

78. Heavy Non- $q\bar{q}$ Mesons

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The constituent quark model describes the observed meson spectrum as bound $q\bar{q}$ states grouped into SU(N) flavor multiplets (see the ‘Quark Model’ in this issue of the *Review of Particle Physics*). However, the self coupling of gluons in QCD suggests that additional mesons made of bound gluons (glueballs), or $q\bar{q}$ -pairs with an excited gluon (hybrids), may exist. Furthermore, multi-quark color singlet states such as $qq\bar{q}\bar{q}$ (tetraquarks as compact diquark-antidiquark systems and ‘molecular’ bound states of two mesons) or $qqq\bar{q}\bar{q}\bar{q}$ (six-quark, ‘baryonium’ or quasinuclear baryon-antibaryon bound states) have also been predicted—they were actually already mentioned in the pre-QCD works on the quark model in Refs. [1, 2]. The focus of this review is on the current understanding of exotic states that apparently do not fit into the most simple realizations of the $q\bar{q}$ constituent quark model and contain at least one heavy quark (charm or bottom — the lifetime of the top quark is too short to provide the formation of hadrons with a sufficiently long life time that allows for spectroscopic investigations). Light non- $q\bar{q}$ candidates (made of u , d and s quarks) are discussed in ‘Scalar Mesons below 1 GeV’ and ‘Spectroscopy of Light Meson Resonances’ in this *Review*.

There are candidates for exotic states with both quantum numbers not allowed and quantum numbers allowed for $q\bar{q}$ states. For the latter class it is up to now unclear to what extent a mixing of conventional and unconventional states occurs. While Refs. [3–6] report a significant mixing, Refs. [7, 8] describe a scenario where the mixing especially between molecular structures and $q\bar{q}$ states is suppressed. Ref. [9] finds a mixing that depends strongly on the pole location of the state studied. A promising path to study mixing of different configurations is the use of few-body Bethe-Salpeter equation within the Dyson-Schwinger framework which is outlined in Refs. [10, 11] for open and hidden-flavor doubly heavy states, respectively.

Theoretically heavy non- $q\bar{q}$ mesons are studied using quark models, see, e.g., Refs. [12, 13], diquark-anti-diquark models, where (anti-)diquarks are treated as compact dynamical degrees of freedom within the hadron, see, e.g., Refs. [14–17], meson-exchange models, see, e.g., [18, 19], as well as different variants of effective field theories (EFTs). For mesons made of one heavy quark the proper non-relativistic EFT is called Heavy Quark Effective Theory (HQET), for reviews see [20, 21]. Doubly heavy systems are characterized by various scales. For $m_Q \gg m_Q v \gg m_Q v^2$, non-relativistic QCD (NRQCD) [22–24] can be employed at the scale $m_Q v$, obtained by integrating out QCD modes associated with the heavy scale m_Q , and potential NRQCD (pNRQCD) [25–27] at the scale $m_Q v^2$, obtained additionally integrating out modes associated with the energy scale $m_Q v$. The most modern development in this family of effective field theories is the Born-Oppenheimer EFT (BOEFT) [28–32] that can be applied to quarkonia, and all the exotics states discovered in the last decades that are all composed of two heavy quarks (or antiquarks): hybrids, tetraquarks, pentaquarks and doubly-heavy baryons.

To investigate the energy range around two-hadron thresholds, one can construct an NREFT with these hadrons as the effective degrees of freedom. In particular, in its most simple realisation for S -wave interactions, the LO interaction Lagrangian contains only constant contact terms, while higher-order terms involve derivatives of the hadron fields. The effective parameters that at the same time serve as counter terms in the renormalisation procedure are then adjusted by fits to experimental or lattice data. In this context the role of the one-pion exchange (OPE) is still under debate. There are three different approaches available in the literature, namely pionless EFT [33, 34], EFT with perturbative pions [35–40], and EFT with non-perturbative pions [41–48]. All those find their analogs in studies of the nucleon-nucleon interaction—see, e.g., Refs. [49–51] for

reviews. Most of those were used to study the spectrum or line shapes of hadronic molecules [33, 35, 38, 41, 48, 52–56] (for a review, see Ref. [57]). Those are states that owe their existence to the QCD residual force between color-neutral hadrons and not to the gluon exchange between the colored quarks or diquarks. Here chiral symmetry and heavy quark spin symmetry typically guide the construction of the interactions.

QCD sum-rules are also employed to investigate QCD exotics. Here the analyticity of physical amplitudes is used in a dispersion integral to relate the integral over hadronic amplitudes to a sum over various perturbative QCD contributions as well as non-perturbative condensates. The proper choice of interpolating currents allows one to access certain configurations, although the construction is significantly more involved for multi-quark than for quark-antiquark systems [58]. For a recent review on the application of this technique to exotics see Ref. [59].

In recent years lattice QCD also became an important tool to investigate exotics in the light and heavy quark sectors. Lattice QCD, explained in detail in a dedicated section of this *Review*, is a numerical solution of QCD on a discretised space-time lattice, often employing unphysical quark masses, not only for numerical reasons but also to address certain physical phenomena as is detailed below. The applications and challenges of lattice QCD with regard to exotics are discussed in the review [60].

The review is split into 3 parts, discussing separately heavy-light systems, heavy-heavy systems, as well as systems with more than two heavy quarks. For a more detailed discussion of the experimental and/or theoretical status on exotic meson candidates in the doubly heavy meson sector we refer to Refs. [60–66]. Also see the “Spectroscopy of Mesons Containing Two Heavy Quarks” in this *Review*. Reviews with main focus on tetraquarks and molecular states are presented in Ref. [67] and in Ref. [57, 68], respectively. The latest reviews dealing with exotic hadrons are given by Refs. [59, 69, 70].

