says a non-trivial Wilson-line is present). Moreover, this Wilson line can not be continuously turned off since, *e.g.*, in the case of $\mathbb{Z}_2$, its square is the unit element of the group. The induced 'Wilson-line breaking', which comes on top of the breaking by non-zero field strengths, may remove certain sub-representations (*e.g.*, the triplet of SU(5) $\to G_{SM}$) while keeping others exactly massless. A simpler and, due to fixed points, singular version of this will appear below in the context of orbifold GUTs.

One technical problem of heterotic constructions is the dependence on the numerous size and shape parameters of M_6 (the so-called moduli), the stabilization of which is poorly understood (see Refs. [123–125] for recent developments). Another is the sheer mathematical complexity of the analysis, involving in particular the study of (non-Abelian) gauge-bundles on CY spaces [126] (see, however, Refs. [127, 128]).

An interesting aspect of heterotic string constructions is represented by orbifold models [97–100]. Here the internal space is given by a six-torus, modded out by a discrete symmetry group (*e.g.*, $T^6/\mathbb{Z}_n$). More recent progress is reported in Refs. [129–137], including in particular the systematic exploration of the phenomenological advantages of so-called 'non-prime' (referring to n) orbifolds. The symmetry breaking to G_{SM} as well as the survival of Higgs doublets without triplet partners is ensured by the appropriate embedding of the discrete orbifold group in $E_8 \times E_8$. String theory on such spaces, which are locally flat but include singularities, is much more calculable than in the CY case. The orbifold geometries can be viewed as singular limits of CYs.

An even simpler approach to unified model building—while still retaining many of the advantages of full-fledged string constructions—is offered by orbifold GUTs [102–110]. These are typically 5d or 6d SUSY field theories with a unified gauge group—such as SU(5) or SO(10)—that is broken through the compactification to 4d. To illustrate the idea with a simple example, consider SU(5) defined on the spacetime $\mathbb{R}^{1,3} \times S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$. The compact space in this case is an interval of

[8] All embeddings of G_{SM} in one E_8 factor which are consistent with a breaking-pattern $E_8 \to SU(5) \to G_{SM}$ are suitable. Other embeddings can change the ratios between the three resulting G_{SM} couplings at the GUT scale by group-theoretic factors. Crucially, due to the single 10d gauge coupling, no continuous tuning is possible.

[9] See, however, Refs. [120, 121].

length $\pi R/2$, and the embedding of $\mathbb{Z}_2'$ in the hypercharge direction of SU(5) leads to the desired breaking to $G_{\rm SM}$. Concretely, the 5d X bosons are assigned Dirichlet boundary conditions (BCs) at one end of the interval, eliminating their Kaluza–Klein (KK) zero modes. Their lightest modes have masses of $\sim 1/R$, setting the KK scale as the effective GUT scale. A key implication is that the boundary theory lacks full SU(5) invariance. Nevertheless, since the SU(5)-symmetric 5d bulk governs the running of the 4d gauge couplings, gauge coupling unification remains a prediction. Furthermore, many challenges inherent to conventional 4d GUTs can be avoided in this framework. In particular, the doublet-triplet splitting problem is resolved.

With the advent of the string-theory flux landscape [138,139], which is best understood in 10d type-IIB supergravity, the focus in string model building has shifted to this framework. While type II string theories have no gauge group in 10d, brane-stacks support gauge dynamics. A particularly appealing setting (see, *e.g.*, Ref. [140]) is provided by type IIB models with D7 branes (defining 8d submanifolds). However, in the SO(10) context the **16** is not available and, for SU(5), the top-Yukawa coupling vanishes at leading order [141]. As a crucial insight, this can be overcome on the non-perturbative branch of type IIB, also known as F-theory [142–144]. This setting allows for more general branes, thus avoiding constraints of the Dp-brane framework. GUT breaking can be realized using hypercharge flux (the VEV of the $\mathrm{U}(1)_Y$ field strength), an option not available in heterotic models. The whole framework combines the advantages of the heterotic or higher-dimensional unification approach with the more recent progress in understanding moduli stabilization. It thus represents at this moment the most active and promising branch of theory-driven GUT model building (see, *e.g.*, Refs. [145–149] and references therein).

As a result of the flux-breaking, a characteristic 'type IIB' or 'F-theoretic' tree-level correction to gauge unification arises [150–154]. The fact that this correction can be rather significant numerically is occasionally held against the framework of F-theory GUTs. However, at a parametric level, this correction nevertheless behaves like a 4d threshold, *i.e.*, it provides $\mathcal{O}(1)$ additive contributions to the inverse 4d gauge couplings $\alpha_i^{-1}(M_G)$.

A final important issue in string GUTs is the so-called string-scale/GUT-scale problem [155]. It arises since, in heterotic compactifications, the Planck scale and the high-scale value of the gauge coupling unambiguously fix the string-scale to about 10^{18} GeV. As the compactification radius R is raised above the string length, the GUT scale (identified with $1/R$) goes down and the string coupling goes up. Within the domain of perturbative string theory, a gap of about a factor ~ 20 remains between the lowest GUT scale achievable in this way and the phenomenological goal of 2×10^{16} GeV. The situation can be improved by venturing into the non-perturbative regime [155], by considering 'anisotropic' geometries with hierarchically different radii R [155,156] or by including GUT scale threshold corrections [157,158].

In F-theory GUTs, the situation is dramatically improved since the gauge theory lives only in four out of the six compact dimensions. This allows for models with a 'decoupling limit', where the GUT scale is parametrically below the Planck scale [143,144]. However, moduli stabilization may not be without problems in such constructions, in part due to a tension between the required large volume and the desirable low SUSY breaking scale.

92.5 Gauge coupling unification

The quantitative unification of the three SM gauge couplings at the energy scale M_G is one of the central pillars of the GUT paradigm, and it holds clear phenomenological significance. Gauge coupling unification is well understood within the framework of effective field theory (EFT) [159–161]. In the simplest scenario, the EFT at energy scales $\mu \gg M_G$ has a unified gauge symmetry and a single running gauge coupling $\alpha_G(\mu)$. At lower energies $\mu \ll M_G$, heavy states with masses around M_G—such as X bosons, GUT-breaking Higgs fields, and color-triplet Higgs fields—are integrated out. The resulting low-energy EFT then features three independent gauge couplings and the matter content of the SM or its SUSY extension. One-loop renormalization group equations (RGEs) provide a means of extrapolating the gauge couplings down to the electroweak scale,

$$\alpha_i^{-1}(m_Z) = \alpha_G^{-1}(M_G) + \frac{b_i}{2\pi}\log\left(\frac{M_G}{m_Z}\right) + \delta_i\ , \quad (i = 1, 2, 3)\ , \tag{92.4}$$

where b_i are the one-loop β-function coefficients (with $b_i^{\rm SM} = \{41/10, -19/6, -7\}$ in the SM and $b_i^{\rm MSSM} = \{33/5, 1, -3\}$ in the minimal SUSY SM (MSSM)), m_Z is the Z-boson mass, and δ_i encode all subleading effects. These include threshold corrections at or near the weak scale—such as those arising from superpartners and the extended Higgs sector in the MSSM—as well as threshold effects at the GUT scale and higher-order loop corrections.

When SUSY is invoked as a solution to the gauge hierarchy problem—that is, the issue of naturalness or fine-tuning associated with the electroweak scale [18–21]—it predicts that the superpartners of SM particles should lie near the weak scale. Assuming this, and adopting a minimal Higgs sector, gauge coupling unification is achieved with remarkable precision in the MSSM [162–168], whereas it fails in the SM, as illustrated in Fig. 92.1.

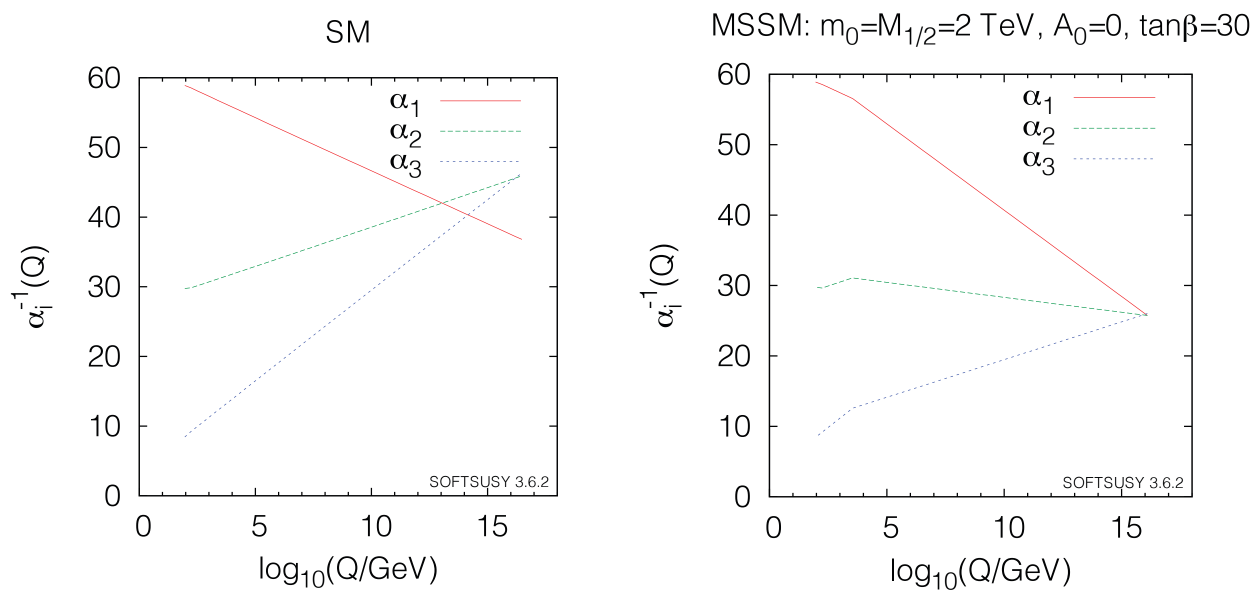

Figure 92.1: Running of the gauge couplings in the SM and MSSM, shown using two-loop RG evolution. The SUSY threshold, set at 2 TeV, is clearly visible on the MSSM side. (We thank Ben Allanach for providing the plots, which were generated using SOFTSUSY [169].)

The three equations contained in Eq. (92.4) can be used to determine the three 'unknowns' $\alpha_3(m_Z)$, $\alpha_G(M_G)$ and M_G, assuming that all other parameters entering the equations are given. Focusing on the SUSY case and using the $\overline{\rm MS}$ coupling constants $\alpha_{\rm EM}^{-1}(m_Z)$ and $\sin^2\theta_W(m_Z)$ taken from Ref. [170] as input, $\alpha_{\rm EM}^{-1}(m_Z) = 127.930 \pm 0.008$ and $\sin^2\theta_W(m_Z) = 0.23122 \pm 0.00004$, one determines $\alpha_{1,2}^{-1}(m_Z)$, which then gives

$$\alpha_G^{-1}(M_G) \simeq 24.3 \qquad \text{and} \qquad M_G \simeq 2\times 10^{16}\ \text{GeV}\ . \tag{92.5}$$

Here we have set $\delta_i = 0$ for simplicity. Crucially, one in addition obtains a prediction for the low-energy observable α_3,

$$\alpha_3^{-1}(m_Z) = -\frac{5}{7}\alpha_1^{-1}(m_Z) + \frac{12}{7}\alpha_2^{-1}(m_Z) + \mathit{\Delta}_3 \ , \tag{92.6}$$

where

$$\mathit{\Delta}_3 = \frac{5}{7}\delta_1 - \frac{12}{7}\delta_2 + \delta_3 \ . \tag{92.7}$$

Here we followed the elegant formulation in Ref. [171] of the classical analyses of Refs. [162–168]. Of course, it is a matter of convention which of the three low-energy gauge coupling parameters one 'predicts' and indeed, early works on the subject discussed the prediction of $\sin^2\theta_W$ in terms of $\alpha_{\rm EM}$ and α_3 [172–174]. Remarkably, the leading order result (*i.e.*, Eq. (92.6) with $\mathit{\Delta}_3 = 0$) is in excellent agreement with experiments [170]:

$$\alpha_3^{\rm LO}(m_Z) = 0.117 \qquad \text{vs.} \qquad \alpha_3^{\rm exp}(m_Z) = 0.1180 \pm 0.0009 \ . \tag{92.8}$$

However, this near perfection is to some extent accidental. To see this, we now discuss the various contributions to the subleading corrections δ_i (and hence to $\mathit{\Delta}_3$).

The two-loop running correction from the gauge sector $\mathit{\Delta}_3^{(2)}$ and the low-scale threshold correction $\mathit{\Delta}_3^{(l)}$ from superpartners can be summarized as [171]

$$\mathit{\Delta}_3^{(2)} \simeq -0.82 \qquad \text{and} \qquad \mathit{\Delta}_3^{(l)} \simeq \frac{19}{28\pi}\log\left(\frac{m_{\rm SUSY}}{m_Z}\right) \ . \tag{92.9}$$

The relevant scale $m_{\rm SUSY}$ can be estimated as [168]

$$m_{\rm SUSY} \simeq m_H^{3/19} m_{\widetilde{H}}^{12/19} m_{\widetilde{W}}^{4/19} \times \left(\frac{m_{\widetilde{W}}}{m_{\widetilde{g}}}\right)^{28/19} \left(\frac{m_{\widetilde{l}}}{m_{\widetilde{q}}}\right)^{3/19} \ , \tag{92.10}$$

where m_H stands for the masses of non-SM Higgs states and superpartner masses are given in self-evident notation. Detailed analyses including the above effects are best done using appropriate software packages, such as SOFTSUSY [169] (or alternatively SuSpect [175] or SPheno [176]). See also Ref. [169] for references for the underlying theoretical two-loop analyses.

To gain a rough estimate of these effects, let us assume that all superpartners are degenerate at $m_{\rm SUSY} = 1$ TeV except for gluino, with a mass ratio $m_{\widetilde{W}}/m_{\widetilde{g}} \simeq 1/3$. Under this assumption, one finds $\mathit{\Delta}_3^{(l)} \simeq -0.35 + 0.22\ln(m_{\rm SUSY}/m_Z) \simeq 0.18$. This leads to a predicted value of $\alpha_3(m_Z) \simeq 0.126$, which significantly deviates from the near-perfect one-loop unification observed earlier. Before exploring this discrepancy in more detail, it is useful to introduce another important class of corrections—those arising from high-scale (GUT-scale) thresholds.

