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Walter H. Barkas and Arthur H. Rosenfeld

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Elementary-particle data and certain other reference information are frequently needed by research workers in high-energy physics in a compact and readily accessible form. For the use of students and staff members in the Radiation Laboratory we have attempted to meet this need. In preparing this summary we have tried to employ units and concepts natural to this field, and to drop those that are irrelevant or obsolete.

The most important improvements are in the presentation of range-energy data, in the addition of a section on relativistic particle formulas, and in reduction of the space allotted to multiple scattering on the pocket cards at the back of this UCRL report.

The tables and graphs are as follows:

Table I. Masses and Mean Lives of Elementary Particles

This table is a compilation of all information on the masses and mean lives of elementary particles available to us at the close of the 1960 Rochester Conference on High-Energy Physics. Both published and unpublished information has been cited to obtain the current best values. This report may not be exhaustive, however. In particular, there may be work from the Soviet Union of which we are unaware.

When systematic as well as statistical errors appear to affect a measurement, we have occasionally been forced to exercise judgment in weighting the data. Otherwise, standard statistical methods were used. To avoid skewed distributions, we have averaged decay rates rather than mean lives. An effort has been made to allow for the interdependence of the masses, but this has not been done in a completely systematic way.

The brief references pertain mainly to very recent work. They, in turn, refer to the earlier publications.

Part of the table was compiled in consultation with Professor George Snow, who was preparing a similar table for the Handbook of the American Institute of Physics.

We have assumed that particle and antiparticle share the same spins, masses, and mean lives.[1,2,3] Conventionally, the negatively charged leptons (e^- and μ^-) and the positively charged mesons (π^+ and K^+) are defined as "particles". We did not, however, want to list as "particles" only negative leptons and positive mesons, since we report a π - μ mass difference which comes from the decay $\pi^+ \longrightarrow \mu^+ + \nu$. Therefore we have adopted the notation $e^\mp$ and $\mu^\mp$ for the leptons, $\pi^\pm$ and $K^\pm$ for the mesons.

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[1] T. D. Lee, R. Oehme, and C. Yang, Phys. Rev. 106, 340 (1957).

[2] S. Okubo, Phys. Rev. 109, 984 (1958).

[3] A. Pais, Phys. Rev. Letters 3, 342 (1959).

TABLES FROM UCRL-8030(rev.). Table I. Masses and mean lives of elementary particles

(The antiparticles are assumed to have the same spins, masses, and mean lives as the particles listed)

	Particle	Spin	Mass (Errors represent standard deviation) (Mev)			Mass difference (Mev)			Mean life (sec)	
Photon	γ	1	0		γ	... ...		γ	Stable	
Leptons	ν	1/2	0		ν	... ...		ν	Stable	
	$e^{\mp}$	1/2	0.510976 ± 0.000007	(a)	$e^{\mp}$	... ...		$e^{\mp}$	Stable	
	$\mu^{\mp}$	1/2	105.655 ± 0.010	(b)	$\mu^{\mp}$, $\pi^{\pm}$	33.93 ± 0.05	(x)	$\mu^{\mp}$	$(2.212 \pm 0.001) \times 10^{-6}$	(r)
Mesons	π^{+}	0	139.59 ± 0.05	(*)	$\pi^{\pm}$, π^{0}	4.59 ± 0.01	(j)	$\pi^{\pm}$	$(2.55 \pm 0.03) \times 10^{-8}$	(w)
	π^{0}	0	135.00 ± 0.05	(*)				π^{0}	$(2.2 \pm 0.8) \times 10^{-16}$	(d)
	$K^{\pm}$	0	493.9 ± 0.2	(k)	K^{+}, K^{0}	3.9 ± 0.6	(i)	K^{+}	$(1.224 \pm 0.013) \times 10^{-8}$	(h)
	K^{0}, K_1, K_2	0	497.8 ± 0.6	(i)				K^{0}	50% K_1, 50% K_2	
					K_1, K_2	$(1.5 \pm 0.5)\hbar/\tau(K_1)$	(z)	K_1	$(1.00 \pm 0.038) \times 10^{-10}$	(e)
								K_2	$6.1(+1.6/-1.1) \times 10^{-8}$	(c)
Baryons	p	1/2	938.213 ± 0.01	(a)	p, n	1.2939 ± 0.0004	(t)	p	Stable	
	n	1/2	939.507 ± 0.01	(t)				n	$(1.013 \pm 0.029) \times 10^{3}$	(y)
	Λ	1/2	1115.36 ± 0.14	(v)	Λ	... ...		Λ	$(2.51 \pm 0.09) \times 10^{-10}$	(u)
	Σ^{+}	1/2	1189.40 ± 0.20	(l)	Σ^{+}, Σ^{-}	6.56 ± 0.22	(n)	Σ^{+}	$0.81(+0.06/-0.05) \times 10^{-10}$	(m)
	Σ^{-}	1/2	1195.96 ± 0.30	(n)	Σ^{-}, Σ^{0}	4.45 ± 0.4	(p)	Σ^{-}	$1.61(+0.1/-0.09) \times 10^{-10}$	(o)
	Σ^{0}	1/2	1191.5 ± 0.5	(*)				Σ^{0}	$< 0.1 \times 10^{-10}$	(s)
	Ξ^{-}	?	1318.4 ± 1.2	(f)	Ξ^{-}	... ...		Ξ^{-}	$1.28(+0.38/-0.30) \times 10^{-10}$	(f)
	Ξ^{0}	?	1311 ± 8	(q)	Ξ^{0}	... ...		Ξ^{0}	1.5×10^{-10} (1 event)	(q)

Walter H. Barkas, Arthur H. Rosenfeld, University of California, Berkeley, Sept. 1960.

(a) From compilations by Cohen, Crowe, and DuMond, Nuovo cimento 5, 541 (1957) and Fundamental Constants of Physics (Interscience, New York, 1957).

(b) L. Lederman, 1960 "Rochester Conference". Also, Lathrop, Lundy, Penman, Telegdi, Yanovitch and Winston, N. C. 17, 2322 (1960).

(c) Bardon, Landé, Lederman, and Chinowsky, Ann. Physik 5, 156 (1958), and Crawford, Cresti, Douglass, Good, Kalbfleisch, and Stevenson, P. R. L. 2, 361 (1959). The weighted average of the two results is given in the second reference.

(d) Glasser, Seeman, and Stiller, private communication. Referred to as a preliminary figure by Ashkin and Tollestrup at 1960 Roch. Conf.

(e) $\tau(K_1)$ is a weighted average of the decay rates corresponding to the mean lives given in Table V of the Proceedings of the 1958 " CERN Conference on High-Energy Physics" with a single exception: The Berkeley result from associated production has been changed to $(0.94 \pm 0.05) \times 10^{-10}$ sec., based on 512 K_1 decays (Crawford, Cresti, Good, Kalbfleisch, Stevenson, and Ticho (LRL), private communication).

(f) $M(\Xi^-)$ is a weighted average of the following results (in Mev):

1320.4 ± 2.2	W. A. Barkas and A. H. Rosenfeld, (UCRL-8030 March, 1958) compilation of 12 Ξ^- found before March 1958.
1318.1 ± 1.9	Fowler, Birge, Eberhard, Ely, Good, Powell, and Ticho, (20 Ξ^- in Berkeley 30-inch propane chamber, unpublished).
1317 ± 2.2	M. I. Soloviev, (11 Ξ^- in Dubna propane chamber; 1960 Roch. Conf.)

$\tau(\Xi^-)$ is taken only from the 20 Ξ^- of Fowler et al., since the other events have considerably larger uncertainties.

(h) $\tau(K^+)$ from weighted average of the decay rates corresponding to the following mean lives: $1.227 \pm 0.015 \times 10^{-8}$ sec (Alvarez, Crawford, Good, and Stevenson (private communication)); $1.211 \pm 0.026 \times 10^{-8}$ sec (V. Fitch and R. Motley, P. R. 101, 496 (1956); P. R. 105, 265 (1957); and private communcation.) The quoted errors are statistical only.

(i) From the compilation by Rosenfeld, Solmitz, and Tripp, P. R. L. 2, 110 (1959).

(j) Haddock, Abashian, Crowe, and Czirr, P. R. L. 3, 478 (1959).

(k) $M(K^+)$ from the mass of three charged pions, quoted in this table, plus the Q value of Reference (a) and an additional allowance of 0.1 Mev for a systematic error in the range-energy relation.

(l) $M(\Sigma^+)$ from the decay mode $\Sigma^+ \rightarrow p + \pi^0$.
The data of M. S. Swami, P. R. 114, 333 (1959), R. S. White, 1957 Roch. Conf., Evans et al., N. C. 15, 873 (1960), and Dyer et al., B. A. P. S. 5, 224 (1960), have been combined using the mass of the π^0 quoted in this table. Only the protonic decay mode has been used, but the mass deduced from the pion mode is consistent with this (Dyer et al.).

(m) $\tau(\Sigma^+)$ comes from combining the bubble chamber result $(0.75 \pm 0.1) \times 10^{-10}$ sec compiled at the 1958 CERN Conf. with the new emulsion results of Evans et al. (N. C. 15, 873 (1960)); Freden, Kornblum, and White (N. C. 16, 611 (1960)) and an unpublished result $0.82(+0.1/-.08) \times 10^{-10}$ sec of Dyer, Barkas, Heckman, Mason, Nickols, and Smith. There is no longer any anomaly in the emulsion measurements of $\tau(\Sigma)$.

(n) $M(\Sigma^-) - M(\Sigma^+)$ is a weighted average of the following mass differences (in Mev):

7.10 ± 0.92	Chupp, Goldhaber, Goldhaber, and Webb.
6.9 ± 1.0	M. S. Swami, P. R. 114, 333 (1959).
7.46 ± 0.56	Evans et al., N. C. 15, 873 (1960).
6.315±0.25	Dyer et al., B. A. P. S. 5, 224 (1960).

To get $M(\Sigma^-)$ we have combined the Σ^+-Σ^- mass difference with $M(\Sigma^+)$. This $M(\Sigma^-)$ is not yet on quite as firm a basis as the others in this table because of an unexplained anomaly, observed in the range of the pions accompanying its production in $K^- + p \Rightarrow \Sigma^- + \pi^+$. All other information on $M(\Sigma^-)$ is consistent with the mass quoted.

(o) $\tau(\Sigma^-)$ obtained from combined bubble chamber mean lives; $1.59(+0.1/-0.09) \times 10^{-10}$ sec (L. W. Alvarez, 1959 Kiev Conf., see also UCRL-9354 Aug. 1960) and an unpublished emulsion mean life of $1.75(+0.39/-0.30) \times 10^{-10}$ sec by Dyer, Barkas, Heckman, Mason, Nickols, and Smith.

(p) Berge, Rosenfeld, Ross, Solmitz, and Tripp have observed the reaction $\Sigma^- + p \Rightarrow \Sigma^0 + n$ and report a Σ^- - Σ^0 mass difference of 4.45 ± 0.4 Mev (private communication). We have not folded in older results with a much larger uncertainty, namely, $M(\Sigma^0) = 1192.6 \pm 3.5$, by Eisler et al., Nevis-60 Report R-198 (1957); $M(\Sigma^0) = 1191.6 \pm 3.3$, by M. Lynn Stevenson, P. R. 111, 1707 (1958).

(q) Alvarez, Eberhard, Good, Graziano, Ticho, and Wojcicki, P. R. L. 2, 215 (1959).

(r) Astbury, Hattersley, Hussain, Kemp, and Muirhead, 1960 Roch. Conf., Fisher, Leontic, Lundby, Meunier, and Stroot, P. R. L. 3, 349, (1959). Reiter, Ramanowski, Sutton, and Chidley, P. R. L. 5, 22 (1960); V. Telegdi, 1960 Roch. Conf.

(s) Alvarez, Bradner, Falk-Vairant, Gow, Rosenfeld, Solmitz, and Tripp, K^- Interactions in Hydrogen, UCRL-3775, May 1957.

(t) Bondelid, Butler, Achilles del Callar, and Kennedy, P. R. L. 5, 182 (1960).

