10. PASSAGE OF PARTICLES THROUGH MATTER

Revised June 1994.

10.1. Notation

Table 10.1: The notation and values given in Table 1.1 are used. The kinematic variables β and γ have the usual meanings. Other variables used in this section are summarized below.

Symbol	Definition	Units or Value
α	Fine structure constant	1/137.035 989 5(61)
M	Incident particle mass	MeV/c^2
E	Incident particle energy γMc^2	MeV
T	Kinetic energy	MeV
$m_e c^2$	Electron mass $\times$ c^2	0.510 999 06(15) MeV
r_e	Classical electron radius $e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 92(38) fm
N_A	Avogadro's number	$6.022\,136\,7(36) \times 10^{23}$ mol^{-1}
ze	Charge of incident particle	
Z	Atomic number of medium	
A	Atomic mass of medium	g mol^{-1}
K/A	$4\pi N_A r_e^2 m_e c^2/A$	0.307 075 MeV g^{-1} cm^2 for $A = 1$ g mol^{-1}
δ	Density effect correction to ionization energy loss	
$\hbar\omega_p$	Plasma energy $\sqrt{4\pi N_e r_e^3}\ m_e c^2/\alpha$	$28.816\sqrt{\rho\langle Z/A\rangle}$ eV$^{(a)}$
w_j	Fraction by weight of the jth element in a compound or mixture	
n_j	$\propto$ number of jth kind of atoms in a compound or mixture	
X_0	Radiation length	g^{-1} cm^2
E_c	Critical energy	MeV
E_s	Scale energy $\sqrt{4\pi/\alpha}\ m_e c^2$	21.2052 MeV
R_M	Molière radius	MeV g^{-1} cm^2

$^{(a)}$ For ρ in g cm^{-3}.

10.2. Ionization energy loss by heavy particles [1–5]

Moderately relativistic charged particles other than electrons lose energy in matter primarily by ionization. If the incident particle velocity βc is larger than that of orbital electrons ($\sim Z\alpha c$) and small enough that radiative effects do not dominate (for example, pion energy smaller than 100–200 GeV in iron), then the mean rate of energy loss (or stopping power) is given by the Bethe-Bloch equation,

$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\rm max}}{I^2} - \beta^2 - \frac{\delta}{2}\right] . \qquad (10.1)$$

Here $T_{\rm max}$ is the maximum kinetic energy which can be imparted to a free electron in a single collision, and the other variables are defined in Table 10.1. The units are chosen so that dx is measured in mass per unit area, *e.g.*, in g cm^{-2}. The function as computed for pions on copper is shown by the solid curve in Fig. 10.1, and for pions on other materials in Fig. 10.2. A minor dependence on M at the highest energies is introduced through $T_{\rm max}$, but for all practical purposes in high-energy physics dE/dx in a given material is a function only of β. Except in hydrogen, particles of the same velocity have very similar rates of energy loss in different materials; there is a slow decrease in the rate of energy loss with increasing Z. The qualitative difference in stopping power behavior at high energies between a gas (He) and the other materials shown in Fig. 10.2 is due to the density effect correction, δ, discussed below. The stopping power functions are characterized by broad minima whose position drops from $\beta\gamma = 3.5$ to 3.0 as Z goes from 7 to 100.

In practical cases, most relativistic particles (*e.g.*, cosmic-ray muons) have energy loss rates close to the minimum, and are said to be minimum ionizing particles, or mip's.

Eq. (10.1) may be integrated to find the total range R for a particle which loses energy only through ionization. Since dE/dx depends only on β, R/M is a function of E/M or pc/M. In practice, range is a useful concept only for low-energy hadrons ($R \lesssim \lambda_I$, where λ_I is the nuclear interaction length), and for muons below a few hundred

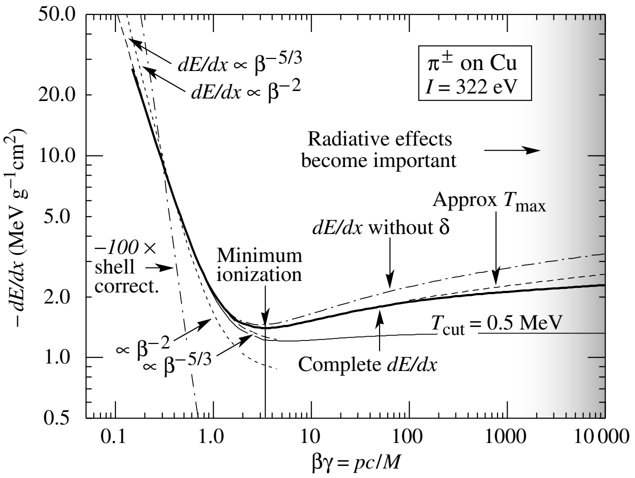

Figure 10.1: Energy loss rate in copper. The function without the density effect correction is also shown, as is the shell correction and two low-energy approximations.

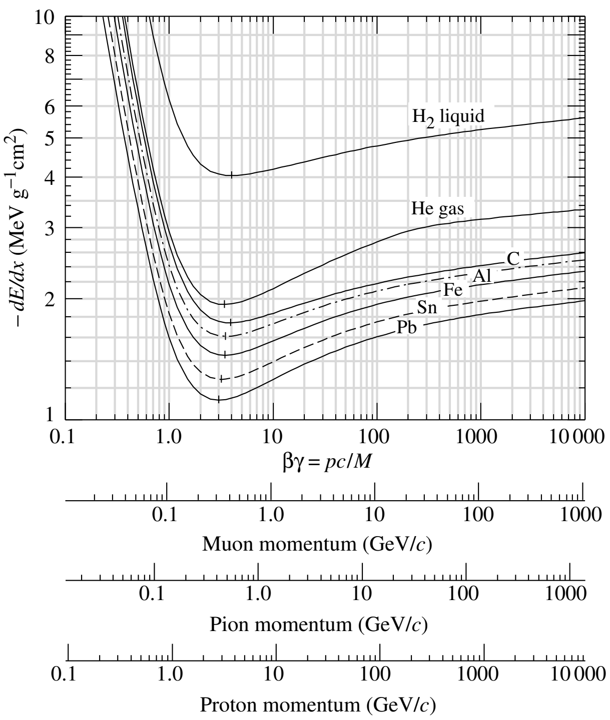

Figure 10.2: Energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, tin, and lead.

GeV (above which radiative effects dominate). R/M as a function of $\beta\gamma = pc/M$ is shown for a variety of materials in Fig. 10.3.

For a point-like charged particle with mass M and momentum $M\beta\gamma$, $T_{\rm max}$ is given by

$$T_{\rm max} = \frac{2m_ec^2\,\beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2} . \qquad (10.2)$$

It is usual [1,2] to make the low-energy approximation $T_{\rm max} = 2m_ec^2\,\beta^2\gamma^2$, valid for $2\gamma m_e/M \ll 1$; this, in fact, is done implicitly in many standard references. For pion in copper, the error thus introduced into dE/dx is greater than 6% at 100 GeV. The correct expression should be used.

At such energies, the maximum 4-momentum transfer to the electron can exceed 1 GeV/c, where structure effects significantly