To discuss high scale thresholds, let us set all other corrections to zero for the moment and write down a version of Eq. (92.4) that captures the running near and above the GUT scale more correctly. The threshold correction at one-loop level can be evaluated accurately by the simple step-function approximation for the β functions in the $\overline{\rm DR}$ scheme [26, 177],[10]

$$\alpha_i^{-1}(m_Z) = \alpha_G^{-1}(\mu) + \frac{1}{2\pi}\left[b_i \ln\frac{\mu}{m_Z} + b_i^C \ln\frac{\mu}{M_C} + b_i^X \ln\frac{\mu}{M_X} + b_i^{\mathit{\Phi}} \ln\frac{\mu}{M_{\mathit{\Phi}}}\right] \ . \tag{92.11}$$

We initiate the RG running at a scale $\mu \gg M_G$, considering the minimal set of states relevant for the transition from the SU(5) GUT to the MSSM. These include the color-triplet Higgs multiplets with

[10] The $\overline{\rm DR}$ scheme is commonly employed for regularization in SUSY theories [178]. The transformation of gauge couplings from the $\overline{\rm MS}$ to the $\overline{\rm DR}$ scheme is provided in Ref. [177]. For an alternative approach based on holomorphic gauge couplings and the NSVZ β-functions, see, for example, Refs. [179–181].

mass M_C, the massive vector multiplets of the X-bosons with mass M_X (along with the associated GUT Higgs degrees of freedom), and the remaining GUT Higgs fields and superpartners with mass M_Φ. The corresponding β-function coefficients $b_i^{C,X,\Phi}$ are given in Refs. [182, 183]. Importantly, the combination of b_i coefficients in Eq. (92.11) is such that the RG evolution preserves GUT universality at high scales, rendering the prediction for α_3 independent of the choice of μ.

To relate this to our previous discussion, we can, for example, define $M_G \equiv M_X$ and then choose $\mu = M_G$ in Eq. (92.11). This gives the high-scale threshold corrections

$$\delta_i^{(h)} = \frac{1}{2\pi}\left[b_i^C \ln\frac{M_G}{M_C} + b_i^\Phi \ln\frac{M_G}{M_\Phi}\right] , \tag{92.12}$$

and a corresponding correction $\Delta_3^{(h)}$. To get some intuition for the magnitude, one can furthermore assume $M_\Phi = M_G$, finding (with $b_i^C = \{2/5, 0, 1\}$)

$$\Delta_3^{(h)} = \frac{9}{14\pi}\ln\left(\frac{M_G}{M_C}\right) . \tag{92.13}$$

To achieve the required effect of $-\Delta_3^{(2)} - \Delta_3^{(l)} \simeq +0.64$, the color-triplet Higgs would need to be roughly a factor of 20 lighter than the GUT scale. Although such a light triplet is excluded in the minimal model by current nucleon decay limits [184]—as will be discussed in Sec. 92.6—this example nonetheless illustrates that threshold corrections of this magnitude can, in principle, be realized through appropriate GUT-scale model building, for instance in specific SU(5) [57–61] or SO(10) [63–72] frameworks. In fact, much larger or even oppositely signed corrections can arise in models that involve extended field content, particularly those employing many higher-dimensional representations, as is often necessary in fully realistic 4d GUTs. Higher-dimensional operators suppressed by M_P^{-1}, involving the GUT gauge fields and the GUT-breaking Higgs field, can also induce threshold corrections of comparable magnitude [185–190]. These corrections can, in particular, be utilized to raise the mass of the color-triplet Higgs. Consequently, there exists substantial freedom in model building.

The analysis above implicitly assumes universal soft SUSY-breaking masses at the GUT scale, which directly affect the spectrum of SUSY particles at the weak scale; in the simplest scenario, motivated by SO(10) unification, one adopts a universal gaugino mass $M_{1/2}$, a common scalar mass m_{16} for squarks and sleptons, and a universal Higgs mass m_{10}. In some cases, threshold corrections to gauge couplings can be traded for threshold effects on the soft SUSY-breaking parameters (see Ref. [191] and references therein). For instance, if gaugino masses are not unified at M_G—in particular, if gluino is lighter than wino at the weak scale (cf. Eq. (92.10))—then weak-scale threshold corrections may allow for a significantly smaller or even slightly negative GUT-scale threshold correction to be consistent with successful gauge coupling unification [192, 193].

It is also noteworthy that precise gauge coupling unification can be achieved without significant GUT-scale threshold corrections simply by increasing $m_{\rm SUSY}$ in Eq. (92.10) [194–196]. This remains possible even if gauginos are kept at the TeV scale, by taking the masses of higgsinos and non-SM Higgs bosons to lie in the multi-TeV range. Such a setup is particularly attractive, as it preserves a viable dark matter candidate—namely, the lightest neutralino. Hierarchical SUSY mass spectra of this type, often motivated by anomaly mediation [197, 198], have been extensively studied in the literature [199–210]. This scenario has the additional merit of accommodating both the observed value of the SM-like Higgs boson mass[11] and the current null results from SUSY searches at the LHC, although it reintroduces a little hierarchy problem associated with the electroweak scale.

[11] In the MSSM, the tree-level Higgs boson mass is bounded from above by m_Z, but radiative corrections—particularly from heavy stop loops—can raise it to the observed value of ~ 125 GeV [211–218].

In non-SUSY GUTs or in SUSY GUTs with a very high SUSY breaking scale, achieving gauge coupling unification typically requires the presence of new light states in incomplete GUT multiplets or multiple symmetry-breaking scales. For example, in non-SUSY models with the breaking chain $SO(10) \to SU(4)_C \times SU(2)_L \times SU(2)_R \to G_{SM}$, where the second stage of symmetry breaking occurs at an intermediate scale corresponding to the mass scale of right-handed neutrinos, gauge couplings can unify at the SO(10) breaking scale [219], while remaining consistent with current bounds on nucleon decay [220]. Similar unification can also be realized in alternative symmetry-breaking chains or in scenarios with extra light states—such as dark matter candidates—present at low energies [221,222]. Alternatively, string-theoretic corrections discussed in Sec. 92.4 can be invoked to offset the effects of a high SUSY-breaking scale. This possibility has been explored concretely, for example, in the context of F-theory GUTs [223]. In principle, large threshold corrections arising from large GUT representations may also enable gauge coupling unification [224,225].

In orbifold GUTs, certain "GUT scale" threshold corrections come from the KK modes between the compactification scale, $M_c \sim 1/R$, and the effective cutoff scale M_*, which is the string scale in string theory. Gauge coupling unification then constrains the values of M_c and M_*. It is interesting to note that a ratio $M_*/M_c \sim 100$, needed for gauge coupling unification to work in orbifold GUTs, is typically the maximum value for this ratio consistent with perturbativity [226]. Often, one finds M_c to be lower than the 4d GUT scale. Since the X-bosons, responsible for nucleon decay, get mass at the compactification scale, this has significant consequences for nucleon decay.

Finally, it has been shown that non-SUSY GUTs in warped 5d orbifolds can be consistent with gauge coupling unification. This assumes (in 4d language) that the right-handed top quark and the Higgs doublets are composite-like objects with a compositeness scale in the TeV range [227].

92.6 Nucleon decay

Quarks and leptons are indistinguishable in any 4d GUT, and both the baryon (B) and lepton number (L) are not conserved. This leads to baryon-number-violating nucleon decay. In addition to baryon-number violation, lepton-number violation is also required for nucleon decay since, in the SM, leptons are the only free fermions which are lighter than nucleons. In the SM, the lowest-dimension operators relevant for nucleon decay are $(B + L)$-violating dimension-six four-fermion terms, and all B-violating operators with dimension less than seven preserve $B - L$ [1,228].

In SU(5) GUTs, the dimension-six baryon-number-violating operators are induced by X boson exchange. These operators are suppressed by $(1/M_X^2)$, where M_X is the X boson mass, and the nucleon lifetime is given by $\tau_N \propto M_X^4/(\alpha_G^2 m_N^5)$, with m_N the nucleon mass. The dominant proton decay mode, as well as the baryon-number-violating decay mode of the neutron, via the X-boson exchange is given by $p \to \pi^0 e^+$ and $n \to \pi^- e^+$, respectively.[12] The present experimental bound on $p \to \pi^0 e^+$ comes from Super-Kamiokande. With 450 kton-years of data they find $\tau_p/\mathrm{Br}(p \to \pi^0 e^+) > 2.4 \times 10^{34}$ years at 90% CL [233].[13] In addition, Hyper-Kamiokande [235] is planned to reach to $\tau_p/\mathrm{Br}(p \to \pi^0 e^+) \sim 10^{35}$ years. For a broader overview of planned experiments and historical developments, the reader is referred to Ref. [236]. The hadronic matrix elements relevant for baryon-number-violating operators are now reliably computed using lattice QCD simulations [237]. In SUSY SU(5) GUTs, the null results from nucleon decay searches place a lower bound on the X boson mass that is approaching 10^{16} GeV [238], which is remarkably close to the GUT scale inferred from gauge coupling unification. In contrast, predictions for nucleon decay rates in non-SUSY GUTs are more uncertain. This is because gauge couplings do not unify with the SM particle content alone. Once additional states or sizable threshold corrections are introduced to achieve precision unification, a broad range of possible unification scales becomes viable.

[12] Other decay modes, such as $p \to \pi^+ \bar{\nu}$, $p \to \pi^0 \mu^+$, and $n \to \pi^0 \bar{\nu}$, can be dominant in extended GUT frameworks, including flipped SU(5) [229,230], SO(10), and E_6 [231,232] models.

[13] For $n \to \pi^- e^+$, the current limit is $\tau_p/\mathrm{Br}(n \to \pi^- e^+) > 5.3 \times 10^{33}$ years [234].

In SUSY GUTs, there exist additional sources of baryon- and/or lepton-number violation in the form of dimension-four/five operators [28, 29]. These arise because, in the SUSY SM, quarks and leptons are accompanied by scalar partners—squarks and sleptons. Note that in the SUSY context, fields are understood as chiral superfields, which include both fermionic and bosonic components. In this framework, baryon- and lepton-number-violating operators of dimension four and five appear as F-terms composed of chiral superfields. These operators involve two fermionic components, with the remaining fields being one or two scalar fields; in SU(5), they have the form:

$$
\begin{aligned}
(\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}) &\supset (u^c\,d^c\,d^c) + (Q\,L\,d^c) + (e^c\,L\,L)\;, \\
(\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\overline{\mathbf{5}}) &\supset (Q\,Q\,Q\,L) + (u^c\,u^c\,d^c\,e^c) + B\text{- and } L\text{-conserving terms}\;. \qquad (92.14)
\end{aligned}
$$

The dimension-four operators in $(\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}})$ violate either B or L. The nucleon lifetime is extremely short if both types of dimension-four operators are present in the SUSY SM since squark or slepton exchange induces the dangerous dimension-six SM operators. Even in the case that they violate B or L only but not both, they are constrained by various phenomena [239, 240]. For example, the primordial baryon number in the Universe is washed out unless the dimensionless coupling constants are less than 10^{-7} [241–243]. Both types of operators can be eliminated by requiring R parity [244], which distinguishes Higgs multiplets from ordinary matter multiplets. R parity or its cousin, matter parity [22–27, 245, 246], act as $F \to -F$, $H \to H$ with $F = \{\mathbf{10}, \overline{\mathbf{5}}\}$, $H = \{\overline{\mathbf{5}}_H,\, \mathbf{5}_H\}$ in SU(5). This R parity forbids the dimension-four operator $(\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}})$, but allows the Yukawa couplings for quark and lepton masses of the form $(\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_H)$ and $(\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H)$. It also forbids the dimension-three, L-violating operator $(\overline{\mathbf{5}}\,\mathbf{5}_H) \supset (L\,H_u)$ as well as the dimension-five, B-violating operator $(\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\overline{\mathbf{5}}_H) \supset (Q\,Q\,Q\,H_d) + \cdots$. In SU(5), the Higgs multiplet $\overline{\mathbf{5}}_H$ and the matter multiplets $\overline{\mathbf{5}}$ have identical gauge quantum numbers. In E_6, Higgs and matter multiplets could be unified within the fundamental **27** representation. Only in SO(10) are Higgs and matter multiplets distinguished by their gauge quantum numbers. The $\mathbb{Z}_4$ center of SO(10) distinguishes **10**s from **16**s and can be associated with R parity [247].

The baryon-number-violating dimension-five operators in SUSY GUTs arise with dimensionful couplings, typically suppressed by $1/M_G$, as they are generated by integrating out the color-triplet Higgs fields with GUT-scale masses. Both triplet higgsinos—due to their fermionic nature—and triplet Higgs scalars—via their mass-enhanced trilinear couplings to matter—contribute to the generation of these operators. Since the resulting operators involve squarks and/or sleptons, nucleon decay requires these scalar superpartners to be converted into light quarks or leptons through the exchange of a gaugino or higgsino in the SUSY SM. The corresponding nucleon lifetime scales as $M_G^2\,\widetilde{m}^2/m_N^5$, where $\widetilde{m}$ denotes the characteristic SUSY scalar mass. Consequently, dimension-five operators can induce nucleon decay rates significantly faster than those from dimension-six operators. In the absence of accidental cancellations, the dominant decay modes from these operators often include a kaon, such as $p \to K^+\bar{\nu}$ and $n \to K^0\bar{\nu}$. This can be understood from symmetry considerations. The relevant superpotential terms take the schematic form $(Q_i\,Q_j\,Q_k\,L_l)$ and $(u_i^c\,u_j^c\,d_k^c\,e_l^c)$, where $i, j, k, l = 1, 2, 3$ label generations, and color and weak indices are implicit. These operators must be invariant under both $SU(3)_C$ and $SU(2)_L$, implying that the color and weak indices must be antisymmetrized. However, as they arise from bosonic superfields, they must be totally symmetric under interchange of all flavor and gauge indices. This leads to the vanishing of the first operator when $i = j = k$, and of the second when $i = j$. As a result, contributions involving second- or third-generation fields dominate unless they are accidentally suppressed [245, 246].

The Super-Kamiokande experiment places stringent constraints on dimension-five proton decay operators. With an exposure of 306 kton · years, the current lower bound on the partial lifetime is $\tau_p/\mathrm{Br}(p \to K^+\bar{\nu}) > 6.61 \times 10^{33}$ years at 90% confidence level [248]. In the minimal SUSY SU(5) model, the predicted partial lifetime is less than about 10^{31} years if the color-triplet Higgs mass is 10^{16} GeV and the SUSY scale is $\widetilde{m} = 1$ TeV [249]. To evade this experimental limit, the color-triplet Higgs would need to be significantly heavier than the GUT scale, which, as discussed in Sec. 92.5, conflicts with the requirement for successful gauge coupling unification. As a result, the minimal SUSY SU(5) model is placed under significant tension by current proton decay constraints [184].