(u) $\tau(\Lambda)$ has not changed from the value given by L. W. Alvarez at the 1959 Kiev Conf. It is a weighted average using some of the data given in Table I of the Proceedings of the 1958 CERN Conf., and some newer ones. In units of 10^{-10} sec they are:

2.95 ± 0.4	Berkeley K^- capture (CERN, 1958).
2.29 ± 0.14	Columbia, Pisa, Bologna (CERN, 1958).
2.75 ± 0.41	Columbia (CERN, 1958).
3.04 ± 0.64	Jungfrau (CERN, 1958).
2.08 ± 0.38	Michigan (CERN, 1958).
2.63 ± 0.21	E. Boldt, D. O. Caldwell, Y. Pal, Phys. Rev. Letters 1, 148 (1958).
2.72 ± 0.16	Crawford, Cresti, Good, Kalbfleisch, Stevenson, and Ticho (private communication).

(v) New data by Mason, Barkas, Dyer, Heckman, Nickols, and Smith in B. A. P. S. 5, 224 (1960) and also C. J. Mason, UCRL-9297, have been combined with that of Bogdanowicz et al. (N. C. 11, 727 (1959)), and with that of A. Pevsner et al. (private communication). All these emulsion data in turn have been combined with the cloud chamber data of D'Andlau et al., N. C. 6, 1135 (1957).

(w) Ashkin, Fazzini, Fidecaro, Goldschmidt-Clermont, Lipman, Merrison, and Paul, N. C. 16, 490 (1960); also, Anderson, Fujii, Miller, and Tau, P. R. L. 5, 86 (1960); and Reference (a).

(x) Barkas, Birnbaum, and Smith, P. R. 101, 778 (1960).

(y) Sosnovskij, Spivak, Prokofiev, Kutikov, and Dobrynin, reported by M. Goldhaber at the 1958 CERN Conf.

(z) Boldt, Caldwell, and Pal, P. R. L. 1, 150 (1958). Muller, Birge, Fowler, Good, Hirsch, Matsen, Oswald, Powell, and White; with Piccioni, P. R. L. 4, 418 (1960). Birge, Ely, Powell, White, Fry, Huzita, Camerini, and Natale (unpublished). Also see U. Camerini, 1960 Roch. Conf.

(*) Calculated using the mass differences given in the next column.

Table II. Atomic and Nuclear Properties of Materials

Atomic and nuclear properties of materials often used as particle absorbers and detectors have been collected for ready reference. The densities given are subject to variations depending on the form in which the material has been prepared. This is an especially important variable for graphite.

The radiation length, as is well known, depends on the approximations made in its calculation. In Table II, for definiteness and consistency, we have preferred simply to take the values quoted by Bethe and Ashkin.[5] These have not been corrected for the failure of the Born approximation, and Wheeler's and Lamb's[6] calculation of the ζ was used (ζ is the efficiency for bremsstrahlung of electrons relative to nuclei in a screened field). Wheeler and Lamb calculated ζ on the basis of a Thomas-Fermi model of the atom and neglected electron exchange. The failure of the Born approximation is known to cause the tabulated radiation length to be about 10% too low for lead,[7] and the error varies approximately with the square of the atomic number, so that the effect in emulsion, for example, is about 3%. The effects of the other approximations are not well known. The calculated radiation length is particularly uncertain in liquid hydrogen. A rough formula useful when the atomic number, Z, exceeds 5 is

$$L_{rad} \approx 166\ Z^{-0.76}\ g/cm^2 .$$

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[5] H. Bethe and J Ashkin, Passage of Radiations through Matter, in Experimental Nuclear Physics, Vol. 1, E. Segrè, Ed. (Wiley, New York, 1953), pp. 166-357.

[6] J. A. Wheeler and W. E. Lamb, Phys. Rev. 55, 858 (1939).

[7] H. Davies, H. A. Bethe, and L. C. Maximon, Phys. Rev. 93, 788 (1954).

Table II. Atomic and nuclear properties (dE/dx, collision mean free path, radiation length, etc.) of materials used as absorbers and detectors

Material	Z	A	Cross section σ [a] (barns)	$-\frac{dE}{dx}$ min [b] $\frac{\text{Mev}}{\text{g/cm}^2}$	Collision[a] length g/cm^2	L_{coll} cm	Radiation[c] length g/cm^2	L_{rad} cm	Density ρ (g/cm^3)
H_2	1	1.01	0.063	4.14	26.5	374	58	819.0	0.0708 } boiling at 1 atmos
Li	3	6.94	0.23	1.72	50.4	94.3	77.5	145	0.534 } boiling at 1 atmos
Be	4	9.01	0.28	1.71	55.0	29.9	62.2	33.8	1.84
C	6	12.00	0.33	1.86	60.4	39.0	42.5	27.4	1.55 (variable)
Al	13	26.97	0.57	1.66	79.2	29.3	23.9	8.86	2.70
Cu	29	63.57	1.00	1.45	105.4	11.8	12.8	1.44	8.9
Sn	50	118.70	1.55	1.27	129.7	17.8	8.54	1.17	7.30
Pb	82	207.21	2.20	1.12	156.2	13.8	5.8	0.51	11.34
U	92	238.07	2.42	1.095	163.6	8.75	5.5	0.29	18.7
Hydrogen (bubble chamber, -27.6°K)				0.243 Mev/cm	26.5	452	58	990	0.0586
Propane (C_3H_8, bubble chamber)				0.935 Mev/cm	48.9	119.3	44.7	109.0	0.41
Freon CF_3Br				2.3	87.1	58.0	17.25	11.5	1.5
Polystyrene (CH scintillator)				2.14 Mev/cm	54.9	52.3	43.4	41.3	~ 1.05
Ilford emulsion				5.49 Mev/cm	103	27.0	11.2	2.91	3.815

Table III. Particle Scattering

An estimate of multiple Coulomb scattering is often made by assuming that the distribution is Gaussian, with a root-mean-square space angle

$$\theta_{rms} \approx (21.2/Pv) \sqrt{L/L_{rad}}, \qquad (1a)$$

where L is the thickness traversed in the scatterer, and L_{rad} is the radiation length of the scatterer.[8] The equivalent formula for the more useful projected rms angle is

$$\theta_{rms-p} \approx (15.0/Pv) \sqrt{L/L_{rad}}. \qquad (1b)$$

Although the formula above is convenient, it has the weakness that the true angular distribution is not strictly Gaussian but has an appreciable "tail" out in the region where a Gaussian distribution has fallen to a few percent of its maximum value.[9] This tail (due to single and plural scattering) causes Eq. (1) to be in error by ~ 20% for thicknesses ~ 1% of a radiation length (it was derived to give correct results for large thicknesses). This error is given in Table III and is discussed below.

Molière has calculated a distribution that fits the experimental facts.[10] Because of the large "tail" the root-mean-square angles θ_{rms} and θ_{rms-p} for the Molière distribution are not meaningful unless an arbitrary cutoff angle is introduced. The theory, however, does define a mean (absolute) projected angle of scattering θ_{mp}.

We have chosen the following way to display the results of Molière's theory. First we have rewritten the familiar Eq. (1) to give the mean projected scattering angle. This was still done on the assumption that the distribution is Gaussian, so that the mean deviation can be obtained from the standard deviation by using the relation $\pi(\theta_{rms-p})^2 = 2(\theta_{mp})^2$. Correcting the 15.0 in Eq. (1b) by $\sqrt{2/\pi}$, we then have

$$\theta_{mp} \approx (12/Pv) \sqrt{L/L_{rad}} \,. \qquad (2)$$

The Molière-theory results are then expressed as correction factors for the crude Eq. (2), i.e., we have expressed the Molière result in the form

$$\theta_{mp} = (12/Pv) \sqrt{L/L_{rad}} \, (1 + \epsilon) \,. \qquad (3)$$

[8] See, for example, Reference 5, Eq. (79b).

[9] See, for example, the experimental work of A. D. Hansen, L. H. Lanzl, E. M. Lyman, and M. B. Scott, Phys. Rev. 84, 634 (1951).

[10] G. Z. Molière, Naturforsch. 3 (a), 78 (1948).

The values of the correction ϵ are compiled in Table III. The root-mean-square formulas, Eq. (1) will also be improved by introducing the factor $(1 + \epsilon)$. The estimates of ϵ in Table III are to be employed with values of L_{rad} taken from Table II.

The screening effect in the Molière theory is derived from the Thomas-Fermi model of the atom. The error introduced in applying these formulas to the scattering by molecular hydrogen is not known (at least to us).

When the thickness of the scatterer becomes comparable to the nuclear interaction free path in that material, the scattering calculated from Molière's theory will be completely wrong, because specific nuclear scattering will by then have become dominant. Also, the high radiation probability makes the theory unusable for electrons except when the foil is thin. Only for muons, therefore, is the formula at all applicable when the absorber is thick.

Table III

Multiple scattering (Coulomb only) calculated from Molière theory.

θ_{mp} is the mean projected angle in radians between tangents to the particle trajectories:

$$|\theta|_{\text{average}} \equiv \theta_{mp} = z \frac{12\ (\text{Mev})}{pv\ (\text{Mev})} \sqrt{\frac{L}{L_{rad}}}\ (1+\epsilon)^{*}$$

L is the thickness, and L_{rad} the radiation length (from Table II) for the absorber (atomic number Z).

For particles of charge ze and velocity βc, the following table for ε applies:

Z \ L/L_{rad}	10^{-3}	10^{-2}	10^{-1}	1	10	
1	-0.20	-0.14	-0.08	-0.03	+0.02	
6	-0.14	-0.06	-0.00	+0.06	+0.12	β/z = 0.1
29	-0.18	-0.10	-0.01	+0.06	+0.13	(4.7-Mev proton)
82	-0.27	-0.16	-0.07	+0.02	+0.10	
1	-0.26	-0.20	-0.14	-0.08	-0.03	
6	-0.20	-0.12	-0.05	+0.01	+0.07	β/z = 0.3
29	-0.20	-0.11	-0.03	+0.05	+0.12	(45-Mev proton)
82	-0.28	-0.17	-0.07	+0.02	+0.09	
1	-0.31	-0.24	-0.18	-0.12	-0.06	
6	-0.26	-0.18	-0.10	-0.03	+0.03	β/z = 0.7
29	-0.25	-0.15	-0.06	+0.02	+0.09	(380-Mev proton)
82	-0.29	-0.17	-0.08	+0.01	+0.09	
1	-0.34	-0.26	-0.20	-0.14	-0.08	
6	-0.33	-0.26	-0.19	-0.14	-0.08	
29	-0.34	-0.23	-0.13	-0.05	+0.03	β/z = 1.0
82	-0.31	-0.19	-0.09	-0.00	+0.08	

*Note that in the Gaussian approximation the root-mean-square projected angle is obtained from the formula above by substituting 15 for the coefficient 12.

Table IIIa: Multiple Coulomb Scattering and Lorentz Transformation

Since Table III does not appear on the wallet card and Table IIIa does; the formula for multiple Coulomb scattering, discussed in connection with Table III, is repeated here.

Comments on Lorentz Transformations

The mnemonic of F. S. Crawford, Jr., appears in Am. Jour. Phys. 26, 376 (1958). Its application is stressed on the wallet card because it gives formulas that avoid the differences of large terms and are accordingly easily handled by slide rule. However, for algebraic manipulations or computer calculations of relativistic problems, it is more convenient to use the following expression for the total energy w (instead of t as given in Eq. 8):

$$w_1 = \frac{\mu^2 + m_1^2 - m_2^2}{2\mu} \; ; \qquad w_2 = \frac{\mu^2 + m_2^2 - m_1^2}{2\mu} \; . \tag{8a}$$

The c.m. momentum p is then given by $p = \sqrt{w^2 - m^2}$. It may also be calculated directly:

$$p = \frac{1}{2\mu} \sqrt{(\mu + m_1 + m_2)(\mu - m_1 - m_2)(\mu + m_1 - m_2)(\mu - m_1 + m_2)} \, . \tag{8b}$$

Another easily obtained relation is the following: in the extreme relativistic limit a particle going backward in the c.m. system approaches a constant momentum in the lab, namely,

$$P_{lab} \to (m_3^2 - m_2^2) / 2m_2 \, ,$$

where particle 2 is the target, particle 3 goes straight backwards in the c.m. (lab direction depends on $m_3 - m_2$); note that the equation is independent of both the beam mass and the number and mass of reaction products in addition to m_3.