The naming scheme for hadrons has been updated by the Particle Data Group in 2023, and now covers the heavy “exotic” tetraquark ($qq\bar{q}\bar{q}$) and pentaquark ($qqqq\bar{q}$) candidates recently discovered. We use the new naming scheme in the following. See “Naming Scheme for Hadrons” in this *Review* for details and a table mapping old and new particle names.

78.1 Heavy-light systems

78.1.1 Positive parity open charm ground states

Two very narrow states, $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$, were observed at B factories [71, 72]. They lie far below the predicted masses for the two expected broad P -wave $c\bar{s}$ mesons. However, strong cusp effects, due to the nearby DK (DK^*) thresholds, could shift their masses downwards and quench the observed widths, still allowing for a conventional explanation of these states [73]. Such an effect was also claimed for the $a_0(980)$ and $f_0(980)$ mesons, which lie just below the $K\bar{K}$ threshold. In contrast to this picture, many authors favor exotic explanations for these lightest positive parity charmed states, such as compact four-quark states with prominent diquark-antidiquark structure [74–77] or DK (DK^*) molecules [78–83]. Both states are isoscalars, and thus strong decays are suppressed, since they are possible only to isovector channels $D_s^{(*)\pm}\pi^0$, leading to very small widths. For compact structures the hadronic decay is driven by $\eta-\pi^0$ mixing and one finds widths below 10 keV [73, 84]. In contrast to this, a hadronic width typically larger than 100 keV would be the unequivocal signature for a prominent molecular nature of $D_{s0}^*(2317)^\pm$ [80, 82, 83, 85, 86], since then meson loops generating enhanced isospin violation due to the energy difference between the $D^{(*)0}K^+$ and the $D^{(*)+}K^0$ thresholds would contribute significantly. The most refined analysis in the molecular scenario [85], where the parameters of the formalism are constrained by a large bulk of lattice QCD data on singly charmed systems, reports a prediction for the width in the range (104–116) keV. The currently measured upper bound for the width is 3.8 MeV.

It should be stressed that – akin to $q\bar{q}$ mesons – multi-quark states also appear in multiplets. An important probe for the structure of hadrons is therefore the spectra, and in particular the SU(3) flavor multiplets within the different scenarios and associated mass splittings. For example, the tetraquark model advocated in Refs. [77, 87] finds that the $D_{s0}^*(2317)^\pm$ should be degenerate with its non-strange partner state $D_0^*(2300)$ in line with the currently available experimental analyses. However, Ref. [88] showed that the recent LHCb data for $B \rightarrow D\pi\pi$ [89, 90] call for a significantly lighter lightest non-strange scalar charmed meson as soon as constraints from the chiral symmetry of QCD are employed in the analysis. This claim is in line with the recent lattice QCD study of Ref. [91]. In fact, experimental as well as lattice data turn out to be consistent with the predictions from unitarised chiral perturbation theory pioneered for these systems in Ref. [79], where the lightest open flavor positive parity states appear as dynamically generated two-hadron states. For more recent calculations see Refs. [85, 92, 93]. Moreover, all the mentioned calculations that are consistent with chiral constraints predict that the interactions in the 15-plet should be repulsive while there should be in addition an about 300 MeV heavier flavor exotic state, formally a member of a flavor sextet and absent for $c\bar{q}$ states. As shown in Ref. [93] there are indeed indications also for this state in the above mentioned LHCb data—why this state was not reported in the lattice study of Ref. [94] is explained in Ref. [95] by its being located on a hidden Riemann-sheet. The pattern described above—with the interactions in the flavor anti-triplet most attractive, in the flavor sextet attractive and in the 15-plet repulsive—was recently also observed in a lattice QCD analysis for πD scattering based on the Lüscher method at a pion mass of about 700 MeV [96]. Thus, in this picture the structure that appears as $D_0^*(2300)$ in the *Listings* originates from the interplay of two poles, similar to the $\Lambda(1405)$ in the baryon sector, see ‘Pole structure of the $\Lambda(1405)$ region’ in this *Review*. Two poles in the non-strange scalar sector are also generated in the tetraquark picture of Ref. [77], but in this work the real parts of the poles are located at 2308 MeV and 2666 MeV, in conflict with the data mentioned above. In the refined analysis of Ref. [87] it is shown, however, that the exact pole locations and even the ordering of the higher multiplets depend on the details of the model. In Ref. [97] it was argued that as soon as one restricts the diquark phenomenology to only scalar diquarks, as a consequence of the Pauli principle also the diquark–anti-diquark picture is consistent with a repulsion in the 15-plet. However, axial-vector diquarks, which would be needed for a consistent hadron phenomenology, if diquarks are indeed relevant degrees of freedom, also produce states in the 15-plet. In particular, while in the scalar D -meson sector those should be rather heavy, in the axial-vector spectrum, they should be even lighter than the states in the sextet [98]. Contrary to this, in the two-hadron picture of unitarised chiral perturbation theory, the scalar and the axial-vector sectors should be equal (up to spin symmetry violations). In this sense the recent lattice study of Ref. [99] is in conflict with the predictions of the diquark-anti-diquark picture, while being consistent with a two-hadron structure for those states.

What is needed in addition to the above quoted strong, mostly theoretical, evidence for the unusual pattern of non-strange D -mesons, is a more direct experimental determination of the pole location of the lightest positive parity D -meson. This could be derived from a high quality measurement of the πD scattering parameters— analogously to the case of the $f_0(500)$, whose pole location was significantly constrained by high quality data on the $\pi\pi$ scattering length. One way to get access to the scattering parameters of unstable particles is to use the femtoscopy technique— here semi-inclusively, the correlations of two (or more) outgoing hadrons produced in a high-energy collision are measured—for reviews see Refs. [100, 101]. Recent studies using this technique find results for the DK and D^*K systems consistent with the above mentioned lattice and chiral effective field theory calculations, however, for $D\pi$ and $D^*\pi$ the extracted scattering parameters are below 0.03 fm and are thus inconsistent with the theoretical investigations [102]¹. It remains to be seen,

¹Ref. [103] shows that there is a systematic uncertainty in the femtoscopy method not yet accounted for.

if these latter values get confirmed by other investigations like in semi-leptonic B decays [104, 105].