Since nucleon decay induced by the color-triplet Higgs is a serious issue in SUSY GUTs, a variety of mechanisms have been proposed to suppress it. One possibility is that an accidental symmetry or structural feature in non-minimal Higgs sectors suppresses the dimension-five operators [52, 53, 63–72, 250, 251]. A central idea behind such constructions is that nucleon decay can be suppressed if the Higgs triplets in the $\mathbf{5}_H$ and $\overline{\mathbf{5}}_H$ do not acquire a common mass term, but instead obtain their masses through couplings to additional SU(5) multiplets. Second, the SUSY-breaking scale may lie in the range $\mathcal{O}(10\text{–}100)$ TeV, as discussed in Sec. 92.5. In such scenarios, nucleon decay is automatically suppressed due to the heavy sfermion masses, while the observed Higgs boson mass can be explained [252–258]. Third, accidental cancellations among contributing diagrams, arising from a finely tuned structure of squark and slepton flavor mixings, may also lead to suppressed decay rates [259]. The predicted upper bounds on the nucleon lifetime in some of these theories lie roughly an order of magnitude above current experimental limits, which may be within the reach of future experiments, such as JUNO [260, 261], Hyper-Kamiokande [235], and DUNE [262].

Orbifold GUTs and string theories contain grand unified symmetries realized in higher dimensions. In the process of compactification and GUT symmetry breaking, the triplet Higgs states may be removed (projected out of the massless sector of the theory). In such models, the nucleon decay due to dimension-five operators can be severely suppressed or eliminated completely. However, nucleon decay due to dimension-six operators may be enhanced, since the gauge-bosons mediating proton decay obtain mass at the compactification scale, M_c, which is typically less than the 4d GUT scale (cf. Sec. 92.5). Alternatively, the same projections which eliminate the triplet Higgs may rearrange the quark and lepton states such that the massless states of one family come from different higher-dimensional GUT multiplets. This can suppress or completely eliminate even dimension-six proton decay. Thus, enhancement or suppression of dimension-six proton decay is model-dependent. In some complete 5d orbifold GUT models [171, 263, 264] the lifetime for the decay $\tau_p/\mathrm{Br}(p \to \pi^0 e^+)$ can be near the bound of 1×10^{34} years with, however, large model-dependence and/or theoretical uncertainties. In other cases, the modes $p \to K^+\bar{\nu}$ and $p \to K^0\mu^+$ may be dominant [171].

In orbifold GUTs or string theory, new discrete symmetries consistent with SUSY GUTs can forbid all dimension-three and four baryon- and lepton-number-violating operators. Even the μ term and dimension-five baryon- and lepton-number-violating operators can be forbidden to all orders in perturbation theory [93–95]. The μ term and dimension-five baryon- and lepton-number-violating operators may then be generated, albeit sufficiently suppressed, via non-perturbative effects. The simplest example of this is a $\mathbb{Z}_4^R$ symmetry which is the unique discrete R symmetry consistent with SO(10) [93–95]. Even though it forbids the dimension-five proton decay operator to the desired level, it allows the required dimension-five neutrino mass term. In this case, proton decay is dominated by dimension-six operators, leading to decays such as $p \to \pi^0 e^+$.

92.7 Yukawa coupling unification

In the SM, the masses and mixings of quarks and leptons arise from their Yukawa couplings to the Higgs doublet; however, the origin and structure of these couplings remain unexplained. GUTs offer at least a partial explanation by embedding each generation into unified multiplets. In this framework, quarks and leptons are treated as components of the same representation, and the GUT symmetry imposes relations among their Yukawa couplings.

In SU(5), there are two types of independent renormalizable Yukawa interactions given by $\lambda_{ij}(\mathbf{10}_i\,\mathbf{10}_j\,\mathbf{5}_H)+\lambda'_{ij}(\mathbf{10}_i\,\overline{\mathbf{5}}_j\,\overline{\mathbf{5}}_H)$. These contain the SM interactions $\lambda_{ij}(Q_i\,u^c_j\,H_u)+\lambda'_{ij}(Q_i\,d^c_j\,H_d+e^c_i\,L_j\,H_d)$, where $i,j=1,2,3$ are family indices. At the GUT scale, this leads to tree-level relations among Yukawa couplings of the charged leptons and down-type quarks. For instance, one obtains $\lambda_b=\lambda_\tau$, where $\lambda_{b,\tau}$ denote the bottom quark and tau lepton Yukawa couplings, respectively [265–268]. In SO(10), there exists only one type of independent renormalizable Yukawa interaction, $\lambda_{ij}(\mathbf{16}_i\,\mathbf{16}_j\,\mathbf{10}_H)$, which implies unified relations among all Yukawa couplings within a single generation [269–282], such as $\lambda_t=\lambda_b=\lambda_\tau$, where λ_t denotes the top quark Yukawa coupling.

In addition to gauge coupling unification, the ratio of the b quark mass to the τ lepton mass has long been a key focus in GUT studies, ever since it was observed that this ratio, after accounting for QCD corrections, is remarkably consistent with experimental data [265, 266]. With the current high-precision determinations of quark masses and gauge couplings, this mass ratio has evolved into a target of detailed and quantitative analyses within the GUT framework, as discussed below.

92.7.1 *The third generation, b–τ or t–b–τ unification*

The third-generation Yukawa couplings are significantly larger than those of the first two generations. As a result, the fermion mass relations predicted by the renormalizable GUT interactions introduced above are expected to be more robust and reliable. To confront these predictions with experimental data, one must account for radiative corrections arising from RG evolution between the GUT scale and the fermion mass scale, as well as threshold corrections from integrating out heavy fields at the GUT and SUSY scales and from weak-scale physics.

Since the unification of Yukawa couplings can only be meaningfully tested in models that also achieve successful gauge coupling unification, we focus here on SUSY GUTs. In the MSSM, the masses of the top and bottom quarks and the τ lepton are related to their respective Yukawa couplings evaluated at the scale m_Z as

$$m_t(m_Z)=\lambda_t(m_Z)\,v_u\left(1+\frac{\delta m_t}{m_t}\right)\,,\qquad m_{b,\tau}(m_Z)=\lambda_{b,\tau}(m_Z)\,v_d\left(1+\frac{\delta m_{b,\tau}}{m_{b,\tau}}\right)\,,\tag{92.15}$$

where $v_u=\sin\beta\,v/\sqrt{2}$ and $v_d=\cos\beta\,v/\sqrt{2}$, with $v\simeq 246$ GeV, and $\delta m_f/m_f$ $(f=t,b,\tau)$ denotes the threshold correction arising from integrating out SUSY particles. For the b quark mass, it has been shown [283, 284] that the dominant corrections originate from gluino-sbottom and Higgsino-stop loop diagrams,

$$\left(\frac{\delta m_b}{m_b}\right)_{g_3}\simeq\frac{g_3^2}{6\pi^2}\frac{m_{\widetilde{g}}\,\mu}{\widetilde{m}^2}\tan\beta\,,\qquad\left(\frac{\delta m_b}{m_b}\right)_{\lambda_t}\simeq\frac{\lambda_t^2}{16\pi^2}\frac{A_t\mu}{\widetilde{m}^2}\tan\beta\,,\tag{92.16}$$

where $m_{\widetilde{g}}$, μ, A_t, and $\widetilde{m}$ denote the gluino mass, higgsino mass, trilinear stop coupling, and the characteristic stop/sbottom mass scale, respectively. It should be noted that Eq. (92.16) captures only the schematic structure of the corrections; the full functional dependence on the soft SUSY-breaking parameters of order $\widetilde{m}$ has been omitted for clarity. For the complete one-loop expressions for the b quark mass corrections, see for example Ref. [285].

Note also that these corrections do not vanish even when SUSY particles are significantly heavier than the electroweak scale. For $\tan\beta=\mathcal{O}(10)$, they can alter the b quark mass by as much as $\mathcal{O}(10)\%$. The overall effect is sensitive to the relative phase between $m_{\widetilde{g}}$ and μ, especially because the infrared fixed-point behavior of the RGE for A_t tends to drive $A_t\simeq -m_{\widetilde{g}}$ [286] in scenarios where SUSY-breaking terms originate from Planck-scale physics, such as gravity mediation. Similar corrections affect the τ lepton mass, although typically only at the few-percent level. Corrections to the top quark mass are not enhanced by $\tan\beta$ and reach at most $\sim 10\%$ [287].

By including one-loop threshold corrections at the electroweak scale and performing the necessary RG evolution, one obtains the masses of the top quark, bottom quark, and τ lepton. In SUSY GUTs, b–τ Yukawa unification allows for two viable solutions corresponding to either small $\tan\beta \sim 1$ or large $\tan\beta = \mathcal{O}(10)$. The small $\tan\beta$ scenario can be realized in the MSSM if the superpartner masses lie at $\mathcal{O}(10)$ TeV [202]. On the other hand, large $\tan\beta$ values, such as $\tan\beta \sim 40$–50, are consistent with the SO(10) symmetry relation [287, 288]. In this regime, threshold corrections to down-type quark masses become significant, as discussed earlier. Consequently, successful Yukawa unification is only achieved within a restricted region of the SUSY parameter space, leading to important implications for SUSY searches [287, 289–293]. More recent studies of Yukawa unification in light of LHC results can be found in Refs. [294–297].

92.7.2 *Beyond leading order: three-family models*

Simple Yukawa unification does not work for the first two generations. In particular, the minimal implementation of SU(5) predicts the relations $\lambda_s = \lambda_\mu$ and $\lambda_d = \lambda_e$, which imply $\lambda_s/\lambda_d = \lambda_\mu/\lambda_e$. This relation is preserved under RG evolution and leads to the prediction $m_s/m_d = m_\mu/m_e$ at the weak scale, which is clearly incompatible with experimental observations, where $m_s/m_d \sim 20$ and $m_\mu/m_e \sim 200$. A compelling solution to this discrepancy was proposed by Georgi and Jarlskog [298]; for a more recent treatment in the context of SUSY, see Ref. [299].

More generally, it is important to recall that all previous discussions of Yukawa couplings were based on the assumption of renormalizable interactions and minimal matter and Higgs content. However, since the GUT scale lies close to the Planck scale, higher-dimensional operators involving GUT-breaking Higgs fields can lead to significant corrections, particularly for the first and second generations. A representative example is the dimension-five operator $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_H\,\mathbf{24}_H$, where $\mathbf{24}_H$ denotes the SU(5) GUT-breaking Higgs field. Such operators can be used to fit the observed fermion masses, albeit at the cost of introducing some degree of fine-tuning. Moreover, the SM Higgs doublet itself may originate, at least in part, from higher-dimensional representations of the GUT group. For instance, the **45** representation of SU(5) contains an $SU(2)_L$ doublet with the correct hypercharge to serve as a SM Higgs field [298]. This **45** can arise from larger representations such as the **120** or **126** of SO(10), once the symmetry is broken down to SU(5) [300–302]. These additional Higgs fields can also have renormalizable couplings to SM fermions, modifying the relations among the Yukawa couplings when the SM Higgs is a linear combination of doublets from different SU(5) multiplets. Finally, the embedding of SM fermions into GUT multiplets may not follow the minimal assignment. In minimal SO(10) models, all quarks and leptons reside in **16**-dimensional representations, but renormalizable Yukawa couplings with a single $\mathbf{10}_H$ cannot account for the observed CKM mixing angles. This issue can be alleviated by introducing additional matter multiplets, such as **10**s. After the breaking of $U(1)_X$—which distinguishes the $\overline{\mathbf{5}}$ components originating from the **16** and **10** representations—the right-handed down-type quarks and left-handed leptons of the SM can be linear combinations of states from **16** and **10**. In this case, the two types of Yukawa couplings in SU(5) need not be identical [303].

To construct realistic three-family models, it is typically necessary to incorporate some or all of the effects discussed above. However, grand unification alone is generally insufficient to yield a predictive framework for fermion masses and mixing angles. Additional ingredients are required—most notably, global family symmetries. In particular, non-Abelian family symmetries can significantly reduce the number of free parameters by restricting the allowed structure of effective higher-dimensional operators that govern fermion masses. Moreover, the sequential breaking of family symmetries can naturally correlate with the observed fermion mass hierarchies [304]. A simple and widely studied approach is to assign each $\mathbf{10}_i$ representation a suppression factor ϵ^{3-i} in its Yukawa couplings, where ϵ is a small expansion parameter. This setup automatically generates a stronger hierarchy in the up-type quark Yukawa couplings compared to those of the down-type quarks and charged leptons, and no strong hierarchy in the neutrino sector—qualitatively consistent with observations. Fully realistic three-family models incorporating these ingredients have been constructed and are capable of reproducing the full spectrum of fermion masses and mixing parameters, including those in the neutrino sector [66–72, 305–313].

Finally, a particularly ambitious variant of unification is to require that the fermions of all three generations come from a single representation of a large gauge group. A somewhat weaker assumption is that the flavor group (*e.g.*, SU(3)) unifies with the SM gauge group in a simple gauge group at some energy scale $M \geq M_G$. Early work on such 'flavor-unified GUTs', see, *e.g.*, Refs. [303, 314–320], has been reviewed in Refs. [321, 322]. For a selection of more recent papers see Refs. [323–327]. In such settings, Yukawa couplings are generally determined by gauge couplings together with symmetry breaking VEVs. This is reminiscent of heterotic string GUTs, where all couplings come from the 10d gauge coupling. However, while the $E_8 \to SU(3) \times E_6$ branching rule $\mathbf{248} = (\mathbf{8}, \mathbf{1}) + (\mathbf{1}, \mathbf{78}) + (\mathbf{3}, \mathbf{27}) + (\overline{\mathbf{3}}, \overline{\mathbf{27}})$ looks very suggestive in this context, the way in which most modern heterotic models arrive at three generations is actually more complicated.