The Usefulness of Eqs. (10) and (11) applied to δ rays

A particle of known momentum P_1 and unknown mass m_1 may collide with an electron $(0, m_e)$ and make a δ ray with energy

$$T_e < 2m_e \eta^2 \, . \tag{11a}$$

This sets a sensitive lower limit on η:

$$\eta^2 > \frac{T_e}{2m_e} = T_e \text{ Mev} \, .$$

Now, since $m_e << m_1$, we have

$$\eta \approx \frac{P_1}{m_1} \, , \tag{12}$$

Combining (11a) and (12) we have

$$m_1^2 < P_1^2 \frac{2m_e}{T_e} \quad . \tag{13}$$

Approximation (12) assumes

$\mu \approx m_1$, from (3) this means $m_e << m_1$

and

$T_1 << \frac{m_1^2}{2m_2}$, i.e., << 10 Bev for a μ, << 20 Bev for a π, etc.

"Dalitz Plots," Properties, and a Generalization

In order to display a three-body reaction in the center of mass, it is convenient to use a coordinate system in which the energy w_1 of one body is plotted along x, and w_2 along y (w_3 is then simply $\mu - w_1 - w_2$). This has the convenient property that unit area $dw_1 dw_2 = dt_1 dt_2$ is proportional to Lorentz-invariant phase space,[11] in the c.m.

A more general pair of variables are the squares μ_{ij}^2 of the effective masses of any two of the three possible diparticles. These have a general meaning, independent of the c.m. energy, but still have the property that unit area is proportional to Lorentz-invariant phase space, or:

$$\mu_{ij}^2 = (w_i + w_j)^2 - (p_i + p_j)^2 .$$

But conservation of energy and momentum gives

$$w_i + w_j = \mu - w_k; \quad \left| p_i + p_j \right| = \left| p_k \right| ,$$

so

$$\mu_{ij}^2 = (\mu - w_k)^2 - p_k^2$$

$$d\mu_{ij}^2 = -2 (\mu - w_k) \, dw_k - 2p_k dp_k$$

$$= -2 (\mu - w_k) \, dw_k - 2w_k dw_k = -2\mu dw_k$$

[11] Appendix C, of "Hyperons and Heavy Mesons", M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 407 (1957).

i. e. $d\mu_{ij}^2$ is linear in dw_k, so that unit area $d\mu_{ij}^2 \, d\mu_{jk}^2 \propto dw_k dw_i \propto$ L. I. phase space. Q. E. D.

Lorentz invarient phase space is appropriate for strong interactions. For weak interaction (e. g. , β-decay) the rate is proportional to the density of states in momentum space i. e. without the factor $(w_1 w_2 w_3)^{-1}$. Thus three-body β-decay with an "energy-independent matrix element" corresponds to a Dalitz plot population $\propto w_1 w_2 w_3$.

Table IIIa. Multiple Coulomb scattering and Lorentz transformation

The rms projected angle θ due to multiple Coulomb scattering (only) of a particle of charge z, momentum P, velocity V is

$$\theta_{proj} = z \frac{15(\mathrm{Mev})}{PV(\mathrm{Mev})} \sqrt{\frac{L}{L_{(rad)}}} (1+\epsilon) \text{ radians;}$$

L = Length in scatterer; L(radiation) from Table II. For $L \geqslant 1/10\ L(rad)$ ϵ is generally $< 1/10$. The distribution of θ is not truly Gaussian. The rms projected displacement is

$$y_{rms} = L\theta_{proj}/\sqrt{3}.$$

Lorentz transformations. Notation: Lower-case type for c.m. 4-momentum $(\vec{p}, w)$ and capitals for lab $(\vec{P}, W)$. ($c \equiv 1$.) To transform from c.m. to lab write

$$\begin{pmatrix} \gamma & 0 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \eta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} p\cos\theta \\ p\sin\theta \\ 0 \\ w \end{pmatrix} = \begin{pmatrix} \gamma p\cos\theta + \eta w \\ p\sin\theta \\ 0 \\ \eta p\cos\theta + \gamma w \end{pmatrix} = \begin{pmatrix} P\cos\Theta \\ P\sin\Theta \\ 0 \\ W \end{pmatrix}$$

If two particles (1 and 2) collide, the invariant "mass" μ of the system is given by

$$\mu^2 = (W_1 + W_2)^2 - (\vec{P}_1 + \vec{P}_2)^2, \tag{1}$$

$$\gamma = \frac{W_1 + W_2}{\mu}; \quad \eta = \left|\frac{\vec{P}_1 + \vec{P}_2}{\mu}\right| = \gamma\beta. \tag{2}$$

Write T for lab kinetic energy, t for c.m.; thus $\mu = m_1 + m_2 + t_1 + t_2 = m_1 + m_2 + Q$. If the target is at rest $(0, m_2)$ μ simplifies:

$$\mu^2 = (m_1 + m_2)^2 + 2T_1 m_2. \tag{3}$$

To get a threshold T_1, set μ = sum of masses of reaction products, then

$$[\Sigma m(\text{products})]^2 = (m_1 + m_2)^2 + 2T_1 m_2. \tag{4}$$

Other invariants are: $w_1 w_2 - p_1 p_2 \cos\theta_{12}$ (5)

and

$$\frac{1}{p}\frac{d^2\sigma}{d\omega dw}. \tag{6}$$

The max. lab angle that a particle of c.m. momentum p_i can have is given by

$$\sin\Theta_i = \frac{\eta_i}{\eta} \quad (\eta_i = \frac{p_i}{m_i} \text{ must be } < \eta); \tag{7}$$

If $\eta_i > \eta$, then of course Θ_i can be π.
Crawford's mnemonic for extending nonrelativistic formulas to relativistic case: "To the rest energy of each moving particle add Q/2" where Q = the total kinetic energy (c.m.) = $\mu - \Sigma m_i$. Thus in the rest frame of a two-body decay the kinetic energy Q is shared between the two particles according to

$$t_1 = Q\frac{m_2 + Q/2}{\mu}, \quad t_2 = Q\frac{m_1 + Q/2}{\mu}. \tag{8}$$

The above of course applies in the c.m. for the production of a two-body final state. To express t in terms of p, apply the mnemonic to a single particle (then $Q = t$). The non-rel. relation $p^2 = 2tm$ becomes

$$p^2 = 2t(m + t/2) = 2tm + t^2. \tag{9}$$

Energy Transfer in elastic collisions of beam (P_1, W_1) with resting target $(0, m_2)$, is

$$T_2 = 2m_2 \frac{P_1^2}{\mu^2} \sin^2(\theta_{c.m.}/2). \tag{10}$$

Note that for max T_2, $\theta_{c.m.} = \pi$, so

$$T_{2max.} = 2m_2 P_1^2/\mu^2 = 2m_2\eta^2. \tag{11}$$

Table IV. Atomic and Nuclear Constants

Atomic and nuclear constants in the directly applicable units of Mev, cm, and sec are tabulated. A few useful formulas and numerical constants are also included.

Table IV. Atomic and nuclear constants in units of Mev, cm, and sec[a]

GENERAL ATOMIC CONSTANTS

$N = 6.0249 \times 10^{23}$ molecules/gram-mole

$c = 2.99793 \times 10^{10}$ cm/sec

$e = 4.80286 \times 10^{-10}$ esu $= 1.6021 \times 10^{-19}$ coulomb.

$1\,\text{Mev} = 1.6021 \times 10^{-6}$ erg [1 ev = e(10⁸/c)]

$\hbar = 6.5817 \times 10^{-22}$ Mev sec $= 1.054 \times 10^{-27}$ erg sec.

$\hbar c = 1.9732 \times 10^{-11}$ Mev cm [$= ƛ$ for p = 1 Mev/c]

$k = 8.6167 \times 10^{-11}$ Mev/°C [Boltzmann constant]

$\alpha = \dfrac{e^2}{\hbar c} = 1/137.037$; $e^2 = 1.44 \times 10^{-13}$ Mev cm

QUANTITIES DERIVED FROM THE ELECTRON MASS, m

Mass and Energy

$m = 0.510976$ Mev $= 1/1836.12\ m_p = 1/273.26\ m_\pi$

Rydberg, $R_\infty = \dfrac{me^4}{2\hbar^2} = mc^2 \times \dfrac{\alpha^2}{2} = 13.605$ ev

Length (1 fermi = 10^{-13} cm; 1 A = 10^{-8} cm)

$r_e = e^2/mc^2 = 2.81785$ fermi

$ƛ_{Compton} = \dfrac{\hbar}{mc} = r_e\alpha^{-1} = 3.8612 \times 10^{-11}$ cm

a_∞ Bohr $= \dfrac{\hbar^2}{me^2} = r_e\alpha^{-2} = 0.52917$ A

Hydrogen-like atom (Non. Rel.; $\mu \equiv$ reduced mass).

$E_n = \dfrac{1}{2}\dfrac{\mu z^2 e^4}{(n\hbar)^2}$; $a_{n=1} = \dfrac{\hbar^2}{\mu z e^2}$; $\left.\dfrac{v}{c}\right)_{rms} = \dfrac{ze^2}{n\hbar c}$

Cross Section

$\sigma_{Thompson} = \dfrac{8}{3}\pi r_e^2 = 0.6652 \times 10^{-24}\,cm^2 = 0.6652$ barn

Magnetic Moment and Cyclotron Angular Frequency

$\mu_{Bohr} = \dfrac{e\hbar}{2mc} = 0.57883 \times 10^{-14}$ Mev/gauss

$\dfrac{1}{2}\omega_{cyclotron} = \dfrac{e}{2mc} = 8.7945 \times 10^6$ rad sec^{-1}/gauss

$g_{electron} = 2[1 + \dfrac{\alpha}{2\pi} - 0.328\,(\dfrac{\alpha}{\pi})^2] = 2[1.0011596]$[b]

$g_{muon} = 2[1 + \dfrac{\alpha}{2\pi} + 0.75\,(\dfrac{\alpha}{\pi})^2] = 2[1.001165]$[b]

QUANTITIES DERIVED FROM THE PROTON MASS, m_p

Rest mass $= 938.211\ \text{Mev}/c^2 = 1836.12\ m_e = 6.719\ m_\pi$

$= 1.007593\ m_1$

where $m_1 = 1$ amu $= \dfrac{1}{16}\ O^{16} = 931.141$ Mev.

Magnetic Moment and Cyclotron Angular Frequency

$\mu_p = \dfrac{e\hbar}{2m_p c} = 3.1524 \times 10^{-18}$ Mev/gauss

$\dfrac{1}{2}\omega_{cyclotron} = \dfrac{e}{2m_p c} = 4.7896 \times 10^3$ rad sec^{-1}/gauss

$\left.\dfrac{\mu}{\mu_p}\right)_{proton} = 2.79275$; $\left.\dfrac{\mu}{\mu_p}\right)_{neutron} = -1.9128$

Table IV (continued)

QUANTITIES DERIVED FROM THE MASS OF THE CHARGED PION, m_π

Rest mass = 139.63 Mev/c^2 = 273.26 m_e = 0.14882 m_p

Length

$\frac{\hbar}{m_\pi c}$ = 1.4132 fermi ($\sim \sqrt{2}$ fermi)

Natural (≈ "geometrical") Nucleon Cross Section

$\pi \left(\frac{\hbar}{m_\pi c}\right)^2 = 62.7344$ mb (1 mb = 10^{-27} cm^2)

(3/2, 3/2)πp Resonance of mass 1237 Mev (Q = 159 Mev).