Lattice QCD allows for predictions of the flavor partner states in the B -sector. The most recent analysis can be found in Ref. [106]. In this work also a comparison with theoretical predictions can be found. Unfortunately an experimental confirmation for those states is difficult with existing facilities. Radiative decays are predicted to be the most prominent decay channels [86], since the above mentioned isospin violation from meson loops is suppressed in the B sector as a consequence of the tiny $B^\pm - B^0$ mass difference.

78.1.2 The T_{cs0}^* (2870) and T_{cs1}^* (2900)

Manifestly exotic candidates around 2.9 GeV were reported in Refs. [107–109] by LHCb in the $\bar{D}K$ system. Since the strong interaction preserves the quark flavor and the final state contains an \bar{s} as well as a \bar{c} , color neutrality calls for a minimum quark content of $ud\bar{c}\bar{s}$. A resonant structure consistent with the T_{cs0}^* (2870) is also observed in the $D^0 K_s^0$ invariant-mass spectrum [110], although with no evidence for the T_{cs1}^* (2900). Charged open-charm tetraquark structures were searched for in $B^0 \rightarrow D^+ D^- K_s^0$ decays, but without any significant signal [111].

The explanation for those states include compact tetraquarks with a structure dominated by a diquark–anti-diquark pair [112–114], hadronic molecules with a prominent $D^* \bar{K}^*$ component [115–120], or kinematic effects such as cusps or triangle singularities (for an explanation of the latter terms we refer to the section ‘Resonances’ in this *Review*) without nearby poles [121, 122]. The different pictures are contrasted and observable differences between them are discussed in Ref. [123].

78.1.3 The T_{cs0}^* (2900)⁰ and T_{cs0}^* (2900)⁺⁺

There is a recent report by LHCb [124] on the observation of a doubly charged open-charm tetraquark candidate together with a neutral partner in the $D_s \pi$ channel, with masses near 2.9 GeV. The results imply two states with $c\bar{s}u\bar{d}$ or $c\bar{s}u\bar{d}$ flavor structures. Although masses and widths are consistent with the T_{cs} states of Refs. [107, 108], the strangeness quantum numbers do not match. The first explanation for those states in coupled-channel studies is based on a kinematic effect such as a cusp [125]. In Ref. [126] a molecular structure with a dominant $D^* K^*$ component is claimed to be excluded. However, since the T_{cs0}^* (2900) is situated near this threshold, some works [119, 127–129] argue that this isospin triplet state should be interpreted as a $D^* K^*$ hadronic molecule. Within this picture, in Ref. [129] predictions for states with other quantum numbers are given. A QCD sum rule study based on diquark–anti-diquark interpolating currents is presented in Ref. [130].

Recently LHCb reported an amplitude analysis on a resonant structure in $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$ decays [131]. One of the analysis models calls for a new $T_{c\bar{s}}^{++}$ state and its isospin partner $T_{c\bar{s}}^0$ with mass and width of about 2327 MeV and 96 MeV. At first sight these $T_{c\bar{s}}$ states can be interpreted as two members of the isotriplet predicted in Ref. [97], although data could also be explained dynamically [132].

78.2 Heavy-heavy systems

With the discovery of the $\chi_{c1}(3872)$ ² in $B^\pm \rightarrow K^\pm X$ ($X \rightarrow J/\psi(1S) \pi^+ \pi^-$) by Belle [133] in 2003, soon confirmed by BaBar [134], many searches for states beyond the standard $q\bar{q}$ quark model were initiated in the charm and bottom sectors. For an updated collection of the currently available experimental information on multi-quark states we refer to ‘Spectroscopy of mesons containing two heavy quarks’ in this *Review*, and in particular to Table 77.2. Moreover, in the decay $A_b^0 \rightarrow J/\psi(1S) K^- p$ the LHCb collaboration has recently reported the observation of new baryons decaying into $J/\psi(1S) p$, which are candidates for heavy pentaquark states [135, 136]. They are discussed in some depth in ‘Pentaquarks’ in this *Review*.

²The $\chi_{c1}(3872)$ is also known as the $X(3872)$. According to the PDG naming scheme, the primary name for a meson expresses its quantum numbers.

In addition to the multiquark scenarios discussed for the heavy-light systems, in double heavy systems two more basic configurations are discussed in the literature. On the one hand there are the hadroquarkonia, which call for a compact quarkonium core surrounded by a light quark cloud [137]. On the other hand there are hybrid mesons, in which an excited gluon field, in combination with the valence quarks, contributes to the overall quantum numbers and properties of the meson.

When restricting ourselves to confirmed states we are faced with several states that do not seem to fit into the mass and quantum number schemes of $q\bar{q}$ quark models. This is clear for the five established charged states ($T_{c\bar{c}1}(3900)^\pm$, $T_{c\bar{c}}(4020)^\pm$ and $T_{c\bar{c}1}(4430)^\pm$ in the charmonium sector, and $T_{b\bar{b}1}(10610)^\pm$ and $T_{b\bar{b}1}(10650)^\pm$ in the bottomonium sector³). The neutral states ($\chi_{c1}(3872)$, $\psi(4230)$, $\psi(4360)$, and $\psi(4660)$ ⁴) also challenge the quark model since their masses and decay properties are in conflict with expectations. Although so far seen just by a single experiment in a single channel [138, 139], the $T_{cc}(3875)$ and its proposed heavy partner states are also discussed.

It should be stressed that mass parameters for resonances located not too far with respect to their width from a pertinent threshold extracted with Breit-Wigner methods should be considered with care, since the lineshape can very well be distorted by the threshold. An example for such a case is reported in Ref. [140], where it was demonstrated that the experimental data by Belle are consistent with a $\bar{D}D$ bound state at 3695 MeV in line with lattice QCD simulations for the same system [141], while from experiment a mass as high as 3860 MeV with a width of 200 MeV was extracted [142].