92.8 Neutrino masses

Neutrino oscillation experiments have firmly established that neutrinos possess nonzero masses. By adding three "sterile" neutrinos ν_i^c with Yukawa couplings $\lambda_{\nu,ij}(\nu_i^c\, L_j\, H_u)$ $(i, j = 1, 2, 3)$, one obtains three massive Dirac neutrinos with mass $m_\nu^D = \lambda_\nu v_u$, analogously to quark and charged lepton masses. However, in order to obtain a τ neutrino with mass of order 0.1 eV, one requires the exceedingly small coupling ratio $\lambda_{\nu_\tau}/\lambda_\tau \lesssim 10^{-10}$. By contrast, in GUTs the seesaw mechanism *naturally* explains such tiny neutrino masses as follows [2–8]: The sterile neutrinos have no SM gauge quantum numbers so that there is no symmetry other than lepton number which forbids the Majorana mass term $\frac{1}{2}M_{ij}\nu_i^c\nu_j^c$. Note also that sterile neutrinos can be identified with the right-handed neutrinos necessarily contained in complete multiplets of SO(10), the flipped SU(5), or $G_{\rm PS}$. Since the Majorana mass term violates these gauge symmetries, one might expect $M_{ij} \sim M_G$. The heavy sterile neutrinos can be integrated out, yielding an EFT with only light active neutrinos with the effective dimension-five operator

$$\mathcal{L}_{\rm eff} = -\frac{1}{2}c_{ij}(L_i\, H_u)(L_j\, H_u) + \text{h.c.}\ , \tag{92.17}$$

where $c = \lambda_\nu^T M^{-1}\lambda_\nu$. This leads to a 3×3 Majorana neutrino mass matrix $m_\nu = (m_\nu^D)^T M^{-1} m_\nu^D$ after the electroweak symmetry breaking. For example, if the neutrino Yukawa coupling λ_{ν_τ} is unified with the top quark Yukawa coupling at the GUT scale, $\lambda_{\nu_\tau} = \lambda_t$, as predicted in SO(10), flipped SU(5), or Pati–Salam models, then the observed value $m_{\nu_\tau} \sim 0.1$ eV implies a right-handed neutrino mass scale $M \sim 10^{14}$ GeV, intriguingly close to the GUT scale.

The seesaw mechanism implemented by right-handed neutrinos is sometimes called the type-I seesaw model. There are variant models in which the dimension-five operator for neutrino masses is induced in different ways: In the type-II model, an $SU(2)_L$ triplet Higgs boson Σ is introduced to have couplings ΣL^2 and also ΣH_u^2 [300, 328–331]. In the type-III model, an $SU(2)_L$ triplet of fermions $\widetilde{\Sigma}$ with a Yukawa coupling $\widetilde{\Sigma} L H_u$ is introduced [332]. In these models, the dimension-five operator is generated by integrating out either the triplet Higgs bosons or fermions. Such mechanisms can be embedded into GUT frameworks by introducing, for example, Higgs bosons in the **15** or fermions in the **24** representations of SU(5), which can originate from the **126** representation of SO(10). It is important to note that the gauge non-singlet fields involved in type-II and type-III seesaw mechanisms typically acquire masses at an intermediate scale. As a result, gauge coupling unification is not automatically preserved when these mechanisms are implemented in SUSY GUTs.

The neutrino sector presents a challenge for GUTs: while the quark mixing angles in the CKM matrix are small, two of the lepton mixing angles in the PMNS matrix are large. Global fits to neutrino masses and mixing parameters can be found in Sec. 14. For reviews of SUSY GUT models that successfully reproduce quark and lepton masses, see Refs. [333, 334]. A compilation of the range of SUSY GUT predictions for neutrino mixing angles is provided in Ref. [335].

It is also worth noting that, both within and beyond the GUT framework, considerable effort has been devoted to constructing models of neutrino masses—and more broadly, flavor structure—based on discrete symmetries (see, *e.g.*, Refs. [334, 336, 337]). A particularly interesting recent development in this context is the application of the (non-linearly realized) modular group $\mathrm{SL}(2,\mathbb{Z})$ as a discrete flavor symmetry [338]. This group plays a central role in string theory as the group of large diffeomorphisms of the two-torus, which appears both as the string worldsheet and—as is especially relevant here—as part of the compactification space in torus orbifold models (cf. Sec. 92.4). For a selection of recent activity in this direction, see, *e.g.*, Refs. [339–342].

92.9 Selected topics

92.9.1 *Global symmetries*

As discussed above, global symmetries are often introduced in GUT models to control higher-dimensional operators. This is particularly crucial in the context of nucleon decay, but such symmetries also play important roles in solutions to the doublet-triplet splitting problem, GUT-based flavor model building, and cosmological applications such as baryogenesis and inflation. However, relying on global symmetries to suppress specific interactions is not always as straightforward. Broadly speaking, two possibilities exist: First, the relevant symmetry may be gauged at a high scale. In this case, the effects of the VEVs responsible for its spontaneous breaking can, in principle, induce dangerous operators, and their impact must be carefully assessed. Second, the symmetry may be truly global, as is necessarily the case for anomalous symmetries whose breaking arises through non-perturbative effects and may therefore be exponentially suppressed. Nonetheless, it is widely believed that exact global symmetries cannot be preserved in the presence of quantum gravity (see, *e.g.*, Refs. [343, 344]). One must then determine the degree of suppression—whether polynomial or exponential—arising from Planck-scale physics. For instance, dimension-five baryon-number-violating operators suppressed by only a single power of the Planck or string scale are highly problematic in SUSY models [345].

In light of the above, it is worth recalling that in string-theoretic models, 4d global symmetries typically originate from higher-dimensional gauge symmetries [111–114,346]. Here, the term 'global' refers to a symmetry whose gauge boson acquires a Stückelberg mass. This mass generation is necessary in the case of anomalous symmetries (via the Green–Schwarz mechanism [347]), but it can also occur for non-anomalous ones. In such scenarios, one generally expects no symmetry violation beyond well-understood non-perturbative effects. Discrete symmetries arise as subgroups of continuous gauge symmetries—for example, $\mathbb{Z}_N \subset \mathrm{U}(1)$. Notably, non-anomalous subgroups of Stückelberg-massive U(1)s correspond to unbroken discrete gauge symmetries, which are exact even non-perturbatively (see, *e.g.*, Refs. [348, 349]). Such discrete gauge symmetries can also emerge as remnants of spontaneously broken continuous gauge symmetries in 4d.

92.9.2 *Anomaly constraints vs. GUT paradigm*

As emphasized at the very beginning, the fact that the SM fermions of one generation fill out the $\mathbf{10}+\overline{\mathbf{5}}$ of SU(5) appears to provide strong evidence for some form of GUT embedding. However, one should be aware that a counterargument can be made which is related to the issue of 'charge quantization by anomaly cancellation' (see Refs. [350–354] for some early papers and Ref. [355] for a more detailed reference list): Imagine we only knew that the low-energy gauge group were G_{SM} and the matter content included $(\mathbf{3},\mathbf{2})_Y$, *i.e.*, a 'quark doublet' with U(1)-charge Y. One can then ask which possibilities exist of adding further matter to ensure the cancellation of all triangle anomalies. It turns out that this problem has only three different, minimal[14] solutions [354]. One of those is precisely a single SM generation, with the apparent 'SU(5)-ness' emerging accidentally. Thus, if one randomly picks models from the set of consistent gauge theories, preconditioning on $G_{\rm SM}$ and $(\mathbf{3},\mathbf{2})_Y$, one may easily end up with '$\mathbf{10}+\overline{\mathbf{5}}$' of an SU(5) that is in no way dynamically present. This is precisely what happens in the context of non-GUT string model building [356–358].

92.9.3 *Topological defects and phase transitions*

Among the various topological defects that can arise in the broken phase of GUT models, magnetic monopoles are the most well-known. These are localized classical solutions carrying magnetic charge under an unbroken U(1) subgroup [359, 360]. Magnetic monopoles with masses of $\mathcal{O}(M_G/\alpha_G)$ can be produced during a GUT phase transition in the early Universe. Experimentally, the flux of such monopoles is constrained to be $\lesssim 10^{-16}\ \text{cm}^{-2}\cdot\text{s}^{-1}\cdot\text{sr}^{-1}$ [361–363]. It was also shown that GUT monopoles can catalyze nucleon decay [364–368], leading to significantly stronger constraints on the monopole flux when considering, for example, X-ray emission from radio pulsars induced by monopole capture and subsequent nucleon decay [369]. Nonetheless, GUTs typically predict a monopole abundance far exceeding these limits—this is the so-called monopole problem.

One of the original motivations for proposing inflation was to resolve the monopole problem by exponentially diluting their density after the GUT phase transition (for a review on inflation, see Sec. 23). Notably, the current upper bound on the inflationary vacuum energy scale is remarkably close to the GUT scale, given by $V_{\rm inf}^{1/4} = (1.88\times10^{16}\ \text{GeV})\times(r/0.10)^{1/4}$, where the scalar-to-tensor ratio is constrained to be $r < 0.036$ [370]. This ensures that reheating does not reach temperatures above the GUT scale M_G, thereby solving the monopole problem—provided M_G is not significantly lower than expected. Alternative mechanisms to address the monopole problem include sweeping monopoles away with domain walls [371], breaking the electromagnetic U(1) symmetry at high temperatures [372], or avoiding GUT symmetry restoration altogether [373].

Topological defects other than monopoles are also common, see, *e.g.*, Refs. [374–376]. They can be cosmologically relevant in GUT models based on a gauge group larger than SU(5), say, SO(10), where an intermediate gauge theory may appear. The breaking of this intermediate gauge symmetry can a priori occur anywhere between the GUT and the electroweak scales. Right-handed neutrino masses favor a value near $\mathcal{O}(10^{14})$ GeV and non-SUSY coupling unification may profit from such a two-step GUT breaking scheme (cf., Sec. 92.5). Crucially for the present context, the last breaking step may occur after inflation, generating topological defects (like cosmic strings) which are not diluted. Their signature can be probed through gravitational waves [377–381], with the metastable string case being of particular phenomenological interest [382–387]. Metastability arises because the nucleation of a monopole-anti-monopole pair, each of which serves as an endpoint for the string, allow the cosmic string to break. The lifetime becomes long and hence the string stable as the ratio between monopole mass and the square root of the string tension grows. This is related to the ratio of the two symmetry-breaking scales. It is of phenomenological interest that metastability allows the string network to evade the recent bound of $G\mu \lesssim 10^{-10}$ (or $\sqrt{\mu} \lesssim 10^{14}$ GeV) [387], where G is the gravitational constant and μ is the string tension. Moreover the spectrum is blue-tilted compared to the stable string case. Finally, we note that if the intermediate gauge symmetry breaking proceeds with a (strong) first order phase transition, this could also generate gravitational waves [387–391]. However, due to the relatively high energy scales involved, the peak frequency tends to be high, with signals hopefully to be explored in future gravitational wave experiments.

[14] Adding extra vector-like sets of fields, *e.g.*, two fermions which only transform under U(1) and have charges Y and $-Y$, is considered to violate minimality.

92.9.4 *Flavor violation*

Yukawa interactions involving GUT-scale particles and SM quarks and leptons can imprint flavor-violating structures onto the soft SUSY-breaking terms [392]. To illustrate this, consider the MSSM with flavor-universal soft terms given at the Planck scale. Working in a basis where the up-type quark and charged lepton Yukawa matrices are diagonal, one finds that the large top Yukawa coupling drives a radiative suppression of the third-generation left-handed squark mass squared via RG evolution. After CKM rotation, this leads to sizable off-diagonal entries in the left-handed down-type squark mass matrix. In GUTs, however, additional sources of flavor violation arise. For instance, in SU(5), the color-triplet Higgs radiatively generates flavor-violating right-handed slepton mass terms via the Yukawa interaction $\lambda_{ij}(\mathbf{10}_i\,\mathbf{10}_j\,\mathbf{5}_H)$ [393, 394]. In the SU(5) extension of the type-I seesaw mechanism—where right-handed neutrinos are introduced as SU(5) singlets with the interaction $\lambda''_{ij}(\mathbf{1}_i\,\overline{\mathbf{5}}_j\,\mathbf{5}_H)$—both the doublet and color-triplet Higgs bosons acquire additional Yukawa couplings. These give rise to radiatively induced flavor violation in the left-handed slepton sector [395–398] and in the right-handed down-type squark sector [399, 400]. Note that even if flavor-universal soft terms are assumed at the GUT scale, sizable flavor violation in the left-handed slepton masses can still be generated through the RG running from the GUT scale down to the right-handed neutrino mass scale, driven by the right-handed neutrino Yukawa couplings. These flavor-violating SUSY-breaking terms generate new contributions to flavor-changing neutral current (FCNC) processes in both the quark and lepton sectors, such as $\mu \to e\gamma$, K^0–$\bar{K}^0$ mixing, and B^0–$\bar{B}^0$ mixing. Flavor violation in the sfermion mass matrices can also enhance nucleon decay induced by the dimension-five baryon-number-violating operators, as not only higgsino and wino exchange but also gluino-mediated processes can contribute in this case [401]. Electric dipole moments (EDMs) may also arise when both left- and right-handed squarks or sleptons receive flavor-violating mass terms with relative complex phases. This has been discussed in the context of SO(10) [402] and SU(5) models with right-handed neutrinos [403]. Thus, low-energy flavor and CP-violating observables serve as important probes of GUT-scale interactions.

92.9.5 *From GUT baryogenesis to leptogenesis and B/L-violating transitions*

During inflation, any conserved quantum number is exponentially diluted. Consequently, the observed baryon asymmetry of the Universe must be generated during or after reheating. The situation, however, is somewhat more subtle: both B and L are classical global symmetries of the SM, but the combination $(B + L)$ is anomalous and violated by thermal fluctuations in the early Universe via sphaleron processes [404]. In addition, $(B + L)$ is also violated in GUTs, as evidenced by nucleon decay. By contrast, the combination $(B - L)$ is anomaly-free and remains conserved in both the SM and in gauge interactions of GUTs such as SU(5) and SO(10).

The classic idea of GUT baryogenesis [405–412] involves generating a $(B + L)$ asymmetry—and hence a baryon asymmetry—via the out-of-equilibrium decay of color-triplet Higgs bosons. However, such an asymmetry produced at the GUT scale is erased by sphaleron processes that become efficient at temperatures below $T \sim 10^{12}$ GeV.[15] This issue can be circumvented [413] by using lepton-number-violating interactions of neutrinos to convert part of the initial $(B + L)$ asymmetry into a $(B - L)$ asymmetry before sphaleron processes become active. The resulting $(B - L)$ asymmetry is preserved throughout the sphaleron era. However, this mechanism is not viable in the minimal SUSY GUT, where the triplet Higgs lies above the GUT scale. In such cases, achieving the required out-of-equilibrium decay would necessitate a reheating temperature exceeding the GUT scale, which—as discussed earlier—is disfavored by cosmological observations.

However, the most widely accepted resolution to this dilemma is to generate a net $(B - L)$ asymmetry dynamically in the early Universe, using right-handed neutrinos. If the decay of heavy

[15] Strictly speaking, if lepton flavor asymmetries L_i $(i = 1, 2, 3)$ are present with $(L_i - L_j) \neq 0$ for $i \neq j$, nonzero values of B and L may persist even when $(B - L) = 0$ [243].

right-handed neutrinos occurs out of equilibrium and violates both L and CP, it can produce a net lepton asymmetry. This lepton asymmetry is then partially converted into a baryon asymmetry through electroweak sphaleron processes. This mechanism is known as leptogenesis [414].