Center-of-mass momentum: p_π = 230 Mev/c

Lab-system momentum: P_π = 303 Mev/c (T_π = 195 Mev)

RADIOACTIVITY

1 curie = 3.7×10^{10} disintegrations/sec

1 r = 87.8 ergs/g air = 5.49×10^7 Mev/g air

Fluxes (per cm^2) to liberate 1 r in carbon:

3×10^7 minimum ionizing singly charged particles
0.9×10^9 photons of 1 Mev energy.

(These fluxes are actually correct to within a factor of two for all materials.)

Natural background: 100 mr/year

"Tolerance". 100 millirem/week [Note, 1 r may produce up to 10 "rem" (r equivalent for man), depending on type of radiation.]

MISCELLANEOUS

Physical Constants

1 year = 3.1536×10^7 sec ($\approx \pi \times 10^7$ sec)

Density of air = 1.205 mg/cm^3 at 20°C

Acceleration by gravity = 980.67 cm/sec^2

1 calorie = 4.184 joules

1 atmosphere = 1033.2 g/cm^2

Numerical Constants

1 radian = 57.29578 deg; e = 2.71828
ln 2 = 0.69315; $\log_{10} e$ = 0.43429;
ln 10 = 2.30259; $\log_{10} 2$ = 0.30103.

Stirling's approximation

$$\sqrt{2\pi n}\,\left(\frac{n}{e}\right)^n < n\,! < \sqrt{2\pi n}\,\left(\frac{n}{e}\right)^n \left(1 + \frac{1}{12n-1}\right)$$

Gaussianlike Distributions

For n > -1 but not necessarily integral:

$$\int_0^\infty x^{2n+1} \exp\left[-\frac{x^2}{2\sigma^2}\right] dx = 2^n\, n!\, \sigma^{2n+2};\ \left(\tfrac{1}{2}\right)! = \sqrt{\pi/2}$$

Relation between standard deviation σ and mean deviation α:

$2\sigma^2 = \pi\alpha^2$; σ = 1.4826 probable error.

Odds against exceeding one standard deviation = 2.15:1; two, 21:1; three, 370:1; four, 16,000:1; five, 1,700,000:1

[a]Based mainly on Cohen, Crowe, and Dumond, The Fundamental Constants of Physics (Interscience, New York, 1957), not on the later corrections of Cohen and Dumond, Phys. Rev. Lett. 1, 291 (1958).

[b]C. Sommerfield, Phys. Rev. 107, 328 (1957) and A. Petermans, Helv. Phys. Acta. 30, 407 (1957).

Table Va, b. Particle Decay and Reaction Dynamics

Energy and momentum conservation have been applied to the possible decay reactions of the unstable particles listed in Table I, and center-of-mass quantities of interest derived from the mass values listed are given in Table Va. Reactions of negative particles with protons and deuterons have also been analyzed and the results are given in Table Vb.

Coulomb binding energies have been neglected.

Table Va

Dynamics of particle decays

For three-body decays (e.g. $\mu \rightarrow e + \nu + \bar{\nu}$) the quantities tabulated for each particle are the maximum values attainable. Deuteron mass, $(H^2)^+ = d = 1875.49$ Mev.[a]

Decaying particle	Decay	Q	Mass (Mev)		Momentum p (Mev/c)	$w=T+Mc^2$ (Mev)	$\eta=p/Mc$	$\gamma=w/Mc^2$	$\beta=pc/w$	Branching ratio
$\mu^+ \rightarrow e^+ + \nu$ (M_{μ^+} = 105.655 Mev)		105.144	0.511	e^+	52.826	52.829	103.3831	103.3879	1.0000	100%[c]
$\pi^+ \rightarrow$ (M_{π^+} = 139.59 Mev)	$\mu^+ + \nu$	33.935	105.655	μ^+	29.810	109.780	0.2821	1.0390	0.2715	~100%
	$e^+ + \nu$		0.511	e^+	69.794	69.796	136.5897	136.5934	1.000	<< 1%[c]
$K^+ \rightarrow$ (M_{K^+} = 493.9 Mev)	$\pi^+ + \pi^0$	219.310	139.59	π^+	205.258	248.226	1.4704	1.7783	0.8269	26%[b]
			135.0	π^0	205.258	245.674	1.5204	1.8198	0.8355	
	$\mu^+ + \nu$	388.245	105.655	μ^+	235.649	258.251	2.2304	2.4443	0.9125	58%[b]
	$\pi^+ + \pi^+ + \pi^-$	75.130	139.59	$\pi^\pm$	125.590	187.772	0.8997	1.3452	0.6688	6%[b]
	$\pi^+ + \pi^0 + \pi^0$	84.310	135.0	π^+	132.371	189.069	0.9805	1.4005	0.7001	2%[b]
			139.590	π^0	133.100	192.876	0.9535	1.3817	0.6901	
	$\pi^0 + \mu^+ + \nu$	253.245	135.0	π^0	215.271	254.099	1.5946	1.8822	0.8472	4%[b]
			105.655	μ^+	215.271	239.801	2.0375	2.2697	0.8977	
			0	ν	188.320	188.320	-0	-0	1.000	
	$\pi^0 + e^+ + \nu$	358.389	135.0	π^0	228.500	265.400	1.6926	1.9659	0.8610	5%[b]
			0.511	e^+	228.500	228.500	447.1826	447.1838	1.0000	
	$\pi^+ + \gamma$	354.310	139.59	π^+	227.224	266.676	1.6278	1.9104	0.8521	<< 1%[b]
			0	γ	227.224	227.224	0	0	1.000	
$K^0 \rightarrow$ (M_{K^0} = 497.8 Mev)	$\pi^0 + \pi^0$	227.800	135.0	π^0	209.108	248.900	1.5489	1.8437	0.8401	31%[c]
	$\pi^+ + \pi^-$	218.620	139.59	π^+	206.072	248.900	1.4763	1.7831	0.8279	69%[c]
	$\pi^0 + \pi^0 + \pi^0$	92.800	135.0	π^0	139.300	193.983	1.0319	1.4369	0.7181	<< 1%[c]
	$\pi^+ + \pi^- + \pi^0$	83.620	139.59	$\pi^\pm$	132.901	192.739	0.9521	1.3807	0.6895	<< 1%[c]
			135.0	π^0	132.158	188.920	0.9789	1.3994	0.6995	
	$\pi^+ + \mu^- + \bar{\nu}$	252.555	134.59	π^+	216.095	257.259	1.5481	1.8430	0.8400	<< 1%[c]
			105.655	μ^-	216.095	240.541	2.0453	2.2767	0.8984	
	$\pi^+ + e^- + \bar{\nu}$	357.699	139.59	π^+	229.328	268.471	1.6429	1.9233	0.8542	<< 1%[c]
			0.511	e^-	229.328	229.329	448.8043	448.8054	1.0000	
$\Lambda \rightarrow$ (M_Λ = 1115.36 Mev)	$p + \pi^-$	37.557	938.213	p	100.174	943.546	0.1068	1.0057	0.1062	64%[c]
			139.59	π^-	100.174	171.814	0.7176	1.2308	0.5830	
	$p + e^- + \bar{\nu}$	176.636	938.213	p	163.079	952.281	0.1738	1.0150	0.1713	< 1%[c]
			0.511	e^-	163.079	163.079	319.1512	319.1528	1.0000	
	$p + \mu^- + \bar{\nu}$	71.492	938.213	p	130.725	947.276	0.1393	1.0097	0.1380	<< 1%[c]
			105.655	μ^-	130.725	168.084	1.2373	1.5909	0.7777	
	$n + \pi^0$	40.853	939.507	n	103.583	945.200	0.1103	1.0061	0.1096	36%[c]
			135.0	π^0	103.583	170.160	0.7673	1.2604	0.6087	

Table Va (continued)

Decay	Mode	Q	Mass (Mev)	Momentum p (Mev/c)	$w=T+Mc^2$ (Mev)	$\eta=p/Mc$	$\gamma=w/Mc^2$	$\beta=pc/w$	Branching ratio
$\Sigma^+ \rightarrow$ (M_{Σ^+} = 1189.4 Mev)	$p + \pi^0$	116.187	p 938.213	189.076	957.075	0.2015	1.0201	0.1976	51%[c]
			π^0 135.0	189.076	232.325	1.4006	1.7209	0.8138	
	$n + \pi^+$	110.303	n 939.507	185.098	957.567	0.1970	1.0192	0.1933	49%[c]
			π^+ 139.59	185.098	231.833	1.3260	1.6608	0.7984	
	$n + \mu^+ + \bar{\nu}$	144.238	n 939.507	202.419	961.066	0.2155	1.0229	0.2106	<< 1%[c] (not observed)
			μ^+ 105.655	202.419	228.334	1.9159	2.1611	0.8865	
	$n + e^+ + \bar{\nu}$	249.382	n 939.507	223.641	965.758	0.2380	1.0279	0.2316	< 1%[c]
			e^+ 0.511	223.641	223.642	437.6747	437.6758	1.0000	
$\Sigma^0 \rightarrow \Lambda + \gamma$ (M_{Σ^0} = 1191.5 Mev)		76.140	Λ 1115.36	73.707	1117.793	0.0661	1.0022	0.0659	100%[c]
			γ 0	73.707	73.707	0	0	1.0000	
$\Sigma^- \rightarrow$ (M_{Σ^-} = 1195.96 Mev)	$n + \pi^-$	116.863	n 939.507	191.658	958.857	0.2040	1.0206	0.1999	≈100%[c]
			π^- 139.59	191.658	237.103	1.3730	1.6986	0.8083	
	$n + \mu^- + \bar{\nu}$	150.798	n 939.507	208.368	962.336	0.2218	1.0243	0.2165	<< 1%[c] (not observed)
			μ^- 105.655	208.368	233.624	1.9722	2.2122	0.8919	
	$n + e^- + \bar{\nu}$	255.942	n 939.507	228.957	967.003	0.2437	1.0293	0.2368	<< 1%[c]
			e^- 0.511	228.957	228.957	448.0769	448.0781	1.0000	
	$\Lambda + e^- + \bar{\nu}$	80.089	Λ 1115.36	77.882	1118.076	0.0698	1.0024	0.0697	≪ 1%[c]
			e^- 0.511	77.882	77.884	152.4190	152.4223	1.0000	
$\Xi^0 \rightarrow \Lambda + \pi^0$ (M_{Ξ^0} = 1311 Mev)		60.640	Λ 1115.36	130.830	1123.007	0.1173	1.0069	0.1165	~100%[c]
			π^0 135.0	130.830	187.993	0.9691	1.3925	0.6959	
$\Xi^- \rightarrow$ (M_{Ξ^-} = 1318.4 Mev)	$\Lambda + \pi^-$	63.450	Λ 1115.36	135.867	1123.605	0.1218	1.0074	0.1209	~ 100%[c]
			π^- 139.59	135.867	194.795	0.9733	1.3955	0.6975	
	$n + \pi^-$	239.303	n 939.507	301.050	986.562	0.3204	1.0501	0.3052	≪ 1%[c]
			π^- 139.59	301.050	331.838	2.1567	2.3772	0.9072	
	$n + \pi^0$	236.493	n 939.507	296.252	985.190	0.3156	1.0486	0.3010	≪ 1%[c]
			π^0 135.0	296.252	325.810	2.1965	2.4134	0.9101	

[a] American Institute of Physics, Handbook (McGraw-Hill, New York, 1957).

[b] R. Ross and W. Humphrey, Lawrence Radiation Laboratory, Berkeley (private communication, July 1961).

[c] G. A. Snow and M. M. Shapiro, Phys. Rev.

Table Vb

Dynamics of particle absorption by H and D

The hyperfragments (Λn), ($\Sigma^- n$), etc., are assumed to have zero binding energy.

Note that the Σ^0 mass was assumed to be 1190.0 Mev for this table. See Ref. (p) of Table I.