78.2.1 The $\chi_{c1}(3872)$

The quantum numbers of the $\chi_{c1}(3872)$ have been determined by LHCb to be $J^{PC} = 1^{++}$, first by assuming zero angular momentum between the $J/\psi(1S)$ and the dipion [143] and then by relaxing this constraint [144]. The $\chi_{c1}(3872)$ cannot be easily identified with the $\chi_{c1}(2^3P_1)$, since the latter is predicted to lie about 100 MeV higher in mass [145]. Instead, the $X(3940)$ reported by Belle in $e^+e^- \rightarrow J/\psi(1S)X$, decaying into $D^*\bar{D}$ but not into $D\bar{D}$ [146] could be the $\chi_{c1}(2^3P_1)$. The $\chi_{c2}(3930)$ 2^3P_2 tensor partner was reported by Belle at 3931 MeV in $\gamma\gamma$ interactions [147].

The $\chi_{c1}(3872)$ lies within 200 keV of the $D^0\bar{D}^{*0}$ threshold and its width is very small — recent Breit-Wigner analyses by LHCb [148, 149] result in an average value of $\Gamma = (1.19 \pm 0.21)$ MeV as indicated in the *Listings*. However, since the respective pole is located very close to an S -wave thresholds to which it couples strongly, the Breit-Wigner lineshape should not be used (see section ‘Resonances’ in this *Review*). Employing the more appropriate Flatté formalism of Ref. [150] under the constraint that the $D^*\bar{D}$ decay channel dominates, results in $\Gamma = 0.22_{-0.19}^{+0.18}$ MeV, where the standard convention is applied to use twice the imaginary part of the pole location as width. A recent determination of the $\chi_{c1}(3872)$ lineshape parameters and the resulting pole location based on BESIII data can be found in Ref. [151]. A combined analysis of the LHCb data and BESIII data is presented in Ref. [152], where the pole location of the $\chi_{c1}(3872)$ was determined with unprecedented accuracy. Moreover, further evidence was presented for the existence of an additional pole in the isovector channel, originally predicted in Ref. [153], which should show up more prominently in e.g. the decay $B^0 \rightarrow K^0 J/\psi \pi^+ \pi^-$. In the future the mass parameters can be refined more by either a direct measurement of the width that should be possible at the planned PANDA experiment [154, 155] or by exploiting the interplay of a triangle singularity and the $\chi_{c1}(3872)$ pole [156–158].

The most natural explanation for the $\chi_{c1}(3872)$ is a $1^{++} D\bar{D}^*$ molecule [159]. In this scenario a strong isospin breaking is predicted [159, 160], since the distance of the pole of the $\chi_{c1}(3872)$ to the $D^0\bar{D}^{*0}$ threshold is significantly smaller than to the D^+D^{*-} threshold. Indeed, the comparable

³In the literature those states are often labeled as Z_c and Z_b , respectively.

⁴The $\psi(4230)$, $\psi(4360)$, and $\psi(4660)$ are also known as the $Y(4230)$, $Y(4360)$, and $Y(4660)$, respectively. Before improved mass measurements, the $\psi(4230)$ was originally called the $\psi(4260)$ or $Y(4260)$.

rates for $\omega J/\psi(1S)$ and $\rho^0 J/\psi(1S)$ are consistent with an interpretation of $\chi_{c1}(3872)$ as an isoscalar $D\bar{D}^*$ molecule, when the different widths of the ρ and ω are taken into account [161]. A molecular component of above 99% was also found in Refs. [152, 162] in sophisticated analyzes that kept the full three-body dynamics of the intermediate state $D\bar{D}\pi$, first studied in Ref. [41].

A dominant molecular $D^0\bar{D}^{*0}$ structure in the $\chi_{c1}(3872)$ with further subleading more compact hadronic components is used to explain strong and radiative decays involving $J/\psi(1S)$ and $\psi(2S)$ in the final states [163, 164]—it should be mentioned that those rates are also consistent with a $c\bar{q}$ structure [165]. The most recent measurement of the ratio of radiative partial decay widths [166] indicates that the $\psi(2S)\gamma$ mode dominates over $J/\psi\gamma$. This observation is claimed to point to a sizable compact component in the $\chi_{c1}(3872)$ (for a compilation of various structure-dependent predictions see [166]). However, from an effective field theory point of view these radiative transitions are not well suited to quantify the molecular admixture in the $\chi_{c1}(3872)$, since there appears a counter term at leading order, diminishing the sensitivity to the long-ranged molecular component [167].

A diquark–anti-diquark structure $(cq)(\bar{c}\bar{q}')$ is another possible interpretation of the $\chi_{c1}(3872)$ [76]. Contrary to the two hadron structure discussed above, here diquark-anti-diquark substructures are assumed to be bound by QCD flux tubes—for a discussion of possible configurations see Ref. [168]. What speaks against this interpretation is that so far no near mass degenerate charged partner of the $\chi_{c1}(3872)$ has been observed (e.g. not in $B^- \rightarrow \bar{K}^0 X^-$ nor in $B^0 \rightarrow K^+ X^-$, where $X^- \rightarrow J/\psi(1S)\pi^-\pi^0$ [169])—see [170] for a possible explanation of this non-observation within the tetraquark approach assuming specific diquark correlations and the more recent discussion in Ref. [171]. Note that the isovector partner state of the $\chi_{c1}(3872)$ discussed in [153] is not consistent with the proposed charged compact tetraquark state, since the pole is located on an unphysical sheet, clearly pointing at a molecular nature of the state [172], and its mass is close to the D^+D^{*-} threshold, 8 MeV above the mass of the $\chi_{c1}(3872)$.

The claim that $\chi_{c1}(3872)$ must be a compact (tetraquark) state, since it is also produced at very high p_T in $\bar{p}p$ collisions [173] and in high multiplicity final states [174], was challenged in [175] and [176], respectively, which in particular stress the importance of rescattering, see also Refs. [177, 178].