If the three heavy Majorana neutrinos have a hierarchical mass spectrum, the net lepton asymmetry is primarily generated by the decay of the lightest one. The resulting asymmetry is proportional to the CP asymmetry in its decay. It is found that in order to account for the observed baryon asymmetry of the Universe, the mass of the lightest heavy neutrino must be $\gtrsim 10^9$ GeV [415,416].[16] Consequently, the reheating temperature after inflation must also be $\gtrsim 10^9$ GeV to ensure thermal production of the heavy neutrinos. In SUSY models, however, there exists a potential tension between the thermal leptogenesis and Big Bang Nucleosynthesis (BBN), particularly if gravitinos are long-lived and decay during or after the BBN epoch. This so-called gravitino problem imposes an upper bound on the reheating temperature, typically in the range $\lesssim 10^{6\text{–}10}$ GeV, depending on the details of the SUSY breaking scenario [419]. This tension can be alleviated in the non-thermal leptogenesis scenario, in which right-handed neutrinos are produced non-thermally via the decay of inflaton [420–422]. For recent reviews of leptogenesis, see Refs. [423–425].

One of the key experimental probes of leptogenesis is the search for neutrinoless double-beta ($0\nu\beta\beta$) decay. In $0\nu\beta\beta$ decay, a nucleus emits two electrons without accompanying (anti)neutrinos, in contrast to the standard double-beta decay, where two antineutrinos are emitted alongside the electrons. This process violates lepton number by two units ($\Delta L = 2$). At the nucleon level, $0\nu\beta\beta$ decay is described by dimension-nine effective operators mediating the process $nn \to ppee$. These operators can originate from combinations of SM weak interactions and lower-dimensional L-violating operators. The leading contribution typically arises from the dimension-five Weinberg operator in Eq. (92.17). Thus, if lepton-number violation originates at energy scales much higher than the electroweak scale, the $0\nu\beta\beta$ decay rate is proportional to the Majorana neutrino masses. The current experimental status of $0\nu\beta\beta$ searches is reviewed in Ref. [170]. For recent studies incorporating all SM effective operators up to dimension nine, see Ref. [426] and references therein.

In addition to lepton-number violation, one can also consider processes that violate B or $(B-L)$. Such processes are of intrinsic theoretical interest and may also play a role in baryogenesis. The relevant operators typically have mass dimensions higher than the well-known dimension-six $(B+L)$-violating operators (cf., Sec. 92.6). Such higher-dimensional operators may arise in SO(10) GUTs featuring an intermediate scale at which baryogenesis is realized, as discussed in Refs. [427–429]. For instance, one may consider interactions that induce nucleon decay with $\Delta(B-L) = 2$, such as $n \to e^-\pi^+$. These processes are mediated by dimension-seven effective operators in the SM, suppressed by the Higgs VEV or derivative interactions. Another important class involves neutron-antineutron (n–$\bar{n}$) oscillations, which violate baryon number by two units ($\Delta B = 2$) and are described by dimension-nine effective operators. The current experimental lower bound on the n–$\bar{n}$ oscillation time is derived using free neutrons [430], while complementary constraints come from limits on the decay of bound neutrons in ^{16}O, obtained by Super-Kamiokande [431]. These bounds are of similar magnitude. Super-Kamiokande has also searched for dinucleon decay modes with $\Delta B = 2$, such as $pp \to \pi^+\pi^+$ and $nn \to \pi^\pm\pi^\mp$ [432].

92.9.6 *The QCD Axion*

The axion is a pseudo-Nambu–Goldstone boson arising from the spontaneous breaking of the Peccei–Quinn symmetry, which is capable of resolving the strong CP problem [433–436] and may potentially be related to GUT physics.[17] In particular, the strength of the direct coupling of the QCD axion to photons, relative to its coupling to gluons, may be seen as a phenomenological

[16] This requirement can be circumvented in resonant leptogenesis [417,418], where the heavy right-handed neutrinos are nearly degenerate in mass.

[17] For alternative approaches to addressing the strong CP problem within the context of GUTs, see Refs. [437–442].

signature of grand unification. It is quantified by the predicted anomaly ratio $E/N = 8/3$ (see Sec. 89 and Refs. [443, 444]). This ratio follows from the GUT normalization of the relevant generators together with the completeness of GUT multiplets contributing to the anomaly.[18] It hence arises in both the DFSZ [445, 446] and KSVZ [447, 448] axion models as long as the Peccei-Quinn symmetry commutes with the GUT group (see, *e.g.*, [449, 450]). We, however, note that, in bottom-up approaches and in the presence of split GUT multiplets, other anomaly ratios can appear (cf., *e.g.*, Refs. [451, 452]). Their embedding in a UV GUT structure, possibly with explicit GUT symmetry breaking, requires case-by-case study. The ratio $E/N = 8/3$ also arises in string-theoretic GUTs [443, 453]. In this case, the axion does not come from the phase of a complex scalar but is a fundamental shift-symmetric real field, coupling through a higher-dimension operator directly to the product of the GUT field-strength and its dual.

92.10 Conclusion

In its most conservative form, grand unification assumes that some or all of the SM gauge interactions—$\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_L$, and $\mathrm{SU}(3)_C$—are embedded within a larger gauge symmetry at high energies. In most models, particularly in the simplest and most compelling realizations based on SU(5) or SO(10), the unification hypothesis goes significantly further: one expects the SM gauge couplings to merge (up to small threshold corrections) at a single unification scale, M_G, the SM matter fields to be (partially) unified into larger multiplets, and the nucleon to become unstable due to exchange of GUT-scale fields. SUSY GUTs provide by far the most predictive and economical framework that achieves perturbative gauge coupling unification. Further details beyond the scope of this article can be found in reviews [321, 454–456] and textbooks [457–460]. See also Refs. [461, 462] for more recent overviews.

Thus, the three classical pillars of GUTs are: gauge coupling unification at $M_G \simeq 2 \times 10^{16}$ GeV, low-energy SUSY (with a large SUSY desert), and nucleon decay. Among these, gauge coupling unification remains robust even if the SUSY particle masses lie somewhat above the weak scale. However, from a conceptual standpoint, the steadily increasing experimental lower bound on the SUSY scale poses a challenge for the SUSY GUT framework. If the original motivation for low-energy SUSY—namely, the stabilization of the electroweak scale—is entirely discarded, then the SUSY scale becomes an arbitrary parameter, and the first two pillars lose their theoretical foundation. This highlights the importance of continuing to improve bounds on nucleon decay, which—though not a universal prediction of all GUT models—is arguably a more generic and robust feature of grand unification than low-energy SUSY.

Whether or not Yukawa couplings unify is highly model dependent. Nevertheless, regardless of the degree to which Yukawa unification is realized, there exists a rich and potentially fruitful interplay between flavor model building and grand unification. This interplay is particularly pronounced in the neutrino sector, where ongoing experimental developments continue to shape theoretical approaches.

It is probably fair to say that, due to limitations of the 4d approach, including especially remaining ambiguities (free parameters or ad hoc assumptions) in models of flavor and GUT breaking, the string theoretic approach has become more important in GUT model building. In this framework, challenges include learning how to deal with the many vacua of the 'landscape' as well as, for each vacuum, developing the tools for reliably calculating phenomenological observables. Finally, due to limitations of space, the present article has barely touched on the interesting cosmological implications of GUTs. They may become more important in the future, especially in the case that a high inflationary energy scale is established observationally.

[18] See, *e.g.*, Eq. (312) of Ref. [444].

References

[1] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[2] P. Minkowski, Phys. Lett. **67B**, 421 (1977).

[3] T. Yanagida, Conf. Proc. C **7902131**, 95 (1979).

[4] S. L. Glashow, NATO Sci. Ser. B **61**, 687 (1980).

[5] M. Gell-Mann, P. Ramond and R. Slansky, Conf. Proc. C **790927**, 315 (1979), [arXiv:1306.4669].

[6] F. Wilczek, eConf **C790823**, 437 (1979).

[7] E. Witten, Phys. Lett. **91B**, 81 (1980).

[8] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980), [,231(1979)].

[9] J. C. Pati and A. Salam, Phys. Rev. **D8**, 1240 (1973).

[10] A. Davidson, Phys. Rev. D **20**, 776 (1979).

[11] R. E. Marshak and R. N. Mohapatra, Phys. Lett. B **91**, 222 (1980).

[12] J. C. Pati, Int. J. Mod. Phys. A **32**, 09, 1741013 (2017), [arXiv:1706.09531].

[13] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[14] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Phys. Lett. **80B**, 360 (1979), [Erratum: Phys. Lett.82B,464(1979)].

[15] E. Golowich, Phys. Rev. **D24**, 2899 (1981).

[16] I. Dorsner, S. Fajfer and N. Kosnik, Phys. Rev. D **86**, 015013 (2012), [arXiv:1204.0674].

[17] I. Doršner and S. Saad, Phys. Rev. D **110**, 7, 075025 (2024), [arXiv:2404.09021].

[18] G. 't Hooft, NATO Sci. Ser. B **59**, 135 (1980).

[19] M. J. G. Veltman, Acta Phys. Polon. **B12**, 437 (1981).

[20] L. Maiani, Conf. Proc. C **7909031**, 1 (1979).

[21] E. Witten, Nucl. Phys. **B188**, 513 (1981).

[22] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. **D24**, 1681 (1981).

[23] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981).

[24] L. E. Ibanez and G. G. Ross, Phys. Lett. **105B**, 439 (1981).

[25] N. Sakai, Z. Phys. **C11**, 153 (1981).

[26] M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196**, 475 (1982).

[27] W. J. Marciano and G. Senjanovic, Phys. Rev. **D25**, 3092 (1982).

[28] S. Weinberg, Phys. Rev. **D26**, 287 (1982).

[29] N. Sakai and T. Yanagida, Nucl. Phys. **B197**, 533 (1982).

[30] H. Georgi, AIP Conf. Proc. **23**, 575 (1975).

[31] H. Fritzsch and P. Minkowski, Annals Phys. **93**, 193 (1975).

[32] R. Slansky, Phys. Rept. **79**, 1 (1981).

[33] H. Georgi, *Lie algebras in particle physics*, volume 54, Perseus Books, Reading, MA, 2nd ed. edition (1999).

[34] N. Yamatsu, PTEP **2016**, 4, 043B02 (2016), [arXiv:1512.05559].

[35] R. Feger, T. W. Kephart and R. J. Saskowski, Comput. Phys. Commun. **257**, 107490 (2020), [arXiv:1912.10969].

[36] S. M. Barr, Phys. Lett. **112B**, 219 (1982).
[37] J. P. Derendinger, J. E. Kim and D. V. Nanopoulos, Phys. Lett. **139B**, 170 (1984).
[38] I. Antoniadis *et al.*, Phys. Lett. **B194**, 231 (1987).
[39] I. Antoniadis *et al.*, Phys. Lett. **B231**, 65 (1989).
[40] F. Gursey, P. Ramond and P. Sikivie, Phys. Lett. **60B**, 177 (1976).
[41] Y. Achiman and B. Stech, NATO Sci. Ser. B 303–314 (1979).
[42] A. De Rujula, H. Georgi and S. L. Glashow, in "Fifth Workshop on Grand Unification," (1984).
[43] B. R. Greene *et al.*, Nucl. Phys. **B278**, 667 (1986).
[44] B. R. Greene *et al.*, Nucl. Phys. **B292**, 606 (1987).
[45] B. R. Greene, C. A. Lutken and G. G. Ross, Nucl. Phys. **B325**, 101 (1989).
[46] J. E. Kim, Phys. Lett. **B591**, 119 (2004), [hep-ph/0403196].
[47] L. O'Raifeartaigh, *GROUP STRUCTURE OF GAUGE THEORIES*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (1988), ISBN 978-0-521-34785-3, 978-1-139-24180-9.
[48] J. Hucks, Phys. Rev. D **43**, 2709 (1991).
[49] D. Tong, JHEP **07**, 104 (2017), [arXiv:1705.01853].
[50] M. T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B **159**, 429 (1979).
[51] E. Witten, Phys. Lett. **105B**, 267 (1981).
[52] A. Masiero *et al.*, Phys. Lett. **115B**, 380 (1982).
[53] B. Grinstein, Nucl. Phys. **B206**, 387 (1982).
[54] S. Dimopoulos and F. Wilczek (1981), print-81-0600 (SANTA BARBARA), NSF-ITP-82-07.
[55] M. Srednicki, Nucl. Phys. **B202**, 327 (1982).
[56] K. Inoue, A. Kakuto and H. Takano, Prog. Theor. Phys. **75**, 664 (1986).
[57] Y. Yamada, Z. Phys. **C60**, 83 (1993).
[58] J. Hisano *et al.*, Phys. Lett. **B342**, 138 (1995), [hep-ph/9406417].
[59] Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B **396**, 150 (1997), [hep-ph/9611277].
[60] G. Altarelli, F. Feruglio and I. Masina, JHEP **11**, 040 (2000), [hep-ph/0007254].
[61] J. Ellis *et al.*, Eur. Phys. J. C **81**, 6, 543 (2021), [arXiv:2103.15430].
[62] K. Hamaguchi, S. Hor and N. Nagata, JHEP **11**, 140 (2020), [arXiv:2008.08940].
[63] K. S. Babu and S. M. Barr, Phys. Rev. **D48**, 5354 (1993), [hep-ph/9306242].
[64] K. S. Babu and S. M. Barr, Phys. Rev. **D50**, 3529 (1994), [hep-ph/9402291].
[65] K. S. Babu, J. C. Pati and Z. Tavartkiladze, JHEP **06**, 084 (2010), [arXiv:1003.2625].
[66] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **74**, 2418 (1995), [hep-ph/9410326].
[67] V. Lucas and S. Raby, Phys. Rev. **D54**, 2261 (1996), [hep-ph/9601303].
[68] T. Blazek *et al.*, Phys. Rev. **D56**, 6919 (1997), [hep-ph/9611217].
[69] Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B **409**, 220 (1997), [hep-ph/9612232].
[70] S. M. Barr and S. Raby, Phys. Rev. Lett. **79**, 4748 (1997), [hep-ph/9705366].
[71] K. S. Babu, J. C. Pati and F. Wilczek, Nucl. Phys. **B566**, 33 (2000), [hep-ph/9812538].
[72] R. Dermisek, A. Mafi and S. Raby, Phys. Rev. **D63**, 035001 (2001), [hep-ph/0007213].