Reaction	Products	Q	Mass (Mev)	Momentum p (Mev/c)	$w=T+Mc^2$ (Mev)	$\eta=p/Mc$	$\gamma=w/Mc^2$	$\beta=pc/w$	Branching fraction
$\pi^- + p \rightarrow$ ($M_{\pi^- + p}$ = 1077.803 (Mev))	$n + \pi^0$	3.296	n 939.507	28.025	939.925	0.0298	1.0004	0.0298	61%[c]
			π^0 135.0	28.025	137.878	0.2076	1.0213	0.2033	
	$n + \gamma$	138.296	n 939.507	129.423	948.380	0.1378	1.0094	0.1365	39%[c]
			γ 0	129.423	129.423	0	0	1.0000	
$K^- + p \rightarrow$ (M_{K^-+p} = 1432.113 (Mev))	$\Lambda + \pi^0$	181.753	Λ 1115.36	254.497	1144.027	0.2282	1.0257	0.2225	6%[b]
			π^0 135.0	254.497	288.086	1.8852	2.1340	0.8834	
	$\Sigma^+ + \pi^-$	103.123	Σ^+ 1189.4	181.472	1203.164	0.1526	1.0116	0.1508	21%[b]
			π^- 139.59	181.472	228.949	1.3000	1.6402	0.7926	
	$\Sigma^0 + \pi^0$	105.613	Σ^0 1191.5	182.199	1205.350	0.1529	1.0116	0.1512	28%[b]
			π^0 135.0	182.199	226.763	1.3496	1.6797	0.8035	
	$\Sigma^- + \pi^+$	96.563	Σ^- 1195.96	174.529	1208.628	0.1459	1.0106	0.1444	45%[b]
			π^+ 139.590	174.529	223.485	1.2503	1.6010	0.7809	
	$\Lambda + \pi^0 + \pi^0$	46.753	Λ 1115.36	146.481	1124.938	0.1313	1.0086	0.1302	≪ 1%[b]
			π^0 135.0	113.826	176.583	0.8432	1.3080	0.6446	
	$\Lambda + \pi^+ + \pi^-$	37.573	Λ 1115.36	132.286	1123.177	0.1186	1.0070	0.1178	≪1%[b]
			$\pi^\pm$ 139.59	102.207	173.008	0.7322	1.2394	0.5908	
$\Sigma^- + p \rightarrow$ (M_{Σ^-+p} = 2134.173 (Mev))	$\Lambda + n$	79.306	Λ 1115.36	287.211	1151.746	0.2575	1.0326	0.2494	
			n 939 507	287.211	982.427	0.3057	1.0457	0.2923	
	$\Sigma^0 + n$	3.166	Σ^0 1191.5	57.696	1192.896	0.0484	1.0012	0.0484	
			n 939.507	57.696	941.277	0.0614	1.0019	0.0613	
$\pi^- + d \rightarrow$ ($M_{\pi+d}$ = 2015.080 (Mev))	$n + n$	136.066	n 939.507	363.955	1007.540	0.3874	1.0724	0.3612	100%
$K^- + d \rightarrow$ (M_{K^-+d} = 2369.390 (Mev)) (continued)	$\Lambda + n$	314.523	Λ 1115.36	588.189	1260.950	0.5274	1.1305	0.4665	< 0.3[a]
			n 939.507	588.189	1108.440	0.6261	1.1798	0.5306	
	$\Sigma^0 + n$	238.383	Σ^0 1191.5	514.947	1298.015	0.4322	1.0894	0.3967	0.3[a]
			n 939.507	514.947	1071.375	0.5481	1.1404	0.4806	
	$\Sigma^- + p$	235.217	Σ^- 1195.96	511.561	1300.775	0.4277	1.0876	0.3933	0.6[a]
			p 938.213	511.561	1068.615	0.5453	1.1390	0.4787	

Table Vb (continued)

		Q	Mass (Mev)	Momentum p (Mev/c)	$w=T+Mc^2$ (Mev)	$\eta=p/Mc$	$\gamma=w/Mc^2$	$\beta=pc/w$	Branching fraction
$K^- + d \to$ ($M_{K^-+d} = 2369.390$ Mev)	$\Lambda + p + \pi^-$	176.227	Λ 1115.36	448.286	1202.077	0.4019	1.0777	0.3729	22[a]
			p 938.213	444.319	1038.105	0.4736	1.1065	0.4280	
			π^- 139.59	264.281	298.881	1.8933	2.1411	0.8842	
	$\Lambda + n + \pi^0$	179.523	Λ 1115.36	452.285	1203.574	0.4055	1.0791	0.3758	11[a]
			n 939.507	448.443	1041.045	0.4773	1.1081	0.4308	
			π^0 135.0	265.099	297.493	1.9637	2.2037	0.8911	
	$\Sigma^- + n + \pi^+$	94.333	Σ^- 1195.96	330.552	1240.800	0.2764	1.0375	0.2664	22[a]
			n 939.507	326.299	994.557	0.3473	1.0586	0.3281	
			π^+ 139.59	178.357	226.488	1.2777	1.6225	0.7875	
	$\Sigma^- + p + \pi^0$	100.217	Σ^- 1195.96	340.446	1243.473	0.2847	1.0397	0.2738	3[a]
			p 938.213	336.188	996.627	0.3583	1.0623	0.3373	
			π^0 135.0	182.976	227.388	1.3554	1.6844	0.8047	
	$\Sigma^0 + n + \pi^0$	103.383	Σ^0 1191.5	345.707	1240.639	0.2901	1.0412	0.2787	19[a]
			n 939.507	341.481	999.641	0.3635	1.0640	0.3416	
			π^0 135.0	186.505	230.237	1.3815	1.7055	0.8101	
	$\Sigma^0 + p + \pi^-$	100.087	Σ^0 1191.5	340.295	1239.142	0.2856	1.0400	0.2746	3[a]
			p 938.213	335.972	996.554	0.3581	1.0622	0.3371	
			π^- 139.59	184.889	231.667	1.3245	1.6596	0.7981	
	$\Sigma^+ + n + \pi^-$	100.893	Σ^+ 1189.4	341.656	1237.498	0.2873	1.0404	0.2761	19[a]
			n 939.507	337.375	998.246	0.3591	1.0625	0.3380	
			π^- 139.59	185.796	232.391	1.3310	1.6648	0.7995	
	$\Lambda+n+\pi^0+\pi^0$	44.523	Λ 1115.36	228.403	1138.506	0.2048	1.0208	0.2006	< 0.1%[a]
			n 939.507	224.500	965.957	0.2390	1.0282	0.2324	
			π^0 135.0	113.803	176.568	0.8430	1.3079	0.6445	
	$\Lambda+n+\pi^++\pi^-$	35.343	Λ 1115.360	203.665	1133.802	0.1826	1.0165	0.1796	< 0.1%[a]
			n 939.507	200.064	960.572	0.2129	1.0224	0.2083	
			$\pi^\pm$ 139.59	101.494	172.587	0.7271	1.2364	0.5881	
	$\Lambda+p+\pi^-+\pi^0$	41.227	Λ 11 15.36	219.851	1136.821	0.1971	1.0192	0.1934	< 0.1%[a]
			p 938.213	216.001	962.757	0.2302	1.0262	0.2244	
			π^- 139.59	110.495	178.029	0.7916	1.2754	0.6207	
			π^0 135.0	109.014	173.519	0.8075	1.2853	0.6283	
$\Sigma^- + d \to$ ($M_{\Sigma+d}$=3071.450 Mev)	$\Lambda + n + n$	77.076	Λ 1115.36	331.145	1163.480	0.2969	1.0431	0.2846	96[a]
			n 939.507	318.542	992.040	0.3391	1.0559	0.3211	
	Σ^0+n+n	0.936	Σ^+ 1191.5	36.949	1192.073	0.0310	1.0005	0.0310	4[a]
			n 939.507	34.941	940.157	0.0372	1.0007	0.0372	

[a] Date of O. Dahl, R. Levine, M. Horowitz, D. Miller, J. Murray, and J. Schwartz (1961, to be published).

[b] W. Humphrey and R. Ross, Lawrence Radiation Laboratory, Berkeley (private communication, July 1961).

[c] B. C. Maglić, Lawrence Radiation Laboratory, Berkeley (private communication, July 1961).

Table VI. Possible Resonances of Strongly Interacting Particles

Table VI lists possible resonances of strongly interacting particles, as of August 1961. Many of the data are very preliminary.

The cited references are made to recent papers, which in turn may contain earlier and sketchier references.

Table VI. Possible resonances of strongly interacting particles (as of August 1961)

	Mass (Mev)	Half-width Γ/2 (Mev)	Spin I	Spin and parity J	Decay properties: Orbital wave	Products	Branching fraction	Q^j (Mev)	k (Mev/c)	Ref.
ρ	750	±50	1	1-	p	π+π	100%	480	350	a
w	790	±<15	0	1-		3π	100%	510	——	b
K^*	885	± 8	1/2?	?	?	K+π	100%	252	282	c
N^*	1238	±45	3/2	3/2+	p	N+π	100%	163	234	d
	1510	±30	1/2	3/2-	d	N+π + others	?	435	449	d
	1680	±50	1/2	5/2+	f+?	N+π + others	?	605	567	d
	1900	±100	3/2	?	?	?	?	-		e
Y^*	1380	±25	1	?	?	Λ+π Σ^0+π	96% 4%	130 54	205 122	f
	1405	±10	0	?	?	$\Sigma^0+\pi^0$ Λ+2π	100%	79 20	153 ——	g
	1525	±20	0	≥ 3/2	?	Σ+π Λ+2π K+p	4 only 1 this ? ratio known	199 130 89	271 —— 246	h
	1815	±60	0	≥ 3/2	?	many	-	-		i

Table VI footnotes

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[a]J. A. Anderson, Vo. X. Bang, P. G. Burke, D. D. Carmony, and N. Schwartz, Phys. Rev. Letters 6, 365 (1961); D. Stonehill, C. Baltay, H. Courant, W. Fickinger, E. C. Fowler, H. Kraybill, J. Sandweiss, J. Sanford, and H. Taft, Phys. Rev. Letters 6, 624 (1961); A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters 6, 628 (1961).

[b](ω) B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters (to be published).

[c](K^*) M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters 6, 300 (1961).

[d](N^*) B. J. Moyer, Revs. Modern Phys. 33, 367 (1961). The entry at 1680 Mev is probably not a simple resonance.

[e](N^*, m = 1900) M. H. Alston and M. Ferro-Luzzi, in Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960).

[f](Y^*, m = 1380) M. H. Alston and M. Ferro-Luzzi, Revs. Modern Phys. 33, 416 (1961); and Pion-Hyperon Resonances, Lawrence Radiation Laboratory Report UCRL-9587, March 7, 1961.

[g](Y^*, m = 1405) M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters 6, 698 (1961); P. Bastien, M. Ferro-Luzzi, and A. H. Rosenfeld, Phys. Rev. Letters 6, 702 (1961).

[h](Y^*, m = 1525) M. Ferro-Luzzi, R. D. Tripp, and M. Watson, Phys. Rev. Letters (to be published).

[i](Y^*, m = 1815) O. Chamberlain, K. M. Crowe, D. Keefe, L. T. Kerth, A. Lemonick, T. M. Maung, and T. F. Zipf, Phys. Rev. (to be published); and L. T. Kerth, Revs. Modern Phys. 3, 389 (1961). This bump may turn out to be not a resonance.

[j]The Q values and momenta calculated are for neutral decays. The others vary slightly from this.

Figure 1. Range, energy-loss rate and momentum-loss rate. The curves are plotted from Aron's calculations for copper,[12] assuming a nominal mean excitation potential of 310 ev. Provided that thicknesses are measured in g/cm^2, the range curves also apply for all other materials (except H_2), with an error usually not exceeding 30%. Ranges are plotted up to 100 g/cm^2, which is about one nuclear mean free path.

More extensive data for specific materials and particles are found in the following:

(a) Ward Whaling, the Energy Loss of Charged Particles in Matter, in Handbuch der Physik, Vol. 34 (Springer-Verlag, Berlin, 1958), pp. 193-217.

(b) R. M. Sternheimer, Phys. Rev. 117, 485 (1960).

(c) Hans Bichsel, Linear Accelerator Group, University of Southern California, Technical Report No. 2 (1961).