The high statistics measurement of the $\chi_{c1}(3872)$ lineshape of Ref. [148] also allowed for an extraction of the effective range for the $D^0\bar{D}^{*0}$ interaction. At face value, the values provided in this publication indicate a large and negative effective range [179], which could be interpreted as a signature of a prominent compact component in the $\chi_{c1}(3872)$ wave function — this follows from the famous Weinberg compositeness criterion [180]. The effect of the one-pion exchange on the effective range turns out to be small [40, 41, 181, 182] and should therefore not invalidate the Weinberg-type analysis. However, it was argued in Ref. [183] that once the effect of the large isospin violation as well as the strong correlations between the effective parameters appearing in Ref. [150] are subtracted, the effective range is consistent with a purely molecular $\chi_{c1}(3872)$. There is no consensus yet how to interpret the line shapes of $\chi_{c1}(3872)$.

78.2.2 The $\psi(4230)$ and $\psi(4360)$

A broad structure, originally called $Y(4260)$, decaying into $J/\psi(1S)\pi^+\pi^-$ was reported by BaBar in initial state radiation $e^+e^- \rightarrow (\gamma_{\text{ISR}})Y(4260)$ [184]. A subsequent measurement with significantly improved precision was reported by BESIII [185], and revealed that the original $Y(4260)$ cannot be described with a simple resonant lineshape. Fitting the BESIII data with two Breit-Wigner distributions leads to a narrower and lighter structure (referred to in the *Listings* as the $\psi(4230)$), but also requires a second state at 4320 MeV. While the $D_1(2420)\bar{D}$ molecular model for the $\psi(4230)$ can describe the same data with just one single exotic pole [186, 187], the coupled

channel analysis of Ref. [188] needs an additional exotic state near 4320 MeV. Both, Ref. [187] and Ref. [188] need the interference of the signal of the $\psi(4230)$ with the $\psi(4160)$ to get a good description of the large amount of available data. How many vector states are in the mass range between 4220 MeV and 4400 MeV is not yet settled, but knowing this is crucial to allow for further progress.

There are no charmonium states expected in this mass region with quantum numbers 1^{--} from quark models using the Cornell type of interaction, although this might not be true for some screened versions thereof—for a recent discussion we refer to Ref. [189]. In addition, a charmonium state at this mass is expected to have significant couplings to one or more of the $\bar{D}^{(*)}D^{(*)}$ channels [190, 191], a feature that is not observed for the $\psi(4230)$. This state could be a hybrid charmonium with a spin-1 [28, 192] or a spin-0 [193, 194] $\bar{c}c$ core—for a recent review on this state see Ref. [195]. However, provided that the observation of $\psi(4230)$ decay into $h_c(1P)\pi\pi$ by BESIII [196] is confirmed, the hybrid hypothesis would be under pressure, since the spin of the heavy quarks (coupled to zero in the $h_c(1P)$) should be conserved in leading order in the expansion in $(\Lambda_{\text{QCD}}/m_c)$. (The individual conservation of the heavy quark spin and the total angular momentum of the light quark cloud is a consequence of the heavy-quark spin symmetry, see ‘Heavy-Quark and Soft-Collinear Effective Theory’ in this issue of the *Review*.) The same criticism applies to the hadrocharmonium interpretation of the $\psi(4230)$. To circumvent the spin-symmetry argument, Ref. [197] suggests that $\psi(4230)$ and $\psi(4360)$ could be mixtures of two hadrocharmonia with spin-triplet and spin-singlet heavy quark pairs. The same kind of mixing could also be at work for hybrid structures. Hybrids can also be treated employing the Born-Oppenheimer EFT originally suggested in Ref. [30, 198]. Those studies revealed that while the $\psi(4230)$ is in principle a good candidate for a hybrid state [199], its decays are incompatible with this assignment [200].

A dominant $D_1(2420)\bar{D}$ component in the $\psi(4230)$ [201] explains naturally why the $T_{c\bar{c}1}(3900)^\pm$ (interpreted by the authors as a $\bar{D}D^*$ bound state) is seen in $\psi(4230) \rightarrow \pi^\mp T_{c\bar{c}1}(3900)^\pm$. A similar mechanism is also found in Ref. [202] linking in addition the $J/\psi(1S)\pi^+\pi^-$ and $\psi(2S)\pi^+\pi^-$ decays of the $\psi(4230)$. Furthermore, a copious production of $\chi_{c1}(3872)$ in $\psi(4230)$ radiative decays was predicted from the prominent $D_1\bar{D}$ component of the $\psi(4230)$ [203], which was confirmed by BESIII [204]. Possible charmonia components both in $\psi(4230)$ and $\chi_{c1}(3872)$ can influence the radiative transition but they are shown to be of subleading order [205]. It was proposed that $\psi(4360)$ as a $D_1\bar{D}^*$ bound state could be the spin partner of the $\psi(4230)$ [206, 207]. In Ref. [208] a dynamical model is presented that identifies the vector states $\psi(4230)$, $\psi(4360)$ and $\psi(4415)$ as DD_1 , $D^*\bar{D}_1$ and $D^*\bar{D}_2$ molecular states, respectively. Moreover, an exotic 0^{--} state is predicted as $D^*\bar{D}_1$ molecule with a mass of 4366 ± 18 MeV and a width of less than 10 MeV.