[73] N. Maekawa, Prog. Theor. Phys. **106**, 401 (2001), [hep-ph/0104200].
[74] N. Maekawa, Prog. Theor. Phys. **107**, 597 (2002), [hep-ph/0111205].
[75] N. Maekawa and T. Yamashita, Prog. Theor. Phys. **108**, 719 (2002), [hep-ph/0205185].
[76] A. A. Anselm and A. A. Johansen, Phys. Lett. B **200**, 331 (1988).
[77] A. A. Anselm, Sov. Phys. JETP **67**, 663 (1988).
[78] Z. G. Berezhiani and G. R. Dvali, Bull. Lebedev Phys. Inst. **5**, 55 (1989).
[79] R. Barbieri, G. R. Dvali and A. Strumia, Nucl. Phys. **B391**, 487 (1993).
[80] Z. Berezhiani, C. Csaki and L. Randall, Nucl. Phys. **B444**, 61 (1995), [hep-ph/9501336].
[81] G. R. Dvali and S. Pokorski, Phys. Rev. Lett. **78**, 807 (1997), [hep-ph/9610431].
[82] Q. Shafi and Z. Tavartkiladze, Phys. Lett. **B522**, 102 (2001), [hep-ph/0105140].
[83] M. Bando and T. Kugo, Prog. Theor. Phys. **109**, 87 (2003), [hep-ph/0209088].
[84] K. Hamaguchi, S. Hor and N. Nagata, Phys. Rev. D **107**, 3, 035016 (2023), [arXiv:2210.09333].
[85] G. F. Giudice and A. Masiero, Phys. Lett. **B206**, 480 (1988).
[86] J. E. Kim and H. P. Nilles, Mod. Phys. Lett. **A9**, 3575 (1994), [hep-ph/9406296].
[87] E. Witten, in "Supersymmetry and unification of fundamental interactions. Proceedings, 10th International Conference, SUSY'02, Hamburg, Germany, June 17-23, 2002," 472–491 (2001), [hep-ph/0201018], URL `http://www-library.desy.de/preparch/desy/proc/proc02-02/Proceedings/susy02/special/hertz_pr.ps`.
[88] L. Randall and C. Csaki, in "Supersymmetry and unification of fundamental interactions. Proceedings, International Workshop, SUSY 95, Palaiseau, France, May 15-19, 1995," 99–109 (1995), [,235(1995)], [hep-ph/9508208].
[89] M. Dine, Y. Nir and Y. Shadmi, Phys. Rev. **D66**, 115001 (2002), [hep-ph/0206268].
[90] A. Hebecker, J. March-Russell and R. Ziegler, JHEP **08**, 064 (2009), [arXiv:0801.4101].
[91] F. Brummer *et al.*, JHEP **08**, 011 (2009), [arXiv:0906.2957].
[92] F. Brummer *et al.*, JHEP **04**, 006 (2010), [arXiv:1003.0084].
[93] H. M. Lee *et al.*, Phys. Lett. **B694**, 491 (2011), [arXiv:1009.0905].
[94] R. Kappl *et al.*, Nucl. Phys. **B847**, 325 (2011), [arXiv:1012.4574].
[95] H. M. Lee *et al.*, Nucl. Phys. **B850**, 1 (2011), [arXiv:1102.3595].
[96] Y. Hosotani, Phys. Lett. **126B**, 309 (1983).
[97] L. J. Dixon *et al.*, Nucl. Phys. **B261**, 678 (1985), [,678(1985)].
[98] L. J. Dixon *et al.*, Nucl. Phys. **B274**, 285 (1986).
[99] L. E. Ibanez, H. P. Nilles and F. Quevedo, Phys. Lett. **B187**, 25 (1987).
[100] L. E. Ibanez *et al.*, Phys. Lett. **B191**, 282 (1987).
[101] P. Candelas *et al.*, Nucl. Phys. **B258**, 46 (1985).
[102] Y. Kawamura, Prog. Theor. Phys. **103**, 613 (2000), [hep-ph/9902423].
[103] Y. Kawamura, Prog. Theor. Phys. **105**, 999 (2001), [hep-ph/0012125].
[104] G. Altarelli and F. Feruglio, Phys. Lett. **B511**, 257 (2001), [hep-ph/0102301].
[105] L. J. Hall and Y. Nomura, Phys. Rev. **D64**, 055003 (2001), [hep-ph/0103125].
[106] A. Hebecker and J. March-Russell, Nucl. Phys. **B613**, 3 (2001), [hep-ph/0106166].
[107] T. Asaka, W. Buchmuller and L. Covi, Phys. Lett. **B523**, 199 (2001), [hep-ph/0108021].

[108] L. J. Hall *et al.*, Phys. Rev. **D65**, 035008 (2002), [hep-ph/0108071].

[109] R. Dermisek and A. Mafi, Phys. Rev. **D65**, 055002 (2002), [hep-ph/0108139].

[110] H. D. Kim and S. Raby, JHEP **01**, 056 (2003), [hep-ph/0212348].

[111] L. E. Ibanez and A. M. Uranga, *String theory and particle physics: An introduction to string phenomenology*, Cambridge University Press (2012), ISBN 978-0-521-51752-2, 978-1-139-22742-1.

[112] R. Blumenhagen *et al.*, Phys. Rept. **445**, 1 (2007), [hep-th/0610327].

[113] K.-S. Choi and J. E. Kim, Lect. Notes Phys. **954**, pp. (2020).

[114] A. Hebecker, *Naturalness, String Landscape and Multiverse: A Modern Introduction with Exercises*, volume 979 (2021), ISBN 978-3-030-65150-3, 978-3-030-65151-0, [arXiv:2008.10625].

[115] D. J. Gross *et al.*, Phys. Rev. Lett. **54**, 502 (1985).

[116] V. Braun *et al.*, JHEP **05**, 043 (2006), [hep-th/0512177].

[117] V. Bouchard and R. Donagi, Phys. Lett. **B633**, 783 (2006), [hep-th/0512149].

[118] L. B. Anderson *et al.*, Phys. Rev. **D84**, 106005 (2011), [arXiv:1106.4804].

[119] L. J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. **B355**, 649 (1991).

[120] G. Aldazabal *et al.*, Nucl. Phys. **B452**, 3 (1995), [hep-th/9410206].

[121] Z. Kakushadze *et al.*, Int. J. Mod. Phys. **A13**, 2551 (1998), [hep-th/9710149].

[122] E. Witten, Nucl. Phys. **B258**, 75 (1985).

[123] S. Gukov *et al.*, Phys. Rev. **D69**, 086008 (2004), [hep-th/0310159].

[124] G. Curio, A. Krause and D. Lust, Fortsch. Phys. **54**, 225 (2006), [hep-th/0502168].

[125] L. B. Anderson *et al.*, Phys. Rev. **D83**, 106011 (2011), [arXiv:1102.0011].

[126] R. Friedman, J. Morgan and E. Witten, Commun. Math. Phys. **187**, 679 (1997), [hep-th/9701162].

[127] R. Blumenhagen, G. Honecker and T. Weigand, JHEP **08**, 009 (2005), [hep-th/0507041].

[128] L. B. Anderson *et al.*, JHEP **01**, 047 (2014), [arXiv:1307.4787].

[129] T. Kobayashi, S. Raby and R.-J. Zhang, Phys. Lett. **B593**, 262 (2004), [hep-ph/0403065].

[130] S. Forste *et al.*, Phys. Rev. **D70**, 106008 (2004), [hep-th/0406208].

[131] T. Kobayashi, S. Raby and R.-J. Zhang, Nucl. Phys. **B704**, 3 (2005), [hep-ph/0409098].

[132] W. Buchmuller *et al.*, Nucl. Phys. **B712**, 139 (2005), [hep-ph/0412318].

[133] W. Buchmuller *et al.*, Phys. Rev. Lett. **96**, 121602 (2006), [hep-ph/0511035].

[134] W. Buchmuller *et al.*, Nucl. Phys. **B785**, 149 (2007), [hep-th/0606187].

[135] O. Lebedev *et al.*, Phys. Lett. **B645**, 88 (2007), [hep-th/0611095].

[136] J. E. Kim, J.-H. Kim and B. Kyae, JHEP **06**, 034 (2007), [hep-ph/0702278].

[137] O. Lebedev *et al.*, Phys. Rev. **D77**, 046013 (2008), [arXiv:0708.2691].

[138] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. **D66**, 106006 (2002), [hep-th/0105097].

[139] S. Kachru *et al.*, Phys. Rev. **D68**, 046005 (2003), [hep-th/0301240].

[140] R. Blumenhagen *et al.*, Nucl. Phys. **B815**, 1 (2009), [arXiv:0811.2936].

[141] R. Blumenhagen *et al.*, Nucl. Phys. **B616**, 3 (2001), [hep-th/0107138].

[142] R. Donagi and M. Wijnholt, Adv. Theor. Math. Phys. **15**, 5, 1237 (2011), [arXiv:0802.2969].

[143] C. Beasley, J. J. Heckman and C. Vafa, JHEP **01**, 058 (2009), [arXiv:0802.3391].
[144] C. Beasley, J. J. Heckman and C. Vafa, JHEP **01**, 059 (2009), [arXiv:0806.0102].
[145] T. Weigand, Class. Quant. Grav. **27**, 214004 (2010), [arXiv:1009.3497].
[146] J. J. Heckman, Ann. Rev. Nucl. Part. Sci. **60**, 237 (2010), [arXiv:1001.0577].
[147] M. Cvetic, I. Garcia-Etxebarria and J. Halverson, JHEP **01**, 073 (2011), [arXiv:1003.5337].
[148] A. Maharana and E. Palti, Int. J. Mod. Phys. **A28**, 1330005 (2013), [arXiv:1212.0555].
[149] S. Krippendorf, S. Schafer-Nameki and J.-M. Wong, JHEP **11**, 008 (2015), [arXiv:1507.05961].
[150] R. Donagi and M. Wijnholt, Adv. Theor. Math. Phys. **15**, 6, 1523 (2011), [arXiv:0808.2223].
[151] R. Blumenhagen, Phys. Rev. Lett. **102**, 071601 (2009), [arXiv:0812.0248].
[152] K.-S. Choi and J. E. Kim, Phys. Rev. **D83**, 065016 (2011), [arXiv:1012.0847].
[153] C. Mayrhofer, E. Palti and T. Weigand, JHEP **09**, 082 (2013), [arXiv:1303.3589].
[154] G. K. Leontaris and Q. Shafi, Phys. Rev. **D96**, 6, 066023 (2017), [arXiv:1706.08372].
[155] E. Witten, Nucl. Phys. **B471**, 135 (1996), [hep-th/9602070].
[156] A. Hebecker and M. Trapletti, Nucl. Phys. **B713**, 173 (2005), [hep-th/0411131].
[157] B. Dundee and S. Raby (2008), [arXiv:0808.0992].
[158] G. G. Ross (2004), [hep-ph/0411057].
[159] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
[160] S. Weinberg, Phys. Lett. **91B**, 51 (1980).
[161] L. J. Hall, Nucl. Phys. **B178**, 75 (1981).
[162] J. R. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. **B260**, 131 (1991).
[163] U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. **B260**, 447 (1991).
[164] P. Langacker and M.-x. Luo, Phys. Rev. **D44**, 817 (1991).
[165] C. Giunti, C. W. Kim and U. W. Lee, Mod. Phys. Lett. A **6**, 1745 (1991).
[166] S. Dimopoulos, S. A. Raby and F. Wilczek, Phys. Today **44N10**, 25 (1991).
[167] P. Langacker and N. Polonsky, Phys. Rev. **D47**, 4028 (1993), [hep-ph/9210235].
[168] M. Carena, S. Pokorski and C. E. M. Wagner, Nucl. Phys. **B406**, 59 (1993), [hep-ph/9303202].
[169] B. C. Allanach, Comput. Phys. Commun. **143**, 305 (2002), [hep-ph/0104145].
[170] S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 3, 030001 (2024).
[171] M. L. Alciati *et al.*, JHEP **03**, 054 (2005), [hep-ph/0501086].
[172] P. Langacker, in "2nd Workshop on Grand Unification," 131–150 (1981).
[173] W. J. Marciano and A. Sirlin, Conf. Proc. C **810424**, 151 (1981).
[174] W. J. Marciano, in "Eighth Workshop on Grand Unification," 185–199 (1987).
[175] A. Djouadi, J.-L. Kneur and G. Moultaka, Comput. Phys. Commun. **176**, 426 (2007), [hep-ph/0211331].
[176] W. Porod and F. Staub, Comput. Phys. Commun. **183**, 2458 (2012), [arXiv:1104.1573].
[177] I. Antoniadis, C. Kounnas and K. Tamvakis, Phys. Lett. B **119**, 377 (1982).
[178] W. Siegel, Phys. Lett. **94B**, 37 (1980).
[179] M. A. Shifman, Int. J. Mod. Phys. **A11**, 5761 (1996), [hep-ph/9606281].
[180] N. Arkani-Hamed and H. Murayama, JHEP **06**, 030 (2000), [hep-th/9707133].

[181] I. Jack, D. R. T. Jones and A. Pickering, Phys. Lett. **B435**, 61 (1998), [hep-ph/9805482].

[182] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. Lett. **69**, 1014 (1992).

[183] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B **402**, 46 (1993), [hep-ph/9207279].

[184] H. Murayama and A. Pierce, Phys. Rev. **D65**, 055009 (2002), [hep-ph/0108104].

[185] C. T. Hill, Phys. Lett. B **135**, 47 (1984).

[186] J. R. Ellis *et al.*, Phys. Lett. B **155**, 381 (1985).

[187] M. Drees, Phys. Lett. B **158**, 409 (1985).

[188] M. Drees, Phys. Rev. D **33**, 1468 (1986).

[189] K. Tobe and J. D. Wells, Phys. Lett. B **588**, 99 (2004), [hep-ph/0312159].

[190] J. Ellis *et al.*, Eur. Phys. J. C **76**, 11, 592 (2016), [arXiv:1608.05370].

[191] G. Anderson *et al.*, eConf **C960625**, SUP107 (1996), [hep-ph/9609457].

[192] L. Roszkowski and M. A. Shifman, Phys. Rev. **D53**, 404 (1996), [hep-ph/9503358].

[193] S. Raby, M. Ratz and K. Schmidt-Hoberg, Phys. Lett. **B687**, 342 (2010), [arXiv:0911.4249].

[194] J. Hisano, T. Kuwahara and N. Nagata, Phys. Lett. **B723**, 324 (2013), [arXiv:1304.0343].

[195] S. Pokorski, K. Rolbiecki and K. Sakurai, Phys. Rev. D **97**, 3, 035027 (2018), [arXiv:1707.06720].

[196] P. N. Bhattiprolu and J. D. Wells, Phys. Rev. D **109**, 1, L011704 (2024), [arXiv:2309.12954].

[197] L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999), [hep-th/9810155].

[198] G. F. Giudice *et al.*, JHEP **12**, 027 (1998), [hep-ph/9810442].

[199] J. D. Wells, in "11th International Conference on Supersymmetry and the Unification of Fundamental Interactions," (2003), [hep-ph/0306127].

[200] J. D. Wells, Phys. Rev. D **71**, 015013 (2005), [hep-ph/0411041].