(d) For emulsion, reference can be made to the tables of Walter H. Barkas, Nuovo cimento 8, 201 (1958), and H. H. Heckman et al., Phys. Rev. 117, 544 (1960).

A simple analytical expression for the range in g/cm^2 for a particle of charge z e, mass number A, and kinetic energy T in a stopping material of atomic number Z (excluding hydrogen) is

$$R = \frac{Z^{0.26} T^{1.7}}{500\, z^2 A^{0.7}} \ g/cm^2 \, ;$$

this is correct to within about 10% for T/A from 1 Mev to 400 Mev. For protons it is simply

$$R = \frac{Z^{0.26} T^{1.7}}{500} \ g/cm^2 \, .$$

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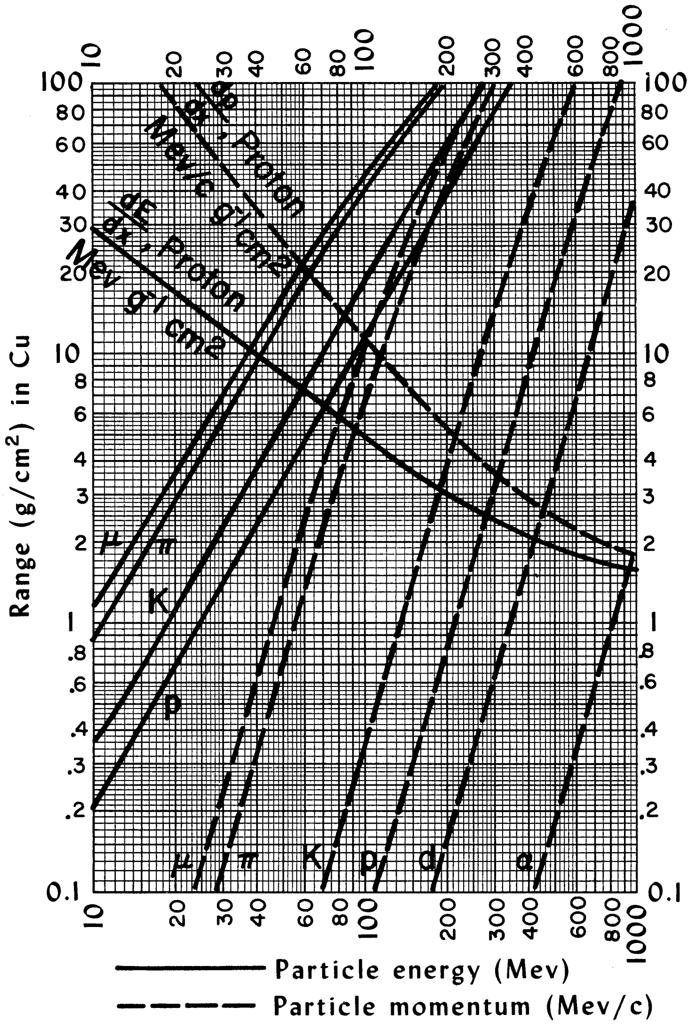
[12] W. A. Aron, The Passage of Charged Particles through Matter (Ph. D. Thesis), University of California Radiation Laboratory Report UCRL-1325, May 1951 (unpublished).


Range (g/cm²) in Cu
dE/dx, Proton Mev g⁻¹ cm2
dp/dx, Proton Mev/c g⁻¹ cm2
μ
π
K
p
d
α
Particle energy (Mev)
Particle momentum (Mev/c)

MU-22623

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There may remain errors and oversights in the tables or text. We should be most grateful to have such faults called to our attention, and to receive suggestions for improving the usefulness of the tables.

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Information Division
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TABLES FROM UCRL-8030(rev.). Table I. Masses and mean lives of elementary particles

(The antiparticles are assumed to have the same spins, masses, and mean lives as the particles listed)

	Particle	Spin	Mass (Errors represent standard deviation) (Mev)		Mass difference (Mev)		Mean life (sec)
Photon	γ	1	0	γ	... ...	γ	Stable
Leptons	ν	1/2	0	ν	... ...	ν	Stable
	$e^{\mp}$	1/2	0.510976 ± 0.000007	$e^{\mp}$	... ...	$e^{\mp}$	Stable
	$\mu^{\mp}$	1/2	105.655 ± 0.010	$\mu^{\mp}$ }	33.93 ± 0.05	$\mu^{\mp}$	$(2.212 \pm 0.001) \times 10^{-6}$
Mesons	π^{+}	0	139.59 ± 0.05	$\pi^{\pm}$ }	4.59 ± 0.01	$\pi^{\pm}$	$(2.55 \pm 0.03) \times 10^{-8}$
	π^{0}	0	135.00 ± 0.05	π^{0} }		π^{0}	$(2.2 \pm 0.8) \times 10^{-16}$
	$K^{\pm}$	0	493.9 ± 0.2	K^{+} }	3.9 ± 0.6	K^{+}	$(1.224 \pm 0.013) \times 10^{-8}$
	K^{0} }			K^{0} }		K^{0}	50% K_1, 50% K_2
	K_1 }	0	497.8 ± 0.6	K_1 }	$(1.5 \pm 0.5)\hbar/\tau(K_1)$	K_1	$(1.00 \pm 0.038) \times 10^{-10}$
	K_2 }			K_2 }		K_2	$6.1(+1.6/-1.1) \times 10^{-8}$
Baryons	p	1/2	938.213 ± 0.01	p }	1.2939 ± 0.0004	p	Stable
	n	1/2	939.507 ± 0.01	n }		n	$(1.013 \pm 0.029) \times 10^{3}$
	Λ	1/2	1115.36 ± 0.14	Λ	... ...	Λ	$(2.51 \pm 0.09) \times 10^{-10}$
	Σ^{+}	1/2	1189.40 ± 0.20	Σ^{+} }	6.56 ± 0.22	Σ^{+}	$0.81(+0.06/-0.05) \times 10^{-10}$
	Σ^{-}	1/2	1195.96 ± 0.30	Σ^{-} }	4.45 ± 0.4	Σ^{-}	$1.61(+0.1/-0.09) \times 10^{-10}$
	Σ^{0}	1/2	1191.5 ± 0.5	Σ^{0} }		Σ^{0}	$< 0.1 \times 10^{-10}$
	Ξ^{-}	?	1318.4 ± 1.2	Ξ^{-}	... ...	Ξ^{-}	$1.28(+0.38/-0.30) \times 10^{-10}$
	Ξ^{0}	?	1311 ± 8	Ξ^{0}	... ...	Ξ^{0}	1.5×10^{-10} (1 event)

Walter H. Barkas, Arthur H. Rosenfeld, University of California, Berkeley, Sept. 1960.

Table II. Atomic and nuclear properties (dE/dx, collision mean free path, radiation length, etc.) of materials used as absorbers and detectors

Material	Z	A	Cross section σ [a] (barns)	$-\frac{dE}{dx}$ min [b] Mev/g/cm²	Collision[a] length g/cm²	L_{coll} cm	Radiation[c] length g/cm²	L_{rad} cm	Density ρ (g/cm³)
H_2	1	1.01	0.063	4.14	26.5	374	58	819.0	0.0708 {boiling at 1 atmos
Li	3	6.94	0.23	1.72	50.4	94.3	77.5	145	0.534
Be	4	9.01	0.28	1.71	55.0	29.9	62.2	33.8	1.84
C	6	12.00	0.33	1.86	60.4	39.0	42.5	27.4	1.55 (variable)
Al	13	26.97	0.57	1.66	79.2	29.3	23.9	8.86	2.70
Cu	29	63.57	1.00	1.45	105.4	11.8	12.8	1.44	8.9
Sn	50	118.70	1.55	1.27	129.7	17.8	8.54	1.17	7.30
Pb	82	207.21	2.20	1.12	156.2	13.8	5.8	0.51	11.34
U	92	238.07	2.42	1.095	163.6	8.75	5.5	0.29	18.7
Hydrogen (bubble chamber, -27.6°K)				0.243 Mev/cm	26.5	452	58	990	0.0586
Propane (C_3H_8, bubble chamber)				0.935 Mev/cm	48.9	119.3	44.7	109.0	0.41
Freon CF_3Br				2.3	87.1	58.0	17.25	11.5	1.5
Polystyrene (CH scintillator)				2.14 Mev/cm	54.9	52.3	43.4	41.3	~ 1.05
Ilford emulsion				5.49 Mev/cm	103	27.0	11.2	2.91	3.815

INCHES 1 2 3

Table IV. Atomic and nuclear constants in units of Mev, cm, and sec[a]

GENERAL ATOMIC CONSTANTS

$N = 6.0249 \times 10^{23}$ molecules/gram-mole

$c = 2.99793 \times 10^{10}$ cm/sec

$e = 4.80286 \times 10^{-10}$ esu $= 1.6021 \times 10^{-19}$ coulomb.

$1\ \text{Mev} = 1.6021 \times 10^{-6}$ erg [1 ev = e(10⁸/c)]

$\hbar = 6.5817 \times 10^{-22}$ Mev sec $= 1.054 \times 10^{-27}$ erg sec.

$\hbar c = 1.9732 \times 10^{-11}$ Mev cm [= ƛ for p = 1 Mev/c]

$k = 8.6167 \times 10^{-11}$ Mev/°C [Boltzmann constant]

$\alpha = \frac{e^2}{\hbar c} = 1/137.037$; $e^2 = 1.44 \times 10^{-13}$ Mev cm

Cross Section

$\sigma_{Thompson} = \frac{8}{3}\pi r_e^2 = 0.6652 \times 10^{-24}\,cm^2 = 0.6652$ barn

Magnetic Moment and Cyclotron Angular Frequency

$\mu_{Bohr} = \frac{e\hbar}{2mc} = 0.57883 \times 10^{-14}$ Mev/gauss

$\frac{1}{2}\omega_{cyclotron} = \frac{e}{2mc} = 8.7945 \times 10^{6}$ rad sec⁻¹/gauss

$g_{electron} = 2[1 + \frac{\alpha}{2\pi} - 0.328(\frac{\alpha}{\pi})^2] = 2[1.0011596]$[b]

$g_{muon} = 2[1 + \frac{\alpha}{2\pi} + 0.75(\frac{\alpha}{\pi})^2] = 2[1.001165]$[b]

QUANTITIES DERIVED FROM THE ELECTRON MASS, m

Mass and Energy

$m = 0.510976$ Mev $= 1/1836.12\, m_p = 1/273.26\, m_\pi$

Rydberg, $R_\infty = \frac{me^4}{2\hbar^2} = mc^2 \times \frac{\alpha^2}{2} = 13.605$ ev

Length (1 fermi = 10^{-13} cm; 1 A = 10^{-8} cm)

$r_e = e^2/mc^2 = 2.81785$ fermi

$ƛ_{Compton} = \frac{\hbar}{mc} = r_e\alpha^{-1} = 3.8612 \times 10^{-11}$ cm

$a_\infty\ \text{Bohr} = \frac{\hbar^2}{me^2} = r_e\alpha^{-2} = 0.52917$ A

Hydrogen-like atom (Non. Rel.; μ ≡ reduced mass).

$E_n = \frac{1}{2}\frac{\mu z e^4}{(n\hbar)^2}$; $a_{n=1} = \frac{\hbar^2}{\mu z e^2}$; $\left.\frac{v}{c}\right)_{rms} = \frac{ze^2}{n\hbar c}$

QUANTITIES DERIVED FROM THE PROTON MASS, m_p

Rest mass $= 938.211$ Mev/c² $= 1836.12\, m_e = 6.719\, m_\pi$

$= 1.007593\, m_1$

where $m_1 = 1$ amu $= \frac{1}{16}O^{16} = 931.141$ Mev.