The tetraquark picture calls for four ground state vector states — once the parameters of the model are fixed from some candidate states in the negative and positive parity sector, states with other quantum numbers can be predicted. Possible scenarios are for instance discussed in Ref. [209] which builds on a tailor-made spin-spin interaction [16] to describe the $\chi_{c1}(3872)$, both $T_{c\bar{c}1}(3900)^{\pm,0}$ and $T_{c\bar{c}}(4020)^\pm$ and even the $T_{c\bar{c}1}(4430)^\pm$ confirmed by Belle [210] and LHCb [211]. This model also explains the copious production of $\chi_{c1}(3872)$ in $\psi(4230)$ radiative decays mentioned above [16, 212]. However, tetraquark models (in most cases based on diquark-antidiquark configurations) tend to predict many additional charged and neutral states which have not yet been discovered. In particular, as for the conventional $\bar{q}q$ structures one should expect nearly degenerate isoscalar and isovector states in analogy to the near degeneracy of $\rho(770)$ and $\omega(782)$. The problem and possible explanations are discussed in some detail in Refs. [170, 171].

Ref. [213] found a sizable SU(3) flavor octet contribution when analyzing the $\pi\pi$ final state from $\psi(4230) \rightarrow J/\psi(1S)\pi^+\pi^-$, which is consistent with both a molecular and a tetraquark interpretation of $\psi(4230)$, but is at odds with a hybrid or a $\bar{c}c$ interpretation.

78.2.3 The $\psi(4660)$

In the mass range above 4600 MeV, the number of poles is also not yet settled. Experimental signals are seen in the $\psi(2S)\pi\pi$, $\Lambda_c^+\bar{\Lambda}_c^-$, $D_s\bar{D}_{s1}$, $D_s\bar{D}_{s2}^*$, and $D^{*0}D^{*-}\pi^+$ final states. In the *Listings* these structures go into one node, $\psi(4660)$, due to their proximity in parameter values. Moreover, various theoretical works describe these states in a combined analysis [214–216]. The signal in the hidden strangeness mode around 4630 MeV still calls for confirmation and might be yet another realization of the same state, but there are already speculations about its nature. Ref. [217] finds a single state in this mass range and identifies it with the $\psi(5S)$. While Ref. [218] argues in favor of a $D_s^{(*)}\bar{D}_{s1}(2536)$ or $D_s^{(*)}\bar{D}_{s2}(2573)$ [219] molecular nature, Ref. [220] does not confirm these claims. Ref. [221] identifies the structure as P -wave $[cs][\bar{c}\bar{s}]$ tetraquark state. Other explanations of the $\psi(4660)$ include a $\psi(2S)f_0(980)$ molecule [222] and a $\Lambda_c^+\bar{\Lambda}_c^-$ baryonium [214]. Also in this mass range studies of the partner states, driven either by spin or flavor symmetry will be very valuable—see e.g. the predictions in Ref. [223].

78.2.4 The $T_{c\bar{c}1}(3900)$, $T_{c\bar{c}}(4020)$, $T_{b\bar{b}1}(10610)$, and $T_{b\bar{b}}(10650)$

The isovector states $T_{c\bar{c}1}(3900)$ and $T_{c\bar{c}}(4020)$, first observed by BESIII [224, 225], decay predominantly into $\bar{D}D^*$ and \bar{D}^*D^* , respectively, while the $T_{b\bar{b}1}(10610)$ and $T_{b\bar{b}}(10650)$, first observed by Belle [226, 227], decay predominantly into $\bar{B}B^*$ and \bar{B}^*B^* [228], respectively, although all four were discovered in a decay into a heavy quarkonium plus a pion. This suggests that these states are close relatives and their interactions are connected via heavy quark flavor symmetry⁵. A molecular interpretation for the bottomonium states was proposed shortly after the discovery of the two $T_{b\bar{b}}$ states [230] and also shortly after that of the $T_{c\bar{c}1}(3900)$ [201]—the up-to-now most precise determination of the pole location of the latter state is reported in Ref. [231]. This picture is confirmed within the meson exchange model of Ref. [232]. Decay patterns of $T_{c\bar{c}1}(3900)$ and the two $T_{b\bar{b}}$ were also shown to be consistent with a molecular interpretation [233–235]. However, some of their properties also appear to be consistent with tetraquark structures [236]. If the molecular picture were correct for the $T_{b\bar{b}}$ states, spin symmetry would lead to the existence of spin partner states [37, 237, 238], which are still to be found. In Ref. [47] it was shown that the actual pole locations of these partner states would be good probes of one-pion exchange in the molecular potential, which makes the experimental search for those states even more interesting. A possible interpretation of $T_{c\bar{c}1}(3900)$ and $T_{c\bar{c}}(4020)$ as crossed channel effects is put forward in Ref. [239]. It remains to be seen, however, if this kind of explanation is also capable of explaining the observations of these lowest $T_{c\bar{c}}$ states, also at other total energies. A recent lattice QCD calculation suggests that the $T_{c\bar{c}1}(3900)$ can be reconciled with a two pole structure in the $J/\psi\pi - D\bar{D}^*$ coupled-channel [240]. In this study the coupling between $D\bar{D}^*$ and $J/\psi\pi$ is significant and plays a crucial role.

78.2.5 The $T_{c\bar{c}1}(4200)$ and $T_{c\bar{c}1}(4430)$

The heaviest confirmed charged state in the charmonium sector is the $T_{c\bar{c}1}(4430)^\pm$ observed by Belle [210]. It is interpreted as hadrocharmonium [137], \bar{D}_1D^* molecule [241], as well as diquark–anti-diquark structure [16]. Alternatively, in [242, 243] the $T_{c\bar{c}1}(4430)^\pm$ is explained as a cross-channel effect enhanced by a triangle singularity from open charm states. These works were criticized in Ref. [244] where an alternative triangle consisting of a K^* , a π , and the $\psi(4230)$, is proposed to generate the $T_{c\bar{c}1}(4430)$. The Argand diagram shows an anticlockwise circle, in line with the experimental analysis [211], while the one of Ref. [243] shows a clockwise motion. By replacing the $\psi(4230)$ with the $\psi(3770)$, and changing the K^* , one can also interpret the $T_{c\bar{c}1}(4200)$ as a kinematic effect [244].

⁵For a critical examination of this claim from an effective field theory point of view see Ref. [229].