[201] N. Arkani-Hamed and S. Dimopoulos, JHEP **06**, 073 (2005), [hep-th/0405159].

[202] G. F. Giudice and A. Romanino, Nucl. Phys. **B699**, 65 (2004), [Erratum: Nucl. Phys.B706,487(2005)], [hep-ph/0406088].

[203] N. Arkani-Hamed *et al.*, Nucl. Phys. B **709**, 3 (2005), [hep-ph/0409232].

[204] L. J. Hall and Y. Nomura, JHEP **01**, 082 (2012), [arXiv:1111.4519].

[205] M. Ibe and T. T. Yanagida, Phys. Lett. B **709**, 374 (2012), [arXiv:1112.2462].

[206] M. Ibe, S. Matsumoto and T. T. Yanagida, Phys. Rev. D **85**, 095011 (2012), [arXiv:1202.2253].

[207] A. Arvanitaki *et al.*, JHEP **02**, 126 (2013), [arXiv:1210.0555].

[208] N. Arkani-Hamed *et al.* (2012), [arXiv:1212.6971].

[209] J. L. Evans *et al.*, Eur. Phys. J. C **73**, 2468 (2013), [arXiv:1302.5346].

[210] J. L. Evans *et al.*, Eur. Phys. J. C **73**, 10, 2611 (2013), [arXiv:1305.7461].

[211] Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991).

[212] Y. Okada, M. Yamaguchi and T. Yanagida, Phys. Lett. B **262**, 54 (1991).

[213] J. R. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. **B257**, 83 (1991).

[214] H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991).

[215] J. R. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B **262**, 477 (1991).

[216] M. Carena *et al.*, Phys. Lett. **B355**, 209 (1995), [hep-ph/9504316].

[217] G. Degrassi *et al.*, Eur. Phys. J. **C28**, 133 (2003), [hep-ph/0212020].

[218] P. Kant *et al.*, JHEP **08**, 104 (2010), [arXiv:1005.5709].

[219] R. N. Mohapatra and M. K. Parida, Phys. Rev. **D47**, 264 (1993), [hep-ph/9204234].

[220] D.-G. Lee *et al.*, Phys. Rev. **D51**, 229 (1995), [hep-ph/9404238].

[221] Y. Mambrini *et al.*, Phys. Rev. D **91**, 9, 095010 (2015), [arXiv:1502.06929].

[222] N. Nagata, K. A. Olive and J. Zheng, JHEP **10**, 193 (2015), [arXiv:1509.00809].

[223] L. E. Ibanez *et al.*, JHEP **07**, 195 (2012), [arXiv:1206.2655].

[224] S. A. R. Ellis and J. D. Wells, Phys. Rev. D **91**, 7, 075016 (2015), [arXiv:1502.01362].

[225] J. Schwichtenberg, Eur. Phys. J. C **79**, 4, 351 (2019), [arXiv:1808.10329].

[226] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Rev. Lett. **91**, 061601 (2003), [hep-th/0210294].

[227] K. Agashe, R. Contino and R. Sundrum, Phys. Rev. Lett. **95**, 171804 (2005), [hep-ph/0502222].

[228] F. Wilczek and A. Zee, Phys. Rev. Lett. **43**, 1571 (1979).

[229] J. Ellis *et al.*, JHEP **05**, 021 (2020), [arXiv:2003.03285].

[230] K. Hamaguchi *et al.*, Universe **10**, 9, 379 (2024), [arXiv:2407.11892].

[231] N. Maekawa and Y. Muramatsu, Phys. Rev. D **88**, 9, 095008 (2013), [arXiv:1307.7529].

[232] N. Maekawa and Y. Muramatsu, Phys. Lett. B **767**, 398 (2017), [arXiv:1601.04789].

[233] A. Takenaka *et al.* (Super-Kamiokande), Phys. Rev. D **102**, 11, 112011 (2020), [arXiv:2010.16098].

[234] K. Abe *et al.* (Super-Kamiokande), Phys. Rev. D **96**, 1, 012003 (2017), [arXiv:1705.07221].

[235] K. Abe *et al.* (Hyper-Kamiokande) (2018), [arXiv:1805.04163].

[236] P. S. B. Dev *et al.* (2022), [arXiv:2203.08771].

[237] J.-S. Yoo *et al.*, Phys. Rev. D **105**, 7, 074501 (2022), [arXiv:2111.01608].

[238] J. Hisano, T. Kuwahara and Y. Omura, Nucl. Phys. **B898**, 1 (2015), [Erratum: Nucl. Phys.B907,476(2016)], [arXiv:1503.08561].

[239] R. Barbier *et al.*, Phys. Rept. **420**, 1 (2005), [hep-ph/0406039].

[240] F. Domingo *et al.*, JHEP **02**, 066 (2019), [arXiv:1810.08228].

[241] B. A. Campbell *et al.*, Phys. Lett. B **256**, 484 (1991).

[242] W. Fischler *et al.*, Phys. Lett. B **258**, 45 (1991).

[243] H. K. Dreiner and G. G. Ross, Nucl. Phys. **B410**, 188 (1993), [hep-ph/9207221].

[244] G. R. Farrar and P. Fayet, Phys. Lett. **76B**, 575 (1978).

[245] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. **112B**, 133 (1982).

[246] J. R. Ellis, D. V. Nanopoulos and S. Rudaz, Nucl. Phys. **B202**, 43 (1982).

[247] C. S. Aulakh *et al.*, Nucl. Phys. **B597**, 89 (2001), [hep-ph/0004031].

[248] S. Mine (Super Kamiokande), J. Phys. Conf. Ser. **718**, 6, 062044 (2016).

[249] T. Goto and T. Nihei, Phys. Rev. **D59**, 115009 (1999), [hep-ph/9808255].

[250] G. D. Coughlan *et al.*, Phys. Lett. **160B**, 249 (1985).

[251] J. L. Chkareuli and I. G. Gogoladze, Phys. Rev. **D58**, 055011 (1998), [hep-ph/9803335].

[252] D. McKeen, M. Pospelov and A. Ritz, Phys. Rev. D **87**, 11, 113002 (2013), [arXiv:1303.1172].

[253] M. Liu and P. Nath, Phys. Rev. D **87**, 9, 095012 (2013), [arXiv:1303.7472].

[254] J. Hisano *et al.*, JHEP **07**, 038 (2013), [arXiv:1304.3651].
[255] J. L. Evans, N. Nagata and K. A. Olive, Phys. Rev. D **91**, 055027 (2015), [arXiv:1502.00034].
[256] S. Pokorski *et al.*, JHEP **04**, 161 (2019), [arXiv:1902.06093].
[257] J. L. Evans, N. Nagata and K. A. Olive, Eur. Phys. J. C **79**, 6, 490 (2019), [arXiv:1902.09084].
[258] J. Ellis *et al.*, Eur. Phys. J. C **80**, 4, 332 (2020), [arXiv:1912.04888].
[259] B. Bajc, P. Fileviez Perez and G. Senjanovic, Phys. Rev. **D66**, 075005 (2002), [hep-ph/0204311].
[260] A. Abusleme *et al.* (JUNO), Prog. Part. Nucl. Phys. **123**, 103927 (2022), [arXiv:2104.02565].
[261] A. Abusleme *et al.* (JUNO), Chin. Phys. C **47**, 11, 113002 (2023), [arXiv:2212.08502].
[262] B. Abi *et al.* (DUNE), Eur. Phys. J. C **81**, 4, 322 (2021), [arXiv:2008.12769].
[263] L. J. Hall and Y. Nomura, Phys. Rev. **D66**, 075004 (2002), [hep-ph/0205067].
[264] H. D. Kim, S. Raby and L. Schradin, JHEP **05**, 036 (2005), [hep-ph/0411328].
[265] M. S. Chanowitz, J. R. Ellis and M. K. Gaillard, Nucl. Phys. B **128**, 506 (1977).
[266] A. J. Buras *et al.*, Nucl. Phys. **B135**, 66 (1978).
[267] K. Inoue *et al.*, Prog. Theor. Phys. **67**, 1889 (1982).
[268] L. E. Ibanez and C. Lopez, Nucl. Phys. **B233**, 511 (1984).
[269] H. Georgi and D. V. Nanopoulos, Nucl. Phys. **B159**, 16 (1979).
[270] J. A. Harvey, P. Ramond and D. B. Reiss, Phys. Lett. **92B**, 309 (1980).
[271] J. A. Harvey, D. B. Reiss and P. Ramond, Nucl. Phys. **B199**, 223 (1982).
[272] T. Banks, Nucl. Phys. **B303**, 172 (1988).
[273] M. Olechowski and S. Pokorski, Phys. Lett. **B214**, 393 (1988).
[274] S. Pokorski, Nucl. Phys. Proc. Suppl. **13**, 606 (1990).
[275] B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Rev. **D44**, 1613 (1991).
[276] Q. Shafi and B. Ananthanarayan, in "Summer School in High-energy Physics and Cosmology," 233–244 (1991).
[277] S. Dimopoulos, L. J. Hall and S. Raby, Phys. Rev. Lett. **68**, 1984 (1992).
[278] S. Dimopoulos, L. J. Hall and S. Raby, Phys. Rev. **D45**, 4192 (1992).
[279] G. W. Anderson *et al.*, Phys. Rev. **D47**, R3702 (1993), [hep-ph/9209250].
[280] B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Lett. **B300**, 245 (1993).
[281] G. Anderson *et al.*, Phys. Rev. **D49**, 3660 (1994), [hep-ph/9308333].
[282] B. Ananthanarayan, Q. Shafi and X. M. Wang, Phys. Rev. **D50**, 5980 (1994), [hep-ph/9311225].
[283] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. **D50**, 7048 (1994), [hep-ph/9306309].
[284] M. Carena *et al.*, Nucl. Phys. **B426**, 269 (1994), [hep-ph/9402253].
[285] A. Anandakrishnan, B. C. Bryant and S. Raby, JHEP **05**, 088 (2015), [arXiv:1411.7035].
[286] M. Lanzagorta and G. G. Ross, Phys. Lett. **B364**, 163 (1995), [hep-ph/9507366].
[287] K. Tobe and J. D. Wells, Nucl. Phys. **B663**, 123 (2003), [hep-ph/0301015].
[288] G. Ross and M. Serna, Phys. Lett. **B664**, 97 (2008), [arXiv:0704.1248].
[289] T. Blazek, R. Dermisek and S. Raby, Phys. Rev. Lett. **88**, 111804 (2002), [hep-ph/0107097].
[290] T. Blazek, R. Dermisek and S. Raby, Phys. Rev. **D65**, 115004 (2002), [hep-ph/0201081].

[291] D. Auto *et al.*, JHEP **06**, 023 (2003), [hep-ph/0302155].
[292] R. Dermisek *et al.*, JHEP **04**, 037 (2003), [hep-ph/0304101].
[293] R. Dermisek *et al.*, JHEP **09**, 029 (2005), [hep-ph/0507233].
[294] A. Anandakrishnan, S. Raby and A. Wingerter, Phys. Rev. **D87**, 5, 055005 (2013), [arXiv:1212.0542].
[295] M. Adeel Ajaib *et al.*, JHEP **07**, 139 (2013), [arXiv:1303.6964].
[296] Z. Poh and S. Raby, Phys. Rev. **D92**, 1, 015017 (2015), [arXiv:1505.00264].
[297] M. Badziak, M. Olechowski and S. Pokorski, JHEP **10**, 088 (2013), [arXiv:1307.7999].
[298] H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979).
[299] S. Antusch and M. Spinrath, Phys. Rev. **D79**, 095004 (2009), [arXiv:0902.4644].
[300] G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. **B181**, 287 (1981).
[301] T. E. Clark, T.-K. Kuo and N. Nakagawa, Phys. Lett. **115B**, 26 (1982).
[302] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993), [hep-ph/9209215].
[303] R. Barbieri and D. V. Nanopoulos, Phys. Lett. **91B**, 369 (1980).
[304] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).
[305] R. Barbieri *et al.*, Nucl. Phys. **B493**, 3 (1997), [hep-ph/9610449].
[306] T. Blazek, S. Raby and K. Tobe, Phys. Rev. **D60**, 113001 (1999), [hep-ph/9903340].
[307] T. Blazek, S. Raby and K. Tobe, Phys. Rev. **D62**, 055001 (2000), [hep-ph/9912482].
[308] Q. Shafi and Z. Tavartkiladze, Phys. Lett. **B487**, 145 (2000), [hep-ph/9910314].
[309] C. H. Albright and S. M. Barr, Phys. Rev. Lett. **85**, 244 (2000), [hep-ph/0002155].
[310] Z. Berezhiani and A. Rossi, Nucl. Phys. **B594**, 113 (2001), [hep-ph/0003084].
[311] C. H. Albright and S. M. Barr, Phys. Rev. **D64**, 073010 (2001), [hep-ph/0104294].
[312] M.-C. Chen and K. T. Mahanthappa, Int. J. Mod. Phys. **A18**, 5819 (2003), [hep-ph/0305088].
[313] R. Dermisek and S. Raby, Phys. Lett. **B622**, 327 (2005), [hep-ph/0507045].
[314] H. Georgi, Nucl. Phys. **B156**, 126 (1979).
[315] P. H. Frampton, Phys. Lett. **88B**, 299 (1979).
[316] P. Frampton and S. Nandi, Phys. Rev. Lett. **43**, 1460 (1979).
[317] J. E. Kim, Phys. Rev. Lett. **45**, 1916 (1980).
[318] J. E. Kim, Phys. Rev. D **23**, 2706 (1981).
[319] J. E. Kim, Phys. Rev. D **26**, 674 (1982).
[320] Y. Fujimoto, Phys. Rev. **D26**, 3183 (1982).
[321] P. Langacker, Phys. Rept. **72**, 185 (1981).
[322] H. Georgi, J. Phys. Colloq. **43**, C3, 705 (1982).
[323] S. M. Barr, Phys. Rev. **D78**, 055008 (2008), [arXiv:0805.4808].
[324] Y. Goto, Y. Kawamura and T. Miura, Phys. Rev. **D88**, 5, 055016 (2013), [arXiv:1307.2631].
[325] J. E. Kim, JHEP **06**, 114 (2015), [arXiv:1503.03104].
[326] C. H. Albright, R. P. Feger and T. W. Kephart, Phys. Rev. D **93**, 7, 075032 (2016), [arXiv:1601.07523].
[327] M. Reig *et al.*, Phys. Lett. B **774**, 667 (2017), [arXiv:1706.03116].
[328] M. Magg and C. Wetterich, Phys. Lett. **94B**, 61 (1980).

[329] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980).

[330] R. N. Mohapatra and G. Senjanovic, Phys. Rev. **D23**, 165 (1981).