Magnetic Moment and Cyclotron Angular Frequency

$\mu_p = \frac{e\hbar}{2m_pc} = 3.1524 \times 10^{-18}$ Mev/gauss

$\frac{1}{2}\omega_{cyclotron} = \frac{e}{2m_pc} = 4.7896 \times 10^{3}$ rad sec⁻¹/gauss

$\left.\frac{\mu}{\mu_p}\right)_{proton} = 2.79275$; $\left.\frac{\mu}{\mu_p}\right)_{neutron} = -1.9128$

Table IIIa. Multiple Coulomb scattering and Lorentz transformation

The rms projected angle θ due to multiple Coulomb scattering (only) of a particle of charge z, momentum P, velocity V is

$$\theta_{proj} = z\,\frac{15(\text{Mev})}{PV(\text{Mev})}\sqrt{\frac{L}{L_{(rad)}}}\,(1+\epsilon)\ \text{radians};$$

L = Length in scatterer; L(radiation) from Table II. For L ≥ 1/10 L(rad) ε is generally < 1/10. The distribution of θ is not truly Gaussian. The rms projected displacement is

$$y_{rms} = L\theta_{proj}/\sqrt{3}.$$

Lorentz transformations. Notation: Lower-case type for c.m. 4-momentum ($\vec{p}$, w) and capitals for lab ($\vec{P}$, W). (c≡1.) To transform from c.m. to lab write

$$\begin{pmatrix} \gamma & 0 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \eta & 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} p\cos\theta \\ p\sin\theta \\ 0 \\ w \end{pmatrix} = \begin{pmatrix} \gamma p\cos\theta + \eta w \\ p\sin\theta \\ 0 \\ \eta p\cos\theta + \gamma w \end{pmatrix} = \begin{pmatrix} P\cos\Theta \\ P\sin\Theta \\ 0 \\ W \end{pmatrix}$$

If two particles (1 and 2) collide, the invariant "mass" μ of the system is given by

$$\mu^2 = (W_1 + W_2)^2 - (\vec{P}_1 + \vec{P}_2)^2, \qquad (1)$$

$$\gamma = \frac{W_1 + W_2}{\mu};\quad \eta = \left|\frac{\vec{P}_1 + \vec{P}_2}{\mu}\right| = \gamma\beta. \qquad (2)$$

Write T for lab kinetic energy, t for c.m.; thus $\mu = m_1 + m_2 + t_1 + t_2 = m_1 + m_2 + Q$. If the target is at rest $(0, m_2)$ μ simplifies:

$$\mu^2 = (m_1 + m_2)^2 + 2T_1m_2. \qquad (3)$$

To get a threshold T_1, set μ = sum of masses of reaction products, then

$$[\Sigma m(\text{products})]^2 = (m_1 + m_2)^2 + 2T_1m_2. \qquad (4)$$

Other invariants are: $w_1w_2 - p_1p_2\cos\theta_{12}$ (5)

and

$$\frac{1}{p}\,\frac{d^2\sigma}{d\omega dw}. \qquad (6)$$

The max. lab angle that a particle of c.m. momentum p_i can have is given by

$$\sin\Theta_i = \frac{\eta_i}{\eta}\ \left(\eta_i = \frac{p_i}{m_i}\ \text{must be} < \eta\right); \qquad (7)$$

If $\eta_i > \eta$, then of course Θ_i can be π. Crawford's mnemonic for extending nonrelativistic formulas to relativistic case: "To the rest energy of each moving particle add Q/2" where Q = the total kinetic energy (c.m.) = $\mu - \Sigma m_i$. Thus in the rest frame of a two-body decay the kinetic energy Q is shared between the two particles according to

$$t_1 = Q\,\frac{m_2 + Q/2}{\mu},\quad t_2 = Q\,\frac{m_1 + Q/2}{\mu}. \qquad (8)$$

The above of course applies in the c.m. for the production of a two-body final state. To express t in terms of p, apply the mnemonic to a single particle (then Q = t). The non-rel. relation $p^2 = 2tm$ becomes

$$p^2 = 2t(m + t/2) = 2tm + t^2. \qquad (9)$$

Energy Transfer in elastic collisions of beam (P_1, W_1) with resting target $(0, m_2)$, is

$$T_2 = 2m_2\,\frac{P_1^2}{\mu^2}\,\sin^2(\theta_{c.m.}/2). \qquad (10)$$

Note that for max T_2, $\theta_{c.m.} = \pi$, so

$$T_{2max.} = 2m_2\,P_1^2/\mu^2 = 2\,m_2\eta^2. \qquad (11)$$

Table IV (continued)

QUANTITIES DERIVED FROM THE MASS OF THE CHARGED PION, m_π

Rest mass $= 139.63$ Mev/c² $= 273.26\, m_e = 0.14882\, m_p$

Length

$\frac{\hbar}{m_\pi c} = 1.4132$ fermi (~ $\sqrt{2}$ fermi)

Natural (≈ "geometrical") Nucleon Cross Section

$\pi\left(\frac{\hbar}{m_\pi c}\right)^2 = 62.7344$ mb (1 mb = 10^{-27} cm²)

(3/2, 3/2)πp Resonance of mass 1237 Mev (Q = 159 Mev).

Center-of-mass momentum: $p_\pi = 230$ Mev/c

Lab-system momentum: $P_\pi = 303$ Mev/c ($T_\pi = 195$ Mev)

RADIOACTIVITY

1 curie $= 3.7 \times 10^{10}$ disintegrations/sec

1 r $= 87.8$ ergs/g air $= 5.49 \times 10^{7}$ Mev/g air

Fluxes (per cm²) to liberate 1 r in carbon:

3×10^{7} minimum ionizing singly charged particles
0.9×10^{9} photons of 1 Mev energy.

(These fluxes are actually correct to within a factor of two for all materials.)

Natural background: 100 mr/year

"Tolerance" 100 millirem/week [Note, 1 r may produce up to 10 "rem" (r equivalent for man), depending on type of radiation.]

MISCELLANEOUS

Physical Constants

1 year $= 3.1536 \times 10^{7}$ sec ($\approx \pi \times 10^{7}$ sec)

Density of air = 1.205 mg/cm³ at 20°C

Acceleration by gravity = 980.67 cm/sec²

1 calorie = 4.184 joules

1 atmosphere = 1033.2 g/cm²

Numerical Constants

1 radian = 57.29578 deg; e = 2.71828
ln 2 = 0.69315; $\log_{10} e = 0.43429$;
ln 10 = 2.30259; $\log_{10} 2 = 0.30103$.

Stirling's approximation

$$\sqrt{2\pi n}\,(\tfrac{n}{e})^n < n! < \sqrt{2\pi n}\,(\tfrac{n}{e})^n\,(1 + \tfrac{1}{12n-1})$$

Gaussianlike Distributions

For n > -1 but not necessarily integral:

$$\int_0^\infty x^{2n+1}\exp\left[-\frac{x^2}{2\sigma^2}\right]dx = 2^n\,n!\,\sigma^{2n+2};\ (\tfrac{1}{2})! = \sqrt{\pi}/2$$

Relation between standard deviation σ and mean deviation α:

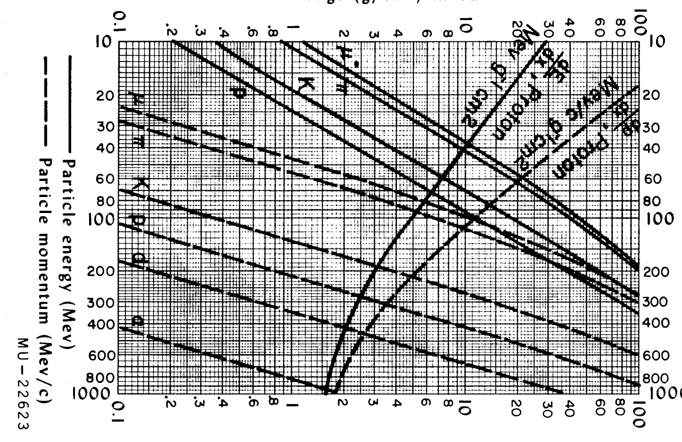
$2\sigma^2 = \pi\alpha^2$; σ = 1.4826 probable error.

Odds against exceeding one standard deviation = 2.15:1; two, 21:1; three, 370:1; four, 16,000:1; five, 1,700,000:1

[a] Based mainly on Cohen, Crowe, and Dumond, The Fundamental Constants of Physics (Interscience, New York, 1957), not on the later corrections of Cohen and Dumond, Phys. Rev. Lett. 1, 291 (1958).

[b] C. Sommerfield, Phys. Rev. 107, 328 (1957) and A. Petermans, Helv. Phys. Acta. 30, 407 (1957).

Range (g/cm²) in Cu



MU-22623

TABLES FROM UCRL-8030(rev.). Table I. Masses and mean lives of elementary particles

(The antiparticles are assumed to have the same spins, masses, and mean lives as the particles listed)

	Particle	Spin	Mass (Errors represent standard deviation) (Mev)		Mass difference (Mev)		Mean life (sec)
Photon	γ	1	0	γ	... ...	γ	Stable
Leptons	ν	1/2	0	ν	... ...	ν	Stable
	$e^{\mp}$	1/2	0.510976 ± 0.000007	$e^{\mp}$	... ...	$e^{\mp}$	Stable
	$\mu^{\mp}$	1/2	105.655 ± 0.010	$\mu^{\mp}$	33.93 ± 0.05 ($\mu^{\mp}$, $\pi^{\pm}$)	$\mu^{\mp}$	$(2.212 \pm 0.001) \times 10^{-6}$
Mesons	π^{+}	0	139.59 ± 0.05	$\pi^{\pm}$	4.59 ± 0.01 ($\pi^{\pm}$, π^0)	$\pi^{\pm}$	$(2.55 \pm 0.03) \times 10^{-8}$
	π^0	0	135.00 ± 0.05	π^0		π^0	$(2.2 \pm 0.8) \times 10^{-16}$
	$K^{\pm}$	0	493.9 ± 0.2	K^{+}	3.9 ± 0.6 (K^+, K^0)	K^{+}	$(1.224 \pm 0.013) \times 10^{-8}$
	K^0			K^0		K^0	50% K_1, 50% K_2
	K_1	0	497.8 ± 0.6 (K^0, K_1, K_2)	K_1	$(1.5 \pm 0.5)\hbar/\tau(K_1)$ (K_1, K_2)	K_1	$(1.00 \pm 0.038) \times 10^{-10}$
	K_2			K_2		K_2	$6.1(+1.6/-1.1) \times 10^{-8}$
Baryons	p	1/2	938.213 ± 0.01	p	1.2939 ± 0.0004 (p, n)	p	Stable
	n	1/2	939.507 ± 0.01	n		n	$(1.013 \pm 0.029) \times 10^{3}$
	Λ	1/2	1115.36 ± 0.14	Λ	... ...	Λ	$(2.51 \pm 0.09) \times 10^{-10}$
	Σ^{+}	1/2	1189.40 ± 0.20	Σ^{+}	6.56 ± 0.22 (Σ^+, Σ^-)	Σ^{+}	$0.81(+0.06/-0.05) \times 10^{-10}$
	Σ^{-}	1/2	1195.96 ± 0.30	Σ^{-}	4.45 ± 0.4 (Σ^-, Σ^0)	Σ^{-}	$1.61(+0.1/-0.09) \times 10^{-10}$
	Σ^0	1/2	1191.5 ± 0.5	Σ^0		Σ^0	$< 0.1 \times 10^{-10}$
	Ξ^{-}	?	1318.4 ± 1.2	Ξ^{-}	... ...	Ξ^{-}	$1.28(+0.38/-0.30) \times 10^{-10}$
	Ξ^0	?	1311 ± 8	Ξ^0	... ...	Ξ^0	1.5×10^{-10} (1 event)

Walter H. Barkas, Arthur H. Rosenfeld, University of California, Berkeley, Sept. 1960.