78.2.6 The $T_{c\bar{c}s1}(4000)$

There is recent evidence for a charged charmonium-like state with strangeness, $T_{c\bar{c}s1}(4000)$, from BESIII [245] and LHCb [246]. The possible existence of a strange partner to the $T_{c\bar{c}}$ near the $D_s\bar{D}^*$ thresholds has been predicted in molecular models [247, 248], for tetraquarks [249, 250], for hadrocharmonium structures [250, 251], and as a coupled-channel effect [252]. Later on this state is interpreted in Refs. [253, 254] as a member of the same multiquark octet and in Refs. [255–257] as a member of the same molecular one as the $T_{c\bar{c}1}(3900)$. Ref. [254] is also able to describe the recent LHCb data in the $J/\psi(1S)K^-$ system [246], although the extracted total width for their lowest $T_{c\bar{c}s}$ state is an order of magnitude larger than that found by BESIII. Refs. [258, 259] claim that both the molecular components and the compact tetraquark core are relevant to describe the $T_{c\bar{c}1}(3900)$ and $T_{c\bar{c}s1}(4000)$ resonances. Again, in [260] the $T_{c\bar{c}s1}(4000)$ can be explained as a coupled-channel effect producing the enhancement close to threshold.

Ref. [255] comes to the conclusion that an understanding of the experimental data for the $T_{c\bar{c}s1}(4000)$ calls for the inclusion of both triangle diagrams as well as a pole term. The authors conclude from their calculations that $T_{c\bar{c}s1}(4000)$ and $T_{c\bar{c}1}(3900)$ are members of the same SU(3) multiplet. This conclusion was confirmed in Ref. [261] and the study extended to higher energies. The fit of the experimental data allowed for two solutions, one where the $T_{c\bar{c}}(4020)$ is a spin partner of the mentioned isovector states and one where it is not. Data e.g. for $e^+e^- \rightarrow K^+D^{*-}D^{*0}$ is claimed to be decisive which scenario is indeed realized.

It should be stressed that the various scenarios, while describing much of the available data, also make decisive predictions, e.g. for states with yet unobserved quantum numbers [209, 262]. The forthcoming data on heavy meson spectroscopy from various facilities should provide a much deeper understanding of how QCD forms matter out of quarks and gluons.

Note that for various states listed above it is still possible that they reflect the presence of cusps and not states—for recent studies see [263, 264].

78.2.7 The $T_{cc}(3875)$

There is a recent report and follow-up by LHCb [138, 139] on the observation of a narrow peak in the $D^0D^0\pi^+$ mass spectrum just below the $D^{*+}D^0$ threshold. The observed state is a candidate for an isoscalar double-charm T_{cc}^+ tetraquark configuration with a quark content of $cc\bar{u}\bar{d}$. Unsuccessful searches in $D^+D^0\pi^+$ and D^+D^+ mass spectra disfavour the isovector assignment. The $D^0\pi^+$ mass distribution indicates that the T_{cc}^+ proceeds through the formation of a D^* in the intermediate state. The additional proximity to the $D^{*+}D^0$ mass threshold favors the $J^P = 1^+$ assignment. The T_{cc}^+ has a remarkable narrow width and lies very close to the $D^{*+}D^0$ threshold [139], comparable to the properties of the $\chi_{c1}(3872)$. Indeed, employing a meson-exchange model [19] or the BOEFT [265] the two states can be described together in the same framework.

Evaluations in the context of QCD sum rules [266, 267] result in mass and width values consistent with the assumption of a compact diquark-antidiquark structure of the T_{cc}^+ . A pole analysis performed with $D^0D^0\pi^+ - D^{*+}D^0$ coupled-channels [268] suggests that the T_{cc}^+ has its origin in a $D^{*+}D^0$ virtual state resulting from the attraction in this channel and the coupling to $D^0D^0\pi^+$. The interpretation of the observed T_{cc}^+ as a D^*D molecular structure was pursued in [48, 56] based on non-relativistic effective field theories without and with pion exchange, respectively, in [269, 270] employing a combination of meson exchange and quark-gluon forces, and in [271] employing the Bethe-Salpeter framework. Predictions depend on fine model details and are usually accompanied by additional partner states, like an isovector molecule or higher radial excitations. A fit to the LHCb data on $T_{cc}(3875)$ [272] is also consistent with a molecular D^0D^{*+} , D^+D^{*0} interpretation in the isoscalar channel, while for the isovector case binding is prevented. A coupled-channel analysis of the $D^0D^0\pi^+$ mass spectrum within a meson-exchange model [19] also concludes the nature of T_{cc}^+

as an isoscalar DD^* hadronic molecule. As a consequence a double-charm isoscalar D^*D^* hadronic molecule candidate is predicted.

In analogy to the case of the $\chi_{c1}(3872)$ in Ref. [273] the production of T_{cc}^+ in high-energy proton-proton collisions at the LHC is studied. Effects of a charm-meson triangle singularity on the inclusive production of T_{cc}^+ from the rescattering of DD lead to a narrow peak in the $T_{cc}^+\pi^+$ invariant mass distribution. A spin 2 partner of the T_{cc}^+ is predicted in Refs. [48,56], whose hadronic decays are studied in Ref. [274].

As for the $\chi_{c1}(3872)$ one can employ the effective range deduced from experiment for the effective range in the DD^* -channel as a measure of the nature of the $T_{cc}(3875)$. Also here isospin violations are significant. It turns out that in a calculation where the one-pion exchange is kept fully non-perturbative, its effect on the effective range is small and positive [48], while it is small and negative when being treated perturbatively [182,275].