[331] G. B. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).

[332] R. Foot *et al.*, Z. Phys. **C44**, 441 (1989).

[333] G. Altarelli, Soryushiron Kenkyu Electron. **116**, A29 (2008), [arXiv:0711.0161].

[334] S. F. King and C. Luhn, Rept. Prog. Phys. **76**, 056201 (2013), [arXiv:1301.1340].

[335] C. H. Albright and M.-C. Chen, Phys. Rev. **D74**, 113006 (2006), [hep-ph/0608137].

[336] G. Altarelli and F. Feruglio, Rev. Mod. Phys. **82**, 2701 (2010), [arXiv:1002.0211].

[337] H. Ishimori *et al.*, Prog. Theor. Phys. Suppl. **183**, 1 (2010), [arXiv:1003.3552].

[338] F. Feruglio, *Are neutrino masses modular forms?*, 227–266 (2019), [arXiv:1706.08749].

[339] T. Kobayashi, K. Tanaka and T. H. Tatsuishi, Phys. Rev. D **98**, 1, 016004 (2018), [arXiv:1803.10391].

[340] J. T. Penedo and S. T. Petcov, Nucl. Phys. B **939**, 292 (2019), [arXiv:1806.11040].

[341] Y. Almumin *et al.*, JHEP **05**, 078 (2021), [arXiv:2102.11286].

[342] H. P. Nilles, S. Ramos-Sánchez and P. K. S. Vaudrevange, JHEP **02**, 045 (2020), [arXiv:2001.01736].

[343] R. Kallosh *et al.*, Phys. Rev. **D52**, 912 (1995), [hep-th/9502069].

[344] T. Banks and N. Seiberg, Phys. Rev. D **83**, 084019 (2011), [arXiv:1011.5120].

[345] M. Dine, P. Draper and W. Shepherd, JHEP **02**, 027 (2014), [arXiv:1308.0274].

[346] H. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. **A19**, 3265 (2004), [hep-th/0304245].

[347] M. B. Green and J. H. Schwarz, Phys. Lett. **149B**, 117 (1984).

[348] L. E. Ibanez and G. G. Ross, Nucl. Phys. **B368**, 3 (1992).

[349] M. Berasaluce-Gonzalez *et al.*, JHEP **12**, 113 (2011), [arXiv:1106.4169].

[350] N. G. Deshpande (1979).

[351] C. Q. Geng and R. E. Marshak, Phys. Rev. **D39**, 693 (1989).

[352] A. Font, L. E. Ibanez and F. Quevedo, Phys. Lett. **B228**, 79 (1989).

[353] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **63**, 938 (1989).

[354] R. Foot *et al.*, Phys. Rev. **D39**, 3411 (1989).

[355] M. Nowakowski and A. Pilaftsis, Phys. Rev. **D48**, 259 (1993), [hep-ph/9304312].

[356] T. P. T. Dijkstra, L. R. Huiszoon and A. N. Schellekens, Phys. Lett. **B609**, 408 (2005), [hep-th/0403196].

[357] F. Gmeiner *et al.*, JHEP **01**, 004 (2006), [hep-th/0510170].

[358] B. Gato-Rivera and A. N. Schellekens, Nucl. Phys. **B883**, 529 (2014), [arXiv:1401.1782].

[359] G. 't Hooft, Nucl. Phys. B **79**, 276 (1974).

[360] A. M. Polyakov, JETP Lett. **20**, 194 (1974).

[361] M. Ambrosio *et al.* (MACRO), Eur. Phys. J. **C25**, 511 (2002), [hep-ex/0207020].

[362] S. Balestra *et al.* (2011), [arXiv:1105.5587].

[363] L. Patrizii and M. Spurio, Ann. Rev. Nucl. Part. Sci. **65**, 279 (2015), [arXiv:1510.07125].

[364] V. A. Rubakov (1981).

[365] V. A. Rubakov, Nucl. Phys. B **203**, 311 (1982).

[366] C. G. Callan, Jr., Phys. Rev. **D26**, 2058 (1982).
[367] F. Wilczek, Phys. Rev. Lett. **48**, 1146 (1982).
[368] S. Dawson and A. N. Schellekens, Phys. Rev. **D27**, 2119 (1983).
[369] K. Freese, M. S. Turner and D. N. Schramm, Phys. Rev. Lett. **51**, 1625 (1983).
[370] P. A. R. Ade *et al.* (BICEP, Keck), Phys. Rev. Lett. **127**, 15, 151301 (2021), [arXiv:2110.00483].
[371] G. R. Dvali, H. Liu and T. Vachaspati, Phys. Rev. Lett. **80**, 2281 (1998), [hep-ph/9710301].
[372] P. Langacker and S.-Y. Pi, Phys. Rev. Lett. **45**, 1 (1980).
[373] G. R. Dvali, A. Melfo and G. Senjanovic, Phys. Rev. Lett. **75**, 4559 (1995), [hep-ph/9507230].
[374] A. Vilenkin, Phys. Rept. **121**, 263 (1985).
[375] M. B. Hindmarsh and T. W. B. Kibble, Rept. Prog. Phys. **58**, 477 (1995), [hep-ph/9411342].
[376] R. Jeannerot, J. Rocher and M. Sakellariadou, Phys. Rev. D **68**, 103514 (2003), [hep-ph/0308134].
[377] B. P. Abbott *et al.* (LIGO Scientific, Virgo), Phys. Rev. D **97**, 10, 102002 (2018), [arXiv:1712.01168].
[378] S. F. King *et al.*, Phys. Rev. Lett. **126**, 2, 021802 (2021), [arXiv:2005.13549].
[379] S. Chigusa, Y. Nakai and J. Zheng, Phys. Rev. D **104**, 3, 035031 (2021), [arXiv:2011.04090].
[380] E. J. Chun and L. Velasco-Sevilla, Phys. Rev. D **106**, 3, 035008 (2022), [arXiv:2112.14483].
[381] R. Maji and Q. Shafi, JCAP **03**, 007 (2023), [arXiv:2208.08137].
[382] A. Vilenkin, Nucl. Phys. B **196**, 240 (1982).
[383] A. Monin and M. B. Voloshin, Phys. Rev. D **78**, 065048 (2008), [arXiv:0808.1693].
[384] W. Buchmuller *et al.*, Phys. Lett. B **809**, 135764 (2020), [arXiv:1912.03695].
[385] J. A. Dror *et al.*, Phys. Rev. Lett. **124**, 4, 041804 (2020), [arXiv:1908.03227].
[386] W. Buchmuller, V. Domcke and K. Schmitz, Phys. Lett. B **811**, 135914 (2020), [arXiv:2009.10649].
[387] A. Afzal *et al.* (NANOGrav), Astrophys. J. Lett. **951**, 1, L11 (2023), [arXiv:2306.16219].
[388] C. Caprini *et al.*, JCAP **04**, 001 (2016), [arXiv:1512.06239].
[389] A. Mazumdar and G. White, Rept. Prog. Phys. **82**, 7, 076901 (2019), [arXiv:1811.01948].
[390] C. Caprini *et al.*, JCAP **03**, 024 (2020), [arXiv:1910.13125].
[391] M. B. Hindmarsh *et al.*, SciPost Phys. Lect. Notes **24**, 1 (2021), [arXiv:2008.09136].
[392] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. **B267**, 415 (1986).
[393] R. Barbieri and L. J. Hall, Phys. Lett. **B338**, 212 (1994), [hep-ph/9408406].
[394] R. Barbieri, L. J. Hall and A. Strumia, Nucl. Phys. **B445**, 219 (1995), [hep-ph/9501334].
[395] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986).
[396] J. Hisano *et al.*, Phys. Lett. **B357**, 579 (1995), [hep-ph/9501407].
[397] J. Hisano *et al.*, Phys. Rev. **D53**, 2442 (1996), [hep-ph/9510309].
[398] J. Hisano and D. Nomura, Phys. Rev. **D59**, 116005 (1999), [hep-ph/9810479].
[399] T. Moroi, JHEP **03**, 019 (2000), [hep-ph/0002208].
[400] D. Chang, A. Masiero and H. Murayama, Phys. Rev. **D67**, 075013 (2003), [hep-ph/0205111].
[401] N. Nagata and S. Shirai, JHEP **03**, 049 (2014), [arXiv:1312.7854].

[402] S. Dimopoulos and L. J. Hall, Phys. Lett. **B344**, 185 (1995), [hep-ph/9411273].

[403] J. Hisano *et al.*, Phys. Lett. **B604**, 216 (2004), [hep-ph/0407169].

[404] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155**, 36 (1985).

[405] A. Yu. Ignatiev *et al.*, Phys. Lett. **76B**, 436 (1978).

[406] M. Yoshimura, Phys. Rev. Lett. **41**, 281 (1978), [Erratum: Phys. Rev. Lett.42,746(1979)].

[407] D. Toussaint *et al.*, Phys. Rev. **D19**, 1036 (1979).

[408] S. Weinberg, Phys. Rev. Lett. **42**, 850 (1979).

[409] M. Yoshimura, Phys. Lett. **88B**, 294 (1979).

[410] S. M. Barr, G. Segre and H. A. Weldon, Phys. Rev. **D20**, 2494 (1979).

[411] D. V. Nanopoulos and S. Weinberg, Phys. Rev. **D20**, 2484 (1979).

[412] A. Yildiz and P. H. Cox, Phys. Rev. **D21**, 906 (1980).

[413] M. Fukugita and T. Yanagida, Phys. Rev. Lett. **89**, 131602 (2002), [hep-ph/0203194].

[414] M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986).

[415] S. Davidson and A. Ibarra, Phys. Lett. **B535**, 25 (2002), [hep-ph/0202239].

[416] K. Hamaguchi, H. Murayama and T. Yanagida, Phys. Rev. **D65**, 043512 (2002), [hep-ph/0109030].

[417] A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B **692**, 303 (2004), [hep-ph/0309342].

[418] A. Pilaftsis and T. E. J. Underwood, Phys. Rev. **D72**, 113001 (2005), [hep-ph/0506107].

[419] M. Kawasaki *et al.*, Phys. Rev. **D78**, 065011 (2008), [arXiv:0804.3745].

[420] G. Lazarides and Q. Shafi, Phys. Lett. B **258**, 305 (1991).

[421] T. Asaka *et al.*, Phys. Lett. B **464**, 12 (1999), [hep-ph/9906366].

[422] T. Asaka *et al.*, Phys. Rev. D **61**, 083512 (2000), [hep-ph/9907559].

[423] W. Buchmuller, R. D. Peccei and T. Yanagida, Ann. Rev. Nucl. Part. Sci. **55**, 311 (2005), [hep-ph/0502169].

[424] S. Davidson, E. Nardi and Y. Nir, Phys. Rept. **466**, 105 (2008), [arXiv:0802.2962].

[425] C. S. Fong, E. Nardi and A. Riotto, Adv. High Energy Phys. **2012**, 158303 (2012), [arXiv:1301.3062].

[426] V. Cirigliano *et al.*, JHEP **12**, 097 (2018), [arXiv:1806.02780].

[427] K. S. Babu and R. N. Mohapatra, Phys. Rev. **D86**, 035018 (2012), [arXiv:1203.5544].

[428] K. S. Babu and R. N. Mohapatra, Phys. Lett. **B715**, 328 (2012), [arXiv:1206.5701].

[429] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **109**, 091803 (2012), [arXiv:1207.5771].

[430] M. Baldo-Ceolin *et al.*, Z. Phys. **C63**, 409 (1994).

[431] K. Abe *et al.* (Super-Kamiokande), Phys. Rev. D **103**, 1, 012008 (2021), [arXiv:2012.02607].

[432] J. Gustafson *et al.* (Super-Kamiokande), Phys. Rev. **D91**, 7, 072009 (2015), [arXiv:1504.01041].

[433] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[434] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).

[435] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).

[436] G. Grilli di Cortona *et al.*, JHEP **01**, 034 (2016), [arXiv:1511.02867].

[437] R. N. Mohapatra and D. Wyler, Phys. Lett. B **89**, 181 (1980).

[438] A. E. Nelson, Phys. Lett. B **136**, 387 (1984).

[439] S. M. Barr, Phys. Rev. Lett. **53**, 329 (1984).

[440] L. J. Hall and K. Harigaya, JHEP **10**, 130 (2018), [arXiv:1803.08119].

[441] Y. Mimura, R. N. Mohapatra and M. Severson, Phys. Rev. D **99**, 11, 115025 (2019), [arXiv:1903.07506].

[442] L. J. Hall and K. Harigaya, JHEP **11**, 033 (2019), [arXiv:1905.12722].

[443] P. Svrcek and E. Witten, JHEP **06**, 051 (2006), [hep-th/0605206].

[444] L. Di Luzio *et al.*, Phys. Rept. **870**, 1 (2020), [arXiv:2003.01100].

[445] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. **104B**, 199 (1981).

[446] A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980), [Yad. Fiz.31,497(1980)].

[447] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979).

[448] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **166**, 493 (1980).

[449] J. Hisano, H. Murayama and T. Yanagida, Phys. Lett. B **291**, 263 (1992).

[450] R. T. Co, F. D'Eramo and L. J. Hall, Phys. Rev. D **94**, 7, 075001 (2016), [arXiv:1603.04439].

[451] G. F. Giudice, R. Rattazzi and A. Strumia, Phys. Lett. B **715**, 142 (2012), [arXiv:1204.5465].

[452] K. J. Bae, H. Baer and H. Serce, JCAP **06**, 024 (2017), [arXiv:1705.01134].

[453] J. P. Conlon, JHEP **05**, 078 (2006), [hep-th/0602233].

[454] K. R. Dienes, Phys. Rept. **287**, 447 (1997), [hep-th/9602045].

[455] P. Nath and P. Fileviez Perez, Phys. Rept. **441**, 191 (2007), [hep-ph/0601023].

[456] T. Fukuyama, Int. J. Mod. Phys. A **28**, 1330008 (2013), [arXiv:1212.3407].

[457] G. G. Ross, *GRAND UNIFIED THEORIES* (1985).

[458] R. N. Mohapatra, *Unification and Supersymmetry. The Frontiers of Quark-Lepton Physics*, Springer, Berlin (1986), ISBN 978-1-4757-1930-7, 978-1-4757-1928-4.

[459] P. Nath, *Supersymmetry, Supergravity, and Unification*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (2016), ISBN 978-0-521-19702-1, 978-1-316-98396-6.

[460] S. Raby, Lect. Notes Phys. **939**, 1 (2017).

[461] P. Nath, Int. J. Mod. Phys. **A33**, 20, 1830017 (2018), [arXiv:1807.05302].

[462] D. Croon *et al.*, Front.in Phys. **7**, 76 (2019), [arXiv:1903.04977].