Table II. Atomic and nuclear properties (dE/dx, collision mean free path, radiation length, etc.) of materials used as absorbers and detectors

Material	Z	A	Cross section σ [a] (barns)	$-\frac{dE}{dx}$ min [b] Mev g/cm²	Collision[a] length g/cm²	L_{coll} cm	Radiation[c] length g/cm²	L_{rad} cm	Density ρ (g/cm³)
H_2	1	1.01	0.063	4.14	26.5	374	58	819.0	0.0708 (boiling at 1 atmos)
Li	3	6.94	0.23	1.72	50.4	94.3	77.5	145	0.534 (boiling at 1 atmos)
Be	4	9.01	0.28	1.71	55.0	29.9	62.2	33.8	1.84
C	6	12.00	0.33	1.86	60.4	39.0	42.5	27.4	1.55 (variable)
Al	13	26.97	0.57	1.66	79.2	29.3	23.9	8.86	2.70
Cu	29	63.57	1.00	1.45	105.4	11.8	12.8	1.44	8.9
Sn	50	118.70	1.55	1.27	129.7	17.8	8.54	1.17	7.30
Pb	82	207.21	2.20	1.12	156.2	13.8	5.8	0.51	11.34
U	92	238.07	2.42	1.095	163.6	8.75	5.5	0.29	18.7
Hydrogen (bubble chamber, -27.6°K)				0.243 Mev/cm	26.5	452	58	990	0.0586
Propane (C_3H_8, bubble chamber)				0.935 Mev/cm	48.9	119.3	44.7	109.0	0.41
Freon CF_3Br				2.3	87.1	58.0	17.25	11.5	1.5
Polystyrene (CH scintillator)				2.14 Mev/cm	54.9	52.3	43.4	41.3	~ 1.05
Ilford emulsion				5.49 Mev/cm	103	27.0	11.2	2.91	3.815

Table IV. Atomic and nuclear constants in units of Mev, cm, and sec[a]

GENERAL ATOMIC CONSTANTS

$N = 6.0249 \times 10^{23}$ molecules/gram-mole

$c = 2.99793 \times 10^{10}$ cm/sec

$e = 4.80286 \times 10^{-10}$ esu $= 1.6021 \times 10^{-19}$ coulomb.

1 Mev $= 1.6021 \times 10^{-6}$ erg [1 ev = e(10⁸/c)]

$\hbar = 6.5817 \times 10^{-22}$ Mev sec $= 1.054 \times 10^{-27}$ erg sec.

$\hbar c = 1.9732 \times 10^{-11}$ Mev cm [= ƛ for p = 1 Mev/c]

$k = 8.6167 \times 10^{-11}$ Mev/°C [Boltzmann constant]

$\alpha = \frac{e^2}{\hbar c} = 1/137.037$; $e^2 = 1.44 \times 10^{-13}$ Mev cm

Cross Section

$\sigma_{Thompson} = \frac{8}{3}\pi r_e^2 = 0.6652 \times 10^{-24}\,cm^2 = 0.6652$ barn

Magnetic Moment and Cyclotron Angular Frequency

$\mu_{Bohr} = \frac{e\hbar}{2mc} = 0.57883 \times 10^{-14}$ Mev/gauss

$\frac{1}{2}\omega_{cyclotron} = \frac{e}{2mc} = 8.7945 \times 10^{6}$ rad sec⁻¹/gauss

$g_{electron} = 2[1 + \frac{\alpha}{2\pi} - 0.328(\frac{\alpha}{\pi})^2] = 2[1.0011596]$[b]

$g_{muon} = 2[1 + \frac{\alpha}{2\pi} + 0.75(\frac{\alpha}{\pi})^2] = 2[1.001165]$[b]

QUANTITIES DERIVED FROM THE ELECTRON MASS, m

Mass and Energy

$m = 0.510976$ Mev $= 1/1836.12\, m_p = 1/273.26\, m_\pi$

Rydberg, $R_\infty = \frac{me^4}{2\hbar^2} = mc^2 \times \frac{\alpha^2}{2} = 13.605$ ev

Length (1 fermi = 10^{-13} cm; 1 A = 10^{-8} cm)

$r_e = e^2/mc^2 = 2.81785$ fermi

$ƛ_{Compton} = \frac{\hbar}{mc} = r_e\alpha^{-1} = 3.8612 \times 10^{-11}$ cm

a_∞ Bohr $= \frac{\hbar^2}{me^2} = r_e\alpha^{-2} = 0.52917$ A

Hydrogen-like atom (Non. Rel.; μ ≡ reduced mass).

$E_n = \frac{1}{2}\frac{\mu z e^4}{(n\hbar)^2}$; $a_{n=1} = \frac{\hbar^2}{\mu z e^2}$; $\left.\frac{v}{c}\right)_{rms} = \frac{ze^2}{n\hbar c}$

QUANTITIES DERIVED FROM THE PROTON MASS, m_p

Rest mass $= 938.211$ Mev/c² $= 1836.12\, m_e = 6.719\, m_\pi$

$= 1.007593\, m_1$

where $m_1 = 1$ amu $= \frac{1}{16} O^{16} = 931.141$ Mev.

Magnetic Moment and Cyclotron Angular Frequency

$\mu_p = \frac{e\hbar}{2m_pc} = 3.1524 \times 10^{-18}$ Mev/gauss

$\frac{1}{2}\omega_{cyclotron} = \frac{e}{2m_pc} = 4.7896 \times 10^{3}$ rad sec⁻¹/gauss

$\left.\frac{\mu}{\mu_p}\right)_{proton} = 2.79275$; $\left.\frac{\mu}{\mu_p}\right)_{neutron} = -1.9128$

Table IIIa. Multiple Coulomb scattering and Lorentz transformation

The rms projected angle θ due to multiple Coulomb scattering (only) of a particle of charge z, momentum P, velocity V is

$\theta_{proj} = z\frac{15(Mev)}{PV(Mev)}\sqrt{\frac{L}{L_{(rad)}}}(1+\epsilon)$ radians;

L = Length in scatterer; L(radiation) from Table II. For L ⩾ 1/10 L(rad) ε is generally < 1/10. The distribution of θ is not truly Gaussian. The rms projected displacement is

$y_{rms} = L\theta_{proj}/\sqrt{3}$.

Lorentz transformations. Notation: Lower-case type for c.m. 4-momentum ($\vec{p}$, w) and capitals for lab ($\vec{P}$, W). (c ≡ 1.) To transform from c.m. to lab write

$$\begin{pmatrix}\gamma & 0 & 0 & \eta\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ \eta & 0 & 0 & \gamma\end{pmatrix}\begin{pmatrix}p\cos\theta\\ p\sin\theta\\ 0\\ w\end{pmatrix} = \begin{pmatrix}\gamma p\cos\theta + \eta w\\ p\sin\theta\\ 0\\ \eta p\cos\theta + \gamma w\end{pmatrix} = \begin{pmatrix}P\cos\Theta\\ P\sin\Theta\\ 0\\ W\end{pmatrix}$$

If two particles (1 and 2) collide, the invariant "mass" μ of the system is given by

$$\mu^2 = (W_1 + W_2)^2 - (\vec{P}_1 + \vec{P}_2)^2, \quad (1)$$

$$\gamma = \frac{W_1 + W_2}{\mu}; \quad \eta = \left|\frac{\vec{P}_1 + \vec{P}_2}{\mu}\right| = \gamma\beta. \quad (2)$$

Write T for lab kinetic energy, t for c.m.; thus $\mu = m_1 + m_2 + t_1 + t_2 = m_1 + m_2 + Q$. If the target is at rest $(0, m_2)$ μ simplifies:

$$\mu^2 = (m_1 + m_2)^2 + 2T_1m_2. \quad (3)$$

To get a threshold T_1, set μ = sum of masses of reaction products, then

$$[\Sigma m(\text{products})]^2 = (m_1 + m_2)^2 + 2T_1m_2. \quad (4)$$

Other invariants are: $w_1w_2 - p_1p_2\cos\theta_{12}$ (5)

and

$$\frac{1}{p}\frac{d^2\sigma}{d\omega dw}. \quad (6)$$

The max. lab angle that a particle of c.m. momentum p_i can have is given by

$$\sin\Theta_i = \frac{\eta_i}{\eta}\ (\eta_i = \frac{p_i}{m_i}\ \text{must be} < \eta); \quad (7)$$

If $\eta_i > \eta$, then of course Θ_i can be π. Crawford's mnemonic for extending nonrelativistic formulas to relativistic case: "To the rest energy of each moving particle add Q/2" where Q = the total kinetic energy (c.m.) = $\mu - \Sigma m_i$. Thus in the rest frame of a two-body decay the kinetic energy Q is shared between the two particles according to

$$t_1 = Q\frac{m_2 + Q/2}{\mu}, \quad t_2 = Q\frac{m_1 + Q/2}{\mu}. \quad (8)$$

The above of course applies in the c.m. for the production of a two-body final state. To express t in terms of p, apply the mnemonic to a single particle (then Q = t). The non-rel. relation $p^2 = 2tm$ becomes

$$p^2 = 2t(m + t/2) = 2tm + t^2. \quad (9)$$

Energy Transfer in elastic collisions of beam (P_1, W_1) with resting target $(0, m_2)$, is

$$T_2 = 2m_2\frac{P_1^2}{\mu^2}\sin^2(\theta_{c.m.}/2). \quad (10)$$

Note that for max T_2, $\theta_{c.m.} = \pi$, so

$$T_{2max.} = 2m_2P_1^2/\mu^2 = 2m_2\eta^2. \quad (11)$$

Table IV (continued)

QUANTITIES DERIVED FROM THE MASS OF THE CHARGED PION, m_π

Rest mass $= 139.63$ Mev/c² $= 273.26\, m_e = 0.14882\, m_p$

Length

$\frac{\hbar}{m_\pi c} = 1.4132$ fermi (~ $\sqrt{2}$ fermi)

Natural (≈ "geometrical") Nucleon Cross Section

$\pi\left(\frac{\hbar}{m_\pi c}\right)^2 = 62.7344$ mb (1 mb = 10^{-27} cm²)

(3/2, 3/2)πp Resonance of mass 1237 Mev (Q = 159 Mev).

Center-of-mass momentum: $p_\pi = 230$ Mev/c

Lab-system momentum: $P_\pi = 303$ Mev/c ($T_\pi = 195$ Mev)

RADIOACTIVITY

1 curie $= 3.7 \times 10^{10}$ disintegrations/sec

1 r $= 87.8$ ergs/g air $= 5.49 \times 10^{7}$ Mev/g air

Fluxes (per cm²) to liberate 1 r in carbon:

3×10^7 minimum ionizing singly charged particles

0.9×10^9 photons of 1 Mev energy.

(These fluxes are actually correct to within a factor of two for all materials.)

Natural background: 100 mr/year

"Tolerance" 100 millirem/week [Note, 1 r may produce up to 10 "rem" (r equivalent for man), depending on type of radiation.]

MISCELLANEOUS

Physical Constants

1 year $= 3.1536 \times 10^{7}$ sec ($\approx \pi \times 10^7$ sec)

Density of air = 1.205 mg/cm³ at 20°C

Acceleration by gravity = 980.67 cm/sec²

1 calorie = 4.184 joules

1 atmosphere = 1033.2 g/cm²

Numerical Constants

1 radian = 57.29578 deg; e = 2.71828

ln 2 = 0.69315; $\log_{10} e = 0.43429$;

ln 10 = 2.30259; $\log_{10} 2 = 0.30103$.

Stirling's approximation

$\sqrt{2\pi n}(\frac{n}{e})^n < n! < \sqrt{2\pi n}(\frac{n}{e})^n(1 + \frac{1}{12n-1})$

Gaussianlike Distributions

For n > -1 but not necessarily integral:

$\int_0^\infty x^{2n+1}\exp\left[-\frac{x^2}{2\sigma^2}\right]dx = 2^n n!\,\sigma^{2n+2}$; $(\frac{1}{2})! = \sqrt{\pi}/2$

Relation between standard deviation σ and mean deviation α:

$2\sigma^2 = \pi\alpha^2$; σ = 1.4826 probable error.

Odds against exceeding one standard deviation = 2.15:1; two, 21:1; three, 370:1; four, 16,000:1; five, 1,700,000:1

[a] Based mainly on Cohen, Crowe, and Dumond, The Fundamental Constants of Physics (Interscience, New York, 1957), not on the later corrections of Cohen and Dumond, Phys. Rev. Lett. 1, 291 (1958).

[b] C. Sommerfield, Phys. Rev. 107, 328 (1957) and A. Petermans, Helv. Phys. Acta. 30, 407 (1957).