First lattice studies with fully dynamical quarks are also available for the $T_{cc}(3875)$ [276–278], all performed at pion masses slightly larger than physical such that the D^* and with this also the $T_{cc}(3875)$ are stable. They all find that at these pion masses the T_{cc} is a virtual state. In Ref. [279] it was stressed, however, that all those analyses should be redone, since the proximity of a left-hand cut invalidates the method used— analogously also the effective range expansion needs to be changed in the proximity of a left-hand cut [280,281]. Possible modifications of the Lüscher method are proposed in Refs. [281–284]. Results point towards a resonance interpretation of the $T_{cc}(3875)$. Chiral extrapolations of lattice simulations for the isoscalar DD^* scattering with [285,286] and without left-hand cut [287] to the physical pion mass lead to a T_{cc} bound states with binding energies compatible with experiment, however, significantly different trajectories.

A first coupled channel isospin-0 DD^* , D^*D^* scattering calculation in lattice QCD [288] also indicates a pole below the DD^* threshold, a further pole corresponding to T'_{cc} is also found just below the D^*D^* threshold. A further lattice QCD study [289] in the doubly charm $cc\bar{q}\bar{q}$ isospin-1 channel concludes that no poles are found in the scattering amplitude near the DD^* threshold consistent with the current experimental status.

Not very long after the discovery of the charm quark it was stressed that, if the ratio of heavy (Q) to light quark (q) masses is well beyond 10, binding should occur for states of the kind $QQ\bar{q}\bar{q}$ [290–292]. These studies were performed within constituent quark models. Accordingly the quoted ratio refers to constituent masses, which are of the order of 300 MeV for the light quarks in contradistinction to the current quark masses of the order of a few MeV. Based on those studies one should expect binding for $bc\bar{q}\bar{q}$ systems and even more so for $bb\bar{q}\bar{q}$ —the transition from a (predominantly) molecular structure to a diquark–anti-diquark structure as the masses of the heavy quarks are increased is discussed e.g. in the phenomenological models of Ref. [269,270]. Nowadays many works investigate doubly heavy tetraquarks including phenomenological models [291–298], lattice QCD either employing static potentials [299–302] or NRQCD for the b -quarks [106,303–307] as well as QCD sum rules [266,308–310]. All those papers agree that a T_{bb} should be bound for the $I = 0$ and $J^P = 1^+$ channel, but there is no consensus about the binding energy relative to the lowest open bottom threshold (BB^*), which ranges from values near zero to 500 MeV. Since in contrast to the states that contain a heavy quark–anti-quark pair this family of states contains two heavy quarks, no strong decay channels are accessible for the heavier partner states of the $T_{cc}(3875)$ and they could only decay weakly. Accordingly, once found, they would provide an exciting laboratory not only to investigate the strong but also the weak interaction.

78.3 Systems containing four heavy quarks

78.3.1 The $T_{cc\bar{c}\bar{c}}$ (6900)

LHCb reported the observation of pronounced structures in a double- $J/\psi(1S)$ invariant mass distribution [311] thus pointing at states with $cc\bar{c}\bar{c}$ quarks contents. First results on di- $J/\psi(1S)$ and $J/\psi(1S)\psi(2S)$ invariant mass distributions in proton-proton collision data by the CMS [312] and ATLAS [313] Collaborations confirm and extend the resonance structures seen by LHCb. In Ref. [314] the quantum numbers of the structures were determined to be $J^{PC} = 2^{++}$. Belle reported on a first search for a double-charmonium state with $e^+e^- \rightarrow \eta_c J/\psi(1S)$ near threshold via the ISR process. No significant signal of the double charmonium state is found in several bins of the invariant mass of $\eta_c J/\psi(1S)$ [315]. Recently the COMPASS [316] Collaboration also studied double $J/\psi(1S)$ production in pion scattering off nuclear targets. No significant signatures that could be associated with tetraquarks are found in the double $J/\psi(1S)$ mass spectrum in the relatively limited data set.

From quark models the possible existence of bound states like this was discussed already long ago [290]. There are now also many model calculations available in the literature. For discussions of those data from the compact tetraquark perspective see Refs. [317–323]. Most of these quark model calculations assign the reported structure to a $cc\bar{c}\bar{c}$ state in the $2S$ multiplet with near-degenerate $J^P = 0^+, 1^+$ and 2^+ configurations. The dominance of a compact tetraquark component in the apparently exotic structure is also supported by the coupled multichannel study of Ref. [324]. The extended coupled-channel study of Ref. [325], where two-meson states are coupled to a potential derived within a quark model, leads to a very rich spectrum of $c\bar{c}-c\bar{c}$ states where parts are candidates for the observed double $J/\psi(1S)$ structures. For direct analyses of the data within the tetraquark approach, see, e.g., Ref. [326]. Also QCD sum rule studies of the system were published, but do not give consistent results: For example Ref. [327] interprets the structure at 6900 MeV as a hybrid state, Ref. [328] as a tetraquark, and Ref. [329] states that both molecular and tetraquark interpretations are possible.

An alternative view on the double- $J/\psi(1S)$ data is provided in Refs. [330–332] where the analyses are performed using coupled channel T -matrices, involving vector-vector channels of quarkonia. In these works the structures in the data emerge from the interplay of thresholds and resonances. While the number of poles that appear above the double J/ψ threshold depends on the number of channels, there is the non-trivial prediction that there should exist, if this dynamical picture were correct, a state located very near the double- $J/\psi(1S)$ threshold. In Ref. [333] it was shown that a $J/\psi(1S)J/\psi(1S)$ scattering potential sufficiently strong to bind is in fact consistent with our current understanding of $\pi J/\psi(1S)$ scattering as well as the strength of the $\pi\pi$ interactions. In Ref. [334] it is shown that the above mentioned data by LHCb, CMS and ATLAS are consistent with the presence of the above mentioned pole, whose pole position could be determined with improved accuracy. In this work it is also stressed that the nature of this $X(6200)$ cannot be determined based on the data currently available.

In contrast, Ref. [335, 336] explains the double- $J/\psi(1S)$ data as cusps without nearby poles. This claim can be tested experimentally: if this explanation were correct, there should not be any narrow near threshold structure in the channel that generates the cusp, for that would call for a non-perturbative interaction in that channel, as pointed out in Ref. [337